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Research Article

Poincaré Map Approach to Global Dynamics of the Integrated Pest Management Prey-Predator Model

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An integrated pest management prey-predator model with ratio-dependent and impulsive feedback control is investigated in this paper. Firstly, we determine the Poincaré map which is defined on the phase set and discuss its main properties including monotonicity, continuity, and discontinuity. Secondly, the existence and stability of the boundary order-one periodic solution are proved by the method of Poincaré map. According to the Poincaré map and related differential equation theory, the conditions of the existence and global stability of the order-one periodic solution are obtained when $\Phi(y_A) < y_A$, and we prove the sufficient and necessary conditions for the global asymptotic stability of the order-one periodic solution when $\Phi(y_A) > y_A$. Furthermore, we prove the existence of the order-k $(k \geq 2)$ periodic solution under certain conditions. Finally, we verify the main results by numerical simulation.

1. Introduction

Differential equations can be widely used in many ways, such as chemostat cultures in ecological systems [1–4], protection of biological resources and management of pests and diseases [5–8], viral power system (HIV) [9–12], and infectious disease research [13–17].

In recent decades, differential equations with a predator-prey model have attracted the attention of many experts. Some researchers have done lots of research studies on several famous predator models including Holling I type [18], Holling II type [19], Holling III type [20], Beddington–DeAngelis type [21], ratio-dependent type [22], and Lotka–Volterra type [23], where the ratio-dependent prey-predator model has an important impact on the interaction between populations in recent years.

$$\begin{cases} x'(t) = rx(t)\left[1 - \dfrac{x(t)}{K}\right] - \dfrac{mx(t)y(t)}{\alpha y(t) + x(t)}, \\ \\ y'(t) = y(t)\left[-d + \dfrac{\beta x(t)}{\alpha y(t) + x(t)}\right], \end{cases} \tag{1}$$

where $x(t)$ are the densities of the pest, $y(t)$ are the densities of the natural enemy, r represents the intrinsic birth rate of pests, K is the carrying capacity for the pest when natural enemy $y = 0$, and β denotes the ratio of biomass conversion. The death rate of the natural enemy is denoted as d. $mx(t)/(\alpha y(t) + x(t))$ represents the ratio-dependent functional response. And r, β, K, d, m, and α are positive constants.

The most common methods of pest management in agriculture are biological control and chemical control. There are usually three methods for biological control using natural enemies to prey on pests, virus control, and releasing bacteria or fungi. Among them, it is common to use natural enemies to prey on pests [24–27]. In addition, chemical control can also be called pesticide control. The main method is to spray insecticides when the pests are flooding [28–33]. But, both methods have drawbacks, for instance, biological control is only suitable for pests with relatively low density, while chemical control is highly toxic and pollutes the environment. Therefore, the researchers proposed an integrated pest management (IPM) method, which is a combination of pesticide control and biological control [34–38]. The IPM method does not make pests become extinct but keep pests below economic thresholds. In recent years, some scholars have studied many IPM models, such as Sun et al. proposed an IPM model and researched its dynamics analysis and control optimization in [39]. Thus, combined with system (1), we shall get system (2) as follows:

$$\begin{cases} \left.\begin{aligned} x'(t) &= rx(t)\left[1-\frac{x(t)}{K}\right]-\frac{mx(t)y(t)}{\alpha y(t)+x(t)} \\ y'(t) &= y(t)\left[-d+\frac{\beta x(t)}{\alpha y(t)+x(t)}\right] \end{aligned}\right\} x<h, \\ \left.\begin{aligned} \Delta x(t) &= -ax(t) \\ \Delta y(t) &= -by(t)+c \end{aligned}\right\} x=h, \end{cases} \tag{2}$$

where h denotes the threshold of the pest, that is, integrated control strategy is adopted when $x=h$. $0<a<1$ and $0<b<1$ denote the proportion of prey and predator killed by spraying pesticides when $x=h$, respectively. c represents the number of predator released.

In fact, many researchers have studied models with ratio-dependent over the past few decades. Berezovskaya et al. investigated parametric analysis on a prey-predator model with ratio-dependent [40]. Bandyopadhyay et al. obtained the condition that the ratio-dependent deterministic model enters the Hopf bifurcation and studied the stochastic stability of the system [41]. Nie et al. studied existence and local asymptotic stability of the positive periodic solution of the system [42]. However, research studies on the existence and global asymptotic stability of the order-k $(k\geq 1)$ periodic solution seem to be rare. Therefore, this paper studies the global dynamics of the order-k $(k\geq 1)$ periodic solution by using the related differential equation theory and properties of Poincaré map.

This consists of the following parts. We study qualitative analysis of system (1) in the next section. In Section 3, some properties of Poincaré map and its expression are given. Moreover, the conditions of the existence and stability of order-k $(k\geq 1)$ periodic solutions of system (2) are obtained. The main results are verified by numerical simulation in Section 4. In Section 5, we give the final conclusion.

2. Qualitative Analysis for System (1)

Model (1) possesses two boundary equilibrium points $O(0,0)$ and $E_1(K,0)$ and an internal equilibrium point $E_*(x_*,y_*)$, where $x_* = (K\alpha\beta r - Km(\beta-d))/\alpha\beta r$ and $y_* = (\beta-d)/\alpha d x_*$.

Lemma 1 (see [43]).

(H_1): if $m\leq\alpha r$ and $\beta\leq d$, then model (1) has no positive equilibrium and $E_1(K,0)$ is globally asymptotically stable

(H_2): $O(0,0)$ is always a saddle point for model (1)

(H_3): if $((md^2+\alpha r\beta^2)/(2md\beta+m\beta^2))\geq 1$ and $m\leq\alpha r$, then $E_*(x_*,y_*)$ is a globally asymptotically stable node or focus (see Figure 1)

In fact, if the corresponding control strategy is not adopted, the pest population may reach the carrying capacity K. Therefore, we only discuss the condition where condition (H_1) is established in this paper, that is, $m\leq\alpha r$ and $\beta\leq d$ hold in the remaining part of the paper.

3. Poincaré Map

In order to facilitate the expression, we make the following regulations.

Two isoclines of system (1) are defined as L_1: $y=(rx(1-x/K))/(m-\alpha r(1-x/K))$ and L_2: $y=(\beta-d)x/\alpha d$. Define two straight lines as L_3: $x=(1-a)h$ and L_4: $x=h$. The intersection point of L_1 and L_3 is expressed as $A((1-a)h, y_A)$ with $y_A = (r(1-a)h[K-(1-a)h]/K)/(m-\alpha r[K-(1-a)h]/K)$, while L_1 intersects L_4 at point $B(h,y_B)$ with $y_B=(rh(K-h)/K)/(m-\alpha r(K-h)/K)$. $\Omega=\{(x,y)\,|\,0<x<h, y>0\}\subset R_+^2$ is defined as the open set in R_+^2. We define set $\{(x,y)\,|\,x=h, 0\leq y\leq y_B\}$ as impulsive set M and set $\{(x^+,y^+)\,|\,x^+=(1-a)h, c\leq y^+\leq (1-b)y_B+c\}$ as phase set N. I is defined as the continuous function which satisfies $N=I(M)$. Assume that the initial point (x_0^+,y_0^+) is on the phase set N.

3.1. Poincaré Map and Its Properties. We define the following two sets:

$$\begin{aligned} S_{ah} &= \{(x,y)\,|\,x=(1-a)h, y\geq 0\}, \\ S_h &= \{(x,y)\,|\,x=h, y\geq 0\}. \end{aligned} \tag{3}$$

We choose to define S_{ah} as Poincaré map. Assuming point $Q_k^+((1-a)h, y_k^+)$ lies in set S_{ah}, the trajectory $\Gamma(t,t_0,(1-a)h,y_k^+) = (x(t,t_0,(1-a)h,y_k^+), y(t,t_0,(1-a)h,y_k^+))$ starting from Q_k^+ reaches at S_h in a limited time t_1 $(x(t_1,t_0,(1-a)h,y_k^+)=h)$, and the intersection point is expressed as $Q_{k+1}=(h,y_{k+1})$. This means that y_{k+1} is determined by y_k^+. Thus we shall get $y_{k+1}=y(t_1,t_0,(1-a)h,y_k^+)=\Psi(y_k^+)$. For convenience, denote that $y((1-a)h,y_k^+)=y(t_1,t_0,(1-a)h,y_k^+)$ throughout this paper. Point Q_{k+1} jumps to point $Q_{k+1}^+((1-a)h,y_{k+1}^+)$ on S_{ah} after impulsive effect, where $y_{k+1}^+=(1-b)y_{k+1}+c$. Thus, the Poincare$'$ map is expressed as:

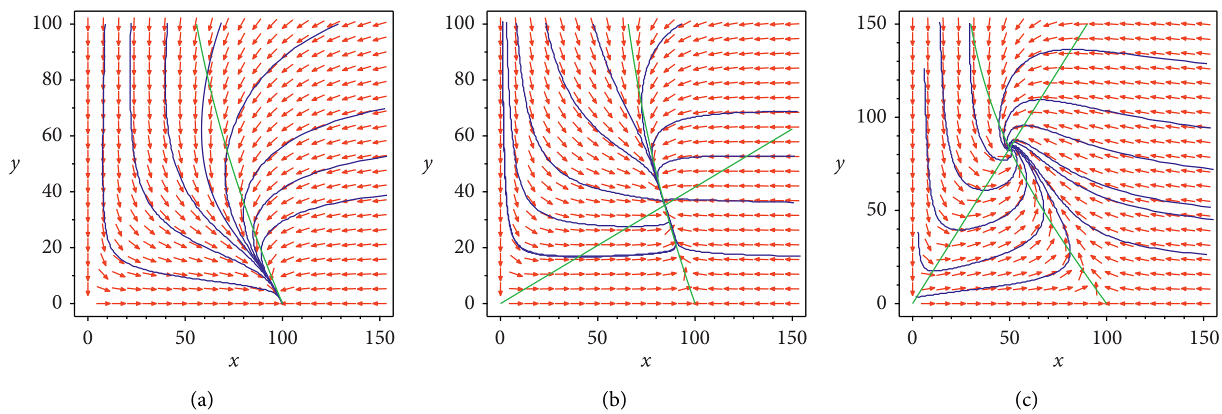

FIGURE 1: Phase diagram for system (1). (a) Boundary equilibria. (b) Internal equilibrium is a node. (c) Internal equilibrium is a focus.

$$y_{k+1}^{+} = (1-b)\Psi(y_k^{+}) + c = (1-b)y((1-a)h, y_k^{+}) + c = \Phi(y_k^{+}). \tag{4}$$

For convenience, we assume

$$\begin{cases} P(x,y) = rx\left[1-\dfrac{x}{K}\right] - \dfrac{mxy}{\alpha y + x}, \\ \\ Q(x,y) = y\left[-d + \dfrac{\beta x}{\alpha y + x}\right]. \end{cases} \tag{5}$$

We can convert model (2) to the following form:

$$\begin{cases} \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{y(-d+((\beta x)/(\alpha y + x)))}{rx(1-(x/K)) - ((mxy)/(\alpha y + x))} = g(x,y), \\ \\ y((1-a)h) = y_0^{+}. \end{cases} \tag{6}$$

For model (6), the region $\Omega\prime = \{(x,y) \mid x > 0, 0 < y < (r/m)(1-(x/K))(x+\alpha y)\}$ is studied.

We set $(x_0^{+}, y_0^{+}) = ((1-a)h, Z)$, where $Z < y_B$ and $Z \in N$, and then $(x_0^{+}, y_0^{+}) \in \Omega\prime$. Therefore, we obtain

$$\begin{aligned} y(x) &= y(x; (1-a)h, Z) \\ &\triangleq y(x,Z). \end{aligned} \tag{7}$$

Moreover, according to model (6), we have that

$$y(x,Z) = Z + \int_{(1-a)h}^{x} g(z, y(z,Z))\mathrm{d}z. \tag{8}$$

Therefore, Poincaré map Φ takes the form in the region $\Omega\prime$:

$$\Phi(Z) = (1-b)y((1-a)h, Z) + c. \tag{9}$$

Theorem 1. *If condition (H_1) holds, then Poincare$'$ map Φ exists, and it has the following properties, see Figures 2 and 3:*

(I) The domain and range of Φ are $[0,+\infty)$ and $[c, (1-b)y((1-a)h, y_A)+c)$, respectively. It increases on $[0, y_A]$ and decreases on $[y_A, +\infty]$

(II) Φ is continuously differentiable

(III) If $d > (\beta x/(\alpha y + x))$ and $c > 0$ hold, then Φ always possesses a unique fixed point

(IV) Φ exists, and it has a horizontal asymptote $y = c$ as $y_k^{+} \longrightarrow \infty$

Proof

(I) For $\forall Z \in [0,+\infty)$, the trajectory $\Gamma(t, t_0, (1-a)h, Z)$ crossing point $C((1-a)h, Z)$ intersects S_h at point $C'(h, \Psi(Z))$. Therefore, the domain of Φ is $[0,+\infty)$. We arbitrarily select two points $U((1-a)h, y_U^{+})$ and $V((1-a)h, y_V^{+})$ with $y_U^{+}, y_V^{+} \in [0, y_A)$ and assume $y_U^{+} < y_V^{+}$. According to the uniqueness of the solution of system (1), $y((1-a)h, y_U^{+}) < y((1-a)h, y_V^{+})$ can be obtained. After one time pulse, then we have

$$\begin{aligned} \Phi(y_V^{+}) - \Phi(y_U^{+}) &= (1-b)y((1-a)h, y_V^{+}) \\ &\quad + c - (1-b)y((1-a)h, y_U^{+}) - c \\ &= (1-b)[y((1-a)h, y_V^{+}) \\ &\quad - y((1-a)h, y_U^{+})] > 0. \end{aligned} \tag{10}$$

For $y_U^{+}, y_V^{+} \in [y_A, +\infty)$ with $y_V^{+} > y_U^{+}$, the orbits that start at these two points will first cross the line L_3 and then hit the line L_4. Due to the vector field of model (1), $y((1-a)h, y_U^{+}) > y((1-a)h, y_V^{+})$ is obtained. After one pulse, we have

$$\begin{aligned} \Phi(y_V^{+}) - \Phi(y_U^{+}) &= (1-b)y((1-a)h, y_V^{+}) + c - (1-b)y \\ &\quad \cdot ((1-a)h, y_U^{+}) - c \\ &= (1-b)[y((1-a)h, y_V^{+}) \\ &\quad - y((1-a)h, y_U^{+})] < 0. \end{aligned} \tag{11}$$

Therefore, Φ increases on $[0, y_A)$ and decreases on $[y_A, +\infty)$, and the range of Φ is $[c, (1-b)y((1-a)h, y_A)+c)$.

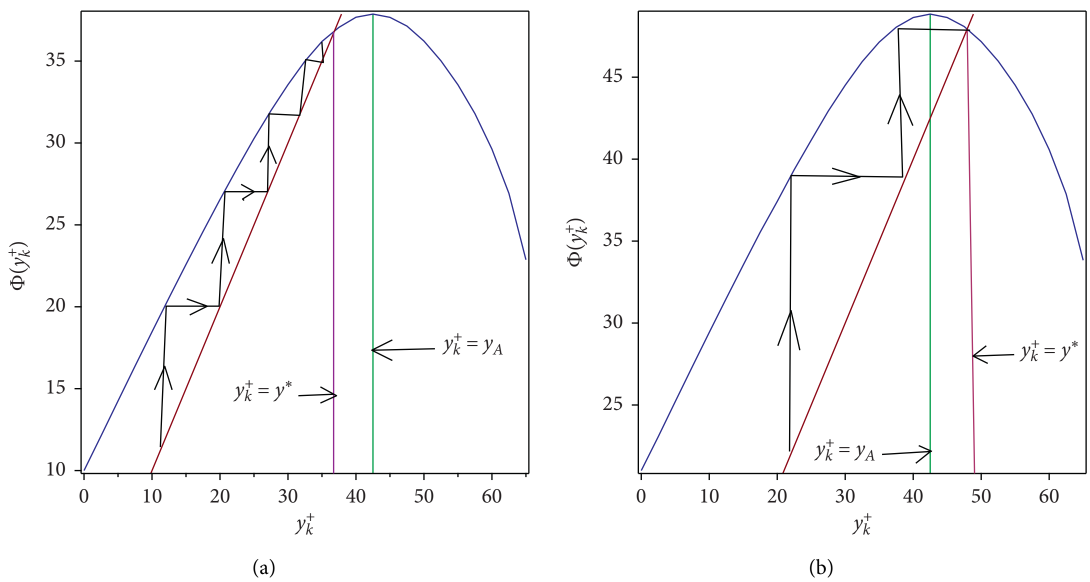

Figure 2: The relationship between Φ and y_k^+ with $r = 1.5$, $K = 100$, $m = 0.9$, $\alpha = 0.1$, $d = 0.5$, $\beta = 0.2$, $a = 0.25$, $b = 0.1$, and $h = 40$, (a) $c = 10$, and (b) $c = 21$.

(II) It is easy to see that $P(x, y)$ and $Q(x, y)$ are continuous and differentiable in the first quadrant. Φ is also continuously differentiable according to Cauchy–Lipschitz theorem with parameters.

(III) Because Φ decreases on $[y_A, +\infty)$, there is $\hat{y} \in [y_A, +\infty)$ that satisfies $\Phi(\hat{y}) < \hat{y}$. Furthermore, we know $\Phi(0) = c \geq 0$. Therefore, Φ has $y^* \in [0, \hat{y})$ satisfying $\Phi(y^*) = y^*$, i.e., there is a fixed point on $[0, \hat{y})$ for Φ. If $c > 0$ and $\Phi(y_A) < y_A$, then $y^* \in [0, y_A)$. Because Φ decreases on $[y_A, \hat{y})$, then $\Phi(y_k^+) < \Phi(y_A) < y_A$ for $y_k^+ \in [y_A, \hat{y})$ which means no fixed point exists for Φ on $[y_A, +\infty)$. In the case $c > 0$ and $\Phi(y_A) > y_A$, no fixed point exists for Φ on $(0, y_A)$ because Φ increases on $(0, y_A)$ and $\Phi(c) > 0$. Since $\Phi(\hat{y}) < \hat{y}$ and $\Phi(y_A) > y_A$, Φ has at least a fixed point on $(y_A, \hat{y})$.

It is assumed that system (2) admits two fixed points $\tilde{y}_1$ and $\tilde{y}_2$, where $\tilde{y}_1, \tilde{y}_2 \in [0, \hat{y})$ and $\tilde{y}_1 > \tilde{y}_2$ such that $\Phi(\tilde{y}_1) = \tilde{y}_1$ and $\Phi(\tilde{y}_2) = \tilde{y}_2$. Then,

$$d'_{\tilde{y}_1\tilde{y}_2}(x) = \tilde{y}_1' - \tilde{y}_2' = \frac{\tilde{y}_1(-d + ((\beta x)/(\alpha\tilde{y}_1 + x)))}{rx(1-(x/K)) - ((mx\tilde{y}_1)/(\alpha\tilde{y}_1 + x))} - \frac{\tilde{y}_2(-d + ((\beta x)/(\alpha\tilde{y}_2 + x)))}{rx(1-(x/K)) - ((mx\tilde{y}_2)/(\alpha\tilde{y}_2 + x))} = g'(\xi)(\tilde{y}_1 - \tilde{y}_2), \tag{12}$$

where $g(y) = (y(-d + (\beta x/(\alpha y + x))))/(rx(1 - (x/K)) - ((mxy)/(\alpha y + x)))$ with

$$g'(y) = \frac{[rx(1-(x/K)) - ((mxy)/(\alpha y + x))]\left[-d + (\beta x/(\alpha y + x)) - \left((\alpha\beta xy)/\left((\alpha y + x)^2\right)\right)\right] + \left((mx^2 y)/\left((\alpha y + x)^2\right)\right)[-d + ((\beta x)/(\alpha y + x))]}{[rx(1-(x/K)) - ((mxy)/(\alpha y + x))]^2}. \tag{13}$$

Since $d > (\beta x/(\alpha y + x))$, we have $g'(y) < 0$. Thus, $d'_{\tilde{y}_1\tilde{y}_2}(x) < 0$ for $x \in [(1-a)h, h]$, which indicates that $d'_{\tilde{y}_1\tilde{y}_2}(x)$ is decreasing on $[0, \hat{y})$ and $d_{\tilde{y}_1\tilde{y}_2}(h) < d_{\tilde{y}_1\tilde{y}_2}((1-a)h)$. Then,

$$\begin{aligned} c_h &= \tilde{y}_1 - (1-b)\Psi(\tilde{y}_1) \\ &= \tilde{y}_2 + d_{\tilde{y}_1\tilde{y}_2}((1-a)h) - (1-b)\left(\Psi(\tilde{y}_2) + d_{\tilde{y}_1\tilde{y}_2}(h)\right) \\ &> \tilde{y}_2 - (1-b)\Psi(\tilde{y}_2), \end{aligned} \tag{14}$$

which leads to a contradiction. Therefore, if conditions $d > (\beta x/(\alpha y + x))$ and $c > 0$ hold, Φ has a unique fixed point on $(0, \widetilde{y})$.

(IV) The closure of Ω is denoted as

$$\overline{\Omega} = \left\{(x, y)\colon\ x \geq 0, 0 \leq y \leq \frac{r(1 - (x/K))x}{m - \alpha r(1 - (x/K))}\right\}. \quad (15)$$

The set $\overline{\Omega}$ is an invariant set of system (1) provided $d > \beta$. Assume

$$L = y - \frac{r(1 - (x/K))x}{m - \alpha r(1 - (x/K))} \quad (16)$$

and $\overline{\Omega}$ is an invariant set of system (1) if the vector field will flow into the boundary $\overline{\Omega}$. This is true if

$$\left[(P(x, y), Q(x, y)) \cdot \left(\frac{\alpha r/K[rx(1 - (x/K))] - r(1 - (2x/K))[m - \alpha r(1 - (x/K))]}{[m - \alpha r(1 - (x/K))]^2}, 1\right)\right]_{L=0} \leq 0, \quad (17)$$

where $\cdot$ represents the scalar product of two vectors; then, we have

$$\begin{aligned} H(x)|_{L=0} &= y\left(-d + \frac{\beta x}{\alpha y + x}\right) \\ &\quad - \left[rx(1 - (x/K)) - \frac{mxy}{\alpha y + x}\right]\frac{r(1 - (2x/K))[m - \alpha r(1 - (x/K))] - (\alpha r/K)[rx(1 - (x/K))]}{[m - \alpha r(1 - (x/K))]^2} \\ &= y\left[-d + \frac{\beta x}{\alpha y + x}\right] < 0. \end{aligned} \quad (18)$$

In addition, we choose (x, y) arbitrarily which satisfies $(x, y) \in \Omega$, and then $x'(t) > 0$ and $y'(t) < 0$ are obtained. Thus, we shall get $\Psi(+\infty) = 0$ with $((1 - a)h, +\infty) \in N$, that is, $\Phi(+\infty) = c$. Furthermore, there is $y_1 > 0$ which satisfies $\Psi(+\infty) = y_1$ with $B_1(h, y_1) \in M$. For any point $B_2(h, y_2)$ with $0 < y_2 < y_1$, the backward orbit initiating B_2 reaches a point $B_0^+((1 - a)h, y_0^+) \in N$ with $y_0^+ > +\infty$ by the uniqueness of the solution of system (2) and the invariance of the set $\overline{\Omega}$, which is a contradiction. Therefore, we shall get $\Psi(+\infty) = 0$ and $\Phi(+\infty) = c$. Therefore, there exists a horizontal asymptote $y = c$ for Φ (Figure 3). □

3.2. Stability of Boundary Order-One Periodic Solution When $c = 0$. We can get the following equation when predatory populations tend to die and predator release is stopped:

$$\begin{cases} x'(t) = rx(t)\left[1 - \dfrac{x(t)}{K}\right], x < h, \\ \Delta x = -ax(t), x = h, \end{cases} \quad (19)$$

where $x(0^+) = (1 - a)h$, and by calculation, we have

$$x(t) = \frac{K}{1 + [(K/(1 - a)h) - 1]\exp(-rt)}. \quad (20)$$

If $x(t)$ reaches the straight line L_4 at time T, then

$$h = \frac{K}{1 + [(K/((1 - a)h)) - 1]\exp(-rT)}. \quad (21)$$

Then, we shall get

$$T = \frac{1}{r}\ln\left[\frac{K - h(1 - a)}{(1 - a)(K - h)}\right]. \quad (22)$$

Thus, system (19) possesses a periodic solution, expressed by $x^T(t)$, and

$$x^T(t) = \frac{K}{1 + [(K/((1 - a)h)) - 1]\exp(-rt)}, \quad (23)$$

with period T, i.e., there exists a boundary order-one periodic solution $(x^T(t), 0)$.

Theorem 2. *If $c = 0$, then the boundary order-one periodic solution $(x^T(t), 0)$ is orbitally asymptotically stable provided*

$$g_2 \triangleq \frac{-d + \beta}{r}\ln\left[\frac{K - (1 - a)h}{(1 - a)(K - h)}\right] < 0. \quad (24)$$

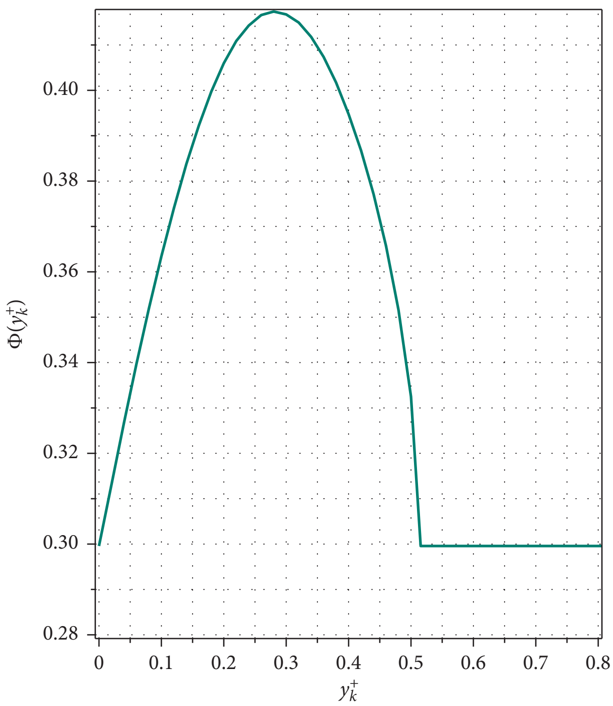

FIGURE 3: The relationship between Φ and y_k^+ with $r = 1$, $K = 1$, $m = 0.8$, $\alpha = 0.2$, $d = 0.5$, $\beta = 0.2$, $a = 0.4$, $b = 0.1$, and $c = 0.3$.

Proof. $\varrho(x, y) = x - h$, $\nu(x, y) = -ax$, $\sigma(x, y) = -by + c$, $(x^T(T), y^T(T)) = (h, 0)$, and $(x^T(T^+), y^T(T^+)) = ((1-a)h, 0)$.

$$
\begin{aligned}
\frac{\partial P}{\partial x} &= r - \frac{2rx}{K} - \frac{\alpha m y^2}{(\alpha y + x)^2},\\
\frac{\partial Q}{\partial y} &= -d + \frac{\beta x^2}{(\alpha y + x)^2},\\
\frac{\partial \nu}{\partial x} &= -a,\\
\frac{\partial \nu}{\partial y} &= 0,\\
\frac{\partial \sigma}{\partial x} &= 0,\\
\frac{\partial \sigma}{\partial y} &= -b,\\
\frac{\partial \varrho}{\partial x} &= 1,\\
\frac{\partial \varrho}{\partial y} &= 0,\\
\Delta_1 &= \frac{P_+((1-a)h, 0)(-b+1)}{P(h,0)}\\
&= \frac{(1-a)(1-b)[K-(1-a)h]}{K-h},\\
&\cdot \int_0^T \left(\frac{\partial P}{\partial x}\left(x^T(t), y^T(t)\right) + \frac{\partial Q}{\partial y}\left(x^T(t), y^T(t)\right)\right) \mathrm{d}t\\
&= \int_0^T \left(r - \frac{2rx^T(t)}{K} - d + \beta \right) \mathrm{d}t\\
&= \int_0^T \left(r - \frac{2re^{rt}}{e^{rt} + (K/(1-a)h) - 1} - d + \beta \right) dt\\
&= \ln\left[\frac{K-h}{(1-a)(K-(1-a)h)}\right] + \frac{-d+\beta}{r}\ln\\
&\cdot \left[\frac{K-(1-a)h}{(1-a)(K-h)}\right],\\
\mu_2 &= \frac{(1-a)(1-b)[K-(1-a)h]}{K-h} \cdot \exp\\
&\cdot \left\{ \ln\left[\frac{K-h}{(1-a)(K-(1-a)h)}\right] + \frac{-d+\beta}{r}\ln \right.\\
&\left. \cdot \left[\frac{K-(1-a)h}{(1-a)(K-h)}\right]\right\}\\
&= (1-b)\exp\left\{\frac{-d+\beta}{r}\ln\left[\frac{K-(1-a)h}{(1-a)(K-h)}\right]\right\}\\
&= (1-b)\exp(g_2).
\end{aligned}
\tag{25}
$$

By a simple calculation, we shall get $|\mu_2| < 1$; therefore, the conclusion of Theorem 2 is proved. □

3.3. Order-k Periodic Solution When $c > 0$. We first give a generalized result of the stability of the order-one periodic solution $(\zeta(t), \delta(t))$ and assume that the period of the order-one periodic solution is T. Then, we shall get

$$
\begin{aligned}
&(\zeta(T), \delta(T)) = (h, \overline{y}),\\
&(\zeta(T^+), \delta(T^+)) = ((1-a)h, (1-b)\overline{y} + c).
\end{aligned}
\tag{26}
$$

Therefore,

$$
\begin{aligned}
\mu_2 &= \Delta_1 \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}\left(x^T(t), y^T(t)\right) + \frac{\partial Q}{\partial y}\left(x^T(t), y^T(t)\right)\right)\mathrm{d}t\right]\\
&= \frac{P_+((1-a)h, (1-b)\overline{y} + c)(-b+1)}{P(h, \overline{y})} \exp\left[\int_0^T \left(r - \frac{2r\zeta(t)}{K} - \frac{\alpha m \delta(t)^2 + \beta \zeta(t)^2}{(\alpha\delta(t) + \zeta(t))^2} - d\right)\mathrm{d}t\right]\\
&= \frac{P_+((1-a)h, (1-b)\overline{y} + c)(-b+1)}{P(h, \overline{y})} \exp\left(\int_0^T \chi(t)\mathrm{d}t\right).
\end{aligned}
\tag{27}
$$

Thus, according to [44], Theorem 2.3, the following theorem is obtained.

Theorem 3. *If* $|\mu_2| < 1$, *then the order-one periodic solution* $(\zeta(t), \delta(t))$ *of system (2) is locally asymptotically stable.*

Theorem 4. *If* $\Phi(y_A) < y_A$, *then the order-one periodic solution of system (2) is globally asymptotically stable.*

Proof. If $\Phi(y_A) < y_A$, according to Theorem 1, Φ possesses a unique fixed y^*, where $y^* \in (0, y_A)$. Then, it can be seen from Theorem 1 that model (2) has a unique local asymptotically stable order-one periodic solution.

For any trajectory $\Gamma(t, t_0, (1-a)h, y_0^+)$, if $y_0^+ \in [0, y^*)$, then $y_0^+ < \Phi(y_0^+) < y^*$ by Theorem 1. After n time pulses, $\Phi^n(y_0^+)$ monotonically increases; thus, $\lim_{n\longrightarrow+\infty} = y^*$.

For $y_0^+ > y^*$, there are two cases which are discussed: (a) for all n, $\Phi^n(y_0^+) > y^*$. When n increases and $\lim_{n\longrightarrow\infty}\Phi^n(y_0^+) = y^*$, the series of $\Phi(y_0^+)$ decreases monotonically by $\Phi(y_0^+) < y_0^+$. (b) $\Phi^n(y_0^+) > y^*$ is not applicable to all n, and we have a minimum positive integer i that satisfies $\Phi^i(y_0^+) < y^*$ according to Theorem 1. Thus, by using a method similar to case (a), we shall get that $\Phi^{i+j}(y_0^+)$ increases monotonically when j increases and $\lim_{j\longrightarrow+\infty}\Phi^{i+j}(y_0^+) = y^*$. Therefore, the result in Theorem 4 is proved. □

Remark 1. Similarly, if $\Phi(y_A) = y_A$, then Φ has a unique fixed point y_A. Therefore, model (2) possesses a unique globally asymptotically stable order-one periodic solution.

Theorem 5. *If* $\Phi(y_A) > y_A$ *and* $\Phi^2(y_A) \geq y_A$ *hold, then model (2) possesses a stable order-one periodic solution or order-two periodic solution.*

Proof. There exists an integer ξ satisfying $y_\xi^+ = \Phi^\xi(y_0^+) \in [y_A, \Phi(y_A)]$ for $((1-a)h, y_0^+) \in N$ with $y_0^+ \in [0, +\infty)$. By Theorem 1, for $y_0^+ \in [0, y_A]$, we shall get that there is no fixed point for Φ on interval $[0, y_A]$, and Φ increases monotonically on $[0, y_A]$. Thus, there exists an integer ξ satisfying $y_{\xi-1}^+ < y_A$ and $y_\xi^+ \geq y_A$. It follows that $y_\xi^+ = \Phi(y_{\xi-1}^+) \leq \Phi(A)$, which means that $y_\xi^+ \in [y_A, \Phi(y_A)]$. If $y_0^+ \in (y_A, +\infty)$, there exists an integer ξ satisfying $y_\xi^+ \in [y_A, \Phi(y_A)]$ because $\Phi(y)$ decreases monotonically on $(y_A, +\infty)$, $y_+ = \Phi(y_0^+) \leq \Phi(y_A)$.

Since Φ is monotonically decreasing on $[y_A, \Phi(y_A)]$ and Φ^2 is monotonically increasing on $[y_A, \Phi(y_A)]$, thus,

$$\Phi([y_A, \Phi(y_A)]) = [\Phi^2(y_A), \Phi(y_A)] \subset [y_A, \Phi(y_A)]. \tag{28}$$

By the above analysis, for any $y_0^+ \in [y_A, \Phi(y_A)]$, we chose that $y_1^+ = \Phi(y_0^+) \neq y_0^+$ and $y_2^+ = \Phi^2(y_0^+) \neq y_0^+$ with $y_i^+ = \Phi^i(y_0^+)$, that is, trajectory starting from point $((1-a)h, y_0^+)$ is not order-one periodic solution or order-two periodic solution. Therefore, there are four cases as follows:

(I) If $\Phi(y_A) \geq y_1^+ > y_2^+ > y_0^+ > y_A$, then $\Phi(y_1^+) = y_2^+ < y_3^+ = \Phi(y_2^+) < \Phi(y_0^+) = y_1^+$ and $\Phi(y_2^+) = y_3^+ > y_4^+ = y_3^+ > \Phi(y_1^+) = y_2^+$, that is, $y_1^+ > y_3^+ > y_4^+ > y_2^+ > y_0^+$ (see Figure 4(a)). By induction, we can obtain

$$\begin{aligned}\Phi(y_A) &\geq y_1^+ > \cdots > y_{2i-1}^+ > y_{2i+1}^+ > \cdots > y_{2i+2}^+ > y_{2i}^+ > \cdots\\ &> y_2^+ > y_0^+ \geq y_A.\end{aligned} \tag{29}$$

(II) If $\Phi(y_A) > y_1^+ > y_0^+ > y_2^+ > y_A$, then $y_3^+ = \Phi(y_2^+) > \Phi(y_0^+) = y_1^+$ and $\Phi(y_3^+) > \Phi(y_1^+) < y_2^+$, Therefore, $y_3^+ > y_1^+ > y_0^+ > y_2^+ > y_4^+$ (see Figure 4(b)). By induction, we have

$$\begin{aligned}\Phi(y_A) &\geq \cdots > y_{2i+1}^+ > y_{2i-1}^+ > \cdots > y_1^+ > y_0^+ > y_2^+ > \cdots\\ &> y_{2i}^+ > y_{2i+2}^+ > \cdots \geq y_A.\end{aligned} \tag{30}$$

(III) If $y_A < y_1^+ < y_0^+ < y_2^+ \leq \Phi(y_A)$ (see Figure 4(c)), then

$$\begin{aligned}y_A &\leq \cdots < y_{2i+1}^+ < y_{2i-1}^+ < \cdots < y_1^+ < y_0^+ < y_2^+ < \cdots\\ &< y_{2i}^+ < y_{2i+2}^+ < \cdots < \Phi(y_A).\end{aligned} \tag{31}$$

(IV) If $y_A \leq y_1^+ < y_2^+ < y_0^+ \leq \Phi(y_A)$ (see Figure 4(d)), then

$$\begin{aligned}y_A &\leq y_1^+ < \cdots < y_{2i-1}^+ < y_{2i+1}^+ < \cdots < y_{2i+2}^+ < y_{2i}^+ < \cdots\\ &< y_2^+ < y_0^+ \leq \Phi(y_A).\end{aligned} \tag{32}$$

By the above analysis, for cases (I) and (IV), there is unique $y^* \in [y_A, \Phi(y_A)]$ satisfying $\lim_{i\longrightarrow\infty} y_{2i+1} = \lim_{i\longrightarrow\infty} y_{2i} = y^*$. For (II) and (III), there are two different values y_1^* and y_2^* with $y_1^* \neq y_2^*$ such that $\lim_{n\longrightarrow\infty} y_{2i+1} = y_1^*$ and $\lim_{i\longrightarrow\infty} y_{2i} = y_2^*$. Therefore, the above conclusion is proved. □

Theorem 6. *When* $\Phi(y_A) > y_A$, *then the sufficient and necessary conditions for the global asymptotic stability of the order-one periodic solution of system (2) are as follows: for any* $y_0^+ \in [y_A, y^*]$ *satisfies* $\Phi^2(y_0^+) > y_0^+$, *where* $\Phi(y^*) = y^*$.

Proof. We get that model (2) possesses a stable order-one periodic solution when $\Phi(y_A) > y_A$ by Theorem 5. Then, for system (2), there exists $y^* \in [y_A, \Phi(y_A)]$ which satisfies $\Phi(y^*) = y^*$. For any $y_0^+ \in [y_A, y^*]$, make $y_n^+ = \Phi^n(y_0^+)$, $n = 1, 2, \ldots$, and according to the monotonicity of Poincare$'$ map Φ, we can get $y_A < y_0^+ < y_2^+ < y^* < y_1^+ < \Phi(y_A)$. Furthermore, $y_A < y_{2n}^+ < y^* < y_{2n+1}^+ < \Phi(y_A)$ is obtained. From the monotonicity of the sequence, we know $\lim_{n\longrightarrow\infty} y_{2n}^+ = \lim_{n\longrightarrow\infty} y_{2n+1}^+ = y^*$. Therefore, the order-one periodic solution of system (2) is globally asymptotically stable.

We assume the order-one periodic solution is globally stable for any $y^+ \in [y_A, y^*]$, $\Phi^2(y^+) > y^+$; if not, there exists minimum $y_0 \in [y_A, y^*]$ which makes $\Phi^2(y_0) \leq y_0$. From the theorem, we get that, for any small $\varepsilon > 0$, there exists $y_1 \in (y^* - \varepsilon, y^* + \varepsilon)$ which makes $\Phi^2(y_1) > y_1$. We know from the continuity of Φ that there exists $y^{**} \in (y_0, y_1)$ which makes $\Phi^2(y^{**}) = y^{**}$, that is, system (2) possesses a

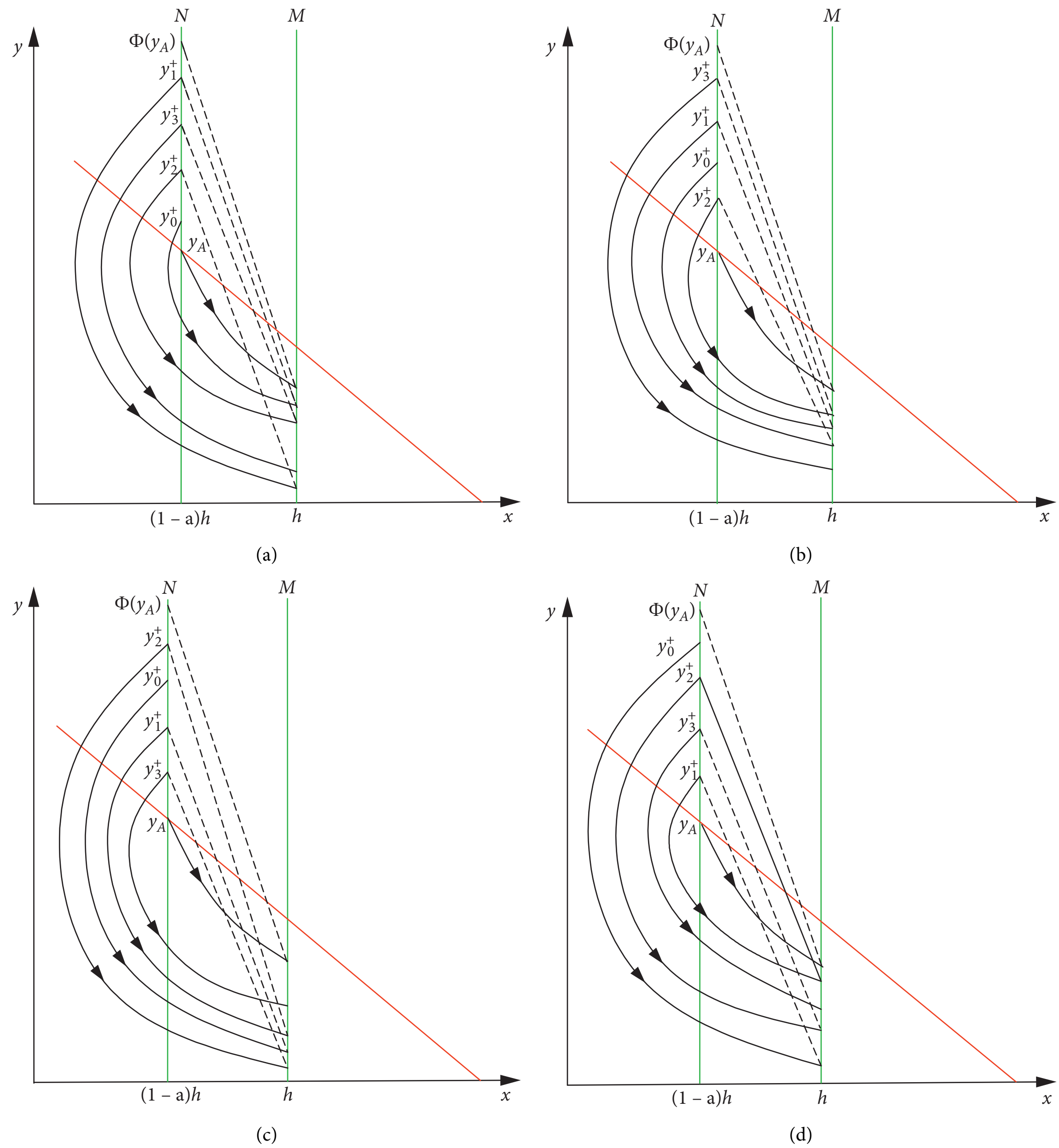

Figure 4: (a) $\Phi(y_A) \geq y_1^+ > y_2^+ > y_0^+ > y_A$. (b) $\Phi(y_A) > y_1^+ > y_0^+ > y_2^+ > y_A$. (c) $y_A < y_1^+ < y_0^+ < y_2^+ \leq \Phi(y_A)$. (d) $y_A \leq y_1^+ < y_2^+ < y_0^+ \leq \Phi(y_A)$.

order-two periodic solution, which is a contradiction with the assumption. □

Theorem 7. *When $\Phi(y_A) > y_A$ and $\Phi^2(y_A) < y_m^+$, where $\Phi(y_m^+) = y_A$, then system (2) has the order-three periodic solution.*

Proof. Since $\Phi(y_A) > y_A$, according to Theorem 1, it is easy to know that there is unique $y^* \in (y_A, \Phi(y_A))$, which makes $\Phi(y^*) = y^*$. Since Poincare′ map is continuous in closed intervals $[0, y^*]$ and $\Phi(0) = c > 0$ and $\Phi(y^*) = y^*$, according to the intermediate value theorem, there is $y_m^+ \in (0, y^*)$ which makes $\Phi(y_m^+) = y_A$; furthermore, $\Phi^3(y_m^+) = \Phi^2(y_A) < y_m^+$, that is, $\Phi^3(y_m^+) - y_m^+ < 0$.

On the contrary, according to the expression of Poincare′ map, we shall get that $\Phi^3(0) = \tau > 0$. So, there is a number $\breve{y} \in (0, y_m^+)$ which makes $\Phi^3(\breve{y}) = \breve{y}$. This means that model (2) has the order-three periodic solution represented by the initial point $((1-a)h, \breve{y})$.

In the same way, we can prove that system (2) has the order-k periodic solution provided $\Phi^{k-1}(y_A) < y_m^+$ and $\Phi(y_m^+) = y_A$. □

4. Numerical Simulation

In this paper, we establish an IPM prey-predator model. We give some special examples to verify the previous theoretical results in this section.

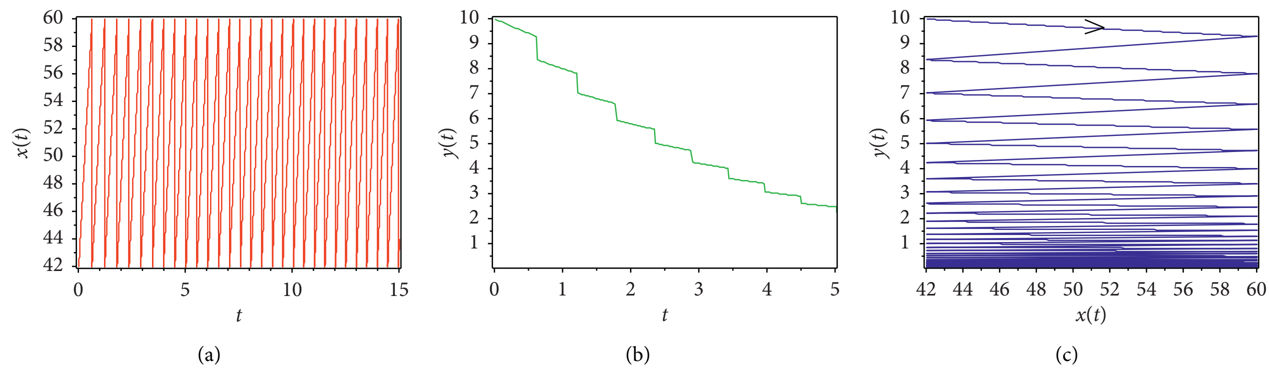

Figure 5: Numerical simulations when $c = 0$. (a) Time series of $x(t)$. (b) Time series of $y(t)$. (c) Phase portrait of $x(t)$ and $y(t)$.

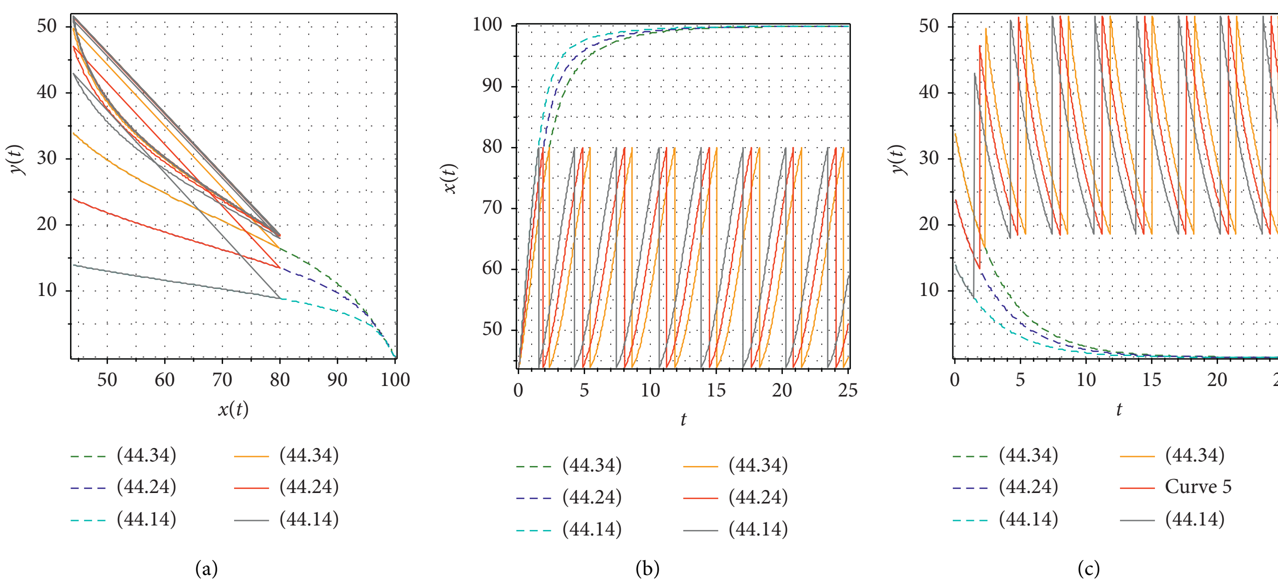

Figure 6: Numerical simulations when $\Phi(y_A) < y_A$. (a) Phase portrait of $x(t)$ and $y(t)$. (b) Time series of $x(t)$. (c) Time series of $y(t)$. The solution of system (2) is presented in full line, and the solution of free system (1) is represented in dotted lines.

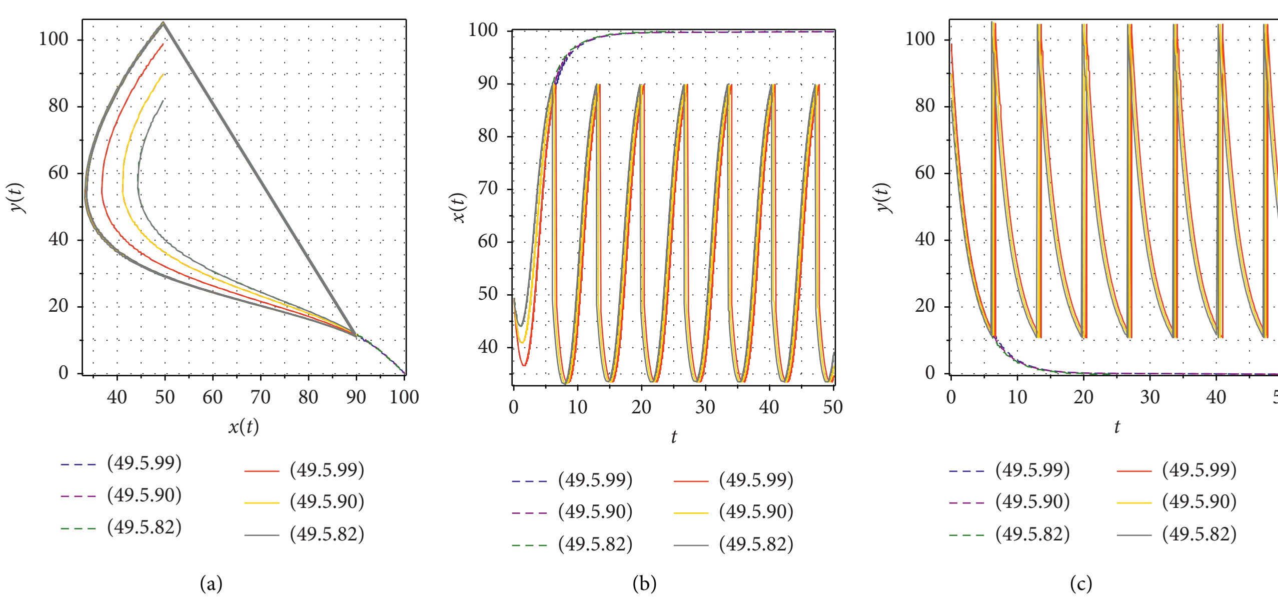

Figure 7: Numerical simulations when $\Phi(y_A) > y_A$. (a) Phase portrait of $x(t)$ and $y(t)$. (b) Time series of $x(t)$. (c) Time series of $y(t)$. The solution of system (2) is presented in full line, and the solution of free system (1) is represented in dotted lines.

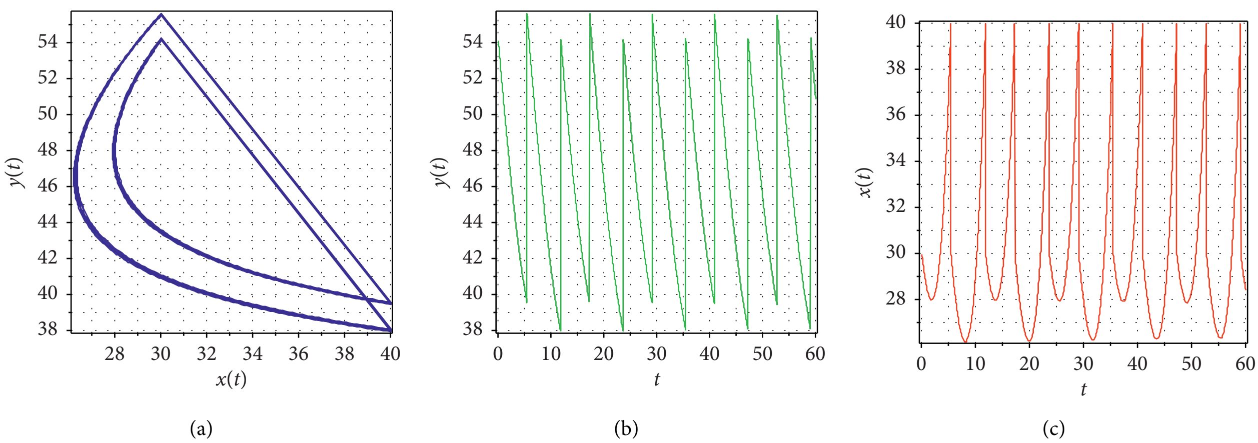

FIGURE 8: Numerical simulation of the order-2 periodic solution. (a) Phase portrait of $x(t)$ and $y(t)$. (b) Time series of $y(t)$. (c) Time series of $x(t)$.

Firstly, we set $r = 1.5$, $\beta = 0.2$, $K = 100$, $d = 0.3$, $m = 0.9$, $\alpha = 0.5$, $a = 0.3$, $b = 0.1$, $h = 60$, and $c = 0$, and then we obtain Figure 5 through numerical simulation. We can know that predator population decreases and becomes extinct eventually provided yield of releases of the predator $c = 0$, while the prey population will periodically oscillate at a relatively high frequency. Secondly, parameter values are assumed as $r = 1.5$, $\beta = 0.2$, $K = 100$, $d = 0.5$, $m = 0.9$, $\alpha = 0.2$, $a = 0.45$, $b = 0.1$, $h = 80$, and $c = 35$, respectively, and obtain Figure 6 that shows model (2) has an order-one periodic solution if $\Phi(y_A) < y_A$. We assume the value of h and c as 90 and 95, respectively, and other values are unchanged. Therefore, we get that model (2) possesses an order-one periodic solution provided $\Phi(y_A) > y_A$ (see Figure 7). It can be seen from Figures 6 and 7 that the trajectories of different initial points eventually tend to be order-one periodic solution when $t \longrightarrow \infty$, which means system (2) has a globally asymptotically stable order-one periodic solution, and the changes of natural enemies and pests are periodic. Therefore, pests are effectively controlled. Meanwhile, we simulate the solution of free system (1), and the results show that natural enemies without impulse will soon become extinct. Finally, we set $r = 1.5$, $K = 100$, $m = 0.9$, $\alpha = 0.1$, $d = 0.5$, $\beta = 0.2$, $a = 0.25$, $b = 0.1$, $h = 40$, $c = 20$, and $\tau = 20$. Then, we get the order-2 periodic solution for system (2) (see Figure 8). Therefore, we verify the existence of the order-2 periodic solution for system (2). The existence of the order-2 periodic solution indicates that natural enemies and pests have periodic changes after two controls. Furthermore, pests are effectively controlled.

5. Conclusion

We construct an IPM model with ratio-dependent in this paper. We take integrated control of biological control and chemical control to keep pest levels below the economic threshold.

Firstly, we conduct a qualitative analysis for model (1) and obtain the conditions for stability of the internal equilibrium point and boundary equilibrium points, respectively. Secondly, we prove some properties of Poincare$'$ map. By using the Poincare$'$ map method, the conditions of stability of the order-one periodic solution are obtained when $c = 0$ and $c > 0$, respectively. Furthermore, we know that system (2) possesses order-k $(k \geq 1)$ periodic solution provided $\Phi^{k-1}(y_A) < y_m^+$ and $\Phi(y_m^+) = y_A$. Finally, the main results are verified by numerical simulation.

In recent years, many scholars have established optimization problems to minimize the cost of pest management [45, 46]. In the future, we will put pest optimization issues into our research work.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors read and approved the final manuscript.

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