Hindawi
Complexity
Volume 2020, Article ID 2476923, 9 pages
https://doi.org/10.1155/2020/2476923

*Research Article*

# Lump-Type Wave and Interaction Solutions of the Bogoyavlenskii–Kadomtsev–Petviashvili Equation

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Received 19 September 2019; Accepted 6 December 2019; Published 8 January 2020

Academic Editor: Átila Bueno



Lump-type wave solution of the Bogoyavlenskii–Kadomtsev–Petviashvili equation is constructed by using the bilinear structure and Hermitian quadratic form. The dynamical behaviors of lump-type wave solution are investigated and presented analytically and graphically. Furthermore, we discuss the interaction between a lump-type wave and a kink wave solution. Absorb-emit interaction between two kinds of solitary wave solutions is shown. This kind of interaction solution can be regarded as a lump-type wave which propagates on the kink wave background.

## 1. Introduction

Lump-type wave solution is a special class of rational localized wave solution which decays algebraically to the background wave in space direction [1]. Since the lump-type wave solution was first discovered in the research of Kadomtsev–Petviashvili (KP) equation [2], seeking for lump-type wave solution of nonlinear evolution equations and exploring its dynamical behavior have attracted more and more attention in the field of nonlinear wave. Over the years, many researchers have found a mass of methods to construct the lump-type wave solution. For example, the inverse scattering transformation method [1], direct algebraic approach [3–6], long wave limit technique [7–9], dressing method [10], Hirota's bilinear method [11, 12], Darboux transformation method [13], Riemann–Hilbert approach [14, 15], and so on. By means of these methods, lump-type wave solutions to many nonlinear wave models have been given, for instance, the Benjamin–Ono equation [9], Veselov–Novikov equation [10], high dimensional Korteweg–de Vries equation [3, 6, 8, 16], Sawada–Kotera equation [17], Jimbo–Miwa equation [4, 18], and so on. Some properties about lump-type wave solution are demonstrated in virtue of the theoretical analysis and graphical representation. In addition, many researchers have also investigated the interaction between lump-type wave and other type of solitary wave solutions, and some interesting interaction phenomena have been shown [16–24].

In this paper, we will devote to investigating the Bogoyavlenskii–Kadomtsev–Petviashvili (BKP) equation [25–32]:

$$u_{xxt} + u_{xxxxy} + 12u_{xx}u_{xy} + 8u_x u_{xxy} + 4u_{xxx}u_y = u_{yyy}, \tag{1}$$

which is an extension of the Bogoyavlenskii–Schiff equation and the KP equation [25]. If we neglect $u_{yyy}$, then it can be reduced to the Bogoyavlenskii–Schiff equation [1]. Through the following transformation [23]:

$$u = (\log f)_x = \frac{f_x}{f}, \tag{2}$$

the BKP equation (1) can be converted into the Hirota bilinear system.

$$\begin{cases} \left(D_x^4 - 3\alpha D_x D_s - 3D_y^2\right) f \cdot f = 0, \\ \left(3D_x D_t + 2D_x^3 D_y + 3\alpha D_y D_s\right) f \cdot f = 0, \end{cases} \tag{3}$$

where $f = f(x, y, t, s)$ is an unknown real function, $s$ is an auxiliary independent variable, and $\alpha$ is a nonzero constant. The differential operator $D$ is defined by [1]

$$D_t^n D_x^m F\cdot G=\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial t'}\right)^n\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)^m[F(x,t)G(x',t')]|_{x'=x,t'=t}. \quad (4)$$

Hence, if $f(x,y,t,s)$ is solved from the Hirota bilinear system (3), then the logarithmic transformation (2) gives rise to a solution of the BKP equation (1). The BKP equation is a member of the high-dimensional KP equations and has been discussed by many researchers [25–31]. The BKP equation is integrable and possesses the Lax pair [25]. The construction of $N$-soliton solutions owes to the work [26]. Some exact solutions, such as period function solution, solitary wave-like solution, and singular rational solution, have been deduced in [27]. Estévez et al. [28] have constructed Lie symmetries by using the Lax pair and the classical Lie group method. The transformation groups and conservation laws of the BKP equation have been presented by the group transformation method and Ibragimov's theorem [29]. By employing the truncated Painlevé method [30], Wang and Fang have also studied the non-auto Bäcklund transformation and consistent Riccati expansion solvable of BKP equation. The bilinear structures and multiple wave solutions have been constructed by means of the binary Bell polynomials method [31]. In [32], Wang and Fang have also investigated various kinds of high-order solitons by employing the perturbation method and Taylor expansion approach. In this paper, we will mainly investigate the lump-type wave solution of the BKP equation (1) by the Hermitian quadratic form. Secondly, we will give the interaction solution of a lump-type wave and a kink wave solutions.

The plan of this work is as follows. Based on the Hermitian quadratic form, Section 2 constructs the lump-type wave and studies its dynamical behavior and symmetrical property further. In Section 3, the hybrid solution consists of a lump-type wave and a kink wave is presented. The absorb-emit interaction between a lump-type wave and a kink wave is discussed. Finally, conclusions are given in section 4.

## 2. Hermitian Quadratic Form and Lump-Type Wave Solution

This section we construct the lump-type wave solution of BKP equation by employing the Hermitian quadratic form. Firstly we introduce the Hermitian matrix and the Hermitian quadratic form.

*Definition 1* (see [33]). If $A$ is a complex square matrix and $A=A^H$, then the matrix $A$ is said to be a Hermitian matrix, where the symbol $H$ denotes the complex conjugate transpose.

*Definition 2* (see [33]). If $X=(x_1,x_2,\cdots,x_n)^T\in\mathbf{C}^n$ is a column matrix and $A=(a_{ij})\in\mathbf{C}^{n\times n}$ is a Hermitian matrix, then the quadratic form,

$$f(X)=X^HAX, \quad (5)$$

is called a Hermitian quadratic form, where the symbol $T$ represents the transpose.

Based on the Hermitian quadratic form, we can obtain the following result:

**Theorem 1.** *The BKP equation (1) has the lump-type wave solution $u=(\log f(x,y,t))_x$ with*

$$f(x,y,t)=X^HAX+R, \quad (6)$$

*where $R$ is a real nonzero constant, $X=(x,y,t,s)^T$ is a real column matrix, $A$ is a four-order Hermitian matrix which is defined by the following matrix:*

$$A=\begin{pmatrix} a_{11} & a_{21}^* & a_{31}^* & a_{41}^* \\ a_{21} & a_{22} & a_{32}^* & a_{42}^* \\ a_{31} & a_{32} & a_{33} & a_{43}^* \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}, \quad (7)$$

*where $^*$ is the complex conjugate and the matrix elements $a_{11},a_{21},a_{22},a_{31},a_{32},a_{33},a_{41},a_{42},a_{43}$ and $a_{44}$ are determined by the following relations:*

$$\begin{aligned} \Re^2(a_{21})&=\frac{a_{11}(Ra_{22}-a_{11}^2)}{R},\\ \Re(a_{31})&=\frac{\Re(a_{21})(Ra_{22}-4a_{11}^2)}{Ra_{11}},\\ \Re(a_{32})&=\frac{a_{22}(Ra_{22}-2a_{11}^2)}{Ra_{11}},\\ \Re(a_{41})&=\frac{2a_{11}^2-Ra_{22}}{R\alpha},\\ \Re(a_{42})&=-\frac{\Re(a_{21})a_{22}}{\alpha a_{11}},\\ \Re(a_{43})&=-\frac{\Re(a_{21})a_{22}^2}{\alpha a_{11}^2},\\ a_{33}&=\frac{a_{22}^3}{a_{11}^2},\\ a_{44}&=\frac{a_{22}^2}{\alpha^2 a_{11}}, \end{aligned} \quad (8)$$

*where $\Re(a_{ij})$ denotes the real part of the complex number $a_{ij}$ and the arbitrary parameters $a_{11},a_{22}$, and $R$ need to satisfy the constraint conditions:*

$$\begin{aligned} Ra_{22}-a_{11}^2&\geq 0,\\ Ra_{11}&>0. \end{aligned} \quad (9)$$

Substituting the solution (6) with (7) and (8) into (3), we can easily verify that this solution is a solution of the Hirota bilinear system (3). In the solution (6), the parameter constraint condition (9) ensures that the solution (6) is well defined and nonzero. In order to reveal the positive definitiveness about the solution (6), let us study further the Hermitian quadratic form $X^HAX$. It is not difficult to find that the Hermitian quadratic form $X^H AX$ can be rewritten as

$$X^H BX, B = \begin{pmatrix} a_{11} & \Re(a_{21}) & \Re(a_{31}) & \Re(a_{41}) \\ \Re(a_{21}) & a_{22} & \Re(a_{32}) & \Re(a_{42}) \\ \Re(a_{31}) & \Re(a_{32}) & a_{33} & \Re(a_{43}) \\ \Re(a_{41}) & \Re(a_{42}) & \Re(a_{43}) & a_{44} \end{pmatrix}. \quad (10)$$

Considering all $k$-order principal minors of the real symmetric matrix $B$, where $k = 1, 2, 3, 4$, then we obtain the following four types of results:

(a) Four first-order principal minors are

$$\begin{aligned} & a_{11}, \\ & a_{22}, \\ & a_{33} = \frac{a_{22}^3}{a_{11}^2}, \\ & a_{44} = \frac{a_{22}^2}{\alpha^2 a_{11}}. \end{aligned} \quad (11)$$

(b) Six second-order principal minors are

$$\begin{aligned} & \frac{a_{11}^3}{R}, \\ & \frac{a_{22}^2 a_{11}}{\alpha^2 R}, \\ & \frac{a_{22}^4}{\alpha^2 R a_{11}}, \\ & \frac{4a_{22}^2 (Ra_{22} - a_{11}^2)}{R^2}, \\ & \frac{4a_{11}^2 (Ra_{22} - a_{11}^2)}{(\alpha R)^2}, \\ & \frac{a_{11} (3Ra_{22} - 4a_{11}^2)^2}{R^3}. \end{aligned} \quad (12)$$

(c) Four third-order principal minors are zero.

(d) One four-order principal minor is zero.

Combining the above principal minors and the method to test positive definiteness of a matrix [33], we can obtain the following two cases:

(1) If $Ra_{22} - a_{11}^2 \geq 0, a_{11} > 0$ and $R > 0$, then all principal minors of $B$ are nonnegative and the matrix $B$ is positive semidefinite

(2) if $Ra_{22} - a_{11}^2 \geq 0, a_{11} < 0$ and $R < 0$, then all odd order principal minors of $B$ are nonpositive, all even order principal minors of $B$ are nonnegative, and the matrix $B$ is negative semidefinite

Therefore, two results above illustrate that the solution (6) is nonzero under condition (9), and the rational polynomial solution (2) with (6) is a nonsingular rational solution.

Based on the above Theorem 1, let

$$a_{ij} = a_i a_j^*, \quad i, j = 1, 2, 3, 4, \quad (13)$$

in the matrix $A$, that is,

$$A = \begin{pmatrix} a_1 a_1^* & a_1 a_2^* & a_1 a_3^* & a_1 a_4^* \\ a_2 a_1^* & a_2 a_2^* & a_2 a_3^* & a_2 a_4^* \\ a_3 a_1^* & a_3 a_2^* & a_3 a_3^* & a_3 a_4^* \\ a_4 a_1^* & a_4 a_2^* & a_4 a_3^* & a_4 a_4^* \end{pmatrix}, \quad (14)$$

then we can obtain the following corollary.

**Corollary 1.** *The BKP equation (1) has the lump-type wave solution $u = (\log f(x, y, t))_x$ with*

$$\begin{aligned} f(x, y, t) = X^H AX + R = & (a_1 x + a_2 y + a_3 t + a_4 s) \\ & \cdot (a_1^* x + a_2^* y + a_3^* t + a_4^* s) + R, \end{aligned} \quad (15)$$

*where the parameters $a_1, a_2, a_3, a_4, R$ are determined by the following relations:*

$$\begin{aligned} R &= -\frac{4(a_1 a_1^*)^3}{(a_1 a_2^* - a_2 a_1^*)^2}, \\ a_3 &= \frac{a_2^3}{a_1^2}, \\ a_4 &= \frac{-a_2^2}{\alpha a_1}, \end{aligned} \quad (16)$$

*and the complex parameters $a_1, a_2$ need to satisfy the nonzero constraint condition:*

$$a_1 a_2^* - a_2 a_1^* \neq 0. \quad (17)$$

In Corollary 1, condition (17) ensures that the parameters $a_3, a_4$ and $R$ are well-defined. Meanwhile, this also suggests that $R > 0$. Consequently, the function (15) is a positive definite function. The rational polynomial solution (2) with $f(x, y, t)$ given by (15) is a nonsingular rational solution.

The above theoretical analysis and results show that the lump-type wave solution can be expressed by (2) with the auxiliary function (6) or (15). However, according to the expressions of the solutions and their constraint conditions, the solution (6) is complicated, because it needs to satisfy ten parameters constraint conditions. The function (15) is more concise and intuitive, and it only contains four parameters constraint conditions. In order to explore and reveal the dynamical properties of the lump-type wave more conveniently, let us represent the complex numbers $a_1$ and $a_2$ as $a_1 = m_1 + in_1, a_2 = v_1 + iw_1$, where $m_1, n_1, v_1, w_1$ are arbitrary real parameters, so the polynomial solution (15) can be rewritten as

$$\begin{aligned} f(x, y, t) = & (m_1 x + v_1 y + r_1 t + \theta_1)^2 \\ & + (n_1 x + w_1 y + d_1 t + \theta_2)^2 + R, \end{aligned} \quad (18)$$

where

$$r_1 = \frac{(v_1^3 - 3v_1w_1^2)(m_1^2 - n_1^2) + 2(3v_1^2w_1 - w_1^3)m_1n_1}{(m_1^2 - n_1^2)^2 + 4m_1^2n_1^2},$$

$$d_1 = \frac{(3v_1^2w_1 - w_1^3)(m_1^2 - n_1^2) - 2(v_1^3 - 3v_1w_1^2)m_1n_1}{(m_1^2 - n_1^2)^2 + 4m_1^2n_1^2},$$

$$\theta_1 = \frac{((w_1^2 - v_1^2)m_1 - 2v_1w_1n_1)}{\alpha(m_1^2 + n_1^2)}s, \tag{19}$$

$$\theta_2 = \frac{((v_1^2 - w_1^2)n_1 - 2v_1w_1m_1)}{\alpha(m_1^2 + n_1^2)}s,$$

$$R = \frac{(m_1^2 + n_1^2)^3}{(m_1w_1 - n_1v_1)^2},$$

and the nonzero constraint condition (17) about the parameters $a_1$ and $a_2$ becomes

$$m_1w_1 - n_1v_1 \neq 0. \tag{20}$$

So, based on (18)–(20), we derive the specific expression of lump-type wave solution:

**Corollary 2.** *The BKP equation has the following lump-type wave solution:*

$$u(\xi,\eta) = \frac{2((m_1^2 + n_1^2)\xi + (m_1v_1 + n_1w_1)\eta + m_1\theta_1 + n_1\theta_2)}{(m_1\xi + v_1\eta + \theta_1)^2 + (n_1\xi + w_1\eta + \theta_2)^2 + R}, \tag{21}$$

*where*

$$\xi = x - \frac{2(v_1^2 + w_1^2)(m_1v_1 + n_1w_1)}{(m_1^2 + n_1^2)^2}t,$$

$$\eta = y - \frac{(m_1w_1 - n_1v_1)^2 - 3(m_1v_1 + n_1w_1)^2}{(m_1^2 + n_1^2)^2}t, \tag{22}$$

*where $\theta_1, \theta_2$, and $R$ are given by (19) and the arbitrary real parameters $m_1, n_1, v_1$, and $w_1$ need to satisfy condition (20).*

The lump-type wave solution (21) is nonsingular and well defined when the parameters $m_1, n_2, v_1$, and $w_1$ satisfy condition (20). In the meantime, the condition $m_1w_1 - n_1v_1 \neq 0$ also indicates that this solution cannot be transformed into the line lump-type wave solution [34]. In the solution (21), the parameters $\theta_1$ and $\theta_2$ can be regarded as two arbitrary phase constants. Figure 1 shows the spatial structure and projection of the lump-type wave solution (21) at $t = 0$. As can be seen from the Figure 1(a), this wave is a localized lump-type wave which has one upper peak and one down hump near the origin. The down hump hides under the background plane wave, and the height of the peak is equal to the depth of the down hump-type wave. In [6], this kind of solution is also known as "bright-dark lump-type wave solution," and its amplitude is $|m_1w_1 - n_1v_1|/m_1^2 + n_1^2$. Figure 1(b) is the contour lines and projection of the lump-type wave solution in the $(x, y)$ plane. The localization features and energy distribution are clearly presented. In the $(x, y)$ plane, the lump-type wave solution is divided into two parts by the line $L$. Here the line $L$ is given by the straight line $(m_1^2 + n_1^2)\xi + (m_1v_1 \quad +n_1w_1)\eta + m_1\theta_1 + n_1\theta_2 = 0$, which represents the contour line with $u = 0$. The right area represents the energy distribution of the bright hump-type wave, and the left area shows the energy distribution of the dark hump-type wave (Figure 1(b)). Moreover, the lump-type wave solution (21) also indicates that it tends to zero as $\xi^2 + \eta^2$ goes to the infinity for any fixed time $t$. These properties demonstrate that the lump-type wave (21) is a rationally decaying solution and localized in all directions in the background space. Furthermore, we can see from (22) that the lump-type wave propagates in the $(x, y)$-plane with the velocity:

$$\mathbf{v} = (v_x, v_y) = \left(\frac{2(w_1^2 + v_1^2)(m_1v_1 + n_1w_1)}{(m_1^2 + n_1^2)^2}, \frac{(m_1w_1 - n_1v_1)^2 - 3(m_1v_1 + n_1w_1)^2}{(m_1^2 + n_1^2)^2}\right). \tag{23}$$

This also means that the lump-type wave moves obliquely along the straight line,

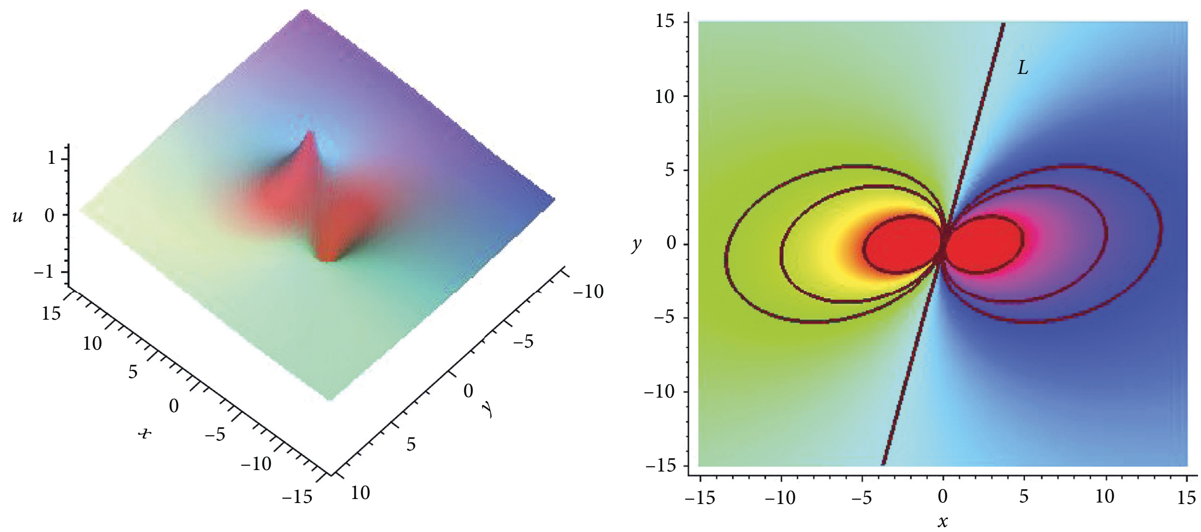
$$\frac{x - x_0}{v_x} = \frac{y - y_0}{v_y}, \tag{24}$$

in the $(x, y)$ plane, where

$$x_0 = \frac{v_1\theta_2 - w_1\theta_1}{m_1w_1 - n_1v_1},$$

$$y_0 = \frac{n_1\theta_1 - m_1\theta_2}{m_1w_1 - n_1v_1}. \tag{25}$$

In Figure 1(b), the demarcation line $L$ between the bright and the dark areas is oblique. This suggests that the shape of lump-type wave is nonaxisymmetric, that is, the lump-type wave is skew. Through the straight line $L$, we can obtain that the angle $\phi$ between the line $L$ and the positive $y$-axis is determined by $\tan\phi = -m_1v_1 + n_1w_1/m_1^2 + n_1^2$. Therefore, if $\phi \neq 0$, i.e., $m_1v_1 + n_1w_1 \neq 0$, the lump-type wave is skew (Figure 1). However, if $m_1v_1 + n_1w_1 = 0$, then the velocity $v_x$ along the $x$-axis is zero. As a result, the propagation velocity of the lump-type wave is completely determined by $v_y = (m_1w_1 - n_1v_1/m_1^2 + n_1^2)^2$, and this implies that the lump-type wave propagates only along the positive $y$-axis. In




(a) (b)

FIGURE 1: 3D profile of lump-type wave solution at $t = 0$ (a) and its contour plot (b). The parameters are selected with $a_1 = 1 + i$, $a_2 = -1.5 + i$, $\alpha = 1$, and $s = 0$. The line $L$ is given by $y = 4x$.

this case, the contour lines of the lump-type wave with $s = 0$ and $t = 0$ are determined by the following equation:

$$\left(x - \frac{1}{h}\right)^2 + \frac{v_1^2 + w_1^2}{m_1^2 + n_1^2} y^2 = \frac{1}{h^2} - \left(\frac{m_1^2 + n_1^2}{m_1 w_1 - n_1 v_1}\right)^2, \quad (26)$$

where $0 < |h| < |m_1 w_1 - n_1 v_1| / m_1^2 + n_1^2$. Specially, when $h = 0$, the contour line becomes the line $x = 0$. Obviously, equation (26) represents the elliptic curve. This elliptic curve is symmetrical with respect to the line $x = 1/h$ and $y = 0$. Therefore, when $m_1 v_1 + n_1 w_1 = 0$, the lump-type wave is symmetric (Figure 2).

## 3. Absorb-Emit Interaction between Lump-Type Wave and Kink Wave Solutions

We investigate the interaction between a lump-type wave and kink wave solutions. The kink wave solution can be obtained by using the following exponential function [35]:

$$f(x, y, t) = 1 + e^{\tau}, \quad \tau = kx + ly + ct + ds, \quad (27)$$

where the nonzero real parameters $k, l, c,$ and $d$ satisfy the following conditions:

$$c = \frac{l(l^2 - k^4)}{k^2}, \qquad d = \frac{k^4 - 3l^2}{3\alpha k}. \quad (28)$$

Substituting (27) with (28) into (2) yields a kink wave solution:

$$u(x, y, t) = \frac{1}{2} k\left(1 + \tanh \frac{1}{2}\tau\right). \quad (29)$$

By the asymptotic analysis, we can find that this solution tends to $k$ as $\tau \longrightarrow +\infty$ and approaches to zero as $\tau \longrightarrow -\infty$. Two different asymptotic states are presented. So the solution (29) is a kink-type solitary wave and has amplitude $|k|$.

In the above discussion, we study the lump-type wave and kink wave solutions. Next, we will consider the interaction solution of a lump-type wave and a kink wave. Based on the structure form of the two-soliton solution [1], we assume that the Hirota bilinear system (3) has the following hybrid solution:

$$f(x, y, t) = X^H A X + R + e^{X^H K}\left(Y^H B Y + R\right), \quad (30)$$

where

$$X = \begin{pmatrix} x \\ y \\ t \\ s \end{pmatrix}, \quad K = \begin{pmatrix} k \\ l \\ c \\ d \end{pmatrix}, \quad Y = \begin{pmatrix} x \\ y \\ t \\ s \\ 1 \end{pmatrix}, \quad B = \begin{pmatrix} a_1 a_1^* & a_1 a_2^* & a_1 a_3^* & a_1 a_4^* & a_1 a_5^* \\ a_2 a_1^* & a_2 a_2^* & a_2 a_3^* & a_2 a_4^* & a_2 a_5^* \\ a_3 a_1^* & a_3 a_2^* & a_3 a_3^* & a_3 a_4^* & a_3 a_5^* \\ a_4 a_1^* & a_4 a_2^* & a_4 a_3^* & a_4 a_4^* & a_4 a_5^* \\ a_5 a_1^* & a_5 a_2^* & a_5 a_3^* & a_5 a_4^* & a_5 a_5^* \end{pmatrix}, \quad (31)$$

where $A$ is given by (14), the parameters $a_1, a_2, a_3,$ and $a_4$ and $R$ satisfy conditions (16) and (17), the relationships between $k, l, c$ and $d$ are given by (28), and $a_5$ is a shift

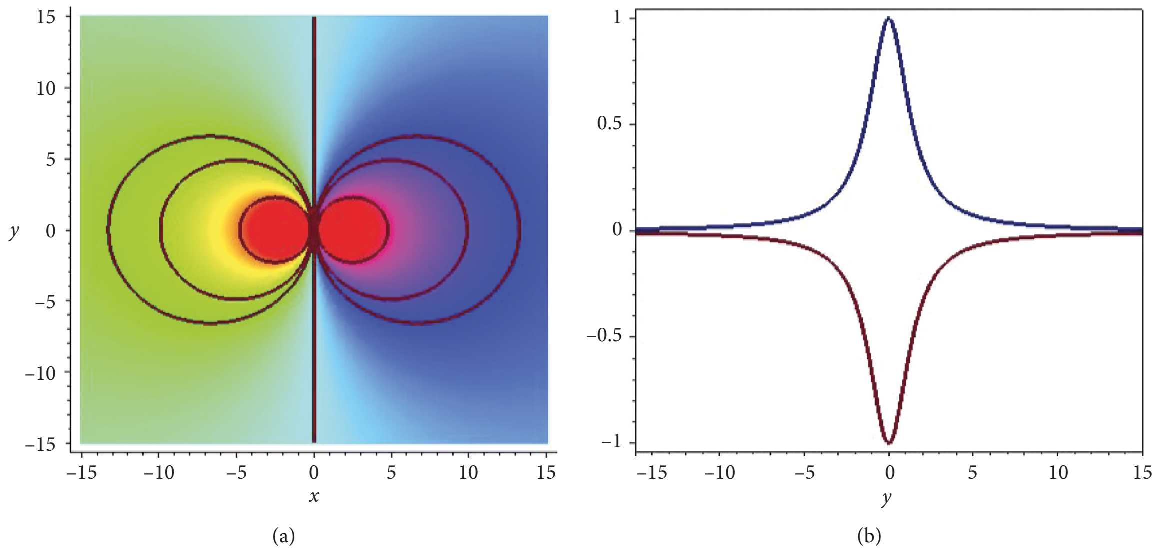



FIGURE 2: (Color online) Contour of symmetrical lump-type wave solution at $t = 0$ (a) and its cross-sectional plot at $|x| = 1$ (b). The parameters are selected with $a_1 = 1 + i, a_2 = -1 + i, \alpha = 1$, and $s = 0$.

parameter to be determined later. Then, substituting (30) with (31) into equation (3) yields

$$a_5 = \frac{-4(ka_1)^3}{a_1^2k^4 + (ka_2 - la_1)^2}. \tag{32}$$

Hence, the interaction solution consists of a lump-type wave, and a kink wave can be obtained as follows.

**Theorem 2.** *The BKP equation (1) has the interaction solution $u = (\log f(x, y, t))_x$ with (30), where $a_1, a_2, a_3, a_4, a_5, k, l, c, d$, and $R$ are given by*

$$\begin{aligned} a_3 &= \frac{a_2^3}{a_1^2}, \\ a_4 &= \frac{-a_2^2}{\alpha a_1}, \\ a_5 &= \frac{-4(ka_1)^3}{a_1^2k^4 + (ka_2 - la_1)^2}, \\ c &= \frac{l(l^2 - k^4)}{k^2}, \\ d &= \frac{k^4 - 3l^2}{3\alpha k}, \\ R &= \frac{-4(a_1a_1^*)^3}{(a_1a_2^* - a_2a_1^*)^2}, \end{aligned} \tag{33}$$

*where the parameters $a_1, a_2$, and $k$ need to satisfy the nonzero constraint conditions:*

$$k \neq 0, a_1a_2^* - a_2a_1^* \neq 0. \tag{34}$$

As can be seen from the hybrid solution (30), it consists of three parts, i.e., two polynomial functions and an exponential function, corresponding to the lump-type wave and kink wave solutions. Hence, this solution describes a kind of interaction between a lump-type wave and a kink wave. Figure 3 shows the interaction phenomenon between a lump-type wave and a kink wave in the $(x, y)$ plane. When $t$ is much less than zero, i.e., far from the interaction region, the resulting interaction solution describes a superposition of a kink wave and a lump-type wave, (Figure 3(a)). The lump-type wave is located at the asymptotic background $u = 0$ of the kink wave. However, when the lump-type wave approaches to the kink wave, the lump-type wave is gradually absorbed by the kink wave, (Figures 3(b) and 3(c)). At the same time, a new lump-type wave is emitted from the top of the kink wave. In particular, when $t = 0$, this solution clearly displays a hybrid structure of two lump-type waves and a kink wave, as presented in Figure 3(c). When $t > 0$, with the development of time, the lump-type wave at the bottom finally disappears in the kink wave. The lump-type wave at the top is separated completely from the kink wave, and it is located at another asymptotic background $u = k$ of the kink wave (Figures 3(d) and 3(e)). In the process of interaction, the lump-type wave seems to exchange the energy by the absorption and emission of the kink wave at the interaction instant. And in this way, the lump-type wave can jump from one asymptotic state of the kink wave to another asymptotic state. Thus, this interaction phenomenon is called absorb-emit interaction of a lump-type wave and a kink wave. Besides, from Figure 3, we can see that, when the distance between the lump-type wave and the kink wave increases, the interaction solution can be expressed as a sum of two isolated lump-type wave and one kink wave solutions:

$$u(x, y, t) = \begin{cases} u_l(x, y, t) + u_k(x, y, t), \\ u_r(x, y, t) + u_k(x, y, t), \end{cases} \tag{35}$$

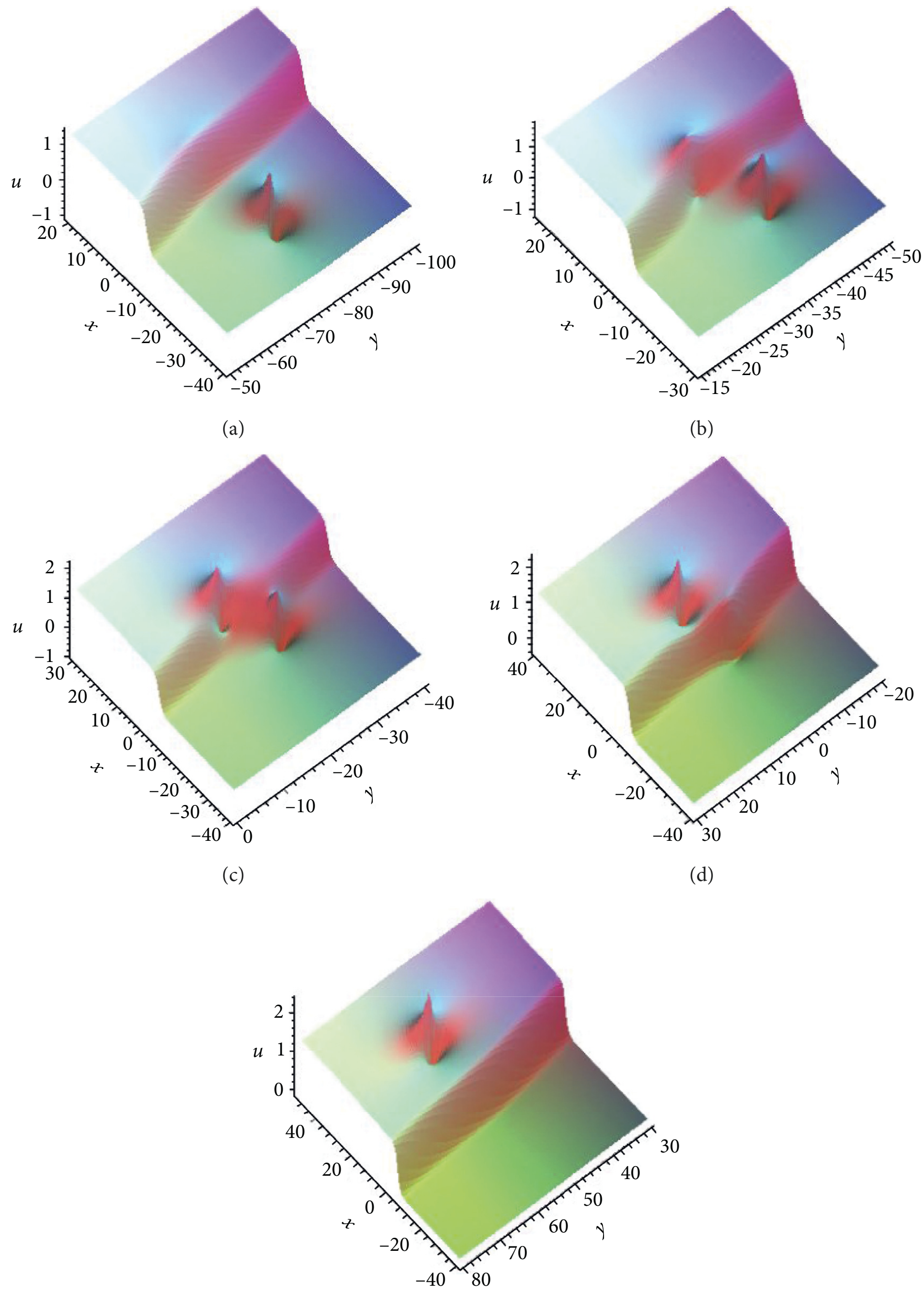



Figure 3: (Color online) Absorb-emit interaction between a lump-type wave and a kink wave. The parameters are selected with $a_1 = 1 + 1 = 1.2i, a_2 = 1.5 - i, \alpha = 1, k = 1.25, l = 0.3$, and $s = 0$. (a) $t = -60$; (b) $t = -30$; (c) $t = -20$; (d) $t = 0$; (e) $t = 40$.

where

$$\begin{aligned} u_l(x, y, t) &= \left(\ln\left(X^H AX + R\right)\right)_x, \\ u_r(x, y, t) &= \left(\ln\left(Y^H BY + R\right)\right)_x, \end{aligned} \tag{36}$$

are two single lump-type waves and $u_k$ is a kink wave given by (29). Indeed, $u_l(x, y, t)$ and $u_r(x, y, t)$ display the lump-type waves before and after the interaction, and they are the same lump-type wave except for a phase shift. This implies that the interaction between a lump-type wave and a kink wave yields a lump-type wave with phase shift. The kink wave remains the initial propagation trajectory and has no phase shift. Furthermore, using the asymptotic behaviors of kink wave, we also note that in terms of the interaction solution, we have the following asymptotic behaviors for $t \longrightarrow \pm\infty$ with $c < 0$

$$u(x, y, t) \longrightarrow \begin{cases} u_l(x, y, t) + k, & \text{as } t \longrightarrow +\infty, \\ u_r(x, y, t), & \text{as } t \longrightarrow -\infty. \end{cases} \tag{37}$$

The asymptotic behaviors show that when $|t| \longrightarrow \infty$, the interaction solution represents only a lump-type wave solution which is located at the asymptotic background of the kink wave.

In short, as can be seen from the above analysis and graphical representations, the interaction solution given by (30) can be regarded as a lump-type wave which propagates on the kink wave background. On the kink wave background, the lump-type wave can move from one asymptotic state to another asymptotic state (Figure 3). In [36, 37], the interaction between a kink solitary wave solution and a lump-type wave solution was investigated. However, the obtained results showed that when the lump-type wave solution collided with the kink solitary wave, the lump-type wave solution was completely absorbed or emitted by the kink solitary wave. The lump-type wave cannot jump from one asymptotic state of the kink wave to another asymptotic state.

## 4. Conclusion

By using the Hermitian quadratic form, we have derived the lump-type waves of the BKP equation (Theorem 1 and Corollaries 1 and 2). The dynamical behavior and symmetrical property of the lump-type wave solution have been investigated and displayed analytically and graphically (Figures 1 and 2). We also showed that this lump-type wave solution propagates along the straight line given by (24) on the constant background wave. Furthermore, we have discussed the interaction solution of a lump-type wave and kink wave solutions (Theorem 2). Absorb-emit interaction between a lump-type wave and a kink wave has been demonstrated (Figure 3). Indeed, Figure 3 has also shown that the lump-type wave can move from one asymptotic state of the kink wave to another asymptotic state. Thus, this kind of interaction solution can be regarded as a lump-type wave which propagates on the kink wave background.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Acknowledgments

This research was supported by the National Natural Science Foundation of China (No. 11801240) and the Fund for Fostering Talents in Kunming University of Science and Technology (No. KKSY201707021).

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