Research Article

Hybrid-Driven Mechanism Based on Uncertain Network for Markov Jump System with Quantizations and Delay

Tian Tan , Jinfeng Gao , Jinxia Wang , Zhen Zhao , and Miao Ma

Faculty of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Hangzhou, Zhejiang 310018, China

Correspondence should be addressed to Jinfeng Gao; gaojf163@163.com

Received 1 July 2019; Revised 18 October 2019; Accepted 24 February 2020; Published 8 April 2020

Academic Editor: Marcio Eisencraft

Copyright © 2020 Tian Tan et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper investigates the hybrid-driven mechanism problem for Markov jump system, where both channel quantization (BCQ) and network-induced delay based on uncertain network are considered. Firstly, comparing with the traditional event-triggered scheme, a hybrid-driven mechanism is employed in networked control systems (NCSs) for the finite capacity of communication bandwidth resources and system performance in equilibrium. Then, the quantization technology is applied in the communication channel from sensor-to-controller and controller-to-actuator. The application of BCQ is for further investigation that mitigate data packet transmission rate. Thirdly, Markov jump system is modeled for the hybrid-driven mechanism and network-induced delay. By constructing the Lyapunov–Krasovskii function, a sufficient condition is derived as the stability criterion, and the controller is designed in which the nonlinear term is rewritten for simplifying the calculation. Finally, two simulation examples are provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

Networked control systems (NCSs), distributed systems in which control components such as sensor, controller, and actuator are connected to each other through the communication network channels, are widely applied in various areas such as industrial-control field, multiagent collaboration systems, and smart grids [1]. Accordingly, much attention of researchers is paid to analyzing the NCSs during the past several decades. In contrast with traditional point-to-point data transmission strategy, great advantages of NCSs are in the physical and economic dimension, including low complexity, flexibility, low cost, high reliability, and economic benefits [2]. However, due to the introduction of control objects, a few issues are drawn into the closed-loop control systems. Numerous transmission data occupy limited communication bandwidth which causes consequences, for instance, network-induced delays, data packet dropout, and disorder [3–5]. These drawbacks may lead to a threat of system performance and even result in network paralysis. Therefore, the investigation that how to promote the

utilization rate of communication bandwidth is studied extensively in [6–11] and references therein.

The stochastic noise and exogenous disturbance affect the stability and performance of the system greatly. To address the imperfections mentioned effectively, researchers model the random variations of system state as Markov chains. In [12], the researchers propose a Markov jump nonlinear system, where they refer to the controller design of adaptive sliding-mode control. Different system research objects can be modeled as Markov jump process, such as sampling process, stochastic time delay, and the combination of guaranteed cost control (GCC) and neural networks in [13–15]. In [16], Dong et al. devise Markov system GCC problem, in which the stochastic quantization is modeled with hidden Markov process. In [17, 18], authors focus on the data packet dropouts problem which is considered as Markov jump process. More systems and research objects which combine with Markov jump system or semi-Markovian jump system are reported in [19–21]. Obviously, as to data packet dropout, network-induced delay, and uncertain system, Markov jump is pretty effective and more intuitive.

In order to tackle the difficulties with regard to control systems information transmission and the communication bandwidth resources constrained, researchers propose a method called quantization. The quantization of the signal can be regarded as mapping that continuous signals are converted to a set of finite discrete signals. Then, only the discrete signals can be transmitted through network communication channel and the objective of relieving the control systems communication burden can be achieved. This method is discussed by many researchers in the literature. In [22–24], the authors investigate the NCSs that the control input is quantified. The application areas of quantization technology are diverse under different network frames. For neural network, a logarithmic static and time-invariant quantizer are considered in [25], and quantization is employed in discrete semi-Markov jump network in [26]. The effects of different logarithmic quantizers are investigated in [27], the so-called convex combination and sector bounded. Multidensity logarithmic quantizer is developed in [28] for the communication resource saving. However, the application of quantization will introduce the quantization error in the meantime, which causes the system to become nonlinear. Inevitably, the stability of the system will be reduced. The system performance should be considered and balanced when selecting the quantization parameters.

For the purpose of solving the problems mentioned above, event-triggered scheme is put forward and arouses increasing research attention in recent years. For the traditional time-triggered scheme, sampler operates in a fixed period. A lot of meaningless sampled signals are transmitted through a shared communication network. Comparatively, the event-triggered scheme is a new update mode of control signal, of which signals transmission will be executed only if a certain performance indicator is violated. Therefore, more network bandwidth resources are released for other control components, and system performance can be guaranteed. Up to now, many previous investigations are reported. In [29–31], the event-triggered control scheme is applied in neural network filter designing, fault detection, and sliding-mode control. In-depth research studies of network-based systems oriented to different objects are published [32–34], in which multiagent systems and Markov jump systems are involved. To balance and optimize the data transmission referring to bandwidth occupancy rate and the loss of significant information, Liu et al. [35] proposes a novel method called hybrid-driven mechanism which is composed of time-triggered and event-triggered, and Bernoulli distribution is utilized to decide which communication scheme will be chosen. Therefore, many significant literature studies are published. For instance, a H_∞ control problem of the nonlinearity system is researched [36], which employs the hybrid-driven mechanism. In [37, 38], NCSs under cyber attacks with hybrid-driven mechanism are developed. To the best knowledge of author, no work has been carried out in the area of hybrid-driven mechanism for the Markov jump system with quantization and delay. This motivates our study.

Inspired by the investigation aforementioned, in this paper, we focus on the hybrid-driven mechanism based on

uncertain networked for Markov jump system with quantizations and delay. The main contributions are given as follows:

- (i) The network state is modeled as the Markov jump system, and the hybrid-driven mechanism is put forward to determining whether the sampling data should be transmitted or not.
- (ii) To optimize the dynamic equilibrium between system performance and communication load contrasting with the existing literature [8], the switching rule between time-triggered scheme and event-triggered scheme will be ensured by stochastic Bernoulli distribution.
- (iii) Dual-quantizer is applied for the purpose of reducing communication resources wastage further. The network-induced delay will be considered by the method of [6].
- (iv) Via Lyapunov–Krasovskii and free-weighting matrix, a sufficient condition is presented and the desired controller will be designed. A numerical example and a practical example are provided to illustrate the effectiveness of the approach proposed in this paper.

The remainder of this paper is organized as follows. In Section 2, the problem is narrated and analyzed in detail. The main results of system stability and controller design are derived in Section 3. In Section 4, the results of simulation examples are given. The conclusion is presented in Section 5.

1.1. Notation. $\mathbb{R}^m$ represents the m -dimensional Euclidean space and $\mathbb{R}^{m \times n}$ means the set of $m \times n$ dimension real matrices. $\mathbb{N}$ denotes the set of nonnegative integers. I denotes the identity matrix of appropriate dimension. Superscript T is the transposition of matrix and $\text{diag}\{\dots\}$ stands for the block diagonal matrix. $\text{sym}\{A\}$ stands for the $A + A^T$. In symmetric matrices, “ $*$ ” denotes a term induced by symmetry. The notation $P > 0$ (< 0) stands for real symmetric positive definite.

2. Problem Formulation

2.1. System Description. The structure of NCSs is shown in Figure 1. Hybrid-driven communication mechanism is the key element for reducing the signal data transmission rate in this system. Consider the Markov jump uncertain system as follows:

$$\dot{x}(t) = [A(r_t) + \Delta A(r_t)]x(t) + B(r_t)u(t), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the system control input, $A(r_t)$ and $B(r_t)$ represent real constant system matrices with appropriate dimensions, and $\Delta A(r_t)$ is unknown item. It denotes time-varying matrices that are bounded parameter uncertainties, and follow the constraint [39] as follows:

$$\Delta A(r_t) = D(r_t)F(t)H(r_t), \quad (2)$$

FIGURE 1: The structure of the Markov jump system with hybrid-driven mechanism.

where $D(r_t)$ and $H(r_t)$ are constant matrices with appropriate dimensions. $F(t)$ is the unknown time-varying matrix and has the following property:

$$F^T(t)F(t) \leq I, \quad (3)$$

r_t is assumed to be the system mode that is a homogeneous Markov jump stochastic process, which obtain values in a finite set $S = \{1, 2, \dots, M\}$. $\Pi = \lambda_{ij}$ ($i, j \in S$) denotes the transition probability matrix, and is given by

$$P\{r_{t+\Delta h} = j | r_t = i\} = \begin{cases} \lambda_{ij}\Delta h + o(\Delta h), & i \neq j, \\ 1 + \lambda_{ii}\Delta h + o(\Delta h), & i = j, \end{cases} \quad (4)$$

where λ_{ij} represents the transition rate that system jumps from mode i to mode j and follows the principles of $\lim_{\Delta h \rightarrow \infty} (o(\Delta h)/\Delta h) = 0$, $\Delta h > 0$, and $\lambda_{ij} \geq 0$, $\lambda_{ii} = -\sum_{j=1, j \neq i}^M \lambda_{ij}$ for $i \neq j$.

The transition probability matrix is defined as follows:

$$\Pi = \begin{bmatrix} \lambda_{11} & \cdots & \lambda_{1M} \\ \vdots & \ddots & \vdots \\ \lambda_{M1} & \cdots & \lambda_{MM} \end{bmatrix}. \quad (5)$$

The following assumptions and simplicity is considered in the paper for further development and obtaining the main results:

- (i) The loss and disorder of data packet are not considered in the system. A single packet is transmitted at each time-stamped in the network channel. The external disturbance is omitted.
- (ii) The logic zero order hold (ZOH) in the actuator is used to hold the control input. In other words, the actuator maintains the last transmitted data until the current sampling data arrive via network.
- (iii) For simplicity, we substitute $A(r_t)$, $\Delta A(r_t)$, $B(r_t)$, $D(r_t)$, and $H(r_t)$ with A_i , ΔA_i , B_i , D_i , and H_i , while $r_t = i$ ($i \in S$).

In this paper, the hybrid-driven mechanism problem for the Markov jump system with quantization and network-induced delay based on uncertain network is developed. The feedback controller expression is given as follows:

$$u(t) = K_i x(t_k h), \quad t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}). \quad (6)$$

Similar to [6], τ_{t_k} is the network-induced communication delay which represents the delayed time with sensor-to-

controller and controller-to-actuator. $t_k h$ is the sampling instant which satisfy $\{t_1, t_2, \dots\} \subset \{1, 2, 3, \dots\}$. K_i is the controller gain to be designed.

Remark 1. The actuator, quantizers, and controllers are driven by event, and the set of sampling sequences is expressed as $\{t_k h | t_k \in \mathbb{N}\}$ while the sensors and the sampler are time-triggering with a constant sampling period h . The set of data transmitted successfully is described by $\{kh | k \in \mathbb{N}\}$.

2.2. Hybrid-Driven Mechanism. In order to reduce the network channel communication burden and save the network resource, comparing with the traditional time-triggered communication scheme and event-triggered communication scheme, inspired by [35], we address the controller problem with hybrid-driven mechanism, which is composed of time-triggered and event-triggered. For the balance of system performance and data transmission frequency, a switching following Bernoulli distribution is used to choose the appropriate communication scheme between two mechanisms.

If the hybrid-driven mechanism is under time-triggering, we define network-induced delay $d(t) = t - t_k h$, where $t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$; then equation, (6) can be rewritten as

$$u(t) = K_i x(t - d(t)), \quad (7)$$

where $0 \leq d_m \leq d(t) \leq d_M$, and d_M is the upper bound of the network-induced communication delay.

For the purpose of reducing data packet transfer rate, inspired by [22], whether the latest sampling data can be transmitted via the communication channels to the controller and actuator will be decided by the condition as follows:

$$e_k^T(t) \Phi_i e_k(t) < \sigma x^T(t - \tau(t)) \Phi_i x(t - \tau(t)), \quad (8)$$

where $e_k(t) = x(t_k h) - x(t_k h + lh)$ indicates the system error vector, $\sigma \in [0, 1]$ is a given scalar triggered parameter, and $\Phi_i > 0$ ($i = 1, 2$) mean symmetric positive triggered matrices. $x(t_k h)$ represents the last transmission data. $x(t_k h + lh)$ is the current sampled signals.

Define $\rho = t_{k+1} - t_k - 1$, the interval $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$ are divided into subintervals as $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) = \cup_{l=0}^{\rho} \Omega_l$, where $\Omega_l = [i_k^l + \tau_{t_{k+1}}, i_k^l + h + \tau_{t_{k+1}}]$, and $i_k^l h = t_k h + lh$, $l = 0, 1, 2, \dots, \rho$. Define $\tau(t) = t - i_k^l h$, which has $0 \leq \tau_m \leq \tau(t) \leq h + \tau_{t_{k+1}} \triangleq \tau_M$. Then, we can get $x(t_k h) = x(t - \tau_t) + e_k(t)$, and equation (6) can be rewritten as

$$u(t) = K_i [x(t - \tau(t)) + e_k(t)]. \quad (9)$$

According to [35], for the switching between time-triggered scheme and event-triggered scheme, Bernoulli distribution is considered to construct hybrid-driven mechanism based on equations (7) and (9). The plant input $u(t)$ can be rewritten as

$$u(t) = \alpha(t)K_i x(t - d(t)) + (1 - \alpha(t))K_i [x(t - \tau(t)) + e_k(t)], \quad (10)$$

where $\alpha(t) \in [0, 1]$. $\alpha(t)$ is a stochastic variable. $\alpha(t) = 1$ means that the NCSs are under the time-driven scheme and the sampling signals are transmitted periodically. $\alpha(t) = 0$ means that the NCSs are in motion under the event-driven scheme, and the period sampling signals transmitted will be constrained unless the condition of event-triggered scheme is violated.

According to the Bernoulli distribution, the mathematical expectation of $\alpha(t)$ satisfy $\mathbb{E}(\alpha(t)) = \bar{\alpha}$, and its mathematical variance $\mu^2 = \bar{\alpha}(1 - \bar{\alpha})$.

Remark 2. The traditional event-triggered scheme is considered in hybrid-driven mechanism of this paper. More investigation about event-triggered scheme is developed in many literature studies, such as adaptive event-triggered scheme [34, 40]. One possible future research can be directed to employ the adaptive event-triggered scheme into hybrid-driven mechanism.

2.3. Quantizer. Based on the limited transmission capability and the aim to decrease the data transmission of the communication channel, for the feature that the network communication quality is seriously affected with high quantization density, while important data packet information will be lost with low quantization density, BCQ is employed in system. As shown in Figure 1, the control signal $x(t_k h)$ is converted into $h(x(t_k h))$ by the quantizer $h(\cdot)$. Inspired by [41], at the side of the sensor, we define the quantizer as $h(x) = [h_1(x_1)h_2(x_2)h_3(x_3) \dots h_n(x_n)]^T$, where the logarithmic quantizers $h_i(x_i)$ ($i = 1, 2, \dots, n$) are given by

$$h_i(x_i) = \begin{cases} \omega_l^i, & \text{if } \frac{\omega_l^i}{1 + \delta_i} < x_i < \frac{\omega_l^i}{1 - \delta_i}, x_i > 0, \\ 0, & \text{if } x_i = 0, \\ -h_i(-x_i), & \text{if } x_i < 0, \end{cases} \quad (11)$$

where

$$\delta_{h_i} = \frac{1 - \rho_{h_i}}{1 + \rho_{h_i}}, \quad (12)$$

$\rho_{h_i} \in (0, 1)$, and ρ_{h_i} denotes the quantization density that is a given constant. Similar to [42], the quantization levels can be described as

$$W = \{\pm \omega_l^i: \omega_l^i = \rho_{h_i}^l \omega_0^i, l = \pm 1, \pm 2, \dots\} \cup \{\pm \omega_0^i\} \cup \{0\}, \quad (13)$$

where $\omega_0^i > 0$. To our best knowledge, the current static quantizer can be summarized as a class of quantizers, of which sector bound condition as the following should be satisfied:

$$|\Delta_q| \leq \delta |u| + (1 - \delta)d, \quad (14)$$

where quantizer design parameters $\delta \in [0, 1)$ and $d > 0$. u represents the input of the quantizer. $q(u)$ is the output of quantizer and $\Delta_q = q(u) - u$ denotes the error of quantization. Without loss of generality, we define $\Delta h = \text{diag}\{\Delta h_1, \Delta h_1, \dots, \Delta h_n\}$, where $\Delta h_i \in [-\delta_{h_i}, \delta_{h_i}]$.

The quantizer $h(x)$ can be expressed as

$$h(x) = (I + \Delta h)x. \quad (15)$$

Similarly, at the side of the actuator, we define the quantizer as $g(x) = [g_1(x_1)g_2(x_2)g_3(x_3) \dots g_n(x_n)]^T$, where $g_i(x_i)$ ($i = 1, 2, \dots, n$). Correspondingly, the quantizer $g(x)$ is expressed as

$$g(x) = (I + \Delta g)x, \quad (16)$$

where $\Delta g_i \in [-\delta_i, \delta_i]$, and

$$\delta_{g_i} = \frac{1 - \rho_{g_i}}{1 + \rho_{g_i}}, \quad (17)$$

where ρ_{g_i} is the quantization density.

Remark 3. Static quantization is adopted in this paper for the advantages of the positive mathematical model and convenient design. By contrast, the dynamic quantizer which is composed of the dynamic parameter and the static quantizer is potential on practical stability and the corresponding research is published in [43, 44].

Based on the abovementioned description, at the side of the sensor, the output of quantizer $h(x)$ can be expressed as

$$u(t) = \alpha(t)K_i(I + \Delta h)x(t - d(t)) + (1 - \alpha(t))K_i(I + \Delta h)[x(t - \tau(t)) + e_k(t)]. \quad (18)$$

Then, at the side of actuator, the output of quantizer $g(x)$ can be expressed as

$$\begin{aligned} u(t) &= \alpha(t)K_i(I + \Delta h)(I + \Delta g)x(t - d(t)) \\ &\quad + (1 - \alpha(t))K_i(I + \Delta h)(I + \Delta g) \\ &\quad \times [x(t - \tau(t)) + e_k(t)] \\ &= \alpha(t)(K_i + \Delta K_i)x(t - d(t)) \\ &\quad + (1 - \alpha(t))(K_i + \Delta K_i)[x(t - \tau(t)) + e_k(t)], \end{aligned} \quad (19)$$

where $\Delta K_i = \Delta_g K_i + K_i \Delta_h + \Delta_g K_i \Delta_h$. Combining (1) with (19), the closed-loop network system can be described as

$$\begin{cases} \dot{x}(t) = [A_i + \Delta A_i]x(t) + B_i \alpha(t)(K_i + \Delta K_i) \times x(t - d(t)) \\ \quad + B_i (1 - \alpha(t))(K_i + \Delta K_i) \times [x(t - \tau(t)) + e_k(t)], & t \in \Omega_t, \\ x(t) = \phi(t), & t \in [-z, 0], \end{cases} \quad (20)$$

where $\phi(t)$ is the initial condition of $x(t)$, and $z = \max\{d_M, \tau_M\}$.

As to simplify the derivation and expression, the following lemmas are employed.

Definition 1. The state feedback closed-loop Markov jump system (1) which considers the hybrid-driven mechanism is asymptotically stable if there exists a controller $u(t)$.

Lemma 1 (see [39]). Given matrices with appropriate dimensions $Q = Q^T, H, E$, then

$$Q + HF(t)E + E^T F^T(t)H^T < 0, \quad (21)$$

where the necessary and sufficient condition for any $F(t)$ satisfying $F^T F(t) \leq I$ is that there exists a positive parameter such that

$$Q + \varepsilon^{-1} H H^T + \varepsilon E^T E < 0. \quad (22)$$

Lemma 2 (see [45]). Suppose appropriate dimensional matrices $\Gamma_1, \Gamma_2, \Omega$ and $0 < \tau_m \leq \tau(t) \leq \tau_M$, the necessary and sufficient condition that made the following inequation

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0, \quad (23)$$

is

$$\begin{aligned} (\tau_M - \tau_m)\Xi_1 + \Omega &< 0, \\ (\tau_M - \tau_m)\Xi_2 + \Omega &< 0. \end{aligned} \quad (24)$$

Lemma 3 (see [46]). For any given vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, the positive definite $Q \in \mathbb{R}^{n \times n}$ satisfy the inequation

$$2\mathbf{x}^T \mathbf{y} \leq \mathbf{x}^T Q \mathbf{x} + \mathbf{y}^T Q^{-1} \mathbf{y}. \quad (25)$$

Lemma 4 (see [47]). For matrices $R > 0, X$ and any real parameter ρ , the inequality holds

$$-X R^{-1} X \leq \rho^2 R - 2\rho X. \quad (26)$$

3. Main Results

In this section, we devise the problem of the Markov jump continuous system based on uncertain network, and the result is given for the stability of the system. The controller is designed with Lyapunov–Krasovskii stability theory.

Theorem 1. For given scalars $\tau_M, d_M, 0 < \sigma < 1$ and probability parameter $\bar{\alpha}$, gain matrix K_i , the closed-loop system (20) is asymptotically stable under the hybrid-driven mechanism, if there exist matrices $P_i > 0, \Phi_i > 0 (i = 1, 2), Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0$, and W, V, M , and N with appropriate dimensions such that inequality (27) satisfying the following LMI:

$$\begin{bmatrix} \Sigma_{11} & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * \\ \Sigma_{31} & 0 & \Sigma_{33} & * \\ \Sigma_{41} & 0 & 0 & \Sigma_{44} \end{bmatrix} < 0, \quad (27)$$

where

$$\Sigma_{11} = \Theta + \Upsilon + \Upsilon^T,$$

$$\Theta = \begin{bmatrix} \Theta_{11} & * & * & * & * & * \\ \Theta_{21} & 0 & * & * & * & * \\ 0 & 0 & -Q_1 & * & * & * \\ \Theta_{41} & 0 & 0 & \sigma \Phi_i & * & * \\ 0 & 0 & 0 & 0 & -Q_2 & * \\ \Theta_{61} & 0 & 0 & 0 & 0 & -\Phi_i \end{bmatrix},$$

$$\Theta_{11} = A_i^T P_i + P_i A_i + \left\{ \sum_{j=1}^M \lambda_{ij} P_j \right\} + Q_1 + Q_2,$$

$$\Theta_{21} = \bar{\alpha} (K_i + \Delta K_i)^T B_i^T P_i,$$

$$\Theta_{41} = \Theta_{61}$$

$$\Upsilon = [M + W \quad N - M \quad -N \quad V - W \quad -V \quad 0], \quad (28)$$

$$\Sigma_{21} = [\Xi_1 \quad \Xi_2 \quad 0 \quad \Xi_3 \quad 0 \quad \Xi_4],$$

$$\Xi_1 = \begin{bmatrix} \tau_M P_i A_i \\ d_M P_i A_i \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} \tau_M \bar{\alpha} P_i B_i (K_i + \Delta K_i) \\ d_M \bar{\alpha} P_i B_i (K_i + \Delta K_i) \end{bmatrix},$$

$$\Xi_3 = \Xi_4$$

$$\Xi_5 = \Xi_6$$

$$\Sigma_{31} = [0 \quad \Xi_5 \quad 0 \quad \Xi_6 \quad 0 \quad \Xi_7],$$

$$\Sigma_{41} = [\tau_M W \quad \tau_M V \quad d_M M \quad d_M N],$$

$$\Sigma_{22} = \Sigma_{33}$$

$$\Sigma_{44} = \text{diag}\{-\tau_M R_1, -\tau_M R_1, -d_M R_2, -d_M R_2\}.$$

Proof. Considering the closed-loop system (20), we construct the following Lyapunov–Krasovskii function:

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad (29)$$

where

$$V_1(t) = x^T(t) P_i x(t),$$

$$V_2(t) = \int_{t-d_M}^t x^T(s) Q_1 x(s) ds + \int_{t-\tau_M}^t x^T(s) Q_2 x(s) ds,$$

$$\begin{aligned} V_3(t) = & \int_{t-\tau_M}^t \int_{t+\theta}^t \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta \\ & + \int_{t-d_M}^t \int_{t+\theta}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta. \end{aligned} \quad (30)$$

Taking the time-derivative of $V(t)$, then we have

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t), \quad (31)$$

where

$$\begin{aligned} \dot{V}_1(t) &= \dot{x}^T(t)P_i x(t) + x^T(t)P_i \dot{x}(t) \\ &\quad + x^T(t) \left\{ \sum_{j=1}^M \lambda_{ij} P_j \right\} x(t), \\ \dot{V}_2(t) &= x^T(t)Q_1 x(t) - x^T(t-d_M)Q_1 x(t-d_M) \\ &\quad + x^T(t)Q_2 x(t) - x^T(t-\tau_M)Q_2 x(t-\tau_M), \end{aligned} \quad (32)$$

$$\begin{aligned} \dot{V}_3(t) &= \tau_M \dot{x}^T(t)R_1 \dot{x}(t) - \int_{t-\tau_M}^t \dot{x}^T(s)R_1 \dot{x}(s)ds \\ &\quad + d_M \dot{x}^T(t)R_2 \dot{x}(t) - \int_{t-d_M}^t \dot{x}^T(s)R_2 \dot{x}(s)ds. \end{aligned}$$

Inspired by the free-weighting matrices, we define

$$2\xi^T(t)W \left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^t \dot{x}(s)ds \right] = 0, \quad (33)$$

$$\begin{aligned} &2\xi^T(t)V \\ &\quad \times \left[x(t-\tau(t)) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds \right] = 0, \end{aligned} \quad (34)$$

$$2\xi^T(t)M \left[x(t) - x(t-d(t)) - \int_{t-d(t)}^t \dot{x}(s)ds \right] = 0, \quad (35)$$

$$\begin{aligned} &2\xi^T(t)N \\ &\quad \times \left[x(t-d(t)) - x(t-d_M) - \int_{t-d_M}^{t-d(t)} \dot{x}(s)ds \right] = 0, \end{aligned} \quad (36)$$

where

$$\begin{aligned} \xi^T(t) &= [\xi_1^T(t) \quad \xi_2^T(t)], \\ \xi_1^T(t) &= [x^T(t) \quad x^T(t-d(t)) \quad x^T(t-d_M)], \\ \xi_2^T(t) &= [x^T(t-\tau(t)) \quad x^T(t-\tau_M) \quad e_k^T(t)], \\ W &= [W_1^T \quad W_2^T \quad W_3^T \quad W_4^T \quad W_5^T \quad W_6^T]^T, \\ V &= [V_1^T \quad V_2^T \quad V_3^T \quad V_4^T \quad V_5^T \quad V_6^T]^T, \\ M &= [M_1^T \quad M_2^T \quad M_3^T \quad M_4^T \quad M_5^T \quad M_6^T]^T, \\ N &= [N_1^T \quad N_2^T \quad N_3^T \quad N_4^T \quad N_5^T \quad N_6^T]^T. \end{aligned} \quad (37)$$

By Lemma 2, we can obtain

$$\begin{aligned} &-2\xi^T(t)W \int_{t-\tau(t)}^t \dot{x}(s)ds \leq \\ &\tau(t)\xi^T(t)WR_1^{-1}W^T\xi(t) + \int_{t-\tau(t)}^t \dot{x}^T(s)R_1\dot{x}(s)ds, \end{aligned} \quad (38)$$

$$\begin{aligned} &-2\xi^T(t)V \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds \leq (\tau_M - \tau(t)) \\ &\xi^T(t)VR_1^{-1}V^T\xi(t) + \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s)R_1\dot{x}(s)ds, \end{aligned} \quad (39)$$

$$\begin{aligned} &-2\xi^T(t)M \int_{t-d(t)}^t \dot{x}(s)ds \leq \\ &d(t)\xi^T(t)MR_2^{-1}M^T\xi(t) + \int_{t-d(t)}^t \dot{x}^T(s)R_2\dot{x}(s)ds, \end{aligned} \quad (40)$$

$$\begin{aligned} &-2\xi^T(t)N \int_{t-d_M}^{t-d(t)} \dot{x}(s)ds \leq (d_M - d(t)) \\ &\xi^T(t)NR_2^{-1}N^T\xi(t) + \int_{t-d_M}^{t-d(t)} \dot{x}^T(s)R_2\dot{x}(s)ds. \end{aligned} \quad (41)$$

Combining (8), (20)–(40), and Lemma 3, we obtain following inequation with taking mathematical expectation into account:

$$\begin{aligned} E\{\dot{V}(t)\} &\leq E\left\{ \dot{x}^T(t)P_i x(t) + x^T(t)P_i \dot{x}(t) \right\} \\ &\quad + x^T(t) \left\{ \sum_{j=1}^M \lambda_{ij} P_j \right\} x(t) + x^T(t)Q_1 x(t) \\ &\quad - x^T(t-d_M)Q_1 x(t-d_M) + x^T(t)Q_2 x(t) \\ &\quad - x^T(t-\tau_M)Q_2 x(t-\tau_M) \\ &\quad + E\left\{ \dot{x}^T(t)(\tau_M R_1 + d_M R_2)\dot{x}(t) \right\} \\ &\quad + 2\xi^T(t)W x(t) - 2\xi^T(t)W x(t-\tau(t)) \\ &\quad + 2\xi^T(t)V x(t-\tau(t)) - 2\xi^T(t)V x(t-\tau_M) \\ &\quad + \tau(t)\xi^T(t)WR_1^{-1}W^T\xi(t) \\ &\quad + (\tau_M - \tau(t))\xi^T(t)VR_1^{-1}V^T\xi(t) + 2\xi^T(t)M x(t) \\ &\quad - 2\xi^T(t)M x(t-d(t)) + 2\xi^T(t)N x(t-d(t)) \\ &\quad - 2\xi^T(t)N x(t-d_M) + \xi^T(t)NR_2^{-1}N^T\xi(t) \\ &\quad + d(t)\xi^T(t)MR_2^{-1}M^T\xi(t) \\ &\quad + (d_M - d(t))\xi^T(t)NR_2^{-1}N^T\xi(t) - e_k^T(t)\Phi_i e_k(t) \\ &\quad + \sigma x^T(t-\tau(t))\Phi_i x(t-\tau(t)), \end{aligned} \quad (42)$$

where

$$\begin{aligned}
& E\left\{\dot{x}^T(t)P_i x(t) + x^T(t)P_i \dot{x}(t)\right\} \\
&= \mathcal{M}^T P_i x(t) + x^T(t)P_i \mathcal{M}, \\
& E\left\{\dot{x}^T(t)(\tau_M R_1 + d_M R_2)\dot{x}(t)\right\} \\
&= \mathcal{M}^T (\tau_M R_1 + d_M R_2) \mathcal{M} \\
&+ \mu^2 \mathcal{N}^T (\tau_M R_1 + d_M R_2) \mathcal{N}, \\
& \mathcal{M} = A_i x(t) + B_i \bar{\alpha} (K_i + \Delta K_i) x(t - d(t)) \\
&+ B_i (1 - \bar{\alpha}) (K_i + \Delta K_i) [x(t - \tau(t)) + e_k(t)], \\
& \mathcal{N} = B_i (K_i + \Delta K_i) x(t - d(t)) \\
&+ B_i (K_i + \Delta K_i) [x(t - \tau(t)) + e_k(t)].
\end{aligned} \tag{43}$$

By employing Schur complement, we have the conclusion that $E\{\dot{V}(t)\} < 0$ [48] if inequation (27) hold. Furthermore, the uncertain Markov jump system (1) is asymptotically stable. This completes the proof.

Based on the analysis and proof above, a controller will be derived and given in the next theorem for system (20). $\square$

Theorem 2. For given scalars $\tau_M, d_M, 0 < \sigma < 1$ and probability parameter $\bar{\alpha}$, the closed-loop system (20) is asymptotically stable under the hybrid-driven mechanism, if there exist matrices $\bar{P}_i > 0, \bar{\Phi}_i > 0 (i = 1, 2), \bar{Q}_1 > 0, \bar{Q}_2 > 0, \bar{R}_1 > 0, \bar{R}_2 > 0$, and $\bar{W}, \bar{V}, \bar{M}, \bar{N}$, and Y_i with appropriate dimensions such that inequality (27) satisfies the following LMI:

$$\begin{bmatrix} \bar{\Sigma}_{11} & * & * & * \\ \bar{\Sigma}_{21} & \bar{\Sigma}_{22} & * & * \\ \bar{\Sigma}_{31} & 0 & \bar{\Sigma}_{33} & * \\ \bar{\Sigma}_{41} & 0 & 0 & \bar{\Sigma}_{44} \end{bmatrix} < 0, \tag{44}$$

where

$$\bar{\Sigma}_{11} = \bar{\Theta} + \bar{Y} + \bar{Y}^T,$$

$$\bar{\Theta} = \begin{bmatrix} \bar{\Theta}_{11} & * & * & * & * & * \\ \bar{\Theta}_{21} & 0 & * & * & * & * \\ 0 & 0 & -\bar{Q}_1 & * & * & * \\ \bar{\Theta}_{41} & 0 & 0 & \sigma \bar{\Phi}_i & * & * \\ 0 & 0 & 0 & 0 & -\bar{Q}_2 & * \\ \bar{\Theta}_{61} & 0 & 0 & 0 & 0 & -\bar{\Phi}_i \end{bmatrix},$$

$$\bar{\Theta}_{11} = \bar{P}_i A_i^T + A_i \bar{P}_i + \lambda_{ii} \bar{P}_i + \bar{Q}_1 + \bar{Q}_2,$$

$$\bar{\Theta}_{21} = \bar{\alpha} (Y_i + \Delta Y_i)^T B_i^T,$$

$$\bar{\Theta}_{41} = \bar{\Theta}_{61} = (1 - \bar{\alpha}) (Y_i + \Delta Y_i)^T B_i^T,$$

$$\bar{Y} = [\bar{M} + \bar{W} \quad \bar{N} - \bar{M} \quad -\bar{N} \quad \bar{V} - \bar{W} \quad -\bar{V} \quad 0],$$

$$\bar{\Sigma}_{21} = [\bar{\Xi}_1 \quad \bar{\Xi}_2 \quad 0 \quad \bar{\Xi}_3 \quad 0 \quad \bar{\Xi}_4],$$

$$\bar{\Sigma}_{31} = [0 \quad \bar{\Xi}_5 \quad 0 \quad \bar{\Xi}_6 \quad 0 \quad \bar{\Xi}_7],$$

$$\bar{\Sigma}_{41} = [\tau_M \bar{W} \quad \tau_M \bar{V} \quad d_M \bar{M} \quad d_M \bar{N}],$$

$$\bar{\Sigma}_{22} = \bar{\Sigma}_{33} = \text{diag}\{\tau_M (\bar{R}_1 - 2\bar{P}_i), d_M (\bar{R}_1 - 2\bar{P}_i)\},$$

$$\bar{\Sigma}_{44} = \text{diag}\{-\tau_M \bar{R}_1, -\tau_M \bar{R}_1, -d_M \bar{R}_2, -d_M \bar{R}_2\},$$

$$\bar{\Xi}_1 = \begin{bmatrix} \tau_M A_i \bar{P}_i \\ d_M A_i \bar{P}_i \end{bmatrix} \bar{\Xi}_2 = \begin{bmatrix} \tau_M \bar{\alpha} B_i (Y_i + \Delta Y_i) \\ d_M \bar{\alpha} B_i (Y_i + \Delta Y_i) \end{bmatrix},$$

$$\bar{\Xi}_3 = \bar{\Xi}_4 = \begin{bmatrix} \tau_M (1 - \bar{\alpha}) B_i (Y_i + \Delta Y_i) \\ d_M (1 - \bar{\alpha}) B_i (Y_i + \Delta Y_i) \end{bmatrix},$$

$$\bar{\Xi}_5 = \bar{\Xi}_6 = \bar{\Xi}_7 = \begin{bmatrix} \tau_M \mu B_i (Y_i + \Delta Y_i) \\ d_M \mu B_i (Y_i + \Delta Y_i) \end{bmatrix}.$$

(45)

Proof. According to Theorem 1, system (20) is asymptotically stable. We define the appropriate dimension matrix $J = \text{diag}\{P_i^{-1}, P_i^{-1}, \dots, P_i^{-1}\}$, then pre- and postmultiply (27) with J and its transposition, respectively. We define

$$K_i = Y_i \bar{P}_i^{-1},$$

$$\bar{P}_i = P_i^{-1},$$

$$\bar{W} = \bar{P}_i W \bar{P}_i,$$

$$\bar{V} = \bar{P}_i V \bar{P}_i,$$

$$\bar{M} = \bar{P}_i M \bar{P}_i,$$

$$\bar{N} = \bar{P}_i N \bar{P}_i,$$

$$\bar{R}_1 = \bar{P}_i R_1 \bar{P}_i,$$

$$\bar{R}_2 = \bar{P}_i R_2 \bar{P}_i,$$

$$\bar{Q}_1 = \bar{P}_i Q_1 \bar{P}_i,$$

$$\bar{Q}_2 = \bar{P}_i Q_2 \bar{P}_i,$$

$$\bar{\Phi}_i = \bar{P}_i \Phi_i \bar{P}_i,$$

$$\Delta Y_i = \Delta K_i \bar{P}_i,$$

(46)

where K_i represents the controller gain of the closed-loop system. For simplifying calculations and derivation, inspired by Lemma 4, we can substitute $-R_1$ and $-R_2$ with $\bar{R}_1 - 2\bar{P}_i$ and $\bar{R}_2 - 2\bar{P}_i$. According to the Schur complement, we obtain the inequality (44). This completes the proof. $\square$

Remark 4. It is worth noting that a sufficient condition is proposed in Theorem 2 referring to the co-design of the controller and hybrid-driven matrix Φ_i . Due to the existence of nonlinear term ΔY_i in Theorem 2, it is scarcely possible for the LMI toolbox to deal with more than two unknown items.

Therefore, we must tackle nonlinear term. In addition, considering the Markov jump continuous uncertain network system, we substitute A_i with $A_i + D_i F(t) H_i$. Then, a new technique is used to rewrite the Theorem 2 in the next section.

Theorem 3. For given scalars $\tau_M, d_M, 0 < \sigma < 1$ and probability parameter $\bar{\alpha}$, the closed-loop system (20) is asymptotically stable under the hybrid-driven mechanism, if there exist matrices $\bar{P}_i > 0, \bar{\Phi}_i > 0 (i = 1, 2), \bar{Q}_1 > 0, \bar{Q}_2 > 0, \bar{R}_1 > 0, \bar{R}_2 > 0$ and $\bar{W}, \bar{V}, \bar{M}, \bar{N}, Y_i$ with appropriate dimensions such that the inequality (27) satisfying the following LMI:

$$\begin{bmatrix} \hat{\Sigma}_1 & * \\ \hat{\Sigma}_2 & \hat{\Sigma}_3 \end{bmatrix} < 0, \quad (47)$$

where

$$\hat{\Sigma}_1 = \begin{bmatrix} \hat{\Sigma}_{11} & * & * & * \\ \hat{\Sigma}_{21} & \bar{\Sigma}_{22} & * & * \\ \hat{\Sigma}_{31} & 0 & \bar{\Sigma}_{33} & * \\ \hat{\Sigma}_{41} & 0 & 0 & \bar{\Sigma}_{44} \end{bmatrix} < 0,$$

$$\hat{\Sigma}_{11} = \hat{\Theta} + \bar{Y} + \bar{Y}^T,$$

$$\hat{\Theta} = \begin{bmatrix} \bar{\Theta}_{11} & * & * & * & * & * \\ \hat{\Theta}_{21} & 0 & * & * & * & * \\ 0 & 0 & -\bar{Q}_1 & * & * & * \\ \hat{\Theta}_{41} & 0 & 0 & \sigma \bar{\Phi}_i & * & * \\ 0 & 0 & 0 & 0 & -\bar{Q}_2 & * \\ \hat{\Theta}_{61} & 0 & 0 & 0 & 0 & -\bar{\Phi}_i \end{bmatrix},$$

$$\hat{\Theta}_{21} = \bar{\alpha} Y_i^T B_i^T,$$

$$\hat{\Theta}_{41} = \hat{\Theta}_{61}$$

$$\hat{\Sigma}_{21} = [\bar{\Xi}_1 \quad \hat{\Xi}_2 \quad 0 \quad \hat{\Xi}_3 \quad 0 \quad \hat{\Xi}_4],$$

$$\hat{\Xi}_2 = \begin{bmatrix} \tau_M \bar{\alpha} B_i Y_i \\ d_M \bar{\alpha} B_i Y_i \end{bmatrix},$$

$$\hat{\Xi}_3 = \hat{\Xi}_4$$

$$\hat{\Sigma}_{31} = [0 \quad \hat{\Xi}_5 \quad 0 \quad \hat{\Xi}_6 \quad 0 \quad \hat{\Xi}_7],$$

$$\hat{\Xi}_5 = \hat{\Xi}_6$$

$$\bar{\Sigma}_2 = [\Psi_1 \quad \Psi_2]^T,$$

$$\Psi_1 = [\nu_1 \bar{L}_D^T \quad \bar{L}_H^T \quad \kappa_1 \bar{S}_{11}^T \quad \bar{S}_{31}^T],$$

$$\Psi_2 = [\kappa_3 \bar{S}_{41}^T \quad \kappa_2 \bar{S}_{12}^T \quad \bar{S}_{32}^T \quad \kappa_4 \bar{S}_{42}^T],$$

$$\bar{\Sigma}_3 = \text{diag}\{\Psi_3 \quad \Psi_4\},$$

$$\Psi_3 = [-\nu_1 I \quad -\nu_1 I \quad -\kappa_1 I \quad -\nu_2 I],$$

$$\Psi_4 = [-\kappa_3 I \quad -\kappa_2 I \quad -\nu_5 I \quad -\kappa_4 I],$$

$$\bar{L}_D^T = \begin{bmatrix} D^T & \underbrace{0 \cdots 0}_5 & \tau_M D^T & d_M D^T & \underbrace{0 \cdots 0}_6 \end{bmatrix},$$

$$\bar{L}_H = \begin{bmatrix} H \bar{P}_i & \underbrace{0 \cdots 0}_{13} \end{bmatrix},$$

$$\bar{S}_{11} = \begin{bmatrix} \bar{\alpha} B_i^T & \underbrace{0 \cdots 0}_5 & \Gamma_1 & \underbrace{0 \cdots 0}_4 \end{bmatrix},$$

$$\Gamma_1 = [\tau_M \bar{\alpha} B_i^T \quad d_M \bar{\alpha} B_i^T \quad \tau_M \mu B_i^T \quad d_M \mu B_i^T],$$

$$\bar{S}_{12} = \begin{bmatrix} (1 - \bar{\alpha}) B_i^T & \underbrace{0 \cdots 0}_5 & \Gamma_2 & \underbrace{0 \cdots 0}_4 \end{bmatrix},$$

$$\Gamma_2 = [\tau_M (1 - \bar{\alpha}) B_i^T \quad d_M (1 - \bar{\alpha}) B_i^T \quad \tau_M \mu B_i^T \quad d_M \mu B_i^T],$$

$$\bar{S}_{31} = \begin{bmatrix} 0 & Y_i & \underbrace{0 \cdots 0}_{12} \end{bmatrix},$$

$$\bar{S}_{41} = \begin{bmatrix} 0 & \bar{P}_i & \underbrace{0 \cdots 0}_{12} \end{bmatrix},$$

$$\bar{S}_{32} = \begin{bmatrix} \underbrace{0 \cdots 0}_4 & Y_i & 0 & Y_i & \underbrace{0 \cdots 0}_7 \end{bmatrix},$$

$$\bar{S}_{42} = \begin{bmatrix} \underbrace{0 \cdots 0}_4 & \bar{P}_i & 0 & \bar{P}_i & \underbrace{0 \cdots 0}_7 \end{bmatrix}.$$

(48)

Proof. On the basis of inequality (44) in Theorem 2, we can rewrite the inequation as follows:

$$\hat{\Sigma}_1 + \Lambda_1 + \Lambda_2 + \Lambda_3 < 0 \quad (49)$$

where

$$\Lambda_1 = \text{sym}\{L_{Di} F(t) L_{Hi}\},$$

$$\Lambda_2 = \text{sym}\{S_{11}^T \Delta g S_{31}\} + \text{sym}\{S_{11}^T K_i S_{21}\}$$

$$+ \text{sym}\{S_{11}^T \Delta g K_i S_{21}\}, \quad (50)$$

$$\Lambda_3 = \text{sym}\{S_{12}^T \Delta g S_{32}\} + \text{sym}\{S_{12}^T K_i S_{22}\}$$

$$+ \text{sym}\{S_{12}^T \Delta g K_i S_{22}\}.$$

We can obtain the following equations by applying Lemma 1:

$$\Lambda_1 = \nu_1^{-1} \bar{L}_D \bar{L}_D^T + \nu_1 \bar{L}_H^T \bar{L}_H,$$

$$\Lambda_2 = \nu_2^{-1} \bar{S}_{11}^T \Delta^2 g \bar{S}_{11} + \nu_2 \bar{S}_{31}^T \bar{S}_{31} + \nu_3^{-1} \bar{S}_{11}^T \bar{S}_{11}$$

$$+ \nu_3 \bar{S}_{21}^T K_i^T K_i \bar{S}_{21} + \nu_4^{-1} \bar{S}_{11}^T \Delta^2 g \bar{S}_{11}$$

$$+ \nu_4 \bar{S}_{21}^T K_i^T K_i \bar{S}_{21}, \quad (51)$$

$$\Lambda_3 = \nu_5^{-1} \bar{S}_{12}^T \Delta^2 g \bar{S}_{12} + \nu_5 \bar{S}_{32}^T \bar{S}_{32} + \nu_6^{-1} \bar{S}_{12}^T \bar{S}_{12}$$

$$+ \nu_6 \bar{S}_{22}^T K_i^T K_i \bar{S}_{22} + \nu_7^{-1} \bar{S}_{12}^T \Delta^2 g \bar{S}_{12}$$

$$+ \nu_7 \bar{S}_{22}^T K_i^T K_i \bar{S}_{22}.$$

Synthesizing the formulas above, we get the inequation of (49):

$$\begin{aligned} & \hat{\Sigma}_1 + \nu_1^{-1} \bar{L}_D \bar{L}_D^T + \nu_1 \bar{L}_H^T \bar{L}_H + \kappa_1 \bar{S}_{11}^T \bar{S}_{11} + \kappa_2 \bar{S}_{12}^T \bar{S}_{12} \\ & + \nu_2 \bar{S}_{31}^T \bar{S}_{31} + \nu_5 \bar{S}_{32}^T \bar{S}_{32} + \kappa_3 \bar{S}_{41}^T \bar{S}_{41} + \kappa_4 \bar{S}_{42}^T \bar{S}_{42} < 0. \end{aligned} \quad (52)$$

Noticed that $\Delta^2 g \leq \delta_g^2 I$ and $\Delta^2 f \leq \delta_f^2 I$, inequality (52) can be rewritten as (47) by applying Schur complement. This completes the proof. $\square$

4. Simulation Examples

In this section, two examples will be provided to illustrate the advantages of the controller designed in this paper.

Example 1. Consider the system parameters as follows.

Mode 1:

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.2 & 0.6 \\ 0.1 & -1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 \\ 0.1 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}. \end{aligned} \quad (53)$$

Mode 2:

$$\begin{aligned} A_2 &= \begin{bmatrix} -1 & 0.6 \\ 0.6 & -1.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.5 & 0.2 \end{bmatrix}. \end{aligned} \quad (54)$$

The transition probability matrix of Markov jump process is supposed to be

$$\Pi = \begin{bmatrix} -3 & 3 \\ 4 & -4 \end{bmatrix}. \quad (55)$$

Then, the initial condition of the system is decided as $x(0) = [-2.5 \quad 2.5]^T$. Comparing the previous investigation [35], we divide the cases into four conditions. Among the four cases, we apply the same Markov jump and the BCQ effectively while different stochastic variables are employed following the Bernoulli distribution.

Setting the corresponding parameters network-induced delay $\tau_M = 0.5$, $d_M = 0.1$, event-trigger scalar $\sigma = 0.05$, the sampling period $h = 0.1$ s, $\nu_1 = \nu_2 = \nu_4 = \nu_5 = 1$, and $\kappa_1 = \kappa_2 = \kappa_3 = \kappa_4 = 35$. For simplicity, we assume that the quantization indicators of the BCQ are identical $\delta_h = \delta_g$. We obtain the following cases.

Case 1. The stochastic variable $\bar{\alpha} = 1$ means that the system is working under the “time-triggered” mode. By the LMI toolbox and Theorem 3, we can obtain the controller feedback gains:

$$\begin{aligned} K_1 &= [-1.4677 \quad 0.7772], \\ K_2 &= [0.3801 \quad -0.2721]. \end{aligned} \quad (56)$$

Figure 2 shows the transition probability of Markov jump between different modes. Figure 3 illustrates the state response trajectories, demonstrating that the system under time-triggered scheme is asymptotically stable.

Case 2. Setting the stochastic variable $\bar{\alpha} = 0$, then system (20) is under the “event-triggered” communication scheme. By utilizing Theorem 3, we can obtain the controller feedback gains:

$$\begin{aligned} K_1 &= [0.5377 \quad -0.3553], \\ K_2 &= [-0.1512 \quad -1.4329], \end{aligned} \quad (57)$$

and the event-triggered matrices

$$\begin{aligned} \Phi_1 &= \begin{bmatrix} 9.1161 & -3.4920 \\ -3.4920 & 1.9255 \end{bmatrix}, \\ \Phi_2 &= \begin{bmatrix} 10.7435 & -4.9141 \\ -4.9141 & 11.4700 \end{bmatrix}. \end{aligned} \quad (58)$$

Figures 4, 5, and 6 represent the probabilities of Markov jump between different modes, the state response trajectories and the “event-triggered” release instants and interval.

Case 3. Utilizing the LMI technology of Matlab, we assume that the stochastic variable $\bar{\alpha} = 0.2$ and communication mechanism is “hybrid-driven” scheme. The controller parameters subject to the following matrices:

$$\begin{aligned} K_1 &= [1.8301 \quad -0.2808], \\ K_2 &= [-0.4947 \quad -4.0302], \end{aligned} \quad (59)$$

and the event-triggered matrices

$$\begin{aligned} \Phi_1 &= \begin{bmatrix} 2.0808 & -52.8944 \\ -52.8944 & 8.9900 \end{bmatrix}, \\ \Phi_2 &= \begin{bmatrix} 17.3391 & 43.7078 \\ 43.7078 & 1.7041 \end{bmatrix}. \end{aligned} \quad (60)$$

Figure 7 illustrates the variation of state response trajectories in Case 3. Figure 8 represents the “event-triggered” release instants and interval in hybrid-driven mechanism. Figure 9 stands for the jump of Bernoulli distribution between “event-triggered” and “time-triggered.”

Case 4. Having a resemblance to Case 3, we obtain the following matrices by solving LMIs of Theorem 3 and setting stochastic variable $\bar{\alpha} = 0.6$:

$$\begin{aligned} K_1 &= [0.2255 \quad -1.5036], \\ K_2 &= [0.36042 \quad 7.912], \end{aligned} \quad (61)$$

and the event-triggered matrices

$$\begin{aligned} \Phi_1 &= \begin{bmatrix} 1.7512 & -17.9441 \\ -17.9441 & 0.8417 \end{bmatrix}, \\ \Phi_2 &= \begin{bmatrix} 23.5141 & 13.3289 \\ 13.3289 & 6.0630 \end{bmatrix}, \end{aligned} \quad (62)$$

FIGURE 2: The state transition of Markov jump in Case 1.

FIGURE 3: The state response in Case 1.

FIGURE 4: The state transition of Markov jump in Case 2.

FIGURE 5: The state response in Case 2.

FIGURE 6: The “event-triggered” release instants and interval in Case 2.

Figure 10 depicts the variation of state response trajectories of Case 4. Figure 11 indicates the “event-triggered” release instants and interval in the hybrid-driven mechanism. Figure 12 stands for the jump of Bernoulli distribution between “event-triggered” and “time-triggered.”

Based on the simulation pictures, controller gains and event-triggered matrix above, we make the following detailed analysis.

As depicted in the pictures, we design the sampling period $h = 0.1s$ and the sampling instants $N = 300$.

During the simulation time of 30 s, the trigger implements only 26 times in 300 times with $\bar{\alpha} = 0.2$. To compare the signal transmission rates of the system under a different Bernoulli probability within 30s, the result of the comparison is given as Table 1.

The trigger times of event-trigger are 37 with $\bar{\alpha} = 0$, as shown in Figure 6. Comparatively, with the increase of value

FIGURE 7: The state response in Case 3.

FIGURE 10: The state response in Case 4.

FIGURE 8: The “event-triggered” of “hybrid-driven” release instants and interval in Case 3.

FIGURE 11: The “event-triggered” of “hybrid-driven” release instants and interval in Case 4.

FIGURE 9: The jump of Bernoulli distribution in Case 3.

FIGURE 12: The jump of Bernoulli distribution in Case 4.

TABLE 1: Transmission rates.

$\bar{\alpha}$	V	Transmission rates
0	37	12.33%
0.2	26	8.67%
0.6	17	5.67%
0.9	3	1%

$\bar{\alpha}$, the system is under hybrid-driven mode, and the frequency of data transmission and trigger count are decreasing. When Bernoulli probability $\bar{\alpha} = 0.2$, $\bar{\alpha} = 0.6$, and $\bar{\alpha} = 0.9$, respectively, the signals transmission rates reduce to 8.67%, 5.67%, and 1%. It is obvious that the data packet transmission rate with hybrid-driven mechanism is less than the one with event-triggered scheme. The method proposed in this paper degrades the waste of network communication resources and mitigate the burden of communication channel in control systems. It is better than the existing event-triggered mechanism investigation under the premise of asymptotically stable systems.

Remark 5. Contrasting with the existing literature about the dealing method of NCSs with quantization and network-induced delay, some results with using extraordinary techniques have shown certain superiorities [8], in which two event-triggers are employed in plant-to-controller channel and controller-to-actuator channel, and the best transmission rate 12% outnumbered our simulation result. Furthermore, the system stabilization period in this paper is superior to [33].

Remark 6. In Figures 7 and 10, the state response trajectories with different Bernoulli probabilities are depicted. We can easily conclude that the number of data packet transmission decreases greatly under the hybrid-driven mechanism, but the system stability time is longer than event-triggered scheme. It proves the standpoint in literature [35] that we should get a balance between the system performance and resource saving.

Example 2. In this section, a system of single-link robot arm is considered [16, 49] and given by

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\frac{glM}{J}\sin(x_1(t)) - \frac{R}{J}x_2(t) + \frac{1}{J}u(t), \end{cases} \quad (63)$$

where $x_1(t)$ is the angle of the robot arm from the vertical and $x_2(t)$ denotes the angular velocity. The system parameters M , J , R , g , and l represent the mass of the load, the moment of inertia, the damping coefficient, the gravity constant, and the length of the arm, respectively.

Being similar to Example 1, the sampling period is set as $T = 0.1$ s and the sampling instants $N = 300$. The stochastic variable is given as $\bar{\alpha} = 0.5$ to illustrate that the hybrid-driven mechanism with BCQ for the Markov jump system is stable when it is introduced in the real system. It is worth mentioning that the real system parameters M_i

TABLE 2: Different modes of M_i and J_i .

Mode	$i = 1$	$i = 2$
M_i	1	4
J_i	1	4

FIGURE 13: The Markov jump of Example 2.

and J_i have two different modes, which are displayed in Table 2, and the transition probability matrix is maintained.

Then, the real system of the single-link robot arm can be presented as following based on hybrid-driven mechanism for the Markov jump system:

$$\dot{x}(t) = [A_i + \Delta A_i]x(t) + B_i u(t), \quad (64)$$

where

$$A_i = -\begin{bmatrix} 1 & T \\ -\frac{TglM_i}{J_i} & 1 - \frac{TR}{J_i} \end{bmatrix}, \quad B_i = -\begin{bmatrix} 0 \\ \frac{T}{J_i} \end{bmatrix}, \quad (65)$$

$$\Delta A_i = D_i F(t) H_i,$$

$$D_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad H_i = \begin{bmatrix} 0 & 0.1 \end{bmatrix},$$

with the calculation of Theorem 3 by LMT toolbox technology, and the real system controller gains are

$$\begin{aligned} K_1 &= [0.0011 \quad -0.0231], \\ K_2 &= [0.0010 \quad -0.0212], \end{aligned} \quad (66)$$

FIGURE 14: The state response of Example 2.

FIGURE 15: The “event-triggered” of “hybrid-driven” release instants and interval of Example 2.

and the corresponding event-triggered matrices

$$\begin{aligned} \Phi_1 &= \begin{bmatrix} 54.6512 & -1.4947 \\ -1.4947 & 14.0670 \end{bmatrix}, \\ \Phi_2 &= \begin{bmatrix} 33.4038 & 4.1097 \\ 4.1097 & 17.3571 \end{bmatrix}. \end{aligned} \quad (67)$$

In this example, Figure 13 expresses the Markov stochastic jump for transition probability. It is apparent from Figure 14 that the real system of the single-link robot arm is asymptotically stable under the approach proposed in this paper. Figure 15 denotes the “event-triggered” release instants and interval in hybrid-driven mechanism of Example 2. As shown in Figures 13–15, the hybrid-driven mechanism with BCQ for the Markov jump system can be employed into the real system and the stability is guaranteed.

5. Conclusion

The hybrid-driven mechanism problem of controller design for the Markov jump system with both channel quantization and network-induced delay based on uncertain networked is developed in this paper. The hybrid-driven mechanism is employed for the balance between data transmission rate and system performance. Both channel quantization technology is applied to reduce network communication resource utilization further. The numerical simulations and the practice example illustrate the effectiveness of the proposed method in this paper.

Nevertheless, with the development of information science and communication technology, the network security issue is attracting extensive attention and interest. How to defend the cyber attacks is the present research focus. Thus, the approach of unifying the different varieties of cyber attacks, for instance, denial of service (DoS) attacks and deception attacks into a Markov jump system remains to further be discussed.

Data Availability

The data used to support the findings of this study can be calculated by Matlab/LMI toolbox. The others are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest in this study.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China under Grant no. 61374083 and Natural Science Foundation of Zhejiang Province under Grant nos. LQ19F030014 and LY20F030015.

References

- [1] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, “A survey of recent results in networked control systems,” *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.
- [2] L. Zhang, H. Gao, and O. Kaynak, “Network-induced constraints in networked control systems-A survey,” *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 403–416, 2013.
- [3] X.-S. Zhan, Z.-H. Guan, X.-H. Zhang, and F.-S. Yuan, “Optimal tracking performance and design of networked control systems with packet dropouts,” *Journal of the Franklin Institute*, vol. 350, no. 10, pp. 3205–3216, 2013.
- [4] X.-S. Zhan, J. Wu, T. Jiang, and X.-W. Jiang, “Optimal performance of networked control systems under the packet dropouts and channel noise,” *ISA Transactions*, vol. 58, pp. 214–221, 2015.
- [5] Z. Li, H. Yan, H. Zhang, X. Zhan, and C. Huang, “Improved inequality-based functions approach for stability analysis of time delay system,” *Automatica*, vol. 108, Article ID 108416, 2019.
- [6] D. Yue, E. Tian, and Q.-L. Han, “A delay system method for designing event-triggered controllers of networked control

- systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.
- [7] X. Zhang, Q. Han, and X. Yu, "Survey on recent advances in networked control systems," *IEEE Transactions on Industrial Informatics*, vol. 12, no. 5, pp. 1740–1752, 2017.
  - [8] D. Liu and G.-H. Yang, "Robust event-triggered control for networked control systems," *Information Sciences*, vol. 459, pp. 186–197, 2018.
  - [9] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 3, pp. 586–601, 2011.
  - [10] X. Yang, S. Xu, and Z. Li, "Consensus congestion control in multirouter networks based on multiagent system," *Complexity*, vol. 2017, pp. 1–10, 2017.
  - [11] X.-S. Zhan, L.-L. Cheng, J. Wu, and H.-C. Yan, "Modified tracking performance limitation of networked time-delay systems with two-channel constraints," *Journal of the Franklin Institute*, vol. 356, no. 12, pp. 6401–6418, 2019.
  - [12] H. Li, P. Shi, and D. Yao, "Adaptive sliding-mode control of Markov jump nonlinear systems with actuator faults," *IEEE Transactions on Automatic Control*, vol. 62, no. 4, pp. 1933–1939, 2017.
  - [13] Y. Xu, H. Su, and Y.-J. Pan, "Output feedback stabilization for Markov-based nonuniformly sampled-data networked control systems," *Systems & Control Letters*, vol. 62, no. 8, pp. 656–663, 2013.
  - [14] H. Li and Y. Shi, "Robust H_∞ filtering for nonlinear stochastic systems with uncertainties and Markov delays," *Automatica*, vol. 48, no. 1, pp. 159–166, 2012.
  - [15] H. Yan, H. Zhang, F. Yang, X. Zhan, and C. Peng, "Event-triggered asynchronous guaranteed cost control for Markov jump discrete-time neural networks with distributed delay and channel fading," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 8, pp. 3588–3598, 2018.
  - [16] S. Dong, Z. Wu, P. Shi, H. Su, and T. Huang, "Quantized control of Markov jump nonlinear systems based on fuzzy hidden Markov model," *IEEE Transactions on Cybernetics*, vol. 49, no. 7, pp. 2420–2430, 2018.
  - [17] D. Wang, J. Wang, and W. Wang, "Output feedback control of networked control systems with packet dropouts in both channels," *Information Sciences*, vol. 221, Article ID 544554, 2013.
  - [18] Y. Xu, Z. Wang, D. Yao, R. Lu, and C. Su, "State estimation for periodic neural networks with uncertain weight matrices and Markovian jump channel states," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 11, pp. 1841–1850, 2017.
  - [19] D. Yao, R. Lu, Y. Xu, and L. Wang, "Robust H_∞ filtering for Markov jump systems with mode-dependent quantized output and partly unknown transition probabilities," *Signal Processing*, vol. 137, pp. 328–338, 2017.
  - [20] D. Yao, M. Liu, R. Lu, Y. Xu, and Q. Zhou, "Adaptive sliding mode controller design of Markov jump systems with time-varying actuator faults and partly unknown transition probabilities," *Nonlinear Analysis: Hybrid Systems*, vol. 28, pp. 105–122, 2018.
  - [21] H. Yan, Y. Tian, H. Li, H. Zhang, and Z. Li, "Input-output finite-time mean square stabilization of nonlinear semi-Markovian jump systems," *Automatica*, vol. 104, pp. 82–89, 2019.
  - [22] S. Hu and D. Yue, "Event-triggered control design of linear networked systems with quantizations," *ISA Transactions*, vol. 51, no. 1, pp. 153–162, 2012.
  - [23] X. Chen, Y. Wang, and S. Hu, "Event-based robust stabilization of uncertain networked control systems under quantization and denial-of-service attacks," *Information Sciences*, vol. 459, pp. 369–386, 2018.
  - [24] X.-H. Chang, C. Yang, and J. Xiong, "Quantized fuzzy output feedback H_∞ control for nonlinear systems with adjustment of dynamic parameters," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 10, pp. 2005–2015, 2019.
  - [25] Z.-M. Li, X.-H. Chang, X.-K. Du, and L. Yu, " H_∞ control of discrete-time uncertain linear systems with quantized feedback," *Neurocomputing*, vol. 174, pp. 790–794, 2016.
  - [26] R. Rakkiyappan, K. Maheswari, G. Velmurugan, and J. H. Park, "Event-triggered H_∞ state estimation for semi-Markov jumping discrete-time neural networks with quantization," *Neural Networks*, vol. 105, pp. 236–248, 2018.
  - [27] T. H. Lee, J. Xia, and J. H. Park, "Networked control system with asynchronous samplings and quantizations in both transmission and receiving channels," *Neurocomputing*, vol. 237, pp. 25–38, 2017.
  - [28] Y. Xu, R. Lu, H. Peng, and S. Xie, "Filtering for fuzzy systems with multiplicative sensor noises and multidensity quantizer," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 1011–1022, 2018.
  - [29] J. Liu, J. Tang, and S. Fei, "Event-triggered H_∞ filter design for delayed neural network with quantization," *Neural Networks*, vol. 82, pp. 39–48, 2016.
  - [30] Y. Pan and G. Yang, "Event-triggered fault detection filter design for nonlinear networked systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 48, no. 11, pp. 1851–1862, 2017.
  - [31] D. Yao, B. Zhang, P. Li, and H. Li, "Event-triggered sliding mode control of discrete-time Markov jump systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 49, no. 10, pp. 2016–2025, 2019.
  - [32] G. Guo, L. Ding, and Q.-L. Han, "A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems," *Automatica*, vol. 50, no. 5, pp. 1489–1496, 2014.
  - [33] L. Zha, J.-A. Fang, X. Li, and J. Liu, "Event-triggered output feedback H_∞ control for networked Markovian jump systems with quantizations," *Nonlinear Analysis: Hybrid Systems*, vol. 24, pp. 146–158, 2017.
  - [34] R. Yang, H. Zhang, G. Feng, H. Yan, and Z. Wang, "Robust cooperative output regulation of multi-agent systems via adaptive event-triggered control," *Automatica*, vol. 102, pp. 129–136, 2019.
  - [35] J. Liu, L. Zha, J. Cao, and S. Fei, "Hybrid-driven-based stabilisation for networked control systems," *IET Control Theory & Applications*, vol. 10, no. 17, pp. 2279–2285, 2016.
  - [36] Y. Tan, M. Xiong, D. Du, and S. Fei, "Hybrid-triggered state feedback H_∞ control for networked control systems with stochastic nonlinearity and quantization," *Peer-to-Peer Networking and Applications*, vol. 12, no. 3, pp. 660–676, 2019.
  - [37] J. Liu, Z.-G. Wu, D. Yue, and J. H. Park, "Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 99, pp. 1–11, 2019.
  - [38] J. Liu, L. Wei, X. Xie, E. Tian, and S. Fei, "Quantized stabilization for T-S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 6, pp. 3820–3834, 2018.
  - [39] I. R. Petersen and C. V. Hollot, "A riccati equation approach to the stabilization of uncertain linear systems," *Automatica*, vol. 22, no. 4, pp. 397–411, 1986.
  - [40] S. Hu, D. Yue, X. Yin, X. Xie, and Y. Ma, "Adaptive event-triggered control for nonlinear discrete-time systems,"

- International Journal of Robust and Nonlinear Control*, vol. 26, no. 18, pp. 4104–4125, 2016.
- [41] N. Elia and S. K. Mitter, “Stabilization of linear systems with limited information,” *IEEE Transactions on Automatic Control*, vol. 46, no. 9, pp. 1384–1400, 2001.
  - [42] C. Peng and Y.-C. Tian, “Networked H_∞ control of linear systems with state quantization,” *Information Sciences*, vol. 177, no. 24, pp. 5763–5774, 2007.
  - [43] X. Chang, R. Huang, H. Wang, and L. Liu, “Robust design strategy of quantized feedback control,” *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 99, p. 1, 2019.
  - [44] X. Chang, C. Yang, and J. H. Park, “Robust guaranteed cost control under digital communication channels,” *IEEE Transactions on Industrial Informatics*, vol. 16, no. 1, pp. 319–327, 2019.
  - [45] D. Yue, Y. Zhang, E. Tian, and C. Peng, “Delay-distribution-dependent exponential stability criteria for discrete-time recurrent neural networks with stochastic delay,” *IEEE Transactions on Neural Networks*, vol. 19, no. 7, pp. 1299–1306, 2008.
  - [46] Y. Wang, L. Xie, and C. Souza, “Robust control of a class of uncertain nonlinear systems,” *Control & Decision Letters*, vol. 19, no. 2, pp. 139–149, 1997.
  - [47] J. Xiong and J. Lam, “Stabilization of networked control systems with a logic ZOH,” *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 358–363, 2009.
  - [48] X. Mao, “Exponential stability of stochastic delay interval systems,” *IEEE Transactions on Automatic Control*, vol. 47, no. 10, pp. 1604–1612, 2002.
  - [49] J. Tao, R. Lu, P. Shi, H. Su, and Z.-G. Wu, “Dissipativity-based reliable control for fuzzy markov jump systems with actuator faults,” *IEEE Transactions on Cybernetics*, vol. 47, no. 9, pp. 2377–2388, 2017.