Research Article

Dynamics Analysis of a Betel Nut Addiction Spreading Model on Scale-Free Networks

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Medical research has shown that overeating betel nut can be addictive and can damage health. More serious cases may cause mouth cancer and other diseases. Even worse, people's behavior habit of chewing betel nut may influence each other through social interaction with direct or indirect ways, such as face-to-face communication, Facebook, Twitter, microblog, and WeChat, which leads to the spreading phenomenon of betel nut addiction. In order to investigate the dynamic spreading characteristics of betel nut addiction, a PMSR (Potential-Mild-Severe-Recovered) betel nut addiction spreading model is presented on scale-free networks. The basic reproductive number R_0 and equilibria are derived. Theoretical results indicate that the basic reproductive number R_0 is significantly dependent on the topology of the underlying networks, and some influence parameters. The existence of equilibria is determined by the basic reproductive number R_0 . Furthermore, we prove that if $R_0 < 1$ the addiction-elimination equilibrium is globally asymptotically stable. If $R_0 > 1$, the betel nut addiction spreading is permanent, and the addiction-prevailing equilibrium is globally attractive. Finally, numerical simulations confirm the theoretical analysis results.

1. Introduction

The habit of chewing betel nut is common in many places around the world [1–8]. It is worth noting that overeating betel nut can be addictive and is harmful to health [8, 9]. It is found that betel nut affects the health of the nervous, gastrointestinal, metabolic, respiratory, and reproductive systems. More serious cases may cause mouth cancer and other diseases. Betel nut is classified in the first category of carcinogens by the International Agency for Research on Cancer [9–11]. Even worse, people's behavior habit of chewing betel nut may influence each other through social contact, which leads to the spreading phenomenon of betel nut addiction. The widespread spread of betel nut addiction, in turn, exacerbates the damage to people's health and even the whole society.

It is very important to control the spreading phenomenon of betel nut addiction. In recent years, the research on

betel nut addiction has attracted the attention of many scholars and researchers. Murphy et al. confirmed that the formation of the “chewing betel nut” habit could be due to exposure to and influence by betel nut chewers through statistical analysis [12]. As indicated in [13], the cultural differences can affect the spreading of betel nut addiction in society. Ghani et al. identified the factors which influence the development of betel nut addiction. They also proposed the health policies to prevent addiction [14]. Moss et al. [15] showed that the critical factors of addiction were contact with addicts and the self-prevention consciousness. Many researchers studied the addictiveness and harmfulness of the “chewing betel nut” habit [16–19], but few focused on the spreading dynamics of betel nut addiction. There are some studies in social information and disease spreading dynamics [20–25]. Liu et al. studied the spreading dynamics of cyber violence [26]. Barabási and Albert studied the impact of neighboring infection on the computer virus spread [27].

These studies can give some help for the analysis of the spreading dynamics of betel nut addiction. Through the study of addiction spreading dynamics, we can comprehensively and systematically learn about the addiction spreading mechanism and influence factors, which is helpful to control the spread of betel nut addiction. Meanwhile, some researchers found that the scale-free property is an important property of social networks [28–32]. Obviously, the spread of betel nut addiction is based on social networks. So, based on scale-free networks, we study the spreading dynamics of betel nut addiction in the paper. Apparently, the marketing strategy of betel nut has a great influence on sales, and at the same time, it affects the spreading of betel nut addiction. Taking into account the influence of betel nut advertising campaigns and the heterogeneity of underlying spreading networks, we present a new comprehensively PMSR (Potential-Mild-Severe-Recovered) betel nut addiction spreading model.

The rest of the paper is as follows. In Section 2, the PMSR betel nut addiction spreading model is proposed and described. In Section 3, the basic reproductive number and equilibriums are derived at first. Then, we analyze the globally asymptotic stability of addiction-elimination equilibrium, the permanence of the addiction spreading, and the global attractivity of addiction-prevailing equilibrium. Section 4 presents the results of our numerical simulation. Finally, we conclude the paper in Section 5.

2. Model Formulation

We present a new comprehensively PMSR (Potential-Mild-Severe-Recovered) betel nut addiction spreading model. The model has the spread sketch in Figure 1. In the model, nodes are used to stand for individuals, and edges are used to stand for the relationships between individuals. The whole population is divided into four distinct classes, namely, potential individuals (P), mild addicts (M), severe addicts (S), and recovered individuals (R). P nodes refer to the people who do not have addictive behavior of chewing betel nut; M nodes refer to the people who have mild addiction of the “chewing betel nut” habit; S nodes refer to the people who have severe addiction of the “chewing betel nut” habit; and R nodes refer to the people who get rid of the addiction.

In the spreading process of betel nut addiction, these states are subjected to the following rules:

- (1) If a potential individual is influenced by a mild addict with direct or indirect ways, such as face-to-face communication, Facebook, Twitter, microblog, and WeChat, then he or she will convert into a mild addict with probability ρ_1 . Similarly, if a potential individual is influenced by a severe addict with direct or indirect ways, then he or she will convert into a mild addict with probability ρ_2 .
- (2) The parameter α is defined as an influence parameter of betel nut advertisements. In real life, the marketing strategy of betel nut will influence people to try to purchase betel nut.

FIGURE 1: The flow diagram of the PMSR model.

- (3) A mild addict may convert into a severe addict with probability ϕ or get rid of the addiction with probability w_1 .
- (4) The severe addict may get rid of the addiction with probability w_2 .
- (5) Considering the relapse of addiction, a recovered individual may convert into a mild addict with probability f .
- (6) Here, we assume that the immigration rate equals the emigration rate, and the rate constant is μ .

We define $P_k(t)$, $M_k(t)$, $S_k(t)$, and $R_k(t)$ as the relative densities of potential individuals, mild addicts, severe addicts, and recovered individuals nodes of degree k at time t , respectively. According to the above description and assumption, we can get the PMSR model as follows:

$$\left\{ \begin{array}{l} \frac{dP_k(t)}{dt} = \mu - k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) - \mu P_k(t), \\ \frac{dM_k(t)}{dt} = k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) + fR_k(t) \\ \quad - (\phi + \mu + w_1)M_k(t), \\ \frac{dS_k(t)}{dt} = \phi M_k(t) - (w_2 + \mu)S_k(t), \\ \frac{dR_k(t)}{dt} = w_2S_k(t) + w_1M_k(t) - (f + \mu)R_k(t), \end{array} \right. \quad (1)$$

where $\theta_1(t)$ and $\theta_2(t)$ denote the probability of a contact pointing to a mild addict and a severe addict, respectively, and

$$\theta_1(t) = \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} kH(k)M_k(t), \quad (2)$$

$$\theta_2(t) = \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} kH(k)S_k(t). \quad (3)$$

Here, $\langle k \rangle = \sum_k kH(k)$ represents the average degree values of the network and $H(k)$ represents the degree distribution. $M(t) = \sum_k H(k)M_k(t)$ is the density of all the mild addicts and $S(t) = \sum_k H(k)S_k(t)$ is the density of all the severe addicts.

According to the normalization conditions, we have

$$P_k(t) + M_k(t) + S_k(t) + R_k(t) = 1. \quad (4)$$

Obviously, the initial values for system (1) are as follows:

$$\begin{aligned} P_k(0) &> 0, \\ M_k(0) &\geq 0, \\ S_k(0) &\geq 0, \\ C_k(0) &\geq 0, \quad k = 1, 2, \dots, n. \end{aligned} \quad (5)$$

Then, we obtain that

$$\begin{aligned} P_k(t) &> 0, \\ M_k(t) &\geq 0, \\ S_k(t) &\geq 0, \\ C_k(t) &\geq 0, \quad k = 1, 2, \dots, n \text{ and } t \geq 0. \end{aligned} \quad (6)$$

3. Stability Analysis of the Model

In this section, we analyze the dynamic properties of the PMSR betel nut addiction spreading model.

Theorem 1. *According to system (1), the basic reproductive number is defined as follows:*

$$R_0 = \frac{\alpha(f + \mu)[\rho_1(\mu + w_2) + \rho_2\varphi]}{[(w_2 + \mu)(w_1 + f + \mu) + \varphi(w_2 + f + \mu)]\mu}. \quad (7)$$

There always exists an addiction-elimination equilibrium $E_0(1, 0, 0, 0)$, and if $R_0 > 1$, system (1) has a unique addiction-prevailing equilibrium $E_+(P_k^*, M_k^*, S_k^*, R_k^*)$.

Proof. It can be easy to find that system (1) satisfies $R_k(t) = 1 - P_k(t) - M_k(t) - S_k(t)$. According to system (1), we can get

$$\begin{cases} \frac{dP_k(t)}{dt} = \mu - k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) - \mu P_k(t), \\ \frac{dM_k(t)}{dt} = k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) \\ \quad + f(1 - P_k(t) - M_k(t) - S_k(t)) - (\varphi + \mu + w_1)M_k(t), \\ \frac{dS_k(t)}{dt} = \varphi M_k(t) - (d + \mu)S_k(t). \end{cases} \quad (8)$$

Obviously, there is an addiction-elimination equilibrium $E_0(1, 0, 0, 0)$ in system (1). It is easy to verify that system (8) satisfies the conditions $(A_1) - (A_5)$ in [33]. By using the next generation matrix method [33], system (8) can get

$$\frac{dx}{dt} = Z(x) - V(x), \quad (9)$$

where

$$\begin{aligned} x &= (P_k, M_k, S_k)^T, \\ Z(x) &= \begin{pmatrix} -k\alpha(\rho_1\theta_1(t) + \rho_2\theta_2(t))P_k(t) \\ k\alpha(\rho_1\theta_1(t) + \rho_2\theta_2(t))P_k(t) \\ 0 \end{pmatrix}, \\ V(x) &= \begin{pmatrix} \mu P_k(t) \\ (w_1 + \mu + \varphi)M_k(t) - fC_k(t) \\ (w_2 + \mu)S_k(t) - \varphi M_k(t) \end{pmatrix}. \end{aligned} \quad (10)$$

At the addiction-elimination equilibrium E_0 , the Jacobian matrices of $Z(x)$ and $V(x)$ can get

$$\begin{aligned} Z &= JZ(E_0) \\ &= \begin{pmatrix} 0 & Z_{12} & Z_{13} \\ 0 & Z_{22} & Z_{23} \\ 0 & 0 & 0 \end{pmatrix}, \\ V &= JV(E_0) \\ &= \begin{pmatrix} V_{11} & 0 & 0 \\ V_{21} & V_{22} & V_{23} \\ 0 & V_{32} & V_{33} \end{pmatrix}, \end{aligned} \quad (11)$$

where

$$\begin{aligned} Z_{12} &= \frac{-\alpha\rho_1}{\langle k \rangle} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix} [1P(1) \ 2P(2) \ \dots \ nP(n)], \\ Z_{13} &= \frac{-\alpha\rho_2}{\langle k \rangle} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix} [1P(1) \ 2P(2) \ \dots \ nP(n)], \\ Z_{22} &= \frac{\alpha\rho_1}{\langle k \rangle} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix} [1P(1) \ 2P(2) \ \dots \ nP(n)], \\ Z_{23} &= \frac{\alpha\rho_2}{\langle k \rangle} \begin{bmatrix} 1 \\ 2 \\ \vdots \\ n \end{bmatrix} [1P(1) \ 2P(2) \ \dots \ nP(n)]. \end{aligned} \quad (12)$$

Here,

$$\begin{aligned} V_{11} &= \mu I, \\ V_{21} &= f I, \\ V_{22} &= (w_1 + \mu + \varphi + f) I, \\ V_{23} &= f I, \\ V_{32} &= -\varphi I, \\ V_{33} &= (w_2 + \mu) I, \end{aligned} \quad (13)$$

where I is the identity matrix. So, we can get the basic reproductive number:

$$\begin{aligned} R_0 &= \rho(ZV^{-1}) \\ &= \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\alpha(f + \mu)[\rho_1(\mu + w_2) + \rho_2\varphi]}{[(w_2 + \mu)(w_1 + f + \mu) + \varphi(w_2 + f + \mu)]\mu}, \end{aligned} \quad (14)$$

where $\langle k^2 \rangle = \sum_k k^2 P(k)$.

Next, it is clear that system (1) has an equilibrium $E_0 = \{(1, 0, 0, 0)\}_k$. To get the addition-prevailing equilibrium $E^*(P_k^*, M_k^*, S_k^*, R_k^*)$, system (1) satisfies

$$\begin{aligned} \frac{dP_k(t)}{dt} &= 0, \\ \frac{dM_k(t)}{dt} &= 0, \\ \frac{dS_k(t)}{dt} &= 0, \\ \frac{dR_k(t)}{dt} &= 0. \end{aligned} \quad (15)$$

So, we can know

$$\begin{cases} \mu - k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) - \mu P_k(t) = 0, \\ k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) + fR_k(t) - (\varphi + \mu + w_1)M_k(t) = 0, \\ \varphi M_k(t) - (w_2 + \mu)S_k(t) = 0, \\ w_2S_k(t) + w_1M_k(t) - (f + \mu)R_k(t) = 0. \end{cases} \quad (16)$$

According to the above equations, we get

$$\begin{cases} P_k = \frac{\mu[w_1(w_2 + \mu) + (f + \mu)(w_2 + \mu) + \varphi(w_2 + f + \mu)]}{k\alpha(\rho_1\theta_1 + \rho_2\theta_2)(w_2 + \mu)(f + \mu)} M_k, \\ S_k = \frac{\varphi}{w_2 + \mu} M_k, \\ R_k = \frac{(w_2 + \mu)w_1 + \varphi w_2}{(w_2 + \mu)(f + \mu)} M_k. \end{cases} \quad (17)$$

By using the normalization condition $P_k^*(t) + M_k^*(t) + S_k^*(t) + R_k^*(t) = 1$, we get

$$\begin{cases} P_k^* = \frac{\mu[w_1(w_2 + \mu) + (f + \mu)(w_2 + \mu) + \varphi(w_2 + f + \mu)]}{k\alpha(\rho_1\theta_1^* + \rho_2\theta_2^*)(w_2 + \mu)(f + \mu)} M_k^*, \\ M_k^* = \frac{(f + \mu)w_1(w_2 + \mu) + \mu[(f + \mu)(w_2 + \mu) + \varphi(w_2 + f + \mu)]}{(w_2 + \mu)(f + \mu)(w_1 + \mu + \varphi)} M_k^*, \\ S_k^* = \frac{\varphi}{w_2 + \mu} M_k^*, \\ R_k^* = \frac{(w_2 + \mu)w_1 + \varphi w_2}{(w_2 + \mu)(f + \mu)} M_k^*. \end{cases} \quad (18)$$

And, for $\theta_1^* = 1/\langle k \rangle \sum_{k=1}^{\infty} kP(k)M_k^*(t)$, $\theta_2^* = 1/\langle k \rangle \sum_{k=1}^{\infty} kP(k)S_k^*(t)$, we get

$$\begin{aligned} \theta_1^* &= \frac{w_2 + \mu}{\varphi} \theta_2^*, \\ \theta_2^* &= \frac{\varphi}{w_2 + \mu} \theta_1^*. \end{aligned} \quad (19)$$

By substituting the second equation of system (16) into equation (2), we get

$$\begin{aligned} \theta_1^* &= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} kP(k)M_k^*(t) \\ &= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} kP(k) \frac{k\alpha(\rho_1\theta_1^* + \rho_2\theta_2^*)(w_2 + \mu)(f + \mu)}{[k\alpha(\rho_1\theta_1^* + \rho_2\theta_2^* + \mu)][(w_2 + \mu)(f + \mu) + \varphi(f + \mu)]} \\ &= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k^2 P(k) \frac{k\alpha(f + \mu)(w_2 + \mu)[\rho_1(w_2 + \mu) + \rho_2\varphi]\theta_1^*}{k\alpha(w_2 + \mu)(f + \mu)[\rho_1(w_2 + \mu) + \rho_2\varphi]\theta_1^* + A_k} \\ &= f(\theta_1^*), \end{aligned} \quad (20)$$

where

$$A_k = \mu(w_2 + \mu)(f + \mu)[(w_2 + \mu) + \varphi + k\alpha\varphi] + k\alpha\varphi(f + \mu)[(w_2 + \mu)\rho_1 + \varphi\rho_2]\theta_1^*. \quad (21)$$

Obviously, $\theta_1^* = 0$ satisfies system (20). System (20) has a unique nontrivial solution provided that

$$\left. \frac{df(\theta_1^*)}{d\theta_1^*} \right|_{\theta_1^*=0} > 1, \quad f(1) \leq 1. \quad (22)$$

So, we get

$$R_0 = \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\alpha(f + \mu)[\rho_1(\mu + w_2) + \rho_2\varphi]}{[(w_2 + \mu)(w_1 + f + \mu) + \varphi(w_2 + f + \mu)]\mu} > 1. \quad (23)$$

Thus, a unique nontrivial solution exists if and only if $R_0 > 1$. According to equation (17), we know

$$\begin{aligned} 0 < P_k^\infty < 1, \\ 0 < M_k^\infty < 1, \\ 0 < S_k^\infty < 1, \\ 0 < R_k^\infty < 1. \end{aligned} \quad (24)$$

Therefore, the addiction-prevailing equilibrium $E^*(P_k^*, M_k^*, S_k^*, R_k^*)$ is well defined. Then, when $R_0 > 1$, there is a unique positive equilibrium $E^*(P_k^*, M_k^*, S_k^*, R_k^*)$. The proof is completed.

Remark 1. The basic reproductive number R_0 is obtained by equation (23), which depends on the fluctuations of the degree distribution $\langle k^2 \rangle / \langle k \rangle$ and some model parameters. When the heterogeneity of the degree distribution $\langle k^2 \rangle / \langle k \rangle$ is larger, the basic reproductive number R_0 is greater, i.e., the larger heterogeneity of the degree distribution can promote the spreading of betel nut addiction. Obviously, as the influence parameter α of betel nut advertisements increases, the basic reproductive number R_0 increases.

Theorem 2. If $R_0 < 1$, the addiction-elimination equilibrium E_0 is globally asymptotically stable. If $R_0 > 1$, the addiction spreading phenomenon is persistent, i.e., there exists a constant $\eta > 0$, such that

$$\lim_{t \rightarrow \infty} \inf (M(t) + S(t)) = \lim_{t \rightarrow \infty} \inf \sum_k H(k)(M_k(t) + S_k(t)) \geq \eta. \quad (25)$$

Proof. For the addiction-elimination equilibrium, system (8) has the Jacobian matrix as follows:

$$J = \begin{pmatrix} A_{11} & \cdots & A_{1n} \\ \vdots & \ddots & \vdots \\ A_{n1} & \cdots & A_{nn} \end{pmatrix}, \quad (26)$$

where

$$\begin{aligned} A_{11} &= \begin{pmatrix} \alpha\rho_1 P_1 - (w_1 + \mu + \varphi) & \alpha\rho_2 P_1 & f \\ \varphi & -(w_2 + \mu) & 0 \\ w_1 & w_2 & -(f + \mu) \end{pmatrix}, \\ A_{1n} &= \begin{pmatrix} \alpha\rho_1 P_n & \alpha\rho_2 P_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ A_{n1} &= \begin{pmatrix} n\alpha\rho_1 P_n & n\alpha\rho_2 P_n & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \\ A_{nn} &= \begin{pmatrix} n\alpha\rho_1 P_n - (w_1 + \mu + \varphi) & n\alpha\rho_2 P_n & f \\ \varphi & -(w_2 + \mu) & 0 \\ w_1 & w_2 & -(f + \mu) \end{pmatrix}. \end{aligned} \quad (27)$$

So, the characteristic polynomial of the addiction-elimination equilibrium is

$$\begin{aligned} &(\lambda + w_1 + \mu + \varphi)^{n-1} (\lambda + w_2 + \mu)^{n-1} (\lambda^2 + p\lambda + q) \\ &\cdot (\lambda + f + \mu)^{n-1} = 0, \end{aligned} \quad (28)$$

where

$$\begin{aligned} p &= (w_2 + \mu)(w_1 + f + \mu) + \varphi(w_2 + f + \mu) \\ &\quad + (f + \mu)(w_1 + \mu) + \mu(w_2 + \mu + \varphi) \\ &\quad - [\rho_1(f + \mu + w_2 + \mu) + \rho_2\varphi\alpha] \sum_{i=1}^n iP_i - fw_1, \\ q &= (w_1 + \mu + \varphi)(w_2 + \mu)(f + \mu) + f\varphi w_2 \\ &\quad - \alpha(f + \mu)[\rho_1(w_2 + \mu) + \rho_2\varphi] \sum_{i=1}^n iP_i - fw_1(w_2 + \mu). \end{aligned} \quad (29)$$

When $R_0 < 1$, we can get

$$R_0 = \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\alpha(f + \mu)[\rho_1(\mu + w_2) + \rho_2\varphi]}{[(w_2 + \mu)(w_1 + f + \mu) + \varphi(w_2 + f + \mu)]\mu} < 1. \quad (30)$$

It also means

$$\begin{aligned} &[(w_2 + \mu)(w_1 + f + \mu) + \varphi(w_2 + f + \mu)]\mu \\ &> \alpha(f + \mu)[\rho_1(\mu + w_2) + \rho_2\varphi] \sum_{i=1}^n iP_i. \end{aligned} \quad (31)$$

In other words, we get $p > 0$ and $q > 0$. According to the above proof, the real eigenvalues λ of matrix J are all negative when $R_0 < 1$. Furthermore, if and only if $R_0 > 1$, there is a unique positive eigen value λ of matrix J . By using the Perron–Frobenius theorem, the maximal real part of all eigenvalues of λ is positive only if $R_0 > 1$. Through the theorem of Lajmanovich and York [34], we can get the results. The proof is completed.

Lemma 1 (see [35]). If $a > 0, b > 0$ and $dx(t)/dt \geq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have $\lim_{t \rightarrow \infty} \inf x(t) \geq b/a$; if $a > 0, b > 0$ and $dx(t)/dt \leq b - ax$, when $t \geq 0$ and $x(0) \geq 0$, we have $\lim_{t \rightarrow \infty} \sup x(t) \leq b/a$.

Next, the global attractivity of the addiction-prevailing equilibrium is discussed. The main result is given in the following theorem.

Theorem 3. Suppose that $(P_k(t), M_k(t), S_k(t))$ is a solution of system (8) satisfying initial conditions $M_k(t) > 0$ or $S_k(t) > 0$. If $R_0 > 1$, then

$$\lim_{t \rightarrow \infty} (P_k(t), M_k(t), S_k(t)) = (P_k^*(t), M_k^*(t), S_k^*(t)), \quad (32)$$

where $(P_k^*(t), M_k^*(t), S_k^*(t))$ is the addiction-prevailing equilibrium of system (8) for $k = 1, 2, \dots, n$.

Proof. In the course of the proving, we followed the methods of reference [35]. In the following, we suppose that k is an integer between 1 and n . According to Theorem 2, there exists a positive constant $0 < \tau < 1/3$ and a large enough constant $T > 0$ such that $M_k(t) \geq \tau$ and $S_k(t) \geq \tau$ for $t > T$. Therefore, $\theta_1(t) > \tau, \theta_2(t) > \tau$ for $t > T$. Substituting this into the first equation of system (8), it is easy to get

$$\begin{aligned} \frac{dP_k(t)}{dt} &= \mu - k\alpha(\rho_1\theta_1 + \rho_2\theta_2)P_k(t) - \mu P_k(t) \\ &\leq \mu - [k\alpha(\rho_1 + \rho_2)\tau + \mu]P_k(t), \quad t > T. \end{aligned} \quad (33)$$

From Lemma 1, according to the standard comparison theorem of differential equation theory, for any given positive constant $0 < \tau_1 < k(\rho_1 + \rho_2)\tau/(\mu + k\alpha(\rho_1 + \rho_2)\tau)$, there exists $t_1 > T$, such that $P_k(t) \leq X_k^{(1)} - \tau_1$ for $t > t_1$, where

$$X_k^{(1)} = \frac{\mu}{\mu + k\alpha(\rho_1 + \rho_2)\tau} + 2\tau_1 < 1. \quad (34)$$

From system (8), it is easy to obtain

$$\begin{aligned} \frac{dM_k}{dt} &= [k\alpha(\rho_1\theta_1 + \rho_2\theta_2) + f](1 - M_k(t)) \\ &\quad - (w_1 + \mu + \varphi)M_k(t), \quad t > t_1. \end{aligned} \quad (35)$$

So, the constant $0 < \tau_2 < \min\{1/2, w_1 + \mu + \varphi/2[k\alpha(\rho_1 + \rho_2)\tau + w_1 + \mu + \varphi + f]\}$, there exists $t_2 > t_1$, so $M_k(t) \leq Y_k^{(1)} - \tau_2$ for $t > t_2$, where

$$Y_k^{(1)} = \frac{2k(\rho_1 + \rho_2)\tau + f}{2k(\rho_1 + \rho_2)\tau + f + w_1 + \mu + \varphi} + 2\tau_2 < 1. \quad (36)$$

From system (8), it is easy to obtain

$$\frac{dS_k(t)}{dt} \leq \varphi(1 - S_k(t)) - (w_2 + \mu)S_k(t), \quad t > t_2. \quad (37)$$

Thus, for constant $0 < \tau_3 < \min\{1/3, \tau_2, w_2 + \mu/2(\varphi + w_2 + \mu)\}$, there exists $t_3 > t_2$, so $S_k(t) \leq Z_k^{(1)} - \tau_3$ for $t > t_3$, where

$$Z_k^{(1)} = \frac{\varphi}{\varphi + w_2 + \mu} + 2\tau_3 < 1. \quad (38)$$

Then, replacing $M_k(t) \leq Y_k^{(1)} - \tau_2$ and $S_k(t) \leq Z_k^{(1)} - \tau_3$ into the first equation of system (8), we get

$$\begin{aligned} \frac{dP_k(t)}{dt} &= \mu - k\alpha(\rho_1\theta_1(t) + \rho_2\theta_2(t))P_k(t) - \mu P_k(t) \\ &\geq \mu - [k\alpha(\rho_1 + \rho_2)\tau + \mu]P_k(t), \quad t > T. \end{aligned} \quad (39)$$

Thus, for constant $0 < \tau_4 < \min\{1/4, \mu/2[k\alpha(\rho_1 + \rho_2)\tau + \mu]\}$, there exists $t_4 > t_3$, such that $P_k(t) \geq x_k^{(1)} + \tau_4$ for $t > t_4$, where

$$x_k^{(1)} = \frac{\mu}{k\alpha(\rho_1 + \rho_2)\tau + \mu} - 2\tau_4 > 0. \quad (40)$$

Then, substituting $\theta_1(t) > \tau, \theta_2(t) > \tau$ into system (8), it follows that

$$\frac{dM_k(t)}{dt} \geq k\alpha(\rho_1 + \rho_2)\tau x_k^{(1)} - (w_1 + \mu + \varphi)M_k(t) + f\tau, \quad t > t_4. \quad (41)$$

Hence, for constant $0 < \tau_5 < \min\{1/5, k\alpha(\rho_1 + \rho_2)\tau x_k^{(1)} + f\tau/2(w_1 + \mu + \varphi)\}$, there exists $t_5 > t_4$, such that $M_k(t) \geq y_k^{(1)} + \tau_5$ for $t > t_5$, where

$$y_k^{(1)} = \frac{k\alpha(\rho_1 + \rho_2)\tau x_k^{(1)} + f\tau}{w_1 + \mu + \varphi} - 2\tau_5 > 0. \quad (42)$$

Therefore,

$$\frac{dS_k(t)}{dt} \geq \varphi y_k^{(1)} - (w_2 + \mu)S_k(t), \quad t > t_5. \quad (43)$$

Hence, for constant $0 < \tau_6 < \min\{1/6, \tau_5, \varphi y_k^{(1)}/2(w_2 + \mu)\}$, there exists $t_6 > t_5$, such that $S_k(t) \geq z_k^{(1)} + \tau_6$ for $t > t_6$, where

$$z_k^{(1)} = \frac{\varphi y_k^{(1)}}{w_2 + \mu} - 2\tau_6 > 0. \quad (44)$$

Because τ is a small positive constant, we can get

$$\begin{aligned} 0 &< x_k^{(1)} < P_k < X_k^{(1)} < 1, \\ 0 &< y_k^{(1)} < M_k < Y_k^{(1)} < 1, \\ 0 &< z_k^{(1)} < S_k < Z_k^{(1)} < 1. \end{aligned} \quad (45)$$

Let

$$\begin{aligned} q^{(j)} &= \sum_{i=1}^n P_i(y_i^{(j)} + z_i^{(j)}), \\ Q^{(j)} &= \sum_{i=1}^n P_i(Y_i^{(j)} + Z_i^{(j)}), \\ j &= 1, 2, \dots, n. \end{aligned} \quad (46)$$

From the above discussion, we have

$$0 < q^{(1)} \leq (\rho_1 \theta_1 + \rho_2 \theta_2) \leq Q^{(1)} < \rho_1 + \rho_2, \quad t > t_6. \quad (47)$$

And, we substitute $M_k(t) \geq y_k^{(1)} + \tau_5$, $S_k(t) \geq z_k^{(1)} + \tau_6$ into system (8), we can get

$$\frac{dP_k(t)}{dt} \leq \mu - (k\alpha q^{(1)} + \mu)P_k(t), \quad t > t_6. \quad (48)$$

Consequently, for constant $0 < \tau_7 < \min\{1/7, \tau_6\}$, there exists $t_7 > t_6$, such that

$$P_k(t) \leq X_k^{(2)} \triangleq \min \left\{ X_k^{(1)} - \tau_1, \frac{\mu}{\mu + k\alpha q^{(1)}} + \tau_7 \right\}, \quad t > t_7. \quad (49)$$

Thus,

$$\begin{aligned} \frac{dM_k(t)}{dt} &\leq k\alpha Q^{(1)} X_k^{(2)} - (w_1 + \varphi + \mu)M_k(t) \\ &\quad + f(1 - M_k(t) - x_k^{(1)} - z_k^{(1)}) \\ &\leq k\alpha Q^{(1)} X_k^{(2)} - (w_1 + \varphi + \mu)M_k(t) \\ &\quad + f(1 - x_k^{(1)} - z_k^{(1)}), \quad t > t_7. \end{aligned} \quad (50)$$

Consequently, for constant $0 < \tau_8 < \min\{1/8, \tau_7\}$, there exists $t_8 > t_7$, such that

$$\begin{aligned} M_k(t) &\leq Y_k^{(2)} \triangleq \min \left\{ Y_k^{(1)} \right. \\ &\quad \left. - \tau_2, \frac{kQ^{(1)} X_k^{(2)} + f(1 - x_k^{(1)} - z_k^{(1)})}{w_1 + \varphi + \mu + f} + \tau_8 \right\}, \quad t > t_8. \end{aligned} \quad (51)$$

As a result, it follows that

$$\frac{dS_k(t)}{dt} \leq \varphi M_k^{(2)} - (w_2 + \mu)S_k(t), \quad t > t_9. \quad (52)$$

Therefore, for constant $0 < \tau_9 < \min\{1/9, t\tau_8\}$, we can get

$$S_k(t) \leq Z_k^{(2)} \triangleq \min \left\{ Z_k^{(1)} - \tau_3, \frac{\varphi Y_k^{(2)}}{w_2 + \mu} + \tau_9 \right\}, \quad t > t_9. \quad (53)$$

Substituting $M_k(t) \leq Y_k^{(2)}$, $S_k(t) \leq Z_k^{(2)}$ and $\rho_1 \theta_1 + \rho_2 \theta_2 \leq Q^{(2)}$ into system (8), where

$$\frac{dP_k(t)}{dt} \geq \mu - (k\alpha Q^{(2)} + \mu)P_k(t), \quad t > t_9. \quad (54)$$

Hence, for constant $0 < \tau_{10} < \min\{1/10, t\tau_9, q\mu/2(k\alpha Q^{(2)} + \mu)\}$, there exists $t_{10} > t_9$, such that $P_k(t) \geq x_k^{(2)} + \tau_{10}$ for $t > t_{10}$, where

$$x_k^{(2)} = \max \left\{ x_k^{(1)} + \tau_4, \frac{\mu}{\mu + k\alpha Q^{(2)}} - 2\tau_{10} \right\}. \quad (55)$$

Thus,

$$\begin{aligned} \frac{dM_k(t)}{dt} &\geq k\alpha q^{(1)} x_k^{(2)} - (\varphi + w_1 + \mu)M_k(t) \\ &\quad + f(1 - M_k(t) - x_k^{(1)} - z_k^{(1)}), \quad t > t_{10}. \end{aligned} \quad (56)$$

So, for constant $0 < \tau_{11} < \min\{1/11, t\tau_{10}n, qk\alpha q^{(1)} x_k^{(2)} + f(1 - x_k^{(1)} - z_k^{(1)})/2(w_1 + \varphi + \mu + f)\}$, there exists $t_{11} > t_{10}$, and $M_k(t) \geq y_k^{(2)} + \tau_{11}$ for $t > t_{11}$, where

$$y_k^{(2)} = \max \left\{ y_k^{(1)} + \tau_5, \frac{k\alpha q^{(1)} x_k^{(2)} + f(1 - x_k^{(1)} - z_k^{(1)})}{w + f + \varphi + \mu} - 2\tau_{11} \right\}. \quad (57)$$

Similarly,

$$\frac{dS_k(t)}{dt} \geq \varphi y_k^{(2)} - (w_2 + \mu)S_k(t), \quad t > t_{11}. \quad (58)$$

Therefore, for constant $0 < \tau_{12} < \min\{1/12, t\tau_{11}n, q\varphi y_k^{(2)}/2(w_2 + \mu)\}$, there exists $t_{12} > t_{11}$, and $S_k(t) \geq z_k^{(2)} + \tau_{12}$ for $t > t_{12}$, where

$$z_k^{(2)} = \max \left\{ z_k^{(1)} + \tau_6, \frac{\varphi y_k^{(2)}}{w_2 + \mu} - 2\tau_{12} \right\}. \quad (59)$$

According to the above discussion and analyses, we can obtain six sequences: $\{X_k^{(l)}\}$, $\{Y_k^{(l)}\}$, $\{Z_k^{(l)}\}$, $\{x_k^{(l)}\}$, $\{y_k^{(l)}\}$, and $\{z_k^{(l)}\}$. We can find that the first three sequences are monotonically increasing, and the last three sequences are strictly monotonically decreasing, and there is a sufficiently large positive integer H such that, for $l \geq H$,

$$\left\{ \begin{aligned} X_k^{(l)} &= \frac{\mu}{\mu + k\alpha q^{(l-1)}} + \tau_{6l-5}, \\ Y_k^{(l)} &= \frac{k\alpha Q^{(l-1)} X_k^{(l)} + f(1 - x_k^{(l-1)} - z_k^{(l-1)})}{w_1 + \varphi + f + \mu} + \tau_{6l-4}, \\ Z_k^{(l)} &= \frac{\varphi Y_k^{(l)}}{w_2 + \mu} + \tau_{6l-3}, \\ x_k^{(l)} &= \frac{\mu}{\mu + k\alpha Q^{(l)}} - 2\tau_{6l-2}, \\ y_k^{(l)} &= \frac{k\alpha q^{(l-1)} x_k^{(l)} + f(1 - X_k^{(h)} - Z_k^{(h)})}{w_1 + \varphi + f + \mu} - 2\tau_{6l-1}, \\ z_k^{(l)} &= \frac{\varphi y_k^{(l)}}{w_2 + \mu} - 2\tau_{6l-1}. \end{aligned} \right. \quad (60)$$

It is easy to find that

$$\begin{aligned}
x_k^{(l)} &\leq P_k(t) \leq X_k^{(l)}, \\
y_k^{(l)} &\leq M_k(t) \leq Y_k^{(l)}, \\
z_k^{(l)} &\leq S_k(t) \leq Z_k^{(l)}, \\
t &> t_{6l}.
\end{aligned} \tag{61}$$

where

$$\begin{aligned}
q &= \sum_{i=1}^n P_i(y_i + z_i), \\
Q &= \sum_{i=1}^n P_i(Y_i + Z_i).
\end{aligned} \tag{63}$$

Since the sequential limits of system (60) exist, let $\lim_{l \rightarrow \infty} \Omega_k^{(l)} = \Omega_k$, where $\Omega_k^{(l)} \in \{X_k^{(l)}, Y_k^{(l)}, Z_k^{(l)}, x_k^{(l)}, y_k^{(l)}, z_k^{(l)}, Q_k^{(l)}, q_k^{(l)}\}$ and $\Omega_k \in \{X_k, Y_k, Z_k, x_k, y_k, z_k, Q_k, q_k\}$.

Since $0 < \tau_l < 1/l$, it has $\tau_l \rightarrow 0$ as $l \rightarrow \infty$. Supposing $l \rightarrow \infty$, it follows from (61) that

$$\begin{aligned}
X_k &= \frac{\mu}{k\alpha q + \mu}, \\
Y_k &= \frac{k\alpha Q X_k + f(1 - x_k - z_k)}{w_1 + f + \mu + \varphi}, \\
Z_k &= \frac{\varphi}{w_2 + \mu} Y_k, \\
x_k &= \frac{\mu}{k\alpha Q + \mu}, \\
y_k &= \frac{k\alpha q x_k + f(1 - X_k - Z_k)}{w_1 + f + \mu + \varphi}, \\
z_k &= \frac{\varphi}{w_2 + \mu} y_k,
\end{aligned} \tag{62}$$

$$\begin{aligned}
Y_k &= \frac{\{kQ\alpha[(w_2 + \mu)(w_2 + \mu)(k\alpha Q\mu + f(k\alpha q + \mu)) + \varphi f(k\alpha q + \mu) - (k\alpha q + \mu)(f(w_2 + \mu)(w_2 + \mu) + \varphi k\alpha q) + f\varphi]\}}{(k\alpha q + \mu)(k\alpha Q + \mu)(w_1 + f + \mu + \varphi)(w_2 + \mu)(w_2 + \mu)}, \\
y_k &= \frac{kq\alpha[\{(w_2 + \mu)(w_2 + \mu)(k\alpha q\mu + f(k\alpha Q + \mu)) + \varphi f(k\alpha Q + \mu)\} - (k\alpha Q + \mu)(f(w_2 + \mu)(w_2 + \mu) + \varphi k\alpha Q) + f\varphi]}{(k\alpha q + \mu)(k\alpha Q + \mu)(w_1 + f + \mu + \varphi)(w_2 + \mu)(w_2 + \mu)}.
\end{aligned} \tag{64}$$

Substituting (62) and (64) into Q and q , respectively, we obtain

$$\begin{aligned}
1 &= \frac{\varphi + w_2 + \mu}{w_2 + \mu} \sum_{i=1}^n i P_i \left\{ \frac{\{k\alpha[(w_2 + \mu)(w_2 + \mu)(k\alpha Q\mu + f(k\alpha q + \mu)) + \varphi f(k\alpha q + \mu)] - (k\alpha q + \mu)[(f(w_2 + \mu)(w_2 + \mu) + \varphi k\alpha q) + f\varphi]\}}{(k\alpha q + \mu)(k\alpha Q + \mu)(w_1 + f + \mu + \varphi)(w_2 + \mu)(w_2 + \mu)} \right\}, \\
1 &= \frac{\varphi + w_2 + \mu}{w_2 + \mu} \sum_{i=1}^n i P_i \left\{ \frac{\{k\alpha[(w_2 + \mu)(w_2 + \mu)(k\alpha q\mu + f(k\alpha Q + \mu)) + \varphi f(k\alpha Q + \mu)] - (k\alpha Q + \mu)[(f(w_2 + \mu)(w_2 + \mu) + \varphi k\alpha Q) + f\varphi]\}}{(k\alpha q + \mu)(k\alpha Q + \mu)(w_1 + f + \mu + \varphi)(w_2 + \mu)(w_2 + \mu)} \right\}
\end{aligned} \tag{65}$$

FIGURE 2: The influence of parameters ρ_1 and ρ_2 on R_0 . FIGURE 3: The time series and orbits of four states with $R_0 < 1$.

From the above two equations, we can get

$$(Q - q) \frac{\varphi + w_2 + \mu}{w_2 + \mu} \sum_{i=1}^n iP_i \left\{ \frac{[(kaq + \mu) - (kaQ + \mu)][(f(w_2 + \mu)(w_2 + \mu) + \varphi kaQ) + f\varphi]}{(kaq + \mu)(kaQ + \mu)(w_1 + f + \mu + \varphi)(w_2 + \mu)(w_2 + \mu)} \right\} \equiv 0. \quad (66)$$

It is clear that $Q = q$. So, $\sum_{i=1}^n P_i [(Y_i - y_i) + (Z_i - z_i)] = 0$, which is equivalent to $Y_i = y_i$ and $Z_i = z_i$ for $1 \leq i \leq n$. Then, from systems (61) and (62), it can be concluded that

$$\begin{aligned} \lim_{t \rightarrow \infty} P_k(t) &= X_k = x_k, \\ \lim_{t \rightarrow \infty} M_k(t) &= Y_k = y_k, \\ \lim_{t \rightarrow \infty} R_k(t) &= Z_k = z_k. \end{aligned} \quad (67)$$

FIGURE 4: The time series and orbits of four states with $R_0 > 1$. FIGURE 5: The prevalence of M_{100} and S_{100} versus t to different α with $R_0 < 1$.

Finally, $Q = q$ is substituted into system (64). For system (62), we can get $X_k = P_k^*$, $Y_k = M_k^*$, and $Z_k = R_k^*$. This proof is completed.

4. Numerical Simulations

This section illustrates the analytical results through numerical simulations. Based on scale-free network, we have

$H(k) = ck^{-3}$ in system (1), and the parameter c satisfies $\sum_{k=1}^n ck^{-3} = 1$, $n = 1000$.

In Figure 2, the parameters are chosen as $\mu = 0.3$, $\alpha = 0.2$, $\varphi = 0.3$, $w_2 = 0.5$, $w_1 = 0.3$, and $f = 0.6$. We can see that the larger the ρ_1 and ρ_2 , the larger the R_0 , i.e., long time contact with mild addicts and severe addicts both can increase the spreading speed of betel nut addiction. This also means that if people's health knowledge is improved, the

FIGURE 6: The prevalence of M_{100} and S_{100} versus t to different α with $R_0 > 1$.

probability of people chewing betel nut is reduced. The spreading of betel nut addiction will be weakened.

In Figure 3, we choose $\mu = 0.3$, $\alpha = 0.2$, $\varphi = 0.4$, $w_2 = 0.4$, $w_1 = 0.3$, $f = 0.6$, $\rho_1 = 0.4$, and $\rho_2 = 0.4$, thus the basic reproductive number $R_0 = 0.8090 < 1$. We can see that there is almost no spreading of betel nut addiction when $R_0 < 1$, which means that betel nut addiction will eventually disappear. It also suggests that the addiction-elimination equilibrium E_0 is globally asymptotically stable when $R_0 < 1$. This also means that restricting the marketing strategy of betel nut and controlling the recurrence rate of betel nut addiction will weaken the spreading of betel nut addiction.

In Figure 4, we choose $\mu = 0.3$, $\alpha = 0.5$, $\varphi = 0.4$, $w_2 = 0.3$, $w_1 = 0.3$, $f = 0.6$, $\rho_1 = 0.7$, and $\rho_2 = 0.7$, thus the basic reproductive number $R_0 = 3.7163 > 1$. We can see that the number of mild addicts and severe addicts will maintain at a positive constant, i.e., the spreading phenomenon of betel nut addiction is permanent on the scale-free networks when $R_0 > 1$.

In Figure 5, we choose $\mu = 0.4$, $\varphi = 0.6$, $w_2 = 0.6$, $w_1 = 0.4$, $f = 0.2$, $\rho_1 = 0.2$, and $\rho_2 = 0.2$, thus the basic reproductive number $R_0 < 1$. We can see that the corresponding M_{100} and S_{100} decrease significantly as the parameter α decreases, i.e., a smaller parameter α can weaken the spreading phenomenon of betel nut addiction. This also means that the marketing strategies of betel nut merchants have a great influence on the spreading of betel nut addiction.

In Figure 6, we choose $\mu = 0.3$, $\varphi = 0.4$, $w_2 = 0.3$, $w_1 = 0.3$, $f = 0.6$, $\rho_1 = 0.7$, and $\rho_2 = 0.7$, thus the basic reproductive number $R_0 > 1$. Figure 6 describes the change of

the mild addicts M_{100} and severe addicts S_{100} with different rate α . We can see that when $R_0 > 1$, M_{100} and S_{100} both grow to positive constants with the increasing of parameter α . This figure shows that a larger α can lead to the larger value of betel nut addiction spreading level. It means that restricting the marketing strategies of betel nut plays a vital role in controlling the spread of betel nut addiction.

5. Conclusions

In this paper, a new comprehensively PMSR betel nut addiction spreading model has been proposed. By the mean-field theory, we have obtained the basic reproduction number R_0 addiction-elimination equilibrium E_0 , and addiction-prevailing equilibrium E_+ . If $R_0 < 1$, the addiction-elimination equilibrium E_0 is globally asymptotically stable, i.e., betel nut addiction spreading behavior will eventually disappear. If $R_0 > 1$, the spreading dynamical behavior of betel nut addiction is persistent and globally asymptotically stable. Moreover, we found that the larger the parameters ρ_1 and ρ_2 , the larger the basic reproductive number R_0 , i.e., long-term and frequent contact with mild addicts and severe addicts can increase the spreading phenomenon of betel nut addiction. Furthermore, a larger α can increase the spread speed and range of betel nut addiction. Therefore, controlling the spread of betel nut addiction requires restricting the marketing strategies adopted by betel nut merchants and widely publicizing the harmfulness of chewing betel nut. The research results have important guiding significance in controlling the spreading of betel nut addiction.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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