

Research Article

A Robust Control Scheme for a PVTOL System Subject to Wind Disturbances

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In this study, a control scheme that allows performing height position regulation and stabilization for an unmanned planar vertical take-off and landing aerial vehicle, in the presence of disturbance due to wind, is presented. To this end, the backstepping procedure together with nested saturation function method is used. Firstly, a convenient change of coordinates in the aerial vehicle model is carried out to dissociate the rotational dynamics from the translational one. Secondly, the backstepping procedure is applied to obtain the height position controller, allowing the reduction of the system and expressing it as an integrator chain with nonlinear disturbance. Therefore, the nested saturation function method is used to obtain a stabilizing controller for the horizontal position and roll angle. The corresponding stability analysis is conducted via the Lyapunov second method. In addition, to estimate the disturbance due to wind, an extended state observer is used. The effectiveness of the proposed control scheme is assessed through numerical simulations, from which convincing results have been obtained.

1. Introduction

The Planar Vertical Take-Off and Landing (PVTOL) unmanned aerial vehicle is a representation of the Harrier Yab-8b aircraft when considering a minimum of inputs and outputs to obtain a vertical short take-off and landing behavior [1], which has been used as a test bed for automatic control applications. In fact, the PVTOL aerial vehicle is a simplified model that embodies the behavior of several actual vertical take-off and landing aircraft, which ultimately makes it a suitable benchmark to test new and existing controllers. Therefore, there is a vast literature on the subject. However, a current and open challenge is the design of

robust controllers for the PVTOL system under wind disturbance in order to pursue outdoor applications. Thus, this paper presents a robust control scheme for a PVTOL system subjected to disturbance due to wind.

The works considered most relevant and closely related to the control problem treated in this study are mentioned as follows. In [1], an input-output linearization to achieve trajectory tracking control for the non-minimum phase nonlinear system is presented. In [2], the authors developed a nonlinear output-feedback controller for the trajectory tracking of a reference model by using a global exponential observer, coordinate transformations, the Lyapunov's method, and an extension of

backstepping. In [3], a global stabilization control derived from nonlinear combinations of linear saturation functions was presented, whereas in [4], a nonlinear controller for taking-off, hovering, tracking of a straight line, and landing of the PVTOL system was introduced. The corresponding experimental implementation of the controller was reported in [5], where a camera was used to estimate the position and orientation of the PVTOL vehicle. Also, in [6], a global configuration stabilization for the VTOL aircraft with a strong input coupling using a smooth static state feedback was reported. An alternative feedback-based stabilization law to the one introduced in [6] was presented in [7]. This law simplifies the term that connects the state vector with the dynamics error. In [8], the authors designed a stabilization control by transforming the system into an integrator chain plus a nonlinear disturbance, after which the saturation technique was used so that neither backstepping/forwarding approaches nor small gain analysis is required. Furthermore, a nonlinear controller for a PVTOL vehicle, based on prediction and partial feedback linearization, was designed in [9]. A robust and linear state-feedback gain-scheduled control to achieve hovering of a PVTOL system with uncertainties in the mass, the momentum of inertia, and the parasitic coupling parameter was introduced in [10], while a nested set stabilization approach to locally solve path following for the PVTOL system was introduced in [11]. The system center of mass was constrained to lie on the path, and the roll angle should be specified at any given point on the path. In [12], the authors developed a bounded backstepping method to achieve input-to-state stability, with respect to the actuator errors, and to force all trajectories of the system to track a reference trajectory for all initial configurations. Also, Zavala-Río et al. [13] introduced a finite-time observer-based output-feedback control for the global stabilization of both taking-off and landing of the PVTOL system. In [14–16], a solution for the regulation of a simplified version of the PVTOL system was reported, which consisted of two control actions that act simultaneously. Aguilar-Ibáñez et al. [14] used, as first action, a feedback linearization along with a saturation function to asymptotically stabilize the vertical position. For the second action, backstepping was exploited to stabilize both the horizontal and the angle positions. Similarly, Aguilar-Ibáñez et al. [15] utilized as first action a feedback linearization in combination with a nonlinear controller to stabilize the vertical variable. The other action stabilizes the horizontal and angular variables to the desired rest position through an energy-control method, whereas Aguilar-Ibáñez [16] employed again a feedback linearization with a saturation function to stabilize the vertical variable, while a PD-controller and a sliding-mode controller were used to stabilize both the horizontal and angular variables. Moreover, Yu-Chan et al. [17] dealt with the stabilization of the PVTOL system with unknown model parameters by applying a sliding-mode technique to design a state feedback control law. Recently, in [18], a controller for the stabilization of the PVTOL vehicle was designed on the basis of the

immersion and invariance control technique. The controller gives priority to the control of the aircraft's altitude before controlling the lateral displacement. More recently, in [19], a cascade active disturbance rejection controller was introduced to counteract the adverse effects caused by an actuator failure in the PVTOL aircraft, while Escobar et al. [20] were focused on finding conditions to determine local asymptotic stability using a feedback linearization control for the PVTOL platform, so that reaching any singularity due to the transformation of the system is prevented. Aguilar-Ibanez et al. [21] introduced an output-feedback regulation control law for a PVTOL aircraft, based on a version of the matching control energy method. Such a control was improved to compensate bounded, smooth, and matching perturbations with a suitable finite time-varying identifier. Finally, Aguilar-Ibanez et al. [22] proposed a robust controller to solve the trajectory-tracking control problem of PVTOL aircraft under crosswind by applying an input-output feedback linearization to the PVTOL model under no crosswind conditions. Thus, the resulting linearized system under the crosswind effects is controlled using an active disturbance rejection control approach to counteract the effects of these perturbations.

Having reviewed the literature, it was found that almost all the works mentioned above were developed to test the PVTOL system indoor, mainly to avoid the undesirable effect produced by the wind (instability and, even, the collapse of the PVTOL system), which is not easy to counteract. Therefore, few controllers are robust under unknown model parameters, actuator failure, and crosswind. Thus, with the intention of contributing to overcome wind undesirable effects, a robust control scheme that combines a backstepping approach and a nested saturation function-based controller is proposed herein to perform taking-off maneuvers in the presence of disturbance due to wind. The backstepping is used to carry out the trajectory tracking task over the vertical position of the PVTOL system and, consequently, to control the height position. Then, from a set of convenient linear transformations, the system is represented as an integrator chain with a nonlinear perturbation, for which a nested saturation function-based controller is developed to stabilize the horizontal position and roll angle. This is carried out by satisfying stability conditions obtained from application of the second method of Lyapunov. Therefore, boundedness of each state and asymptotic convergence to the origin are ensured. Lastly, to estimate the disturbance due to the wind, an extended state observer is used.

The remaining of the paper is organized as follows. In Section 2, the PVTOL system and its dynamics are introduced. In Section 3 the design of the backstepping controller to perform trajectory tracking for the height position of the system is presented. In Section 4, the controller based on nested saturation functions for stabilization of the horizontal position and the roll angle is designed. The extended state observer is introduced in Section 5. In Section 6, the outcome of numerical simulations that show the behavior of the proposed control scheme is reported. Finally, Section 7 is devoted to the concluding remarks.

2. PVTOL System

Here, the PVTOL system and its dynamic model considering a disturbance due to wind are presented.

The PVTOL system emulates the vertical take-off and landing of an aerial vehicle, whereas it is automatically stabilized. Hence, in practice, this system has a rigid structure and two motors collocated at the ends of the structure, as can be seen in Figure 1, where θ_1 is the roll angle, x_1 is the horizontal position in the x axis, y_1 is the vertical position in the y axis, f_1 and f_2 are the forces produced by the motors, L is the distance between the center of the rigid structure to the center of the motors, m is the mass of the system, and g is the gravitational acceleration.

The representation in state variables of the PVTOL dynamic model, when L , m , and g are normalized, has been previously reported in [1] and used in [14, 23, 24], which is given by

$$\begin{aligned}\dot{x}_1 &= \dot{x}_2, \\ \dot{x}_2 &= -u_1 \sin \theta_1 + \epsilon u_2 \cos \theta_1, \\ \dot{y}_1 &= y_2, \\ \dot{y}_2 &= u_1 \cos \theta_1 + \epsilon u_2 \sin \theta_1 - 1, \\ \dot{\theta}_1 &= \theta_2, \\ \dot{\theta}_2 &= u_2 + A_L,\end{aligned}\quad (1)$$

where x_1 , $x_2 = \dot{x}_1$, y_1 , $y_2 = \dot{y}_1$, θ_1 , and $\theta_2 = \dot{\theta}_1$ are the state variables, $u_1 = f_1 + f_2$ and $u_2 = f_1 - f_2$ are the control inputs, ϵ is the coefficient giving the coupling between the rolling moment and the lateral acceleration, and A_L is the rolling moment due to the air, defined by Gomes and Ramos [25] as

$$A_L = \left(\frac{1}{2}\right) \rho C_l U^2 V_a, \quad (2)$$

with ρ being the air density, C_l being the nondimensional coefficient of the rolling moment in the standard convention for airships, U being the air speed, and V_a being the airship model volume. That is, A_L is considered as a disturbance due to wind.

3. Height Position Control

To obtain the height position control, we apply the following global coordinate change [26] to model (1):

$$\begin{aligned}\bar{x}_1 &= x_1 - \epsilon \sin \theta_1, \\ \bar{x}_2 &= x_2 - \epsilon \theta_2 \cos \theta_1, \\ \bar{y}_1 &= y_1 + \epsilon (\cos \theta_1 - 1), \\ \bar{y}_2 &= y_2 - \epsilon \theta_2 \sin \theta_1, \\ \bar{\theta}_1 &= \theta_1, \\ \bar{\theta}_2 &= \theta_2.\end{aligned}\quad (3)$$

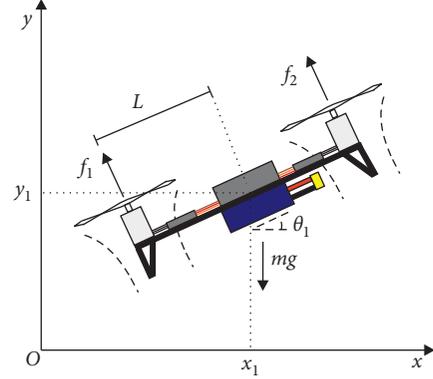


FIGURE 1: Diagram of the PVTOL system.

Also, we introduce $\bar{A}_L = A_L - \hat{A}_L$, where \hat{A}_L is an estimation of the disturbance due to wind carried out by an extended state observer, which is described later in Section 5. Thus, model (1) is transformed into the following system:

$$\begin{aligned}\dot{\bar{x}}_1 &= \bar{x}_2, \\ \dot{\bar{x}}_2 &= -\bar{u}_1 \sin \bar{\theta}_1 - \epsilon \bar{A}_L \cos \bar{\theta}_1, \\ \dot{\bar{y}}_1 &= \bar{y}_2, \\ \dot{\bar{y}}_2 &= \bar{u}_1 \cos \bar{\theta}_1 - \epsilon \bar{A}_L \sin \bar{\theta}_1 - 1, \\ \dot{\bar{\theta}}_1 &= \bar{\theta}_2, \\ \dot{\bar{\theta}}_2 &= u_2 + \bar{A}_L,\end{aligned}\quad (4)$$

where $\bar{u}_1 = u_1 - \epsilon \bar{\theta}_2^2$ is a new control input.

From this point, the backstepping procedure can be applied to force the system to track a desired trajectory and, consequently, to reach a desired height position. To this end, an error is defined as follows:

$$e_y = \bar{y}_{1_d} - \bar{y}_1, \quad (5)$$

with \bar{y}_{1_d} being the desired height position. Then, the method of Lyapunov is used, considering the following candidate function:

$$V(e_y) = \left(\frac{1}{2}\right) e_y^2, \quad (6)$$

which is positive definite and whose time derivative results in

$$\dot{V}(e_y) = e_y (\dot{\bar{y}}_{1_d} - \bar{y}_2). \quad (7)$$

To ensure the stabilization of e_y , the auxiliary control, \bar{y}_2 , is proposed as

$$\bar{y}_2 = \dot{\bar{y}}_{1_d} + \alpha_1 e_y, \quad (8)$$

with $\alpha_1 > 0$, so that (7) results in the following negative semidefinite expression:

$$\dot{V}(e_y) = -\alpha_1 e_y^2. \quad (9)$$

Then, it is proceeded with the following change of variables:

$$e_{2y} = \bar{y}_2 - \dot{\bar{y}}_{1_d} - \alpha_1 e_y. \quad (10)$$

So, the augmented Lyapunov function is given by

$$V(e_y, e_{2y}) = \left(\frac{1}{2}\right)(e_y^2 + e_{2y}^2), \quad (11)$$

whose time derivative is determined by

$$\begin{aligned} \dot{V}(e_y, e_{2y}) = & -e_y e_{2y} - \alpha_1 e_y^2 + e_{2y} \\ & \cdot (\bar{u}_1 \cos \bar{\theta}_1 - 1 - \epsilon \bar{A}_L \sin \bar{\theta}_1) \\ & - e_{2y} [\ddot{\bar{y}}_{1_d} + \alpha_1 (-e_{2y} - \alpha_1 e_y)]. \end{aligned} \quad (12)$$

To facilitate the proposal of \bar{u}_1 that ensures stabilization of e_{2y} , let us make (12) equal to zero for a moment and solve for \bar{u}_1 . Since $\ddot{\bar{y}}_{1_d} = 0$ is considered because it is the desired acceleration of the height position, \bar{u}_1 is found as follows:

$$\bar{u}_1 = \frac{1}{\cos \bar{\theta}_1} \left[e_y + \alpha_1 \left(\frac{e_y^2}{e_{2y}} \right) + \alpha_1 (-e_{2y} - \alpha_1 e_y) + 1 + \epsilon \bar{A}_L \sin \bar{\theta}_1 \right]. \quad (13)$$

Note that in order to avoid indeterminate (13), $-(\pi/2) < \bar{\theta}_1 < +(\pi/2)$ is required.

Taking into account the previous result and proposing $\alpha_1 (e_y^2/e_{2y}) = -(\alpha_2 e_{2y})$, it is clear that the stabilization of the control system is accomplished if \bar{u}_1 is selected as follows:

$$\bar{u}_1 = \frac{1}{\cos \bar{\theta}_1} [e_y - \alpha_2 e_{2y} + \alpha_1 (-e_{2y} - \alpha_1 e_y) + 1 + \epsilon \bar{A}_L \sin \bar{\theta}_1], \quad (14)$$

because it achieves

$$\dot{V}(e_y, e_{2y}) = -(\alpha_1 e_y^2) - (\alpha_2 e_{2y}^2) < 0, \quad (15)$$

with $\alpha_1 > 0$, $\alpha_2 > 0$ and $-(\pi/2) < \bar{\theta}_1 < +(\pi/2)$. Hence, $\bar{y}_1 \rightarrow \bar{y}_{1_d}$, that is, the PVTOL system reaches the desired height position.

4. Control of the Horizontal Position and Roll Angle

In this section, a nested saturation function-based controller for the stabilization of the horizontal position, \bar{x}_1 , and roll angle, $\bar{\theta}_1$, is developed [27]. This technique has been used for the stabilization of nonlinear systems that can be approximately expressed as integrator chain [28–30]. To solve the PVTOL system stability problem, first a linear transformation to propose the stabilizing controller is used. Then, it is showed that the proposed controller guarantees the

boundedness of all states and, after a finite time, the closed-loop system is asymptotically stable.

Before developing the control strategy, the definition of a saturation function is introduced.

Definition 1 (see [31]). A linear saturation function $\sigma_b(s): \mathbb{R} \rightarrow \mathbb{R}$ is defined as

$$\sigma_b(s) = \begin{cases} s, & \text{if } |s| \leq b, \\ b \cdot \text{sign}(s), & \text{if } |s| > b, \end{cases} \quad (16)$$

with $b > 0$ being the upper bound of the function.

4.1. System as an Integrator Chain. After applying the controller \bar{u}_1 , the transformed system (4) can be reduced to the subsystem $(\bar{x}_1, \bar{\theta}_1)$, that is:

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{x}_2, \\ \dot{\bar{x}}_2 &= -\tan \bar{\theta}_1 - \epsilon \bar{A}_L \sec \bar{\theta}_1, \\ \dot{\bar{\theta}}_1 &= \bar{\theta}_2, \\ \dot{\bar{\theta}}_2 &= u_2 + \bar{A}_L. \end{aligned} \quad (17)$$

To express system (17) as an integrator chain, with a nonlinear perturbation, and to propose a controller for the stabilization of the subsystem $(\bar{x}_1, \bar{\theta}_1)$, it is proceeded similarly as in [32], so that the following global nonlinear transformation is defined:

$$\begin{aligned} w_1 &= -\tan \bar{\theta}_1, \\ w_2 &= -\bar{\theta}_2 \sec^2 \bar{\theta}_1, \\ v &= -u_2 \sec^2 \bar{\theta}_1 + 2\bar{\theta}_2^2 \tan \bar{\theta}_1 \sec^2 \bar{\theta}_1. \end{aligned} \quad (18)$$

Hence, the transformed system as an integrator chain is given by

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{x}_2, \\ \dot{\bar{x}}_2 &= w_1 - \epsilon \bar{A}_L \sec \bar{\theta}_1, \\ \dot{w}_1 &= w_2, \\ \dot{w}_2 &= v + \bar{A}_L \sec^2 \bar{\theta}_1, \end{aligned} \quad (19)$$

whose matrix representation can be expressed as

$$\dot{\xi} = (A\xi + Bv + \omega), \quad (20)$$

where $\xi = (\bar{x}_1, \bar{x}_2, w_1, w_2)$ is the new state vector,

$$\begin{aligned}
A &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
B &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \\
\omega &= \begin{bmatrix} 0 \\ -\epsilon \bar{A}_L \sec^2 \bar{\theta}_1 \\ 0 \\ \bar{A}_L \sec^2 \bar{\theta}_1 \end{bmatrix}.
\end{aligned} \tag{21}$$

4.2. Nested Saturation Function-Based Controller. In order to obtain the stabilizing controller for system (20) and inspired by [27], the linear transformation $q = S\xi$ is used, in which S must satisfy

$$\begin{aligned}
SAS^{-1} &= \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \\
SB &= \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.
\end{aligned} \tag{22}$$

The matrix S that achieves the aforementioned equalities is given by [31]

$$S = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{23}$$

Thus, q results in

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 3 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xi, \tag{24}$$

which transforms system (20) into

$$\begin{aligned}
\dot{q}_1 &= q_2 + q_3 + q_4 + \nu + \bar{A}_L \sec^2 \bar{\theta}_1 - 3\epsilon \bar{A}_L \sec^2 \bar{\theta}_1, \\
\dot{q}_2 &= q_3 + q_4 + \nu + \bar{A}_L \sec^2 \bar{\theta}_1 - \epsilon \bar{A}_L \sec^2 \bar{\theta}_1, \\
\dot{q}_3 &= q_4 + \nu + \bar{A}_L \sec^2 \bar{\theta}_1, \\
\dot{q}_4 &= \nu + \bar{A}_L \sec^2 \bar{\theta}_1,
\end{aligned} \tag{25}$$

for which, the following nested saturation function-based stabilizing controller is proposed:

$$\nu = -q_4 - \sigma_\alpha(q_3 + \sigma_\beta(q_2 + \sigma_\gamma(q_1))), \tag{26}$$

where $\sigma_\alpha(\cdot)$, $\sigma_\beta(\cdot)$, and $\sigma_\gamma(\cdot)$ are linear saturation functions as defined in (16) and α , β , and γ are the upper bounds of each nested saturation function.

Finally, departing from (18) and using (26), u_2 can be constructed as follows:

$$u_2 = -\left(\frac{1}{\sec^2 \bar{\theta}_1}\right) \nu + \left(2\bar{\theta}_2^2 \tan \bar{\theta}_1\right). \tag{27}$$

4.3. Boundedness of All States. Now, it is proved that the proposed closed-loop system, (25) with (26), ensures that all the states are bounded and that the bound of each of them directly depends on the design parameters of the controller.

Step 1: to show that the state q_4 is bounded, the following positive definite function is defined:

$$V_4 = \left(\frac{1}{2}\right) q_4^2, \tag{28}$$

whose time derivative is expressed as

$$\dot{V}_4 = -q_4^2 - q_4 [\sigma_\alpha(q_3 + \sigma_\beta(q_2 + \sigma_\gamma(q_1))) - \bar{A}_L \sec^2 \bar{\theta}_1]. \tag{29}$$

It is clear that $\dot{V}_4 < 0$ is accomplished when $|q_4| \geq \alpha + (\bar{A}_L \sec^2 \bar{\theta}_1)$. Therefore, there exists a finite time $T_1 > 0$, such that

$$|q_4(t)| < \alpha + \bar{A}_L \sec^2 \bar{\theta}_1, \quad \forall t > T_1. \tag{30}$$

Step 2: now, the behavior of q_3 is analyzed. For this, a positive definite function is introduced as follows:

$$V_3 = \left(\frac{1}{2}\right) q_3^2. \tag{31}$$

Differentiating it with respect to time and after substituting (26) into \dot{q}_3 , the following is obtained:

$$\dot{V}_3 = -q_3 [\sigma_\alpha(q_3 + \sigma_\beta(q_2 + \sigma_\gamma(q_1))) - \bar{A}_L \sec^2 \bar{\theta}_1]. \tag{32}$$

To ensure $\dot{V}_3 < 0$ is achieved, the following conditions must be satisfied:

$$\alpha > 2\beta + \bar{A}_L \sec^2 \bar{\theta}_1, \quad |q_3| > \beta + \bar{A}_L \sec^2 \bar{\theta}_1. \tag{33}$$

Then, there exists a finite time $T_2 > T_1$ after which

$$|q_3(t)| < \beta + \bar{A}_L \sec^2 \bar{\theta}_1, \quad \forall t > T_2. \quad (34)$$

Thus, when conditions in (33) are satisfied, q_3 is bounded and the stabilization controller (26) takes the following structure:

$$\nu = -q_4 - q_3 - \sigma_\beta(q_2 + \sigma_\gamma(q_1)), \quad \forall t > T_2. \quad (35)$$

Step 3: substituting (35) into the second differential equation of (25), the following is obtained:

$$\dot{q}_2 = -\sigma_\beta(q_2 + \sigma_\gamma(q_1)) - \epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1. \quad (36)$$

Then, the following definite positive function is defined:

$$V_2 = \left(\frac{1}{2}\right) q_2^2, \quad (37)$$

whose first time derivative is obtained using (36), as follows:

$$\dot{V}_2 = -q_2 [\sigma_\beta(q_2 + \sigma_\gamma(q_1)) + \epsilon \bar{A}_L \sec \bar{\theta}_1 - \bar{A}_L \sec^2 \bar{\theta}_1]. \quad (38)$$

With the purpose of performing $\dot{V}_2 < 0$, it is required that β and γ satisfy the below conditions:

$$\begin{aligned} \beta &> 2\gamma - \epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1, \\ |q_2| &> \gamma - \epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1. \end{aligned} \quad (39)$$

Hence, there exists a finite time $T_3 > T_2$, after which

$$|q_2(t)| < \gamma - \epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1. \quad (40)$$

Consequently, q_2 is bounded and the control ν turns out to be

$$\nu = -q_4 - q_3 - q_2 - \sigma_\gamma(q_1), \quad \forall t > T_3. \quad (41)$$

Step 4: substituting (41) into the first equation of (25), the following is obtained:

$$\dot{q}_1 = -\sigma_\gamma(q_1) - 3\epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1. \quad (42)$$

To demonstrate that q_1 is bounded, a definite positive function is defined as follows:

$$V_1 = \left(\frac{1}{2}\right) q_1^2. \quad (43)$$

Differentiating V_1 along the trajectories of (42), the following is obtained:

$$\dot{V}_1 = -q_1 [\sigma_\gamma(q_1) + 3\epsilon \bar{A}_L \sec \bar{\theta}_1 - \bar{A}_L \sec^2 \bar{\theta}_1], \quad (44)$$

where γ must be selected so that $\gamma > -3\epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1$ and $|q_1| > -3\epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1$ to achieve $\dot{V}_1 < 0$. Therefore, there exists a finite time $T_4 > T_3$, such that

$$|q_1(t)| < -3\epsilon \bar{A}_L \sec \bar{\theta}_1 + \bar{A}_L \sec^2 \bar{\theta}_1, \quad \forall t > T_4. \quad (45)$$

Consequently, q_1 is also bounded. Finally, the values of the parameters α , β , and γ can be determined as follows:

$$\begin{aligned} \alpha &> 2\beta + \bar{A}_L \sec \bar{\theta}_1 (\sec \bar{\theta}_1), \\ \beta &> 2\gamma + \bar{A}_L \sec \bar{\theta}_1 (\sec \bar{\theta}_1 - \epsilon), \\ \gamma &> \bar{A}_L \sec \bar{\theta}_1 (\sec \bar{\theta}_1 - 3\epsilon). \end{aligned} \quad (46)$$

Since $\sec \bar{\theta}_1 > \sec \bar{\theta}_1 - \epsilon > \sec \bar{\theta}_1 - 3\epsilon$, $r > |\bar{A}_L \sec^2 \bar{\theta}_1| > 0$ can be introduced, which is directly related to the magnitude of the system disturbance, to select the upper bounds of the saturation functions as

$$\alpha = 7r, \quad \beta = 3r, \quad \gamma = r. \quad (47)$$

4.4. Convergence of All States to Zero. Here, we prove that the closed-loop system, provided by (25) and (26) satisfying (47), is asymptotically stable.

Note that after $t > T_4$, controller (26) is no longer saturated. That is,

$$\nu = -q_1 - q_2 - q_3 - q_4, \quad (48)$$

and the closed-loop system turns out to be

$$\begin{aligned} \dot{q}_1 &= -q_1 + \bar{A}_L \sec^2 \bar{\theta}_1 - 3\epsilon \bar{A}_L \sec \bar{\theta}_1, \\ \dot{q}_2 &= -q_1 - q_2 + \bar{A}_L \sec^2 \bar{\theta}_1 - \epsilon \bar{A}_L \sec \bar{\theta}_1, \\ \dot{q}_3 &= -q_1 - q_2 - q_3 + \bar{A}_L \sec^2 \bar{\theta}_1, \\ \dot{q}_4 &= -q_1 - q_2 - q_3 - q_4 + \bar{A}_L \sec^2 \bar{\theta}_1. \end{aligned} \quad (49)$$

To demonstrate convergence to zero of all the states, the following Lyapunov function is used:

$$V = \left(\frac{1}{2}\right) q^T q, \quad (50)$$

and differentiating it along the trajectories of (49), the following is obtained:

$$\begin{aligned} \dot{V} &= -q^T M q + (q_1 + q_2 + q_3 + q_4) \bar{A}_L \sec^2 \bar{\theta}_1 \\ &\quad - (3q_1 + q_2) \epsilon \bar{A}_L \sec \bar{\theta}_1, \end{aligned} \quad (51)$$

where

$$M = \begin{bmatrix} 1 & 1/2 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1/2 & 1 \end{bmatrix}, \quad (52)$$

is positive definite with $\lambda_{\min} = (1/2)$. Therefore, (51) is strictly negative, when $\bar{A}_L \rightarrow 0$. Then, the vector of states q exponentially converges to zero after some time $t > T_4$.

5. Extended State Observer

In this section, the extended state observer needed to estimate the disturbance due to wind, A_L , is introduced [33].

Consider only the disturbed coordinate:

$$\dot{\theta}_2 = u_2 + A_L. \quad (53)$$

The following extended state observer is designed:

$$\begin{aligned} \dot{\hat{\theta}}_2 &= u_2 + \hat{A}_L - \lambda_1(\hat{\theta}_2 - \theta_2), \\ \dot{\hat{A}}_L &= -\lambda_2(\hat{\theta}_2 - \theta_2), \end{aligned} \quad (54)$$

where $\hat{\theta}_2$ is the estimate of the roll velocity, \hat{A}_L is the estimate of the disturbance due to wind, λ_1 and λ_2 are the gains of the observer, which must satisfy the following condition: $\lambda_1 \ll \lambda_2$, and lastly u_2 is redefined and proposed as

$$u_2 = \bar{u}_2 - \hat{A}_L, \quad (55)$$

where $\bar{u}_2 = -(1/\sec^2\bar{\theta}_1)\nu + 2\bar{\theta}_2^2 \tan\bar{\theta}_1$ represents a fictitious controller, acting on the coordinate θ_2 .

6. Simulation Results

In this section, the outcomes of some numerical tests are presented in order to validate that the proposed control scheme successfully achieves that $(x_1, x_2, y_1, y_2, \theta_1, \theta_2) \rightarrow (0, 0, y_{1_d}, 0, 0, 0)$. That is, the control scheme carries out height position regulation, through performing trajectory tracking task, and stabilization of the horizontal position and roll angle for the PVTOL system in the presence of random disturbance due to wind.

The simulations were performed with the normalized model (1) in MATLAB-Simulink, using Euler's numerical method with fixed step and a sample time of 1 ms. In that direction, the coefficient giving the coupling between the rolling moment and the lateral acceleration was chosen as in [1], i.e., $\epsilon = 0.001$. Also, the desired trajectory, y_{1_d} , was proposed as the following Bézier polynomial:

$$y_{1_d} = v_1 + (v_2 - v_1)P_d(t), \quad (56)$$

where $v_1 = 0$ m and $v_2 = 2$ m are the constant values, $P_d(t)$ is defined by

$$P_d(t) = \begin{cases} 0, & \text{if } t \leq t_i, \\ \left(\frac{t-t_i}{t_f-t_i} \right)^5 \left[r_1 + r_2 \left(\frac{t-t_i}{t_f-t_i} \right) + r_3 \left(\frac{t-t_i}{t_f-t_i} \right)^2 + r_4 \left(\frac{t-t_i}{t_f-t_i} \right)^3 + r_5 \left(\frac{t-t_i}{t_f-t_i} \right)^4 + r_6 \left(\frac{t-t_i}{t_f-t_i} \right)^5 \right], & \text{if } t_i < t < t_f, \\ 1, & \text{if } t \geq t_f, \end{cases} \quad (57)$$

which smoothly interpolates between v_1 and v_2 in the interval $[t_i, t_f]$, with t_i being the initial time, t_f being the final one, and r_1, r_2, r_3, r_4, r_5 , and r_6 selected as

$$\begin{aligned} r_1 &= 252, \\ r_2 &= -1050, \\ r_3 &= 1800, \\ r_4 &= -1575, \\ r_5 &= 700, \\ r_6 &= -126. \end{aligned} \quad (58)$$

Regarding the disturbance due to wind given in (2), the air density of the Mexico City was used for ρ and the aerodynamic coefficient C_l was characterized as the following linear approximation:

$$C_l = m_1\theta_1, \quad (59)$$

with m_1 being the slope and θ_1 being the roll angle of the PVTOL system in degrees. It is important to mention that this linear approximation was determined from the results of C_l , obtained with respect to the variation of the roll angle of an aircraft similar to the PVTOL system, by using a wind

chamber [34]. Furthermore, the air speed U was generated as follows:

$$U = A_U \cos\left(\frac{t}{P}\right) + \phi + a, \quad (60)$$

where A_U is a random amplitude, t is time, P is a random period, ϕ is a random phase, and a is a random offset. Therefore, (2) is random and can be rewritten as

$$A_L = \left(\frac{1}{2}\right)\rho(m_1\theta_1)\left[A_U \cos\left(\frac{t}{P}\right) + \phi + a\right]^2 V_a. \quad (61)$$

The whole parameters to construct A_L are shown in Table 1.

On the other hand, the tuning parameters of u_1 were set at

$$\begin{aligned} \alpha_1 &= 8, \\ \alpha_2 &= 8. \end{aligned} \quad (62)$$

The parameter $r = 1$ was chosen for u_2 so that

$$\begin{aligned} \alpha &= 7, \\ \beta &= 3, \\ \gamma &= 1. \end{aligned} \quad (63)$$

The gains implemented for the extended state observer were selected as

$$\begin{aligned} \lambda_1 &= 5, \\ \lambda_2 &= 20. \end{aligned} \quad (64)$$

The initial conditions of the PVTOL system were set as indicated in Table 2.

The corresponding simulation results are shown in Figure 2. With the intention of comparing the performance of the proposed control scheme with a classical controller, Figure 2 also presents simulation results when using a PID structure for u_1 and u_2 to carry out regulation of the system. For the simulation of the normalized system in closed loop with the PID controllers, (61) with parameters in Table 1 was preserved, the initial conditions in Table 2 were set, the gains of the observer were maintained, and the gains of the controllers were tuned as

$$\begin{aligned} k_{p1} &= 1.5, \\ k_{i1} &= 1.5, \\ k_{d1} &= 3, \end{aligned} \quad (65)$$

TABLE 1: Parameters of A_L .

Parameter	Value
ρ	0.908906 kg/m ²
m_1	-0.0013
A_U	[4, 12]
P	[1.6, 4.8] s
ϕ	[12°, 21°]
a	[-2, 2]
V_a	0.1 m ³

TABLE 2: Initial conditions.

State	Value
x_1	0.1 m
x_2	0 m/s
y_1	0 m
y_2	0 m/s
θ_1	0.15 rad
θ_2	0 rad/s

for u_1 and

$$\begin{aligned} k_{p2} &= 0.1, \\ k_{i2} &= 0, \\ k_{d2} &= 0.5, \end{aligned} \quad (66)$$

for u_2 . To distinguish the results, the ones associated with the PID controllers are denoted with the subscript *PID*.

In Figure 2, it can be observed that the proposed control scheme allows achieving successfully the height position regulation, through the trajectory tracking task, and stabilization of the horizontal position and roll angle when the system is subjected to random disturbance due to wind. That is, $(x_1, x_2, y_1, y_2, \theta_1, \theta_2) \rightarrow (0, 0, y_{1d}, 0, 0, 0)$ is accomplished. Note that the execution of the trajectory tracking task in the vertical position provides maneuverability when taking-off the PVTOL system. However, with the PID controllers, the regulation of state x_1 cannot be achieved, but only for the positions y_1 and θ_1 . That is, stabilization of the whole system is not achieved. Although PID structure can achieve the height position regulation when the system is far from the desired height, it requires excessive values for u_1 and u_2 and does not allow the taking-off of the system in a controlled way. Thus, advantages of the proposal presented herein are maneuverability and whole stabilization, when the system is under the undesired effect caused by wind.

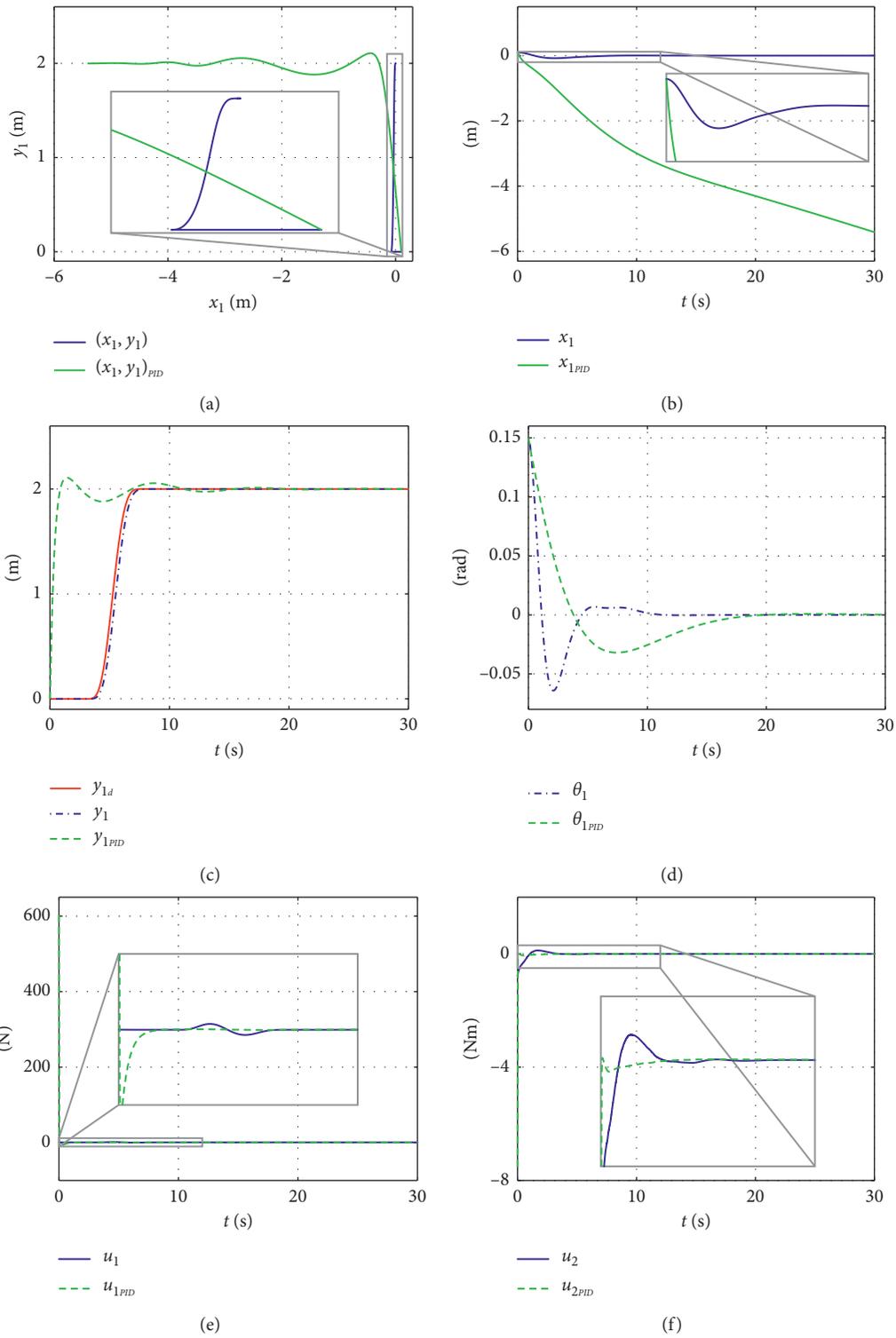


FIGURE 2: Continued.

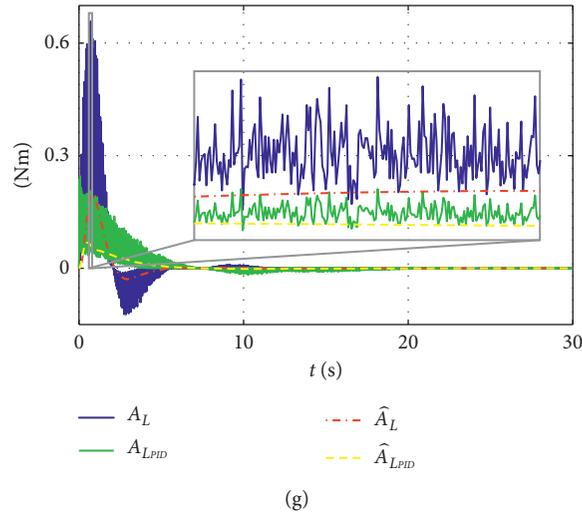


FIGURE 2: Simulation results.

7. Conclusions

In this study, a nested saturation function-based controller, in combination with a backstepping controller, for stabilizing the PVTOL system under a disturbance due to wind was used. With this approach, the control design complexity of a higher-order system is reduced to design a control for a lower-order nonlinear subsystem of the original system. Thus, the proposed control approach allows designing a controller based on nested saturation functions, which contemplates perturbations, guaranteeing the convergence of the roll angle to zero within a finite time and, consequently, the convergence to zero of the horizontal state. The stability analysis of the closed-loop system was based on the second method of Lyapunov, using a simple candidate function. It is important to remark that the controller, based on backstepping and nested saturation functions, allows performing take-off maneuvers in the presence of exogenous disturbances, which are found when aircraft carries out actual maneuvers. Furthermore, an extended state observer is used to estimate the disturbance due to wind. Numerical simulations were carried out to test the effectiveness of the proposed controller, having obtained convincing results. Finally, the proposed scheme was compared with a classical controller, finding that the controller based on backstepping and nested saturation functions presented herein has better performance.

It is worth mentioning that an experimental platform that allows configuring the PVTOL system has been designed, whose construction is in process. Thus, experimental implementation of the control scheme proposed herein is considered as a future work.

Data Availability

The data used to support the conclusion of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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