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Research Article

The W/S Test for Data Having Neutrosophic Numbers: An Application to USA Village Population

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The W/S test under neutrosophic statistics is proposed in this paper. The Monte Carlo simulation under the neutrosophic statistical interval method is proposed and applied to study the sensitivity of various neutrosophic statistical distributions. The power of test curves for neutrosophic distributions is presented. The efficiency of the proposed W/S test under neutrosophic statistics is compared with that of the W/S test under classical statistics. The proposed test is explained with the aid of an example.

1. Introduction

The statistical tools are very important to achieve useful information from the data. Before discovering the useful information from the data in hand, it is necessary to check the behavior of the data. Several statistical tests including *t*-test, analysis of variance test, and Kruskal–Wallis test have been widely applied for testing the symmetry or asymmetry assumptions of the data. For the earlier case, the tests are applied to assume the normality, see [1]. On the other hand, for the latter case, the tests based on nonnormal distributions or nonparametric tests are selected. The sample size has a significant effect on the selection of the test for the data analysis. The tests under the normality assumptions cannot be applied for small sample size [2]. More details about the statistical analysis can be seen in [3, 4].

The W/S test for testing the normality assumption of the data was proposed by Shapiro and Wilk and later on modified by Shapiro and Francia, see [5–9] for more details. Note here that W and S are the range and standard deviation of the data, respectively. Although the W/S test is much simpler than Fisher's cumulant test, it is seldom applied for the testing of normality of the data because of its sensitivity for nonnormal statistical distributions. A detailed study on the W/S test can be seen in [9].

The existing W/S test under classical statistics (CS) is applied for testing the normality of the data when all observations in the data are determined, precise and exact. As mentioned by [10, 11], "quite often the results of an experiment are imprecisely observed because of the significant measurement error or are so uncertain that they are recorded as intervals containing the precise outcomes. Moreover, sometimes the exact value of a variable is hidden deliberately for some confidentiality reasons." The various problems related to interval data in regression are given by [12], in time series by [13], in principle components by [14], in classification by [15], in the analysis of variance by [16], and testing of the hypothesis by [17]. The fuzzy logic is an alternative to analyze the data given in interval. The authors of [18–20] applied the fuzzy sets for modeling the interval data.

Recently, neutrosophic logic developed by [21] attracted researchers due to its flexibility and generalization over the fuzzy logic. Several authors worked using neutrosophic logic in a variety of fields, see [22–26]. Based on the neutrosophic logic, Smarandache [27] introduced the idea of neutrosophic statistics (NS). The NS is an extension of CS and applied if observations are imprecise, indeterminate, and neutrosophic. The authors of [28, 29] analyzed the neutrosophic data in rock measuring problems. Aslam introduced the NS in statistical quality control (SQC), the statistical test using NS the first time, the analysis of variance method under NS, the test of outliers using the NS, and the tests of homogeneity of variance for uncertainty environment [30–34]. For information on NS, the reader may refer to [35, 36].

As mentioned above, several authors worked on the W/S test using CS and fuzzy logic. To the best of our knowledge, there is no work on the W/S test under NS. In this paper, we will propose the W/S test and introduce the Monte Carlo simulation under the neutrosophic statistical interval method (NSIM). We will discuss the sensitivity of the proposed W/S test for various neutrosophic statistical distributions. We expect that the proposed W/S test will be more effective than the W/S test under CS in indeterminacy.

2. The W/S Test under NS

The tests under CS are applied under the assumption that the random sample having determined observations is drawn from the normal population. However, in practice, as mentioned above, it may not be possible that all observations are determined or exact. In such cases, the tests under CS cannot be applied for testing the normality of the data. Therefore, the tests under CS can be replaced with the tests under NS. In the next section, we present the methodology of the proposed W/S test under NS.

2.1. Method of the W/S Test under NS. The main aim of the proposed W/S test is to check whether the data follows the neutrosophic normal distribution or not. Furthermore, we will discuss the sensitivity of the proposed W/S test under various statistical distributions. The proposed test will be an application under the assumption that the random variable is selected from the neutrosophic normal distribution. Furthermore, it is assumed that the data contain the neutrosophic numbers. Based on this information, the neutrosophic null, say $H_{0N}\epsilon[H_{0L}, H_{0U}]$, and alternative hypothesis, say $H_{1N}\epsilon[H_{1L}, H_{1U}]$, for the proposed test are stated as follows:

$H_{0N}\epsilon[H_{0L}, H_{0U}]$: the neutrosophic normal distribution is suitable for the neutrosophic data

$H_{1N}\epsilon[H_{1L}, H_{1U}]$: the neutrosophic normal distribution is not suitable for the neutrosophic data

The null hypothesis $H_{0N}\epsilon[H_{0L}, H_{0U}]$ will be accepted if the calculated values fall within the indeterminacy interval of critical values; otherwise, $H_{1N}\epsilon[H_{1L}, H_{1U}]$ will be accepted. Let $W_N\epsilon[W_L, W_U]$ and $S_N\epsilon[S_L, S_U]$ are the neutrosophic range (NR) and neutrosophic standard deviation (NSD) of the neutrosophic random sample $X_{Ni}\epsilon[X_{Li}, X_{Ui}]$, $i = 1, 2, 3, \ldots, n_N$, where $n_N\epsilon[n_L, n_U]$ be the size of $X_{Ni}\epsilon[X_{Li}, X_{Ui}]$. The NR is defined as follows:

$$\begin{aligned} W_N &= X_{mN} - X_{0N},\\ W_N &\epsilon[W_L, W_U],\\ X_{mN} &\epsilon[X_{mL}, X_{mU}],\\ X_{0N} &\epsilon[X_{0L}, X_{0U}], \end{aligned} \tag{1}$$

where $X_{mN}\epsilon[X_{mL}, X_{mU}]$ and $X_{0N}\epsilon[X_{0L}, X_{0U}]$ represent the neutrosophic maximum and minimum values of the data.

The neutrosophic form of $W_N\epsilon[W_L, W_U]$ can be given by

$$W_N = W_L + W_U I_N; \quad I_N\epsilon[I_L, I_U]. \tag{2}$$

Note here that W_L and $W_U I_N$ are determinate and indeterminate parts of $W_N\epsilon[W_L, W_U]$, where $I_N\epsilon[I_L, I_U]$ denotes the indeterminacy interval. $W_N\epsilon[W_L, W_U]$ reduces to range under CS when $I_L = 0$.

The NSD is defined as follows:

$$\begin{aligned} S_N &= \sqrt{\frac{\sum_{i=1}^{n_N}(X_{Ni} - \overline{X}_{Ni})}{n_N - 1}},\\ S_N &\epsilon[S_L, S_U],\\ \overline{X}_{Ni} &\epsilon[\overline{X}_{Li}, \overline{X}_{Ui}],\\ n_N &\epsilon[n_L, n_U]. \end{aligned} \tag{3}$$

The neutrosophic form of $S_N\epsilon[S_L, S_U]$ can be given by

$$S_N = S_L + S_U I_N; \quad I_N\epsilon[I_L, I_U]. \tag{4}$$

Note here that S_L and $S_U I_N$ are determinate and indeterminate parts of $S_N\epsilon[S_L, S_U]$, where $I_N\epsilon[I_L, I_U]$ denotes the indeterminacy interval. $S_N\epsilon[S_L, S_U]$ reduces to standard deviation under CS when $I_L = 0$. Also, let $\overline{X}_{Ni}\epsilon[\overline{X}_{Li}, \overline{X}_{Ui}]$ be the neutrosophic sample mean and is given as follows:

$$\begin{aligned} \overline{X}_{Ni} &= \frac{\sum_{i=1}^{n_N} X_{Ni}}{n_N},\\ \overline{X}_{Ni} &\epsilon[\overline{X}_{Li}, \overline{X}_{Ui}],\\ n_N &\epsilon[n_L, n_U]. \end{aligned} \tag{5}$$

The neutrosophic form of $\overline{X}_{Ni}\epsilon[\overline{X}_{Li}, \overline{X}_{Ui}]$ can be given by

$$\overline{X}_{Ni} = \overline{X}_{Li} + \overline{X}_{Ui} I_N; \quad I_N\epsilon[I_L, I_U]. \tag{6}$$

Note here that S_L and $S_U I_N$ are determinate and indeterminate parts of $\overline{X}_{Ni}\epsilon[\overline{X}_{Li}, \overline{X}_{Ui}]$, where $I_N\epsilon[I_L, I_U]$ denotes the indeterminacy interval. $\overline{X}_{Ni}\epsilon[\overline{X}_{Li}, \overline{X}_{Ui}]$ reduces to sample mean under CS when $I_L = 0$.

Based on this information, we define the test statistic, say $Q_N\epsilon[Q_L, Q_U]$, for the W/S test under neutrosophic statistics as follows:

$$\begin{aligned} Q_N &= \frac{W_N}{S_N},\\ Q_N &\epsilon[Q_L, Q_U],\\ W_N &\epsilon[W_L, W_U],\\ S_N &\epsilon[S_L, S_U]. \end{aligned} \tag{7}$$

The neutrosophic form of statistic $Q_N\epsilon[Q_L, Q_U]$ can be given by

$$Q_N = Q_L + Q_U I_N; \quad I_N \epsilon [I_L, I_U]. \tag{8}$$

Note here that S_L and $S_U I_N$ are determinate and indeterminate parts of $Q_N \epsilon [Q_L, Q_U]$, where $I_N \epsilon [I_L, I_U]$ denotes the indeterminacy interval. $Q_N \epsilon [Q_L, Q_U]$ reduces to statistic under CS when $I_L = 0$. $H_{0N} \epsilon [H_{0L}, H_{0U}]$ will be accepted if the calculated neutrosophic values of $Q_N \epsilon [Q_L, Q_U]$ are within the indeterminacy interval of critical values; otherwise, $H_{0N} \epsilon [H_{0L}, H_{0U}]$ will be rejected.

2.2. Error Rate of the Proposed Test. As $X_{Ni} \epsilon [X_{Li}, X_{Ui}]$ is drawn from the unknown distribution, the sensitivity of the proposed test will be studied using the error rate. The error rate is defined as the ratio of wrong conclusions to the number of replications of the neutrosophic sample. The error rate is denoted by $\beta_N \epsilon [\beta_L, \beta_U]$ which is defined as the probability of accepting $H_{0N} \epsilon [H_{0L}, H_{0U}]$ when it is actually false. The sensitivity of the proposed test will be discussed with the help of the power of the test. The power of the test is defined as follows:

$$\text{Power} = 1 - \beta_N = P_{rN} \left(\text{reject } H_{0N} \epsilon [H_{0L}, H_{0U}] \cdot \middle| H_{1N} \epsilon [H_{1L}, H_{1U}] \text{ is true} \right); \quad \beta_N \epsilon [\beta_L, \beta_U]. \tag{9}$$

2.3. Neutrosophic Monte Carlo Simulation. The following neutrosophic Monte Carlo (NMC) simulation will be used to evaluate the performance of the proposed W/S test:

Step 1: fix the neutrosophic parameters for the specified distribution

Step 2: generate random numbers of size $n_N \epsilon [n_L, n_U]$ from the specified distributions

Step 3: calculate $W_N \epsilon [W_L, W_U]$ and $S_N \epsilon [S_L, S_U]$ and compute the test statistic $Q_N \epsilon [Q_L, Q_U]$

Step 4: compute $\beta_N \epsilon [\beta_L, \beta_U]$ by the ratio of the wrong conclusion to the number of replicates, 100

Step 5: calculate the power of the test and plot it

3. Data Analysis

Now, we will explain the methodology of the proposed test with the help of NMC simulation data. The data are generated from the Weibull distribution with neutrosophic shape parameter [2.5, 3] and neutrosophic scale parameter [5, 7]. The neutrosophic data from the neutrosophic Weibull distribution are given in Table 1. The neutrosophic range $W_N \epsilon [W_L, W_U]$ and standard deviation $S_N \epsilon [S_L, S_U]$ for each sample are shown in Table 2. The values of the test statistics $Q_N \epsilon [Q_L, Q_U]$, the tabulated values of the test, the level of significance, 0.05, from [37], and the conclusion about the acceptance $H_{0N} \epsilon [H_{0L}, H_{0U}]$ are also shown in Table 2.

The error rate for the Weibull distribution for several of $n_N \epsilon [n_L, n_U]$ is calculated and shown in Table 3. Similarly, we applied the proposed NMC on neutrosophic chi-square distribution, neutrosophic normal distribution, neutrosophic student t-distribution, and neutrosophic Cauchy distribution. The error rate of all these neutrosophic distributions for various neutrosophic shape and scale parameters is shown in Table 3.

Table 1: The neutrosophic data of size $n_N \epsilon [5, 5]$.

Sr#	1	2	...	100
1	[0.856, 0.789]	[1.061, 0.489]	...	[0.815, 0.598]
2	[1.563, 1.116]	[0.683, 1.01]		[0.921, 0.565]
3	[1.113, 0.325]	[0.735, 0.644]		[1.621, 1.043]
4	[0.48, 0.461]	[0.444, 0.923]		[1.22, 1.166]
5	[0.722, 0.957]	[0.935, 0.407]		[0.847, 0.913]

From Table 3, it can be seen that the neutrosophic Weibull distribution has a smaller error rate among other neutrosophic distributions when $n_N \epsilon [5, 5]$. For the neutrosophic sample $n_N \epsilon [n_L, n_U] > [5, 5]$, the neutrosophic Cauchy distribution has a smaller error rate among other distributions. From Table 3, it can be noted that the neutrosophic sample size plays an important role to determine the suitable distribution. The power of the proposed test for various neutrosophic statistical distributions is shown in Table 4. From Table 4, we can note that neutrosophic Weibull distribution has the larger power of test when $n_N \epsilon [5, 5]$ and neutrosophic Cauchy distribution has a higher power of test when $n_N \epsilon [n_L, n_U] > [5, 5]$. From this study, it can be concluded that the W/S test under neutrosophic statistics will be applied to the neutrosophic Weibull distribution when $n_N \epsilon [5, 5]$ and on the neutrosophic Cauchy distribution when $n_N \epsilon [n_L, n_U] > [5, 5]$. The behavior of the power of the test is also shown with the help of curves for all distributions in Figures 1–5. From Figures 1–5, it can be observed that there is no specific trend in neutrosophic Weibull distribution, neutrosophic chi-square distribution, and neutrosophic normal distribution. On the other hand, we can note the increasing trend in power as the neutrosophic sample size increases for neutrosophic t-distribution and neutrosophic Cauchy distribution. Among these curves, the neutrosophic Cauchy distribution is the best power curve.

4. Comparative Study

We compare the efficiency of the proposed W/S test under neutrosophic statistics over the existing W/S test under CS proposed by [9]. Note here that the proposed test reduces to [9] test if no uncertain or imprecise observations are recorded in the data. To show the performance of the proposed W/S test under neutrosophic statistics over the existing W/S test, we set the same values of all parameters. We plotted the values of power test for the proposed test and the existing test in Figures 6–9. The red lines in Figures 6–9 show the power curve under the test proposed by [9]. Figure 6 is displayed for the neutrosophic Weibull distribution. From Figure 6, it can be noted that there is an irregular trend in power curves. The proposed test is better than the [9] test when $n_N \epsilon [n_L, n_U] < [60, 60]$. On the other

Table 2: The calculation of the proposed test statistics.

Sr#	1	2	3	4	5	Range	SD	R/SD	Tabl1	Conclusion
1	[0.856, 0.789]	[1.563, 1.116]	[1.113, 0.325]	[0.48, 0.461]	[0.722, 0.957]	[1.084, 0.791]	[0.414, 0.332]	[2.62, 2.383]	[2.15, 2.753]	[Accept, Accept]
2	[1.061, 0.489]	[0.683, 1.01]	[0.735, 0.644]	[0.444, 0.923]	[0.935, 0.407]	[0.616, 0.603]	[0.238, 0.264]	[2.59, 2.283]	[2.15, 2.753]	[Accept, Accept]
...										
100	[0.815, 0.598]	[0.921, 0.565]	[1.621, 1.043]	[1.22, 1.166]	[0.847, 0.913]	[0.807, 0.601]	[0.34, 0.267]	[2.373, 2.249]	[2.15, 2.753]	[Accept, Accept]

Table 3: Error rate for the proposed test.

n_N	Asymmetric		Symmetric		
	Weibull ([5, 7], [2.5, 3])	Chi-square ([4, 6])	Normal ([5, 7], [0.5, 1])	t ([8, 10])	Cauchy ([5, 7], [2.5, 3])
[5, 5]	[0.94, 0.84]	[0.88, 0.85]	[0.89, 0.94]	[0.9, 0.92]	[0.92, 0.91]
[10, 10]	[0.92, 0.88]	[0.91, 0.9]	[0.92, 0.89]	[0.91, 0.92]	[0.79, 0.83]
[15, 15]	[0.91, 0.87]	[0.93, 0.95]	[0.94, 0.94]	[0.9, 0.88]	[0.55, 0.58]
[20, 20]	[0.91, 0.83]	[0.9, 0.92]	[0.92, 0.87]	[0.87, 0.9]	[0.23, 0.32]
[25, 25]	[0.97, 0.87]	[0.87, 0.87]	[0.87, 0.88]	[0.86, 0.92]	[0.18, 0.19]
[30, 30]	[0.86, 0.88]	[0.85, 0.82]	[0.95, 0.92]	[0.81, 0.75]	[0.08, 0.13]
[35, 35]	[0.85, 0.83]	[0.82, 0.86]	[0.9, 0.92]	[0.77, 0.74]	[0.11, 0.03]
[40, 40]	[0.87, 0.88]	[0.82, 0.84]	[0.88, 0.79]	[0.79, 0.74]	[0.06, 0.06]
[45, 45]	[0.89, 0.91]	[0.88, 0.83]	[0.9, 0.91]	[0.74, 0.83]	[0.03, 0.02]
[50, 50]	[0.93, 0.92]	[0.8, 0.8]	[0.93, 0.88]	[0.71, 0.76]	[0, 0.01]
[55, 55]	[0.91, 0.91]	[0.78, 0.79]	[0.89, 0.91]	[0.72, 0.64]	[0.01, 0.01]
[60, 60]	[0.86, 0.9]	[0.8, 0.89]	[0.86, 0.92]	[0.73, 0.7]	[0.01, 0]
[65, 65]	[0.89, 0.87]	[0.77, 0.89]	[0.92, 0.91]	[0.7, 0.65]	[0.01, 0]
[70, 70]	[0.91, 0.91]	[0.82, 0.79]	[0.93, 0.92]	[0.73, 0.67]	[0, 0.01]
[75, 75]	[0.93, 0.89]	[0.83, 0.84]	[0.91, 0.91]	[0.63, 0.69]	[0, 0.01]
[80, 80]	[0.88, 0.9]	[0.74, 0.75]	[0.95, 0.85]	[0.71, 0.66]	[0, 0]
[85, 85]	[0.88, 0.81]	[0.78, 0.73]	[0.93, 0.89]	[0.72, 0.66]	[0, 0]
[90, 90]	[0.89, 0.93]	[0.76, 0.88]	[0.91, 0.95]	[0.61, 0.59]	[0, 0.01]
[95, 95]	[0.86, 0.92]	[0.8, 0.88]	[0.97, 0.9]	[0.68, 0.72]	[0, 0]
[100, 100]	[0.91, 0.86]	[0.71, 0.82]	[0.89, 0.85]	[0.57, 0.64]	[0, 0]

Table 4: Power of test for the proposed test.

n_N	n	Asymmetric		Symmetric		
		Weibull ([5, 7], [2.5, 3])	Chi-square ([4, 6])	Normal ([5, 7], [0.5, 1])	t ([8, 10])	Cauchy ([5, 7], [2.5, 3])
[5, 5]	5	[0.06, 0.16]	[0.12, 0.15]	[0.11, 0.06]	[0.1, 0.08]	[0.08, 0.09]
[10, 10]	10	[0.08, 0.12]	[0.09, 0.1]	[0.08, 0.11]	[0.09, 0.08]	[0.21, 0.17]
[15, 15]	15	[0.09, 0.13]	[0.07, 0.05]	[0.06, 0.06]	[0.1, 0.12]	[0.45, 0.42]
[20, 20]	20	[0.09, 0.17]	[0.1, 0.08]	[0.08, 0.13]	[0.13, 0.1]	[0.77, 0.68]
[25, 25]	25	[0.03, 0.13]	[0.13, 0.13]	[0.13, 0.12]	[0.14, 0.08]	[0.82, 0.81]
[30, 30]	30	[0.14, 0.12]	[0.15, 0.18]	[0.05, 0.08]	[0.19, 0.25]	[0.92, 0.87]
[35, 35]	35	[0.15, 0.17]	[0.18, 0.14]	[0.1, 0.08]	[0.23, 0.26]	[0.89, 0.97]
[40, 40]	40	[0.13, 0.12]	[0.18, 0.16]	[0.12, 0.21]	[0.21, 0.26]	[0.94, 0.94]
[45, 45]	45	[0.11, 0.09]	[0.12, 0.17]	[0.1, 0.09]	[0.26, 0.17]	[0.97, 0.98]
[50, 50]	50	[0.07, 0.08]	[0.2, 0.2]	[0.07, 0.12]	[0.29, 0.24]	[1, 0.99]
[55, 55]	55	[0.09, 0.09]	[0.22, 0.21]	[0.11, 0.09]	[0.28, 0.36]	[0.99, 0.99]
[60, 60]	60	[0.14, 0.1]	[0.2, 0.11]	[0.14, 0.08]	[0.27, 0.3]	[0.99, 1]
[65, 65]	65	[0.11, 0.13]	[0.23, 0.11]	[0.08, 0.09]	[0.3, 0.35]	[0.99, 1]
[70, 70]	70	[0.09, 0.09]	[0.18, 0.21]	[0.07, 0.08]	[0.27, 0.33]	[1, 0.99]
[75, 75]	75	[0.07, 0.11]	[0.17, 0.16]	[0.09, 0.09]	[0.37, 0.31]	[1, 0.99]
[80, 80]	80	[0.12, 0.1]	[0.26, 0.25]	[0.05, 0.15]	[0.29, 0.34]	[1, 1]
[85, 85]	85	[0.12, 0.19]	[0.22, 0.27]	[0.07, 0.11]	[0.28, 0.34]	[1, 1]
[90, 90]	90	[0.11, 0.07]	[0.24, 0.12]	[0.09, 0.05]	[0.39, 0.41]	[1, 0.99]
[95, 95]	95	[0.14, 0.08]	[0.2, 0.12]	[0.03, 0.1]	[0.32, 0.28]	[1, 1]
[100, 100]	100	[0.09, 0.14]	[0.29, 0.18]	[0.11, 0.15]	[0.43, 0.36]	[1, 1]

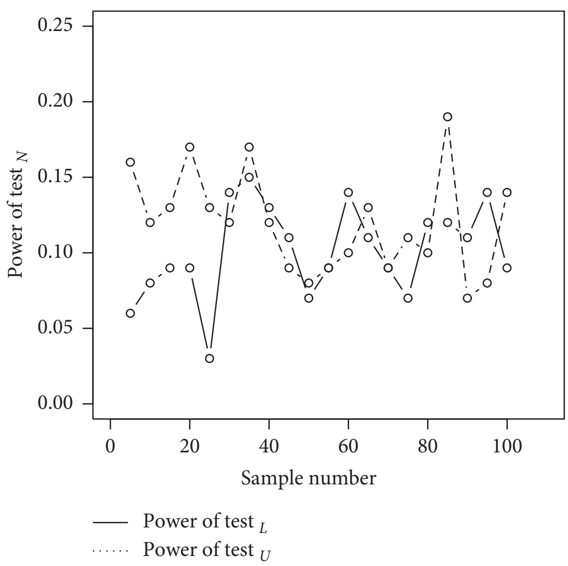

Figure 1: Power curve for the neutrosophic Weibull distribution.

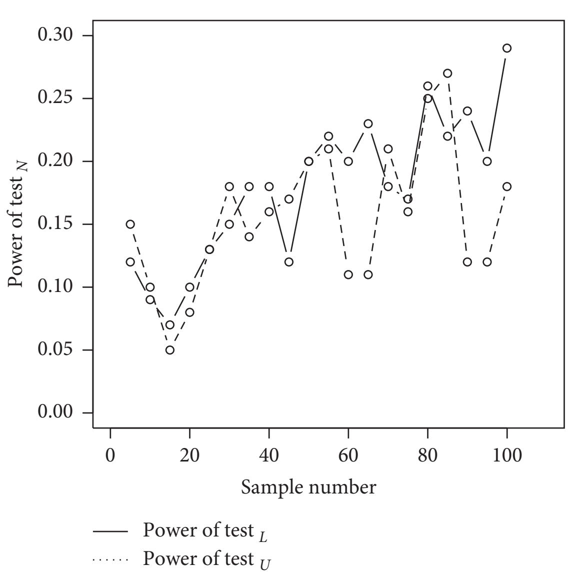

Figure 2: Power curve for the neutrosophic chi-square distribution.

hand, [9] test performs better when $n_N \epsilon [n_L, n_U] > [60, 60]$. Figure 7 is presented for neutrosophic chi-square distribution. From Figure 7, it can be noted that the proposed test is better for all values of $n_N \epsilon [n_L, n_U]$. Figure 8 is shown for neutrosophic t-distribution. From Figure 8, it is observed that overall the proposed test performs better than the existing test. Figure 9 is given for neutrosophic Cauchy distribution. Again, the proposed test is better than the existing test when $n_N \epsilon [n_L, n_U] > [15, 15]$. From Figures 6–9, we conclude that the proposed test is an efficient test for all distributions [9]. In addition, the proposed test provides the power curves in indeterminacy interval which is required under uncertainty environment.

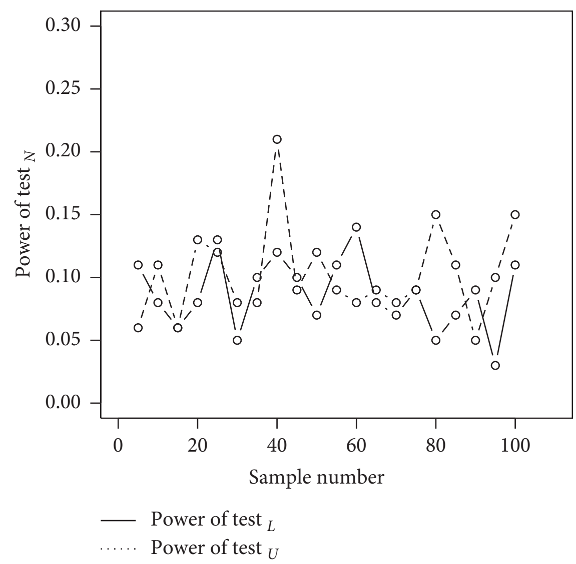

Figure 3: Power curve for the neutrosophic normal distribution.

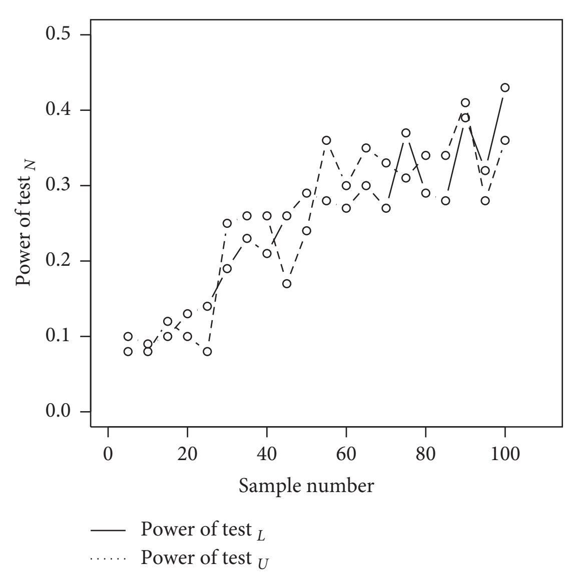

Figure 4: Power curve for the neutrosophic t-distribution.

5. Example

In this example, we consider the approximate population density of some villages of the USA. In practice, it may not be possible to record the exact value of the population density. Therefore, the use of CS on such data may mislead the practitioners. Let us consider the population density of 17 villages. The neutrosophic data are shown in Table 5. From the neutrosophic data, it can be clear that we can apply the proposed test under neutrosophic statistics. We want to test either this data follows the neutrosophic normal distribution. The tabulated value at 0.05 level of significance is [3.06, 3.06] and [4.31, 4.31]. We computed the values of the test statistic as follows:

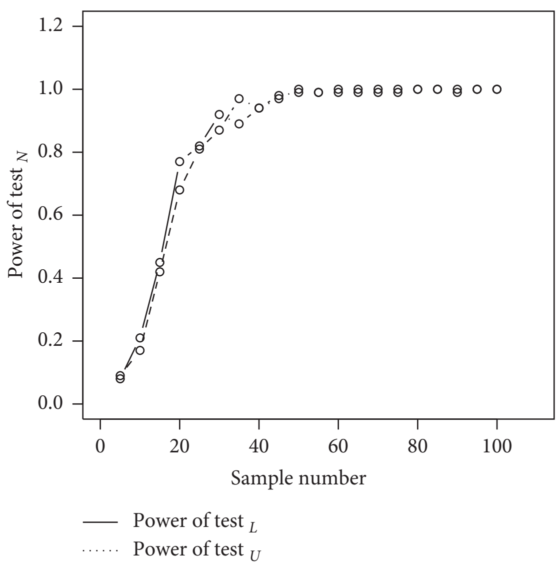

FIGURE 5: Power curve for the neutrosophic Cauchy distribution.

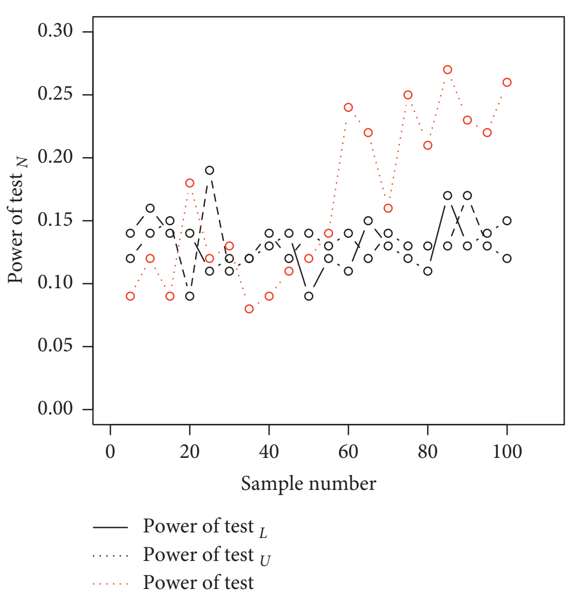

FIGURE 6: Comparison of the proposed test and the existing test for neutrosophic Weibull distribution.

$$Q_N \epsilon \frac{W_N}{S_N} = \frac{[3.6, 3.84]}{[0.8657, 0.9075]} \epsilon [4.15, 4.23]. \tag{10}$$

The neutrosophic form of the statistic $Q_N \epsilon [Q_L, Q_U]$ for the given data is given by

$$Q_N = 4.15 + 4.23 I_N; \quad I_N \epsilon [0, 0.0189]. \tag{11}$$

Note here that the value 4.14 presents the test value under CS. From this neutrosophic form, the change of accepting the null hypothesis is 0.95, the probability of

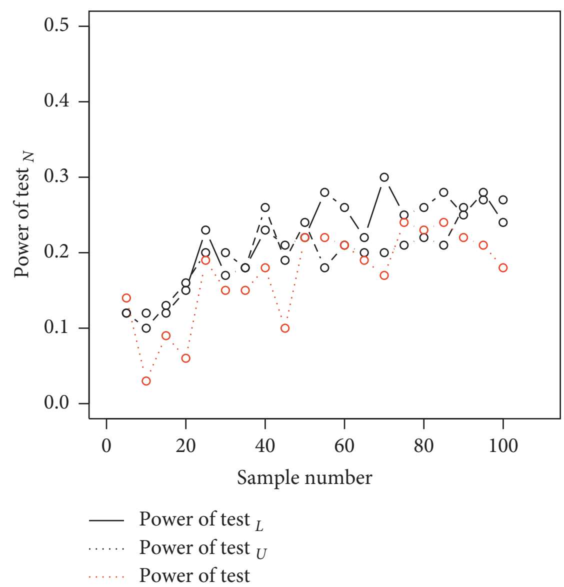

FIGURE 7: Comparison of the proposed test and the existing test for neutrosophic chi-square distribution.

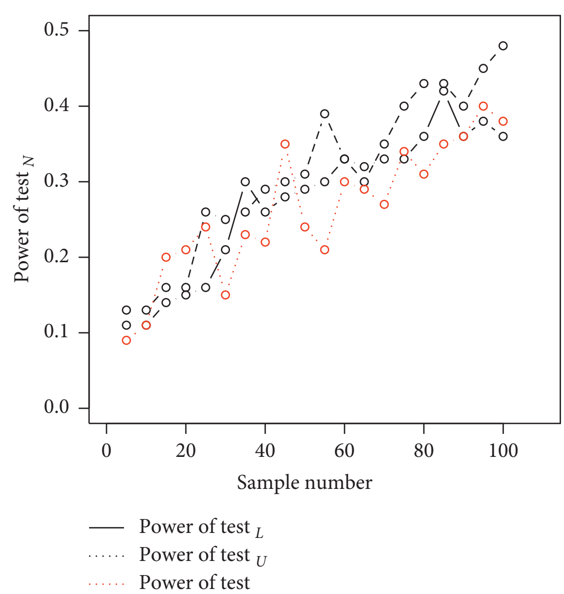

FIGURE 8: Comparison of the proposed test and the existing test for neutrosophic t-distribution.

rejecting the null hypothesis $H_{0N} \epsilon [H_{0L}, H_{0U}]$ is 0.05, and the probability of indeterminacy about the test is 0.0189. We note that the values of $Q_N \epsilon [4.15, 4.23]$ are within the critical values. Therefore, the null hypothesis $H_{0N} \epsilon [H_{0L}, H_{0U}]$ is accepted, and we conclude that the given data follows the neutrosophic normal distribution. From this study, it can be seen that the proposed test provides information about the three probabilities. The proposed test provides information about the measure of

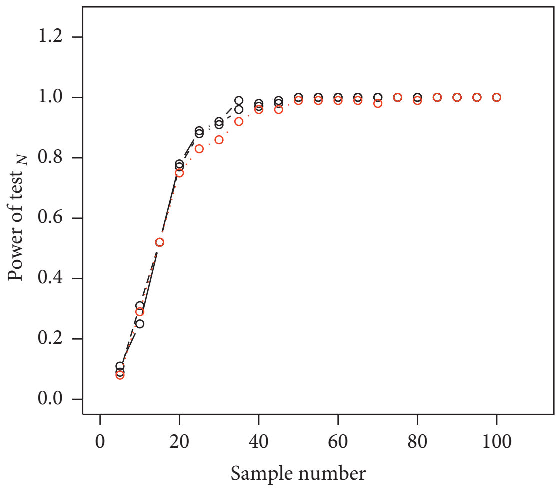

FIGURE 9: Comparison of the proposed test and the existing test for neutrosophic Cauchy distribution.

TABLE 5: The neutrosophic data.

Villages	Population density	Villages	Population density
Aranza	[4.13, 4.14]	Charapan	[5.10, 5.12]
Corupo	[4.53, 4.55]	Comachuen	[5.25, 5.27]
San Lorenzo	[4.69, 4.70]	Pichataro	[5.36, 5.38]
Cheranatzicurin	[4.76, 4.78]	Quinceo	[5.94, 5.96]
Nahuatzen	[4.77, 4.79]	Nurio	[6.06, 6.08]
Pomacuaran	[4.96, 4.98]	Turicuaro	[6.19, 6.21]
Sevina	[4.97, 4.99]	Urapicho	[6.30, 6.32]
Arantepacua	[5.00, 5.02]	Capacuaro	[7.73, 7.98]
Cocucho	[5.04, 5.06]		

indeterminacy. Hence, the proposed test is more informative and adequate to be applied in an indeterminacy environment.

6. Concluding Remarks

In this paper, we presented the designing and application of the W/S test under neutrosophic statistics. We applied the proposed W/S test under neutrosophic statistics for various statistical distributions. We note that the proposed test is quite simple to apply in an uncertain environment. The comparison of the proposed test with the existing test under CS is presented. It is concluded from the comparisons that the proposed test performs better in the power of test than the existing test for all distributions. The application of the proposed test is also given for the example. From the study, it is concluded that the proposed test provides the results in an interval than the exact value as in CS. Therefore, the proposed test is quite effective to be applied in an indeterminate environment. In addition, under uncertainty, the proposed test is more informative than the test under CS. Based on the comparative study, we recommend applying the proposed test when the data are recorded from the complex system or the data having imprecise or fuzzy observations. The proposed test can be applied for testing the data in regression, time series, and marketing problems.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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