Research Article

Event-Triggered H_∞ Filtering for Markovian Jump Neural Networks under Random Missing Measurements and Deception Attacks

Jinxia Wang , Jinfeng Gao , Tian Tan, Jiaqi Wang, and Miao Ma

Faculty of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Hangzhou, Zhejiang 310018, China

Correspondence should be addressed to Jinfeng Gao; gaojf163@163.com

Received 14 May 2020; Revised 3 December 2020; Accepted 11 December 2020; Published 29 December 2020

Academic Editor: Sergey Dashkovskiy

Copyright © 2020 Jinxia Wang et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper concentrates on the event-triggered H_∞ filter design for the discrete-time Markovian jump neural networks under random missing measurements and cyber attacks. Considering that the controlled system and the filtering can exchange information over a shared communication network which is vulnerable to the cyber attacks and has limited bandwidth, the event-triggered mechanism is proposed to relieve the communication burden of data transmission. A variable conforming to Bernoulli distribution is exploited to describe the stochastic phenomenon since the missing measurements occur with random probability. Furthermore, seeing that the communication networks are vulnerable to external malicious attacks, the transferred information via the shared communication network may be changed by the injected false information from the attackers. Based on the above consideration, sufficient conditions for the filtering error system to maintain asymptotically stable are provided with predefined H_∞ performance. In the end, three numerical examples are given to verify the proposed theoretical results.

1. Introduction

Neural networks (NNs) have been attached increasing importance by many researchers on account of the wide applications in robotization, deep learning, optimization problem, and pattern recognition in recent decades. Motivated by the extensive applications, many achievements about NNs have been delivered [1–4]. For instance, by using the universal approximation ability of NNs, an adaptive dynamic surface control method is provided for the nonlinear systems [3, 4]. Nevertheless, in practical application, NNs face many challenges, such as information interruption, random interference, and variations of the network environment. These impulsive effects can be simulated by a Markovian jump chain since the stochastic Markovian jump process can effectively reduce the conservatism. Markovian jump neural networks (MJNNs), which are firstly introduced in [5], have attracted many researchers to make great effort on this issue in recent years [6–12]. Researchers in [6, 7] both pay attention to the state estimation for delayed MJNNs

concretely. The synchronization for MJNNs by employing a detector based on a hidden Markovian model (HMM) is studied in [9]. Researchers in [10–12] are concerned with the performance control of MJNNs based on the event-triggered mechanism (ETM). Among them, the issue of H_∞ state estimation is investigated for a class of semi-Markovian jump NNs [11]. By using a novel distributed ETM, Vadivela et al. discuss the robust H_∞ synchronization for MJNNs in detail [12].

With the development and integration of communication engineering, computer technology, and control theory, the networked control systems (NCSs) exchange information through the shared communication network. Periodic sampling, namely, time-triggered mechanism (TTM), acts as a traditional method of signal processing. If the sampling period is relatively small, a large amount of redundant sampling data will be transmitted to the network channels with limited bandwidth, which will unavoidably cause network congestion. From the perspective of resource utilization, the ETM is introduced to control the signal

transmission by setting a threshold value of signal variation. ETM can vastly decrease the transmission rates and alleviate the pressure of the communication channels on the premise of preserving the system performance [10–14]. Dai et al. ensure the passive synchronization of the MJNNs with gain varying in a random way [10]. In [14], Xu et al. discuss the design method of nonsynchronous H_∞ filter for the singular Markovian jump systems, in which multiple redundant channels are utilized to enhance the quality of data transmission. Since the threshold value of ETM cannot adjust the sampling interval dynamically according to the changes of the system, the adaptive ETM is introduced for the nonlinear multiagent systems [15, 16]. And, the decentralized ETM is proposed in some results such as [12, 17–19] to ensure the ample utilization of network resource. A more optimized stochastic event-triggered strategy is developed in [20, 21], which ensures that only the innovational information is transmitted and the nontriggered information is also utilized. Of course, compared with single event-triggering mechanism or event-triggering mechanism, the advantages of hybrid driving mechanism are more obvious. Bernoulli distribution is used to unify the TTM and the ETM [22, 23]. On the premise of reducing the transmission rate of “invalid” signals and saving the network communication resources, it records the system state changes as much as possible and retains as much detailed information as possible. Therefore, the stochastic event-triggered based filter with better numerical stability and higher accuracy greatly meets the requirements of many practical systems.

Owing to the limited communication bandwidth, the missing measurements that occur stochastically in NCSs are usually unavoidable. Frequent data packet loss will lead to a significant reduction in system performance and even lead to instability of the systems. Generally, missing measurements are described by variables meeting the laws of Bernoulli distribution [24–27] or a Markovian jump chain [28, 29]. In [25], Hu et al. focus on the time-varying nonlinear systems existing the phenomena of multiple packet dropout, in which a combination of independent variables conforming to Bernoulli distribution is exploited to reflect the uncertain probabilities of multiple missing measurements. And, a novel algorithm is put forward for the networked cascade control system with the purpose of suppressing the adverse effects of delays, finite channel, and missing measurements in [30]. For the sake of simulating a more general network environment, taking the randomly occurring missing measurement into account is indispensable.

In practical communication, due to the openness, sharing, interconnection, and universality of the network, the communication network is vulnerable to external cyber-attacks. By definition, cyber attacks refer to the aggressive behaviors of destroying data transmission systems, authentic sampling data, communication infrastructure, and networked equipment. As a consequence, the system

performance decreases seriously, and even the system may collapse. Cyber attacks are classified into three types: repeated attack, denial of service attack, and deception attack, in which the biggest threat to network security is deception attacks. Some interesting results associated with deception attacks have been given [31–34]. For instance, a secure filter is devised for the delayed stochastic nonlinear systems with a novel multiple-channel attack model [33]. For a nonlinear physical system subject to data injection attacks, an elastic filter is designed by employing a new ETM in reference [34]. The goal of [35] is to devise a filter which can guarantee the system security in the presence of stochastic sensor saturation and stochastic deception attacks. The stochastically occurring deception attacks are discussed for NNs, in which the attack signals are presumed to be norm bounded [23, 36]. With the assistance of an adaptive sliding mode controller, Chen et al. discuss the security of a Markovian jump system with injected spurious signals and semi-known transition probabilities in [37]. It should be noted that the MJNNs concerning with deception attacks and stochastic missing measurements have not been discussed in depth yet, which motivates this article.

Based on above discussions, the issue of H_∞ filtering for MJNNs under randomly occurring missing measurements and deception attacks is discussed. The contributions of this study are condensed into three major points: (1) an event-triggered communication strategy is proposed to reduce the redundant information exchange and relieve the pressure of network bandwidth. (2) In the design of filter, randomly occurring missing measurements and cyber attacks are considered to make it closer to the practical communication environment. (3) Based on the constructed mathematical model, sufficient conditions are derived to guarantee that the system is asymptotically stable.

The main framework of this paper is generalized: Section 2 presents the description of the system and the elaboration of all problems. The sufficient conditions to guarantee the asymptotic stability are given, and then a filter is derived for the MJNNs with the deception attacks and random missing measurements in Section 3. In Section 4, three numerical simulations are used to demonstrate the correctness and effectiveness of the analysis.

Notations: in this paper, the superscripts “ T ” and “ -1 ” represent the transpose and the inverse of a matrix; $\mathbb{R}^n$ denotes n -dimensional Euclidean space; $\mathbb{R}^{n \times m}$ denotes the set of real matrices with m rows and n columns; $P > 0$ ($P \in \mathbb{R}^{n \times m}$) means that P is a real symmetric positive definite matrix; $\mathcal{L}_2[0, \infty)$ denotes the space of square-integrable vector functions over $[0, \infty)$; I is a identity matrix; and $*$ represents the symmetric term of a symmetric matrix.

2. Problem Elaboration

2.1. System Description. Consider the MJNNs described as follows:

$$\begin{cases} x(k+1) = A(r_k)x(k) + B(r_k)f(x(k)) + E(r_k)g(x(k-\eta(k))) \\ \quad + D(r_k)\omega(k), \\ y(k) = C(r_k)x(k), \\ z(k) = L(r_k)x(k), \end{cases} \quad (1)$$

where $x(k) = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$ stands for the state vector, $y(k) \in \mathbb{R}^m$ is the measured output, and $z(k) \in \mathbb{R}^p$ denotes the output to be estimated. The nonlinear vector-valued functions $f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \dots, f_n(x_n(k))]^T \in \mathbb{R}^n$ and $g(x(k-\eta(k))) = [g_1(x_1(k-\eta(k))), g_2(x_2(k-\eta(k))), \dots, g_n(x_n(k-\eta(k)))]^T \in \mathbb{R}^n$ are the neuron activation functions. $\omega(k) \in \mathbb{R}^q$ represents the exogenous disturbance with $\omega(k) \in \mathcal{L}_2[0, \infty)$. $\eta(k) \in [\eta_m, \eta_M]$ denotes the time-varying bounded delay of neural network. η_M and η_m represent the upper and lower bounds, respectively. $A(r_k)$, $B(r_k)$, $C(r_k)$, $D(r_k)$, $E(r_k)$, and $L(r_k)$ are known real constant matrices with appropriate dimension. The parameter r_k is constrained by a homogeneous Markovian jump process and generally takes values from the finite set $M = \{1, 2, \dots, N\}$. $\Pi = [\pi_{ij}]$ ($i, j \in M$) represents the transition probability matrix, which is given by

$$\Pr\{r_{k+1} = j | r_k = i\} = \pi_{ij}, \quad \forall i, j \in M, \quad (2)$$

where $\pi_{ij} \in [0, 1]$, ($\forall i, j \in M$) and $\sum_{j=1}^N \pi_{ij} = 1$, ($\forall i \in M$).

Construct the following H_∞ filter:

$$\begin{cases} x_f(k+1) = A_f(r_k)x_f(k) + B_f(r_k)\bar{y}(k), \\ z_f(k) = C_f(r_k)x_f(k), \end{cases} \quad (3)$$

where $x_f(k) \in \mathbb{R}^n$ denotes state vector of filter, $z_f(k) \in \mathbb{R}^p$ represents the output of the filter, $\bar{y}(k) \in \mathbb{R}^m$ represents the actual input of filter, and $A_f(r_k)$, $B_f(r_k)$, and $C_f(r_k)$ are appropriate matrices to be designed.

For convenience, $r_k = i$, ($i \in M$), $A(r_k)$ is denoted by A_i , $B(r_k)$ by B_i , and so on.

2.2. Event-Triggered Mechanism. An event trigger is exploited in this work, by which the data releasing events are generated according to whether the variation of the immediate sampling data and the latest publishing data exceeds the set threshold. The following judgement algorithm is applied widely [11–14, 27]:

$$\begin{aligned} & [y((k+j)h) - y(kh)]^T \Psi_i [y((k+j)h) - y(kh)] \\ & \leq \sigma_i y^T((k+j)h)(k) \Psi_i y((k+j)h), \end{aligned} \quad (4)$$

where $\Psi_i \in \mathbb{R}^m$ are the positive definite weighting matrices that will be determined later, $\sigma_i \in [0, 1)$ are presupposed scalars, $y((k+j)h)$ represents the immediate sampled sensor output, and $y(kh)$ is the latest transmitted data. It should be noted that the immediate sampled measurement output $y((k+j)h)$ satisfying the inequality (4) will not be conveyed to the communication channels.

Remark 1. In the actual NCSs, the limited data transmission capacity usually cannot meet the transmission requirements

of a large number of data. ETM can judge whether the currently sampled data is valid. Only the one that exceeds the threshold in (4) will be conveyed to the filter via the network channels; otherwise, it will be discarded.

Remark 2. Based on the similar threshold screening principle, a stochastic event-triggered strategy is proposed in [38, 39], which establishes an innovational set to judge whether the measured outputs are transferred to the network channels. Nevertheless, it is noted that this method improves the system performance by not only selecting the sampled data with more innovation but also making full use of the information contained in the innovational set. As discussed in [40], the ETM in algorithm (4) only monitors the difference between states sampled at discrete time and does not care what happens between updates. In addition, because the event trigger acts between the sensor and the filter, the expensive work of modifying the existing system is avoided.

According to the algorithm (4), suppose that the data releasing instants are $\kappa_0 h, \kappa_1 h, \kappa_2 h, \dots$, ($\kappa_0 = 0$). The release period of the event trigger is given as $k_q h = (\kappa_{q+1} - \kappa_q)h$.

Remark 3. The set of data release instants meets $\{\kappa_0, \kappa_1, \kappa_2, \dots\} \subseteq \{0, 1, 2, \dots\}$. The amount of $\{\kappa_0, \kappa_1, \kappa_2, \dots\}$ is decided by the value of the variation of the measured output. Setting $\sigma_i = 0$ implies that all sampled signals are released and conveyed to the filter smoothly.

Refer to [40], suppose that ζ_q is the time delay in the network communication at the moment k_q with $\zeta_q \in [0, \zeta^M]$, where ζ^M is a positive real number.

Case 1. If $\kappa_q h + h + \zeta^M \geq \kappa_{q+1} h + \zeta_{q+1}$, introduce a function:

$$\zeta(k) = k - \kappa_q, \quad k \in [\kappa_q h + \zeta_q, \kappa_{q+1} h + \zeta_{q+1}]. \quad (5)$$

Apparently,

$$\zeta_q \leq \zeta(k) \leq (\kappa_{q+1} - \kappa_q)h + \zeta_{q+1} \leq h + \zeta^M. \quad (6)$$

Case 2. If $\kappa_q h + h + \zeta^M < \kappa_{q+1} h + \zeta_{q+1}$, consider the two intervals: $[\kappa_q h + \zeta_q, \kappa_q h + h + \zeta^M)$, $[\kappa_q h + h + \zeta^M, \kappa_{q+1} h + \zeta_{q+1})$.

Since $\zeta_q \geq \zeta^M$, obviously there exists positive integer m that

$$\kappa_q h + mh + \zeta^M < \kappa_{q+1} h + \zeta_{q+1} \leq \kappa_q h + mh + h + \zeta^M, \quad (7)$$

and $y(\kappa_q h)$, $y(\kappa_q h + jh)$ ($j = 1, 2, \dots, m$) meets the condition (4).

Let

$$\begin{cases} I_0 = [\kappa_q h + \zeta_q, \kappa_q h + h + \zeta^M], \\ I_j = \cup_{j=0}^{m-1} [\kappa_q h + jh + \zeta^M + t, \kappa_q h + jh + h + \zeta^M], \\ I_M = [\kappa_q h + mh + \zeta^M, \kappa_{q+1} h + \zeta_{q+1}]. \end{cases} \quad (8)$$

Define the function,

$$\zeta(k) = \begin{cases} k - \kappa_q h, & k \in I_0, \\ k - \kappa_q h - jh, & k \in I_j \ (j = 1, \dots, m-1), \\ k - \kappa_q h - mh, & k \in I_M. \end{cases} \quad (9)$$

From the (9), we can get

$$\begin{cases} 0 \leq \zeta_q \leq \zeta(k) \leq h + \zeta^M, & k \in I_0, \\ 0 \leq \zeta_q \leq \zeta^M \leq \zeta(k) \leq h + \zeta^M, & k \in I_j, \\ 0 \leq \zeta_q \leq \zeta^M \leq \zeta(k) \leq h + \zeta^M, & k \in I_M. \end{cases} \quad (10)$$

Because $\zeta_{q+1} h + \zeta_{q+1} \leq \zeta_q h + (m+1)h + \zeta^M$, the third row of (10) holds. Then, we have $0 \leq \zeta_q \leq \zeta(k) \leq h + \zeta^M \triangleq \zeta_M$.

For Case 1, $k \in [\kappa_q h + \zeta_q, \kappa_{q+1} h + \zeta_{q+1}]$, and the error of measurement output $e_y(k) = 0$ is defined.

For Case 2, define

$$e_y(k) = \begin{cases} 0, & k \in I_0, \\ y(\kappa_q h + jh) - y(\kappa_q h), & k \in I_j, \\ y(\kappa_q h + mh) - y(\kappa_q h), & k \in I_M. \end{cases} \quad (11)$$

Combining the algorithm (4) and the definition (11), it can be concluded that, for $k \in [\kappa_q h + \zeta_q, \kappa_{q+1} h + \zeta_{q+1}]$,

$$e_y^T(k) \Psi_i e_y(k) \leq \sigma_i y^T(k - \zeta(k)) \Psi_i y(k - \zeta(k)). \quad (12)$$

2.3. Deception Attacks and Missing Measurements. In the process of data transmission, the missing measurements that

occur with random probability are taken into consideration. Attackers attempt to decrease the stability and reliability of the system by injecting false signals into the measurement data in the communication network. It is assumed that the injected attack signals are not involved with the missing sensor measurements.

Then, the actual input $\bar{y}(k)$ of the filter is expressed as follows:

$$\bar{y}(k) = \varrho(k) y(\kappa_q) + a(k - d(k)), \quad (13)$$

where $a(k)$ represents the time-discrete signal injected to the measured outputs by attackers. $d(t) \in [0, d_M]$ stands for the delay of cyber attacks. Bernoulli variable $\varrho(k)$ is shown as follows:

$$\text{Prob}\{\varrho(k)\} = \begin{cases} \varrho, & \varrho(k) = 1, \\ 1 - \varrho, & \varrho(k) = 0. \end{cases} \quad (14)$$

Remark 4. According to equality (14), the measurement output can be conveyed to the filter smoothly with the probability ϱ in case of $\varrho(k) = 1$. while $\varrho(k) = 0$ implies that the filter fails to receive the transmitted measurement output.

Remark 5. The occurrence of missing measurements is unavoidable during the data transmission, which makes the actual input of the filter not equivalent to the measured output of the system. The data injection attacks are presumed to be conveyed to the filter via the network channels successfully, which are not in the consideration of packet dropout.

2.4. The Overall Model. Define $\bar{x}(k+1) = [x^T(k), x_f^T(k)]^T$ and $\bar{z}(k) = z(k) - z_f(k)$, we can obtain the augmented filtering error system as follows:

$$\begin{cases} \bar{x}(k+1) = \bar{A}_i \bar{x}(k) + \bar{B}_i f(H\bar{x}(k)) + \bar{E}_i g(H\bar{x}(k - \eta(k))) + (\bar{B}_{11i} + (\varrho(k) - \varrho)\bar{B}_{12i})\bar{x}(k - \zeta(k)) + \bar{B}_{fi}a(k - d(k)) \\ + (\bar{B}_{21i} + (\varrho(k) - \varrho)\bar{B}_{22i})e_y(k) + \bar{D}_i \omega(k), \\ \bar{z}(k) = \bar{L}_i \bar{x}(k), \end{cases} \quad (15)$$

where

$$\begin{aligned} \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & A_{fi} \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \bar{B}_{11i} = \begin{bmatrix} 0 & 0 \\ \varrho B_{fi} C_i & 0 \end{bmatrix}, \\ \bar{B}_{12i} &= \begin{bmatrix} 0 & 0 \\ B_{fi} C_i & 0 \end{bmatrix}, \bar{B}_{21i} = \begin{bmatrix} 0 \\ -\varrho B_{fi} \end{bmatrix}, \bar{B}_{22i} = \begin{bmatrix} 0 \\ -B_{fi} \end{bmatrix}, \\ \bar{B}_{fi} &= \begin{bmatrix} 0 \\ B_{fi} \end{bmatrix}, \bar{E}_i = \begin{bmatrix} E_i \\ 0 \end{bmatrix}, \bar{D}_i = \begin{bmatrix} D_i \\ 0 \end{bmatrix}, H = [I \ 0], \\ \bar{L}_i &= [L_i \ -C_{fi}]. \end{aligned} \quad (16)$$

The design problems of H_∞ filter can be summarized the conditions as follows:

- (i) The augmented system (15) with $\omega(k) = 0$ is asymptotically stable for any initial conditions.
- (ii) Given a scalar $\gamma > 0$ and $\omega(k) \in \mathcal{L}_2[0, \infty)$, the filtering error $\bar{z}(k)$ satisfies

$$\sum_{k=0}^{+\infty} \|\bar{z}(k)\|_2 \leq \gamma^2 \sum_{k=0}^{+\infty} \|\omega(k)\|_2. \quad (17)$$

Next, some assumptions and lemmas which are conducive to deriving the theorem are introduced.

Assumption 1 (see [34]). The deception attacks $a(k)$ meets the following criterion:

$$\|a(k)\|_2 \leq \|Wx(k)\|_2, \quad (18)$$

where W is a given constant matrix.

Assumption 2 (see [41]). The neural functions $f(\cdot)$ and $g(\cdot)$ in (1) meet the initial value setting $f(0) = g(0) = 0$ and the following sector bounded condition:

$$\begin{aligned} [f(x) - f(y) - U_1(x - y)]^T [f(x) - f(y) - U_2(x - y)] &\leq 0, \\ [g(x) - g(y) - V_1(x - y)]^T [g(x) - g(y) - V_2(x - y)] &\leq 0, \end{aligned} \quad (19)$$

$\forall x, y \in \mathbb{R}^n$, where U_1 , U_2 , V_1 , and V_2 are constant real matrices satisfying $U_2 \geq U_1$ and $V_2 \geq V_1$.

Lemma 1 (see [42]). For any symmetric positive-definite matrix $G \in \mathbb{R}^{n \times n}$, scalars γ_1 and γ_2 ($\gamma_2 > \gamma_1$), vector function $\chi(i): \{\gamma_1, \gamma_1 + 1, \dots, \gamma_2\} \rightarrow \mathbb{R}^n$, such that the following inequality holds:

$$-(\gamma_2 - \gamma_1 + 1) \sum_{i=\gamma_1}^{\gamma_2} \chi^T(i) G \chi(i) \leq - \left(\sum_{i=\gamma_1}^{\gamma_2} \chi^T(i) \right) G \left(\sum_{i=\gamma_1}^{\gamma_2} \chi(i) \right). \quad (20)$$

Lemma 2 (see [43]). For any matrix $M > 0$ and constant ϵ , the following inequality holds:

$$-X^T M^{-1} X \leq \epsilon^2 X - 2\epsilon X. \quad (21)$$

3. Main Results

3.1. Asymptotical Stability Analysis. First, the asymptotic stability of the filtering error system (15) with $\omega(k) = 0$ is discussed.

Theorem 1. For given delay bounds $\eta_m, \eta_M, d_M, \zeta_M$, trigger parameters σ_i , and matrix W , system (15) is asymptotically stable with an H_∞ performance index γ , if there exist matrices $P_i > 0$, $\Psi_i > 0$ ($i \in M$), $Q_s > 0$, and $R_s > 0$ ($s = 1, 2, 3, 4$) with appropriate dimensions and scalars $\alpha_1 > 0$, $\alpha_2 > 0$, satisfying the following equation:

$$\Sigma = \begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} < 0, \quad (22)$$

$$\sum_{j=1}^N \pi_{ij} P_j \leq P_i, \quad (23)$$

where

$$\begin{aligned} \Omega_1 &= \begin{bmatrix} \Xi_1 & \Xi_2 \\ * & \Xi_3 \end{bmatrix}, \\ \Omega_2 &= [\Phi_1^T P_i \quad \Phi_3^T P_i \quad \bar{\eta} \Phi_2^T R_1 \quad \bar{\eta} \Phi_3^T R_1 \quad \eta_m \Phi_2^T R_2 \quad \eta_m \Phi_3^T R_2 \quad \zeta_M \Phi_2^T R_3 \quad \zeta_M \Phi_3^T R_3 \quad d_M \Phi_2^T R_4 \quad d_M \Phi_3^T R_4], \\ \Omega_3 &= \text{diag}\{-P_i, -P_i, -R_1, -R_1, -R_2, -R_2, -R_3, -R_3, -R_4, -R_4\}, \\ \Xi_1 &= \begin{bmatrix} \Theta_1 & 0 & R_2 & 0 & R_3 & 0 & R_4 \\ * & \Theta_2 & R_1 + R_2 & 0 & 0 & 0 & 0 \\ * & * & \Theta_3 & R_1 & 0 & 0 & 0 \\ * & * & * & \Theta_4 & 0 & 0 & 0 \\ * & * & * & * & \Theta_5 & R_3 & 0 \\ * & * & * & * & * & \Theta_6 & 0 \\ * & * & * & * & * & * & \Theta_7 \end{bmatrix}, \\ \Xi_2 &= \begin{bmatrix} 0 & -\alpha_1 \bar{U}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\alpha_2 \bar{V}_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ R_4 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Xi_3 &= \text{diag}\{-Q_4 - R_4, -\alpha_1 I, -\alpha_2 I, -\Psi_i, -I\}, \end{aligned} \quad (24)$$

$$\begin{aligned}
\Theta_1 &= -P_i + Q_1 + Q_2 + Q_3 + Q_4 - R_2 - R_3 - R_4 - \alpha_1 \bar{U}_1, \\
\Theta_2 &= -Q_1 - R_1 - R_2, \Theta_3 = -2R_1 - 2R_2 - \alpha_2 \bar{V}_1, \\
\Theta_4 &= -Q_2 - R_1, \Theta_5 = -2R_3 + \sigma_i \hat{\Psi}_i, \\
\hat{\Psi}_i &= \begin{bmatrix} C_i^T \Psi_i C_i & 0 \\ 0 & 0 \end{bmatrix}, \Theta_6 = -Q_3 - R_3, \\
\Theta_7 &= -2R_4 + H^T W H, \bar{\eta} = \eta_M - \eta_m, c^2 = \varrho(1 - \varrho), \\
\Phi_1 &= [\bar{A}_i \ 0 \ 0 \ 0 \ \bar{B}_{11i} \ 0 \ 0 \ 0 \ \bar{B}_i \ \bar{E}_i \ \bar{B}_{21i} \ \bar{B}_{fi}], \\
\Phi_2 &= [\bar{A}_i - I \ 0 \ 0 \ 0 \ \bar{B}_{11i} \ 0 \ 0 \ 0 \ \bar{B}_i \ \bar{E}_i \ \bar{B}_{21i} \ \bar{B}_{fi}], \\
\Phi_3 &= [0 \ 0 \ 0 \ 0 \ c\bar{B}_{12i} \ 0 \ 0 \ 0 \ 0 \ 0 \ c\bar{B}_{22i} \ 0].
\end{aligned} \tag{25}$$

Proof. Construct the following Lyapunov functional as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{26}$$

where

$$\begin{aligned}
V_1(k) &= \bar{x}^T(k) P_i \bar{x}(k), \\
V_2(k) &= \sum_{i=k-\eta_m}^{k-1} \bar{x}^T(i) Q_1 \bar{x}(i) + \sum_{i=k-\eta_M}^{k-1} \bar{x}^T(i) Q_2 \bar{x}(i) + \sum_{i=k-\zeta_M}^{k-1} \bar{x}^T(i) Q_3 \bar{x}(i) + \sum_{i=k-d_M}^{k-1} \bar{x}^T(i) Q_4 \bar{x}(i), \\
V_3(k) &= (\eta_M - \eta_m) \sum_{j=k-\eta_M}^{k-\eta_m-1} \sum_{\lambda=j}^{k-1} \mu^T(\lambda) R_1 \mu(\lambda) + \eta_m \sum_{j=k-\eta_m}^{k-1} \sum_{\lambda=j}^{k-1} \mu^T(\lambda) R_2 \mu(\lambda) + \zeta_M \sum_{j=k-\zeta_M}^{k-1} \sum_{\lambda=j}^{k-1} \mu^T(\lambda) R_3 \mu(\lambda) \\
&\quad + d_M \sum_{j=k-d_M}^{k-1} \sum_{\lambda=j}^{k-1} \mu^T(\lambda) R_4 \mu(\lambda), \\
\mu(\lambda) &= \bar{x}(\lambda+1) - \bar{x}(\lambda),
\end{aligned} \tag{27}$$

under the condition of $P_i > 0$, $Q_s > 0$, and $R_s > 0$ ($s = 1, 2, 3, 4$).

We can get

$$\begin{aligned}
\Delta V_1(k) &= V_1(k+1) - V_1(k) = \bar{x}^T(k+1) \sum_{j=1}^N \pi_{ij} P_j \bar{x}(k+1) \\
&\quad - \bar{x}^T(k) P_i \bar{x}(k),
\end{aligned} \tag{28}$$

$$\begin{aligned}
\Delta V_2(k) &= V_2(k+1) - V_2(k) \\
&= \bar{x}^T(k) Q_1 \bar{x}(k) - \bar{x}^T(k - \eta_m) Q_1 \bar{x}(k - \eta_m) + \bar{x}^T(k) Q_2 \bar{x}(k) - \bar{x}^T(k - \eta_M) Q_2 \bar{x}(k - \eta_M) \\
&\quad + \bar{x}^T(k) Q_3 \bar{x}(k) - \bar{x}^T(k - \zeta_M) Q_3 \bar{x}(k - \zeta_M) + \bar{x}^T(k) Q_4 \bar{x}(k) - \bar{x}^T(k - d_M) Q_4 \bar{x}(k - d_M),
\end{aligned} \tag{29}$$

$$\begin{aligned}
\Delta V_3(k) &= V_3(k+1) - V_3(k) \\
&= (\eta_M - \eta_m)^2 \mu^T(k) R_1 \mu(k) - (\eta_M - \eta_m) \sum_{\lambda=k-\eta_M}^{k-\eta_m-1} \mu^T(\lambda) R_1 \mu(\lambda) \\
&\quad + \eta_m^2 \mu^T(k) R_2 \mu(k) - \eta_m \sum_{\lambda=k-\eta_m}^{k-1} \mu^T(\lambda) R_2 \mu(\lambda) + \zeta_M^2 \mu^T(k) R_3 \mu(k) - \zeta_M \sum_{\lambda=k-\zeta_M}^{k-1} \mu^T(\lambda) R_3 \mu(\lambda) \\
&\quad + d_M^2 \mu^T(k) R_4 \mu(k) - d_M \sum_{\lambda=k-d_M}^{k-1} \mu^T(\lambda) R_4 \mu(\lambda).
\end{aligned} \tag{30}$$

Letting

$$\xi^T(k) = [\bar{x}^T(k) \quad \bar{x}^T(k-\eta_m) \quad \bar{x}^T(k-\eta(k)) \quad \bar{x}^T(k-\eta_M) \quad \bar{x}^T(k-\zeta(k)) \quad \bar{x}^T(k-\zeta_M) \quad \bar{x}^T(k-d(k)) \quad \bar{x}^T(k-d_M) \quad f^T(H\bar{x}(k)) \quad g^T(H\bar{x}(k-\eta(k))) \quad e_y^T(k) \quad a(k-d(k))]. \tag{31}$$

So, it is obvious that

$$\begin{aligned}
\bar{x}(k+1) &= \hat{\Phi} \xi(k), \\
\mu(k) &= \bar{x}(k+1) - \bar{x}(k) = (\hat{\Phi} - I) \xi(k),
\end{aligned} \tag{32}$$

where

$$\begin{aligned}
\hat{\Phi} &= [\bar{A}_i \quad 0 \quad 0 \quad 0 \quad \bar{B}_{1i} \quad 0 \quad 0 \quad 0 \quad \bar{B}_i \quad \bar{E}_i \quad \bar{B}_{2i} \quad \bar{B}_{fi}], \\
\bar{B}_{1i} &= \bar{B}_{11i} + (\varrho(k) - \varrho) \bar{B}_{12i}, \\
\bar{B}_{2i} &= \bar{B}_{21i} + (\varrho(k) - \varrho) \bar{B}_{22i}, \\
I_1 &= [I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0].
\end{aligned} \tag{33}$$

Then, one can get the following equation:

$$\mu^T(k) R_i \mu(k) = \xi^T(k) (\hat{\Phi} - I)^T R_i (\hat{\Phi} - I) \xi(k) = \xi^T(k) \Phi_2^T R_i \Phi_2 \xi(k) + \xi^T(k) \Phi_3^T R_i \Phi_3 \xi(k). \tag{34}$$

By using Assumption 2, for scalars $\alpha_1 > 0$, $\alpha_2 > 0$, we can get

$$\begin{aligned}
-\alpha_1 \left[\begin{array}{c} \bar{x}(k) \\ f(H\bar{x}(k)) \end{array} \right]^T \left[\begin{array}{cc} \bar{U}_1 & \bar{U}_2 \\ * & I \end{array} \right] \left[\begin{array}{c} \bar{x}(k) \\ f(H\bar{x}(k)) \end{array} \right] &\geq 0, \\
-\alpha_2 \left[\begin{array}{c} \bar{x}(k-\eta(k)) \\ g(H\bar{x}(k-\eta(k))) \end{array} \right]^T \left[\begin{array}{cc} \bar{V}_1 & \bar{V}_2 \\ * & I \end{array} \right] \left[\begin{array}{c} \bar{x}(k-\eta(k)) \\ g(H\bar{x}(k-\eta(k))) \end{array} \right] &\geq 0,
\end{aligned} \tag{35}$$

where

$$\begin{aligned}
\bar{U}_1 &= H^T \hat{U}_1 H, \hat{U}_1 = \frac{(U_1^T U_2 + U_2^T U_1)}{2}, \bar{U}_2 = -H^T \hat{U}_2, \\
\hat{U}_2 &= \frac{(U_1^T + U_2^T)}{2}, \bar{V}_1 = H^T \hat{V}_1 H, \bar{V}_2 = -H^T \hat{V}_2, \\
\hat{V}_1 &= \frac{(V_1^T V_2 + V_2^T V_1)}{2}, \hat{V}_2 = \frac{(V_1^T + V_2^T)}{2}.
\end{aligned} \tag{36}$$

Recalling the restrictive condition of deception attacks in (18), there exists an inequality as follows:

$$a^T(k-d(k))a(k-d(k)) \leq \bar{x}^T(k-d(k))H^T W H \bar{x}(k-d(k)). \tag{37}$$

According to Lemma 1 and combining conditions (12) and (26)–(38), the following inequality is obtained:

$$\Delta V(k) \leq \xi^T(k) \Pi \xi(k), \tag{38}$$

where

$$\begin{aligned}
\Pi &= \Omega_1 + \Phi_1^T P_i \Phi_1 + \Phi_3^T P_i \Phi_3 + (\eta_M - \eta_m)^2 \Phi_2^T R_1 \Phi_2 + (\eta_M - \eta_m)^2 \Phi_3^T R_1 \Phi_3 + \eta_m^2 \Phi_2^T R_2 \Phi_2 + \eta_m^2 \Phi_3^T R_2 \Phi_3 \\
&\quad + \zeta_M^2 \Phi_2^T R_3 \Phi_2 + \zeta_M^2 \Phi_3^T R_3 \Phi_3 + d_M^2 \Phi_2^T R_4 \Phi_2 + d_M^2 \Phi_3^T R_4 \Phi_3.
\end{aligned} \tag{39}$$

By employing Schur Complement lemma, the asymptotic stability of system (15) with $\omega(k) = 0$ is guaranteed if condition (22) holds.

3.2. H_∞ Performance Analysis. The H_∞ performance of the augmented system (15) is analysed in the second theorem.

Theorem 2. For given delay bounds $\eta_m, \eta_M, \zeta_M, d_M$, trigger parameters σ_i , and matrix W , the system (15) is asymptotically stable with an H_∞ performance index γ , if there exist matrices $P_i > 0, \Psi_i > 0 (i \in M), Q_s > 0$, and

$R_s > 0 (s = 1, 2, 3, 4)$ with appropriate dimensions and scalars $\alpha_1 > 0, \alpha_2 > 0$, satisfying the following:

$$\bar{\Sigma} = \begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 \\ * & \bar{\Omega}_3 \end{bmatrix} < 0, \quad (40)$$

$$\sum_{j=1}^N \pi_{ij} P_j \leq P_i, \quad (41)$$

where

$$\begin{aligned} \bar{\Omega}_1 &= \begin{bmatrix} \Omega_1 & 0 \\ * & -\gamma^2 I \end{bmatrix}, \bar{\Omega}_3 = \begin{bmatrix} \Omega_3 & 0 \\ * & -I \end{bmatrix}, \\ \bar{\Omega}_2 &= [\bar{\Phi}_1^T P_i \quad \bar{\Phi}_3^T P_i \quad \bar{\eta} \bar{\Phi}_2^T R_1 \quad \bar{\eta} \bar{\Phi}_3^T R_1 \quad \eta_m \bar{\Phi}_2^T R_2 \quad \eta_m \bar{\Phi}_3^T R_2 \quad \zeta_M \bar{\Phi}_2^T R_3 \quad \zeta_M \bar{\Phi}_3^T R_3 \quad d_M \bar{\Phi}_2^T R_4 \quad d_M \bar{\Phi}_3^T R_4 \quad \Gamma^T], \\ \bar{\Phi}_1 &= [\Phi_1 \quad \bar{D}_i], \bar{\Phi}_2 = [\Phi_2 \quad \bar{D}_i], \bar{\Phi}_3 = [\Phi_3 \quad 0], \\ \Gamma &= [\bar{L}_i \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]. \end{aligned} \quad (42)$$

Proof 2. Based on the same Lyapunov functional as Theorem 1, one can derive the following inequality:

$$\Delta V(k) - \gamma^2 \omega^T(k) \omega(k) + \bar{z}(k) \bar{z}(k) \leq \bar{\xi}^T(k) \bar{\Pi} \bar{\xi}(k), \quad (43)$$

where

$$\begin{aligned} \bar{\xi}(k) &= [\xi^T(k) \quad \omega^T(k)]^T, \\ \bar{\Pi} &= \bar{\Omega}_1 + \bar{\Phi}_1^T P_i \bar{\Phi}_1 + \bar{\Phi}_3^T P_i \bar{\Phi}_3 + (\eta_M - \eta_m)^2 \bar{\Phi}_2^T R_1 \bar{\Phi}_2 + (\eta_M - \eta_m)^2 \bar{\Phi}_3^T R_1 \bar{\Phi}_3 \\ &\quad + \eta_m^2 \bar{\Phi}_2^T R_2 \bar{\Phi}_2 + \eta_m^2 \bar{\Phi}_3^T R_2 \bar{\Phi}_3 + \zeta_M^2 \bar{\Phi}_2^T R_3 \bar{\Phi}_2 + \zeta_M^2 \bar{\Phi}_3^T R_3 \bar{\Phi}_3 + d_M^2 \bar{\Phi}_2^T R_4 \bar{\Phi}_2 + d_M^2 \bar{\Phi}_3^T R_4 \bar{\Phi}_3 + \Gamma^T \Gamma. \end{aligned} \quad (44)$$

According to Schur Complement lemma, (40) and (41) are sufficient conditions to guarantee $\bar{\Pi} < 0$, and the following inequality can be ensured:

$$\Delta V(k) - \gamma^2 \omega^T(k) \omega(k) + \bar{z}(k) \bar{z}(k) < 0, \quad (45)$$

under the zero initial condition, one can get

$$\sum_{k=0}^{\infty} \|\bar{z}(k)\|_2 \leq \gamma^2 \sum_{k=0}^{\infty} \|\omega(k)\|_2. \quad (46)$$

The proof is complete.

3.3. H_∞ Filter Design. Based on the derivation of Theorems 1 and 2, the H_∞ filter is designed.

Theorem 3. For given delay bounds $\eta_m, \eta_M, \zeta_M, d_M$, trigger parameters σ_i , and matrix W , the filtering error system (15) is asymptotically stable with an H_∞ performance index γ , if there exist real matrices $P_{1i} > 0, X_i > 0, \Psi_i > 0 (i \in M), \bar{Q}_s > 0, \bar{R}_s > 0 (s = 1, 2, 3, 4), \hat{A}_{fi}, \hat{B}_{fi}$, and $\hat{C}_{fi}$ with appropriate dimension, and scalars $\alpha_1 > 0, \alpha_2 > 0$ satisfying $P_{1i} - X_i > 0$, such that

$$\tilde{\Sigma} = \begin{bmatrix} \tilde{\Omega}_1 & \tilde{\Omega}_2 \\ * & \tilde{\Omega}_3 \end{bmatrix}, \quad (47)$$

$$\sum_{j=1}^N \pi_{ij} (P_{1j} - X_j) \leq P_{1i} - X_i, \quad (48)$$

where

$$\begin{aligned}
\tilde{\Omega}_1 &= \begin{bmatrix} \tilde{\Xi}_1 & \tilde{\Xi}_2 \\ * & \tilde{\Xi}_3 \end{bmatrix}, \tilde{\Omega}_2 = \begin{bmatrix} \tilde{\Xi}_4^{(1)} \\ \tilde{\Xi}_4^{(2)} \end{bmatrix}, \\
\tilde{\Omega}_3 &= \text{diag}\{-\tilde{P}_i, -\tilde{P}_i, -2\tilde{P}_i + \tilde{R}_1, -2\tilde{P}_i + \tilde{R}_1, -2\tilde{P}_i + \tilde{R}_2, -2\tilde{P}_i + \tilde{R}_2, -2\tilde{P}_i + \tilde{R}_3, -2\tilde{P}_i + \tilde{R}_3, -2\tilde{P}_i + \tilde{R}_4, -2\tilde{P}_i + \tilde{R}_4, -I\}\%, \\
\tilde{\Xi}_1 &= \begin{bmatrix} \tilde{\Theta}_1 & 0 & \tilde{R}_2 & 0 & \tilde{R}_3 & 0 & \tilde{R}_4 \\ * & \tilde{\Theta}_2 & \tilde{R}_1 + \tilde{R}_2 & 0 & 0 & 0 & 0 \\ * & * & \tilde{\Theta}_3 & \tilde{R}_1 & 0 & 0 & 0 \\ * & * & * & \tilde{\Theta}_4 & 0 & 0 & 0 \\ * & * & * & * & \tilde{\Theta}_5 & \tilde{R}_3 & 0 \\ * & * & * & * & * & \tilde{\Theta}_6 & 0 \\ * & * & * & * & * & * & \tilde{\Theta}_7 \end{bmatrix}, \\
\tilde{\Xi}_2 &= \begin{bmatrix} 0 & \tilde{\Theta}_8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{\Theta}_9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{R}_4 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
\tilde{\Xi}_3 &= \text{diag}\{-\tilde{Q}_4 - \tilde{R}_4, -\alpha_1 I, -\alpha_2 I, -\Psi_i, -I, -\gamma^2 I\}, \\
\tilde{\Theta}_1 &= -\tilde{P}_i + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 + \tilde{Q}_4 - \tilde{R}_2 - \tilde{R}_3 - \tilde{R}_4 - \alpha_1 \bar{U}_1, \\
\tilde{\Theta}_2 &= -\tilde{Q}_1 - \tilde{R}_1 - \tilde{R}_2, \tilde{\Theta}_3 = -2\tilde{R}_1 - 2\tilde{R}_2 - \alpha_2 \bar{V}_1, \\
\tilde{\Theta}_4 &= -\tilde{Q}_2 - \tilde{R}_1, \tilde{\Theta}_5 = -2\tilde{R}_3 + \sigma_i \hat{\Psi}_i, \\
\tilde{\Theta}_6 &= -\tilde{Q}_3 - \tilde{R}_3, \tilde{\Theta}_7 = -2\tilde{R}_4 + \bar{W}, \tilde{\Theta}_8 = \begin{bmatrix} \alpha_1 \hat{U}_2 \\ 0 \end{bmatrix}, \\
\tilde{\Theta}_9 &= \begin{bmatrix} \alpha_2 \hat{V}_2 \\ 0 \end{bmatrix}, \hat{\Psi}_i = \begin{bmatrix} C_i^T \Psi_i C_i & 0 \\ 0 & 0 \end{bmatrix}, \bar{W} = \begin{bmatrix} W & 0 \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Xi}_4^{(1)} &= [\tilde{\Theta}_{41} \ \tilde{\Theta}_{11} \ \tilde{\Theta}_{21} \ \tilde{\Theta}_{31} \ \tilde{\Theta}_{51} \ \tilde{\Theta}_{61} \ \tilde{\Theta}_{71} \ \tilde{\Theta}_{81} \ \tilde{\Theta}_{91} \ \tilde{\Theta}_{01} \ \tilde{\Theta}_{111}], \\
\tilde{\Xi}_4^{(2)} &= [\tilde{\Theta}_{12} \ \tilde{\Theta}_{22} \ \tilde{\Theta}_{32} \ \tilde{\Theta}_{42} \ \tilde{\Theta}_{52} \ \tilde{\Theta}_{62} \ \tilde{\Theta}_{72} \ \tilde{\Theta}_{82} \ \tilde{\Theta}_{92} \ \tilde{\Theta}_{02} \ 0], \\
\tilde{\Theta}_{11} &= [\Upsilon_{11}^T \ 0 \ 0 \ 0 \ \Upsilon_{12}^T \ 0 \ 0]^T, \\
\Upsilon_{11} &= \begin{bmatrix} A_i^T P_{1i} & A_i^T X_i \\ \hat{A}_{fi}^T & \hat{A}_{fi}^T \end{bmatrix}, \Upsilon_{12} = \begin{bmatrix} \mathfrak{q} C_i^T \hat{B}_{fi}^T & \mathfrak{q} C_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{12} &= [0 \ \Upsilon_{13}^T \ \Upsilon_{14}^T \ \Upsilon_{15}^T \ \Upsilon_{16}^T \ \Upsilon_{17}^T]^T, \\
\Upsilon_{13} &= [B_i^T P_{1i} \ B_i^T X_i], \Upsilon_{14} = [E_i^T P_{1i} \ E_i^T X_i], \\
\Upsilon_{15} &= [-\mathfrak{q} \hat{B}_{fi}^T \ -\mathfrak{q} \hat{B}_{fi}^T], \Upsilon_{16} = [\hat{B}_{fi}^T \ \hat{B}_{fi}^T], \\
\Upsilon_{17} &= [D_i^T P_{1i} \ D_i^T X_i], \\
\tilde{\Theta}_{21} &= [0 \ 0 \ 0 \ 0 \ \Upsilon_{21}^T \ 0 \ 0]^T, \\
\Upsilon_{21} &= \begin{bmatrix} c C_i^T \hat{B}_{fi}^T & c C_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{22} &= [0 \ 0 \ 0 \ \Upsilon_{22}^T \ 0 \ 0]^T, \Upsilon_{22} = [-c \hat{B}_{fi}^T \ -c \hat{B}_{fi}^T],
\end{aligned} \tag{49}$$

$$\begin{aligned}
\tilde{\Theta}_{11} &= [\Upsilon_{11}^T \ 0 \ 0 \ 0 \ \Upsilon_{12}^T \ 0 \ 0]^T, \\
\Upsilon_{11} &= \begin{bmatrix} A_i^T P_{1i} & A_i^T X_i \\ \hat{A}_{fi}^T & \hat{A}_{fi}^T \end{bmatrix}, \Upsilon_{12} = \begin{bmatrix} \mathfrak{q} C_i^T \hat{B}_{fi}^T & \mathfrak{q} C_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{12} &= [0 \ \Upsilon_{13}^T \ \Upsilon_{14}^T \ \Upsilon_{15}^T \ \Upsilon_{16}^T \ \Upsilon_{17}^T]^T, \\
\Upsilon_{13} &= [B_i^T P_{1i} \ B_i^T X_i], \Upsilon_{14} = [E_i^T P_{1i} \ E_i^T X_i], \\
\Upsilon_{15} &= [-\mathfrak{q} \hat{B}_{fi}^T \ -\mathfrak{q} \hat{B}_{fi}^T], \Upsilon_{16} = [\hat{B}_{fi}^T \ \hat{B}_{fi}^T], \\
\Upsilon_{17} &= [D_i^T P_{1i} \ D_i^T X_i], \\
\tilde{\Theta}_{21} &= [0 \ 0 \ 0 \ 0 \ \Upsilon_{21}^T \ 0 \ 0]^T, \\
\Upsilon_{21} &= \begin{bmatrix} c C_i^T \hat{B}_{fi}^T & c C_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{22} &= [0 \ 0 \ 0 \ \Upsilon_{22}^T \ 0 \ 0]^T, \Upsilon_{22} = [-c \hat{B}_{fi}^T \ -c \hat{B}_{fi}^T],
\end{aligned} \tag{50}$$

$$\begin{aligned}
\tilde{\Theta}_{31} &= [\Upsilon_{31}^T \ 0 \ 0 \ 0 \ \Upsilon_{32}^T \ 0 \ 0]^T, \\
\Upsilon_{31} &= \begin{bmatrix} \bar{\eta}(A_i^T P_{1i} - P_{1i}) & \bar{\eta}(A_i^T X_i - X_i) \\ \bar{\eta}(\hat{A}_{fi}^T - X_i) & \bar{\eta}(\hat{A}_{fi}^T - X_i) \end{bmatrix}, \\
\Upsilon_{32} &= \begin{bmatrix} \bar{\eta} \mathfrak{C}_i^T \hat{B}_{fi}^T & \bar{\eta} \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{32} &= [0 \ \Upsilon_{33}^T \ \Upsilon_{34}^T \ \Upsilon_{35}^T \ \Upsilon_{36}^T \ \Upsilon_{37}^T]^T, \\
\Upsilon_{33} &= [\bar{\eta} B_i^T P_{1i} \ \bar{\eta} B_i^T X_i], \\
\Upsilon_{34} &= [\bar{\eta} E_i^T P_{1i} \ \bar{\eta} E_i^T X_i], \\
\Upsilon_{35} &= [-\bar{\eta} \mathfrak{C}_i^T \hat{B}_{fi}^T \ -\bar{\eta} \mathfrak{C}_i^T \hat{B}_{fi}^T], \\
\Upsilon_{36} &= [\bar{\eta} \hat{B}_{fi}^T \ \bar{\eta} \hat{B}_{fi}^T], \\
\Upsilon_{37} &= [\bar{\eta} D_i^T P_{1i} \ \bar{\eta} D_i^T X_i], \\
\tilde{\Theta}_{41} &= [0 \ 0 \ 0 \ 0 \ \Upsilon_{41}^T \ 0 \ 0]^T, \\
\Upsilon_{41} &= \begin{bmatrix} \bar{\eta} c \mathfrak{C}_i^T \hat{B}_{fi}^T & \bar{\eta} c \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{42} &= [0 \ 0 \ 0 \ \Upsilon_{42}^T \ 0 \ 0]^T, \\
\Upsilon_{42} &= [-\bar{\eta} c \hat{B}_{fi}^T \ -\bar{\eta} c \hat{B}_{fi}^T], \\
\tilde{\Theta}_{51} &= [\Upsilon_{51}^T \ 0 \ 0 \ 0 \ \Upsilon_{52}^T \ 0 \ 0]^T, \\
\Upsilon_{51} &= \begin{bmatrix} \eta_m(A_i^T P_{1i} - P_{1i}) & \eta_m(A_i^T X_i - X_i) \\ \eta_m(\hat{A}_{fi}^T - X_i) & \eta_m(\hat{A}_{fi}^T - X_i) \end{bmatrix}, \\
\Upsilon_{52} &= \begin{bmatrix} \eta_m \mathfrak{C}_i^T \hat{B}_{fi}^T & \eta_m \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{52} &= [0 \ \Upsilon_{53}^T \ \Upsilon_{54}^T \ \Upsilon_{55}^T \ \Upsilon_{56}^T \ \Upsilon_{57}^T]^T, \\
\Upsilon_{53} &= [\eta_m B_i^T P_{1i} \ \eta_m B_i^T X_i], \Upsilon_{54} = [\eta_m E_i^T P_{1i} \ \eta_m E_i^T X_i], \\
\Upsilon_{55} &= [-\eta_m \mathfrak{C}_i^T \hat{B}_{fi}^T \ -\eta_m \mathfrak{C}_i^T \hat{B}_{fi}^T], \Upsilon_{56} = [\eta_m \hat{B}_{fi}^T \ \eta_m \hat{B}_{fi}^T], \\
\Upsilon_{57} &= [\eta_m D_i^T P_{1i} \ \eta_m D_i^T X_i], \\
\tilde{\Theta}_{61} &= [0 \ 0 \ 0 \ 0 \ \Upsilon_{61}^T \ 0 \ 0]^T, \\
\Upsilon_{61} &= \begin{bmatrix} \eta_m c \mathfrak{C}_i^T \hat{B}_{fi}^T & \eta_m c \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{62} &= [0 \ 0 \ 0 \ \Upsilon_{62}^T \ 0 \ 0]^T, \\
\Upsilon_{62} &= [-\eta_m c \hat{B}_{fi}^T \ -\eta_m c \hat{B}_{fi}^T], \\
\tilde{\Theta}_{71} &= [\Upsilon_{71}^T \ 0 \ 0 \ 0 \ \Upsilon_{72}^T \ 0 \ 0]^T,
\end{aligned} \tag{51}$$

$$\begin{aligned}
Y_{71} &= \begin{bmatrix} \zeta_M(A_i^T P_{1i} - P_{1i}) & \zeta_M(A_i^T X_i - X_i) \\ \zeta_M(\hat{A}_{fi}^T - X_i) & \zeta_M(\hat{A}_{fi}^T - X_i) \end{bmatrix}, \\
Y_{72} &= \begin{bmatrix} \zeta_M \mathfrak{C}_i^T \hat{B}_{fi}^T & \zeta_M \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{72} &= [0 \ Y_{73}^T \ Y_{74}^T \ Y_{75}^T \ Y_{76}^T \ Y_{77}^T]^T, \\
Y_{73} &= [\zeta_M B_i^T P_{1i} \ \zeta_M B_i^T X_i], Y_{74} = [\zeta_M E_i^T P_{1i} \ \zeta_M E_i^T X_i], \\
Y_{75} &= [-\zeta_M \mathfrak{C}_i^T \hat{B}_{fi}^T \ -\zeta_M \mathfrak{C}_i^T \hat{B}_{fi}^T], Y_{76} = [\zeta_M \hat{B}_{fi}^T \ \zeta_M \hat{B}_{fi}^T], \\
Y_{77} &= [\zeta_M D_i^T P_{1i} \ \zeta_M D_i^T X_i],
\end{aligned}$$

(52)

$$\begin{aligned}
\tilde{\Theta}_{81} &= [0 \ 0 \ 0 \ 0 \ Y_{81}^T \ 0 \ 0]^T, \\
Y_{81} &= \begin{bmatrix} \zeta_M c \mathfrak{C}_i^T \hat{B}_{fi}^T & \zeta_M c \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{82} &= [0 \ 0 \ 0 \ Y_{82}^T \ 0 \ 0]^T, \\
Y_{82} &= [-\zeta_M c \hat{B}_{fi}^T \ -\zeta_M c \hat{B}_{fi}^T], \\
\tilde{\Theta}_{91} &= [Y_{91}^T \ 0 \ 0 \ 0 \ Y_{92}^T \ 0 \ 0]^T, \\
Y_{91} &= \begin{bmatrix} d_M(A_i^T P_{1i} - P_{1i}) & d_M(A_i^T X_i - X_i) \\ d_M(\hat{A}_{fi}^T - X_i) & d_M(\hat{A}_{fi}^T - X_i) \end{bmatrix}, \\
Y_{92} &= \begin{bmatrix} d_M \mathfrak{C}_i^T \hat{B}_{fi}^T & d_M \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{92} &= [0 \ Y_{93}^T \ Y_{94}^T \ Y_{95}^T \ Y_{96}^T \ Y_{97}^T]^T, \\
Y_{93} &= [d_M B_i^T P_{1i} \ d_M B_i^T X_i], Y_{94} = [d_M E_i^T P_{1i} \ d_M E_i^T X_i], \\
Y_{95} &= [-d_M \mathfrak{C}_i^T \hat{B}_{fi}^T \ -d_M \mathfrak{C}_i^T \hat{B}_{fi}^T], Y_{96} = [d_M \hat{B}_{fi}^T \ d_M \hat{B}_{fi}^T], \\
Y_{97} &= [d_M D_i^T P_{1i} \ d_M D_i^T X_i], \\
\tilde{\Theta}_{101} &= [0 \ 0 \ 0 \ 0 \ Y_{101}^T \ 0 \ 0]^T, \\
Y_{101} &= \begin{bmatrix} d_M c \mathfrak{C}_i^T \hat{B}_{fi}^T & d_M c \mathfrak{C}_i^T \hat{B}_{fi}^T \\ 0 & 0 \end{bmatrix}, \\
\tilde{\Theta}_{102} &= [0 \ 0 \ 0 \ Y_{102}^T \ 0 \ 0]^T, \\
Y_{102} &= [-d_M c \hat{B}_{fi}^T \ -d_M c \hat{B}_{fi}^T], \\
\tilde{\Theta}_{111} &= [Y_{111}^T \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T Y_{111} = \begin{bmatrix} L_i^T \\ -C_{fi}^T \end{bmatrix}.
\end{aligned}$$

(53)

The parameter matrices of the filter are given by

$$\begin{cases} A_{fi} = \hat{A}_{fi}X_i^{-1}, \\ B_{fi} = \hat{B}_{fi}, \\ C_{fi} = \hat{C}_{fi}X_i^{-1}. \end{cases} \quad (54)$$

Proof. Firstly, pre- and postmultiplying (40) by $\mathcal{K} = \text{diag}\left\{\overbrace{I, \dots, I}^{15}, t\mathcal{K}_1n, q\mathcal{K}_2h, \mathcal{K}_3x, 7\mathcal{K}_4C, ; I\right\}$ and $\mathcal{K}^T$,

$$\begin{aligned} \hat{\Omega}_2 &= [\bar{\Phi}_1^T P_i \quad \bar{\Phi}_3^T P_i \quad \bar{\eta}\bar{\Phi}_2^T P_i \quad \bar{\eta}\bar{\Phi}_3^T P_i \quad \eta_m\bar{\Phi}_2^T P_i \quad \eta_m\bar{\Phi}_3^T P_i \quad \zeta_m\bar{\Phi}_2^T P_i \quad \zeta_m\bar{\Phi}_3^T P_i \quad d_m\bar{\Phi}_2^T P_i \quad d_m\bar{\Phi}_3^T P_i \quad \Gamma^T], \\ \hat{\Omega}_3 &= \text{diag}\{-P_i, -P_i, -P_iR_1^{-1}P_i, -P_iR_1^{-1}P_i, -P_iR_2^{-1}P_i, -P_iR_2^{-1}P_i, -P_iR_3^{-1}P_i, -P_iR_3^{-1}P_i, -P_iR_4^{-1}P_i, -P_iR_4^{-1}P_i, -I\}. \end{aligned} \quad (56)$$

Partition the matrix P_i as $P_i = \begin{bmatrix} P_{1i} & P_{2i} \\ P_{2i}^T & P_{3i} \end{bmatrix}$ and define $X_i = P_{2i}P_{3i}^{-1}P_{2i}^T$, $J_i = \begin{bmatrix} I & 0 \\ 0 & P_{2i}^T P_{3i}^{-1} \end{bmatrix}$.

Since $P_i > 0$, by Schur Complement, it follows that $P_{1i} > 0$ and $P_{1i} - X_i > 0$.

Define

$$\mathcal{J} = \text{diag}\left\{\overbrace{J_i, \dots, J_i}^8, \overbrace{I, \dots, I}^5, \overbrace{J_i, \dots, J_i}^{10}, I\right\}. \quad (57)$$

Multiply $\mathcal{J}$ and $\mathcal{J}^T$ on both sides of (55), respectively, (47) is obtained by defining

$$\begin{aligned} \tilde{P}_i &= J_i P_i J_i^T = \begin{bmatrix} P_{1i} & X_i \\ X_i & P_{3i} \end{bmatrix}, \\ \tilde{Q}_s &= J_i Q_s J_i, \tilde{R}_s = J_i R_s J_i \quad (s = 1, 2, 3, 4), \\ \begin{cases} \hat{A}_{fi} &= P_{2i} A_{fi} P_{3i}^{-1} P_{2i}^T, \\ \hat{B}_{fi} &= P_{2i}^T B_{fi}, \\ \hat{C}_{fi} &= C_{fi} P_{3i}^{-1} P_{2i}^T. \end{cases} \end{aligned} \quad (58)$$

On the basis of the linear transformation above and according to Lemma 2, $-P_i R_s^{-1} P_i$ can be replaced as $-2\tilde{P}_i + \tilde{R}_s$ ($s = 1, 2, 3, 4$) for simplicity. It can be seen that (40) is equal to (47). This proof is complete.

4. Numerical Simulation

In this part, three examples are presented to verify the effectiveness of the proposed method.

Example 1. Consider the discrete-time system (1) with the relevant parameters as follows:

$$\tilde{\Pi} = \begin{bmatrix} 0.5 & 0.2 \\ 0.3 & 0.6 \end{bmatrix}. \quad (59)$$

in which $\mathcal{K}_1 = \text{diag}\{P_i R_1^{-1}, P_i R_1^{-1}\}$, $\mathcal{K}_2 = \{P_i R_2^{-1}, P_i R_2^{-1}\}$, $\mathcal{K}_3 = \{P_i R_3^{-1}, P_i R_3^{-1}\}$, and $\mathcal{K}_4 = \{P_i R_4^{-1}, P_i R_4^{-1}\}$, one can have

$$\hat{\Sigma} = \mathcal{K}^T \hat{\Sigma} \mathcal{K} = \begin{bmatrix} \bar{\Omega}_1 & \hat{\Omega}_2 \\ * & \hat{\Omega}_3 \end{bmatrix} < 0, \quad (55)$$

with

$$\begin{aligned} \text{Mode 1} \\ A_1 &= \begin{bmatrix} 0.7 & 0 \\ 0 & -0.6 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 & -0.3 \\ -0.1 & 0.2 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 & 0.15 \\ 0.2 & 0.1 \end{bmatrix}, \\ C_1 &= [0.5 \quad 0.3], D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, L_1 = [0.15 \quad 0.3]. \end{aligned} \quad (60)$$

$$\begin{aligned} \text{Mode 2} \\ A_2 &= \begin{bmatrix} -0.5 & 0 \\ 0 & 0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0.2 & -0.1 \\ 0.2 & 0.2 \end{bmatrix}, E_2 = \begin{bmatrix} 0.1 & -0.3 \\ 0.3 & 0.2 \end{bmatrix}, \\ C_2 &= [0.3 \quad -0.2], D_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, L_2 = [-0.1 \quad 0.2]. \end{aligned} \quad (61)$$

Referring to [27], the activation functions of neural network are set as follows:

$$\begin{aligned} f(x) &= \begin{bmatrix} 0.4x_1 - \tanh(0.1x_1 + 0.2x_2) \\ 0.8x_2 - \tanh(0.6x_2) \end{bmatrix}, \\ g(x) &= \begin{bmatrix} 0.5x_1 - \tanh(0.2x_1) + 0.1x_2 \\ 0.7x_2 - \tanh(0.1x_1) + 0.2x_1 \end{bmatrix}. \end{aligned} \quad (62)$$

U_1 , U_2 and V_1 , V_2 conforming to Assumption 1 are acquired easily as follows:

$$\begin{aligned} U_1 &= \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}, U_2 = \begin{bmatrix} 0.4 & 0.2 \\ 0 & 0.8 \end{bmatrix}, \\ V_1 &= \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.7 \end{bmatrix}, V_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.7 \end{bmatrix}. \end{aligned} \quad (63)$$

Assuming the sampling period $h = 0.1s$, the event-triggered parameters $\sigma_1 = 0.05$ and $\sigma_2 = 0.1$. Set the maximum delay in communication and deception attack $\zeta_M = 1$ and $d_M = 1$, respectively, the upper and lower bounds of

neural network delay $\eta_M = 4$, $\eta_m = 1$, and H_∞ performance level is $\gamma = 5$ referring to Table 1.

We set some uncertain parameters to simulate with the initial states are assumed as $x(0) = [0, 0.8]^T$, $\hat{x}(0) = [-0.1, 0.2]^T$, the delay of neural network $\eta(k) = 4|\sin(2\pi k)|$, and the communication delay and deception attacks delay are considered as $\zeta(k) = |\sin(4\pi k)|$ and $d(k) = |\sin(2\pi k)|$. The exogenous disturbance is

$$\omega(k) = \begin{bmatrix} 0.4e^{-0.2k} \cos(0.6k) \\ 0.4e^{-0.15k} \cos(0.4k) \end{bmatrix}. \quad (64)$$

It can be proved that the occurring probability of the missing measurements is bound to influence the H_∞ performance of the system. Table 1 lists the obtained minimal allowable H_∞ performance level γ when changing the Bernoulli distribution probability ρ . One can see that the smaller ρ , the bigger is the H_∞ performance index γ .

The function of deception attacks $a(k) = \tanh(0.5x_1(k) + 0.2x_2(k))$. According to Assumption 1, it satisfies $\|a(k)\|_2 \leq \|Wx(k)\|_2$, where $W = \begin{bmatrix} 0.5 & 0 \\ 0 & 1 \end{bmatrix}$.

Next, the influence of different missing measurements probabilities on the designed H_∞ filter is discussed.

Case 1. Set the Bernoulli distribution parameter $\rho = 1$, which implies that the measured output is conveyed to the filter smoothly. The matrices of filter parameters and event-trigger parameters Ψ_i are acquired as follows:

$$\begin{aligned} A_{f1} &= \begin{bmatrix} 0.6799 & -0.2231 \\ -0.1039 & 0.1291 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.1356 \\ 0.5154 \end{bmatrix}, \\ C_{f1} &= [-0.3346 \quad -0.5468], \Psi_1 = 9.2731, \\ A_{f2} &= \begin{bmatrix} -0.1572 & 0.1410 \\ 0.7812 & -1.0439 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.3482 \\ 0.7943 \end{bmatrix}, \\ C_{f2} &= [0.1640 \quad -0.4777], \Psi_2 = 15.5416. \end{aligned} \quad (65)$$

Figure 1 presents the responses of output to be estimated $z(k)$ and the output of the filter $z_f(k)$. Corresponding filtering error $\bar{z}(k)$ is shown in Figure 2, from which we can get the conclusion that the filter performs well even when the system suffers from the exogenous deception attacks. The releasing instants and its interval of ETM are displayed in Figure 3. In the simulation time, 49.01% of all the sampled signals are triggered in this period.

Case 2. Set the Bernoulli distribution parameter $\rho = 0.6$; it is easily obtained the following matrices.

$$\begin{aligned} A_{f1} &= \begin{bmatrix} 0.3799 & -0.9141 \\ 0.4039 & 0.3291 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.4208 \\ 0.3154 \end{bmatrix}, \\ C_{f1} &= [-0.3255 \quad -0.4962], \Psi_1 = 7.5949, \\ A_{f2} &= \begin{bmatrix} -0.1572 & 0.5463 \\ 1.0527 & 0.6634 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.2325 \\ 0.6743 \end{bmatrix}, \\ C_{f2} &= [-0.2532 \quad -0.6764], \Psi_2 = 13.0506. \end{aligned} \quad (66)$$

As shown in Figure 4, the jump of Bernoulli distribution reflects the state of randomly occurring packet dropout. Figure 5 depicts the responses of output to be estimated $z(k)$ and the actual output of the filter $z_f(k)$, while the filtering error $\bar{z}(k)$ is shown in Figure 6. This proposed method can still work well when the system suffers from deception attacks and partial measurements missing according to the simulations. Figure 7 displays the data releasing instants and intervals, and 42.15% of all the sampled signals are triggered in the simulated period. It reveals the fact that the method proposed in this paper reduces the frequency of data transmission and degrades the waste of network communication resources effectively.

Example 2. Consider the system (1) with following parameters given in [11]:

Mode 1

$$\begin{aligned} A_1 &= \begin{bmatrix} 5.1 & 0 \\ 0 & 4.7 \end{bmatrix}, B_1 = \begin{bmatrix} 1.1 & -0.7 \\ 0.9 & 1.2 \end{bmatrix}, E_1 = \begin{bmatrix} 1.2 & 0.6 \\ 0.8 & 1 \end{bmatrix}, \\ C_1 &= [0.9 \quad 0.8], D_1 = [0.1 \quad 0.2], L_1 = [0.2 \quad 0]. \end{aligned} \quad (67)$$

Mode 2

$$\begin{aligned} A_2 &= \begin{bmatrix} 1.1 & 0 \\ 0 & 1.2 \end{bmatrix}, B_2 = \begin{bmatrix} -0.8 & 0.9 \\ 0.9 & 0.8 \end{bmatrix}, E_2 = \begin{bmatrix} 0.6 & 0.6 \\ 0.65 & 0.6 \end{bmatrix}, \\ C_2 &= [0.9 \quad 0.8], D_2 = [0.1 \quad 0.2], L_2 = [0.2 \quad 0]. \end{aligned} \quad (68)$$

Set $\rho = 1$ and $W = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, which means that the missing measurements and deception attacks are not considered temporarily.

The neuron activation functions and external disturbances are given:

$$\begin{aligned} f(x) &= g(x) = \begin{bmatrix} 0.5x_1 - \tanh(0.5x_1 + 0.2x_2) \\ 0.95x_2 - \tanh(0.75x_2) \end{bmatrix}, \\ \omega(k) &= \begin{cases} 0.5, & k \in [5, 10), \\ -0.5, & k \in [15, 20), \\ 0, & \text{else.} \end{cases} \end{aligned} \quad (69)$$

TABLE 1: The $\gamma_{\min}$ for different ϱ ($\eta_m = 1$, $\eta_M = 4$, $\zeta_M = 1$, $d_M = 1$, $\sigma_1 = 0.05$, $\sigma_2 = 0.1$).

ϱ	1	0.9	0.8	0.7	0.5
$\gamma_{\min}$	1.4882	1.6139	1.8931	2.1770	4.0975

FIGURE 1: $z(k)$ and $z_f(k)$ in Case 1.

FIGURE 2: The filtering error in Case 1.

For given parameters $\gamma = 5$, $\zeta_M = 0.1$, $\eta_M = 0.2$, $\eta_m = 0.1$, and $\sigma_1 = \sigma_2 = 0.1$, we can obtain the corresponding matrices of the filter as follows:

$$\begin{aligned}
 A_{f1} &= \begin{bmatrix} 0.2948 & -0.0564 \\ -0.0324 & 0.3008 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.1508 \\ -0.0854 \end{bmatrix}, \\
 C_{f1} &= [-0.0964 \quad 0.1449], \Psi_1 = 4.6552, \\
 A_{f2} &= \begin{bmatrix} 0.1475 & -0.0303 \\ 0.0014 & 0.1880 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.2204 \\ -0.1870 \end{bmatrix}, \\
 C_{f2} &= [-0.0971 \quad -0.0590], \Psi_2 = 4.1070.
 \end{aligned} \tag{70}$$

Figures 8 and 9 depict the responses of $z(k)$ and $z_f(k)$ under the initial conditions $x(0) = [0.2, -0.1]^T$ and

FIGURE 3: The release instants and their interval in Case 1.

FIGURE 4: The jump of Bernoulli distribution in Case 2.

FIGURE 5: $z(k)$ and $z_f(k)$ in Case 2.

FIGURE 6: The filtering error in Case 2.

FIGURE 9: The filtering error in Example 2.

FIGURE 7: The release instants and their interval in Case 2.

FIGURE 10: The release instants and their interval in Example 2.

FIGURE 8: The response of $z(k)$ and $z_f(k)$ in Example 2.

$\hat{x}(0) = [0.5, 0]^T$. The data releasing instants and their intervals are displayed in Figure 10, and the corresponding triggered times and transmission rates obtained by the method proposed in literature [11] and this paper are listed in Table 2. Obviously, the rate of data transmission in this paper is lower than that in [11], so more communication resources are saved. Besides, the minimum allowable H_∞ performance index $\gamma_{\min} = 0.8482$ is smaller than $\gamma_{\min} = 1.2247$ in [11], which indicates that, by comparison with [11], the method in this paper guarantees the system stability with better H_∞ performance.

Example 3. In this example, the numerical verification will be strengthened by clarifying the physical meaning of the system under the test. Referring to [44, 45], a synthetic genetic regulatory network is provided to demonstrate the application of the proposed approach. The biological network is described as

TABLE 2: The transmission rates and $\gamma_{\min}$.

	This paper	Literature [11]
Triggered times (v)	36	47
Transmission rate (v/N)	35.3%	47%
$\gamma_{\min}$	0.8492	1.2247

TABLE 3: Parameters with different modes.

Parameters	A_{1i}	A_{2i}	A_{3i}	B_{2i}	E_{2i}
$i = 1$	0.2I	0.09I	0.1I	$-0.5\mathcal{X}$	$-0.2\mathcal{X}$
$i = 2$	0.1I	0.08I	0.09I	$-0.8\mathcal{X}$	$-0.1\mathcal{X}$

$$\begin{cases} m(k+1) = A_1(r_k)m(k) + B_1(r_k)f(p(k)) \\ + E_1(r_k)g(p(k - \eta(k))) + D_1(r_k)\omega(k), \\ p(k+1) = A_2(r_k)p(k) + A_3(r_k)m(k) + D_2(r_k)\omega(k), \end{cases} \quad (71)$$

where $m(k)$ and $p(k)$ denote the concentrations of mRNA and protein, $f(k)$ represents the feedback regulation of the protein, $A_1(r_k)$, $A_2(r_k)$, and $A_3(r_k)$ stand for the decay rates of mRNA, decay rates of protein, and the translation rate, respectively. $B_1(r_k)$ and $E_1(r_k)$ are the coupling matrices of the genetic network. Define

$$x(k+1) = \begin{bmatrix} m(k+1) \\ p(k+1) \end{bmatrix}, f(x(k)) = \begin{bmatrix} f(m(k)) \\ f(p(k)) \end{bmatrix}. \quad (72)$$

According to the form of (1), the following matrices can be obtained:

$$A_i = \begin{bmatrix} A_{1i} & 0 \\ A_{2i} & A_{3i} \end{bmatrix}, B_i = \begin{bmatrix} 0 & B_{2i} \\ 0 & 0 \end{bmatrix}, E_i = \begin{bmatrix} 0 & E_{2i} \\ 0 & 0 \end{bmatrix}, D_i = \begin{bmatrix} D_{1i} \\ D_{2i} \end{bmatrix}. \quad (73)$$

The parameters cited from [45] are listed in Table 3.

$$\begin{aligned} \mathcal{X} &= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} -0.7 & 0 & 0.5 & 0 & 0.6 & 0 \\ 0 & 0.2 & 0.3 & 0 & 0 & 0.1 \end{bmatrix}^T, \\ C_1 &= [1 \ 0 \ -1 \ 0 \ 0 \ 1], L_1 = [1 \ 1 \ 0 \ 0 \ 1 \ 0], \\ D_2 &= \begin{bmatrix} 0.3 & 0 & -0.5 & 0.5 & 0 & 0 \\ 0 & 0.3 & 0 & 0.5 & 0.2 & 0 \end{bmatrix}^T, \\ C_2 &= [0.3 \ 0 \ -0.5 \ 0.5 \ 0 \ 0], L_2 = [1 \ 0 \ 0 \ 1 \ 0 \ 0]. \end{aligned} \quad (74)$$

As discussed in [45], the regulation function is set as $f(x) = x^2/(1+x^2)$, $\eta_M = 5$, $\eta_m = 1$, $\sigma_1 = \sigma_2 = 0.01$, and $\gamma = 1$, and the transition probability matrix is given as

$$\tilde{\Pi} = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}. \quad (75)$$

Set $\varrho = 1$, and the constraint matrix of deception attacks is $W = \text{diag}\{1, 0.5, 0.6, 0.5, 1, 0.5\}$. The desired H_∞ filter parameters are obtained:

$$A_{f1} = \begin{bmatrix} 0.0745 & -0.0404 & -0.0503 & -0.0080 & -0.0213 & -0.0102 \\ 0.0079 & 0.0207 & -0.0171 & -0.0023 & -0.0046 & -0.0011 \\ -0.0303 & 0.0195 & 0.0516 & 0.0002 & 0.0018 & 0.0015 \\ -0.0469 & 0.0478 & 0.0327 & 0.0510 & 0.0197 & 0.0109 \\ 0.1070 & -0.0860 & -0.0917 & -0.0158 & 0.0013 & -0.0230 \\ 0.0346 & -0.0360 & 0.0185 & -0.0163 & -0.0172 & 0.0318 \end{bmatrix}, \quad (76)$$

$$A_{f2} = \begin{bmatrix} 0.0561 & 0.0474 & 0.0026 & 0.0122 & 0.0227 & 0.0040 \\ 0.0093 & 0.0513 & -0.0073 & -0.0025 & -0.0024 & 0.0007 \\ 0.0048 & -0.0302 & 0.0412 & -0.0090 & -0.0238 & -0.0045 \\ -0.0317 & -0.0740 & 0.0155 & 0.0037 & -0.0493 & -0.0191 \\ 0.0150 & 0.0383 & -0.0065 & 0.0291 & 0.0793 & 0.0097 \\ -0.0359 & -0.0877 & 0.0193 & -0.0170 & -0.0331 & 0.0411 \end{bmatrix},$$

$$B_{f1} = \begin{bmatrix} 0.5516 \\ -0.6524 \\ -0.4486 \\ -0.6260 \\ 0.7289 \\ 0.6551 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.5119 \\ -0.4167 \\ 0.4677 \\ 0.6712 \\ -0.6903 \\ -0.5975 \end{bmatrix}, \quad (77)$$

$$C_{f1} = [-0.8350 \ -0.8461 \ 0.8040 \ -0.8107 \ -0.8275 \ -0.7850],$$

$$C_{f2} = [-0.9312 \ 1.0065 \ 0.8115 \ -0.9155 \ -1.0017 \ -1.0012],$$

$$\Psi_1 = 96.5800, \Psi_2 = 395.2375.$$

FIGURE 11: The response of $z(k)$ and $z_f(k)$ in Example 3.

FIGURE 12: The filtering error in Example 3.

In the simulation, the initial conditions are assumed as $x(0) = [-0.2, 0.1, -0.3, 0.2, -0.1, 0.2]^T$ and $\hat{x}(0) = [-0.1, -0.2, 0.1, -0.1, 0, -0.1]^T$, and the disturbance is $\omega(k) = [0.4e^{-0.2k} \cos(0.6k), 0.4e^{-0.15k} \cos(0.4k)]^T$. Based on these parameters, the responses of the H_∞ filter and the filtering error are shown in Figures 11 and 12, respectively.

Remark 6. By reviewing the existing research achievements on the filter design of MJNN, the dissipative filtering of MJNN under deception attacks and incomplete measurements is discussed in paper [46]. The incompleteness of actual measurements considered in [46] is dominated by randomly occurring deception attacks. In contrast, this paper considers a more complex and practical network environment including the stochastic packet dropouts due to the limited communication bandwidth and the malicious attack signals injected by attackers through the communication network. Furthermore, this paper discusses in detail the influence of changing the probability of packet dropouts

on system performance and further demonstrates the effectiveness of the proposed method.

5. Conclusion

The issue of event-triggered H_∞ filtering for the discrete-time MJNNs under deception attacks and randomly occurring missing measurements has been investigated in this work. In order to be closer to the real network communication environment, this paper introduces deception attacks and randomly occurring missing measurements. The event-triggered communication strategy is proposed to deal with the problems caused by frequent information exchange. According to the judgement algorithm, the effective sampling data is transmitted, and the purpose of maintaining system performance and saving network resources is achieved. Based on the mathematical model, sufficient conditions are given to guarantee that the filtering augmented system is asymptotically stable. Finally, three numerical simulations are provided to clarify the influence of changing the probability of missing measurements on the system performance, the advantage of event-triggered strategy, and the physical meaning of system under test, respectively. In the next research, we will focus on the detection of cyber-attacks based on the existing research.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants 62073296 and 61374083 and Zhejiang Province Natural Science Foundation of China under Grants LY20F030015 and LQ19F030014.

References

- [1] H. Zhang, Z. Wang, and D. Liu, "A comprehensive review of stability analysis of continuous-time recurrent neural networks," *IEEE Transactions on Neural Networks & Learning Systems*, vol. 25, no. 7, pp. 1229–1262, 2017.
- [2] X.-M. Zhang, Q.-L. Han, X. Ge, and D. Ding, "An overview of recent developments in Lyapunov-Krasovskii functionals and stability criteria for recurrent neural networks with time-varying delays," *Neurocomputing*, vol. 313, no. 3, pp. 392–401, 2018.
- [3] X. Wang, X. Li, Q. Wu, and X. Yin, "Neural network based adaptive dynamic surface control of nonaffine nonlinear systems with time delay and input hysteresis nonlinearities," *Neurocomputing*, vol. 333, pp. 53–63, 2019.
- [4] X. Shi, Y. Cheng, C. Yin, X. Huang, and S.-M. Zhong, "Design of adaptive backstepping dynamic surface control method with RBF neural network for uncertain nonlinear system," *Neurocomputing*, vol. 330, pp. 490–503, 2019.

- [5] Z. Wang, Y. Liu, L. Yu, and X. Liu, "Exponential stability of delayed recurrent neural networks with Markovian jumping parameters," *Physics Letters A*, vol. 356, no. 4-5, pp. 346–352, 2006.
- [6] P. Shi, Y. Zhang, and R. K. Agarwal, "Stochastic finite-time state estimation for discrete time-delay neural networks with Markovian jumps," *Neurocomputing*, vol. 151, no. 1, pp. 168–174, 2015.
- [7] Z. Wu, H. Su, and J. Chu, "State estimation for discrete Markovian jumping neural networks with time delay," *Neurocomputing*, vol. 73, no. 10–12, pp. 2247–2254, 2010.
- [8] W.-J. Lin, Y. He, M. Wu, and Q. Liu, "Reachable set estimation for Markovian jump neural networks with time-varying delay," *Neural Networks*, vol. 108, pp. 527–532, 2018.
- [9] F. Li, S. Song, J. Zhao, S. Xu, and Z. Zhang, "Synchronization control for Markov jump neural networks subject to HMM observation and partially known detection probabilities," *Applied Mathematics and Computation*, vol. 360, pp. 1–13, 2019.
- [10] M. Dai, J. Xia, H. Xia, and H. Shen, "Event-triggered passive synchronization for Markov jump neural networks subject to randomly occurring gain variations," *Neurocomputing*, vol. 331, pp. 403–411, 2019.
- [11] R. Rakkiyappan, K. Maheswari, G. Velmurugan, and J. H. Park, "Event-triggered H_{∞} state estimation for semi-Markov jumping discrete-time neural networks with quantization," *Neural Networks*, vol. 105, pp. 236–248, 2018.
- [12] R. Vadivel, M. Syed Ali, and F. Alzahrani, "Robust H_{∞} synchronization of Markov jump stochastic uncertain neural networks with decentralized event-triggered mechanism," *Chinese Journal of Physics*, vol. 60, pp. 68–87, 2019.
- [13] H. Wang, D. Zhang, and R. Lu, "Event-triggered H_{∞} filter design for Markovian jump systems with quantization," *Nonlinear Analysis: Hybrid Systems*, vol. 28, pp. 23–41, 2018.
- [14] Y. Xu, Y. Wang, G. Zhuang, Y. Wang, and J. Lu, "An event-triggered asynchronous H_{∞} filtering for singular Markov jump systems with redundant channels," *Journal of the Franklin Institute*, vol. 356, no. 16, pp. 10076–10101, 2019.
- [15] Y. Tan, S. Fei, J. Liu, and D. Zhang, "Asynchronous adaptive event-triggered tracking control for multi-agent systems with stochastic actuator faults," *Applied Mathematics and Computation*, vol. 355, pp. 482–496, 2019.
- [16] Y. Wang, G. Song, J. Zhao, J. Sun, and G. Zhuang, "Reliable mixed H_{∞} and passive control for networked control systems under adaptive event-triggered scheme with actuator faults and randomly occurring nonlinear perturbations," *ISA Transactions*, vol. 89, pp. 45–57, 2019.
- [17] L. Zha, E. Tian, X. Xie, Z. Gu, and J. Cao, "Decentralized event-triggered H_{∞} control for neural networks subject to cyber-attacks," *Information Sciences*, vol. 457–458, no. 458, pp. 141–155, 2018.
- [18] Y. Qi, Z. Cao, and X. Li, "Decentralized event-triggered H_{∞} control for switched systems with network communication delay," *Journal of the Franklin Institute*, vol. 356, no. 3, pp. 1424–1445, 2019.
- [19] S. Senan, M. Syed Ali, R. Vadivel, and S. Arik, "Decentralized event-triggered synchronization of uncertain Markovian jumping neutral-type neural networks with mixed delays," *Neural Networks*, vol. 86, pp. 32–41, 2017.
- [20] S. Li, Y. Hu, L. Zheng et al., "Stochastic event-triggered cubature Kalman filter for power system dynamic state estimation," *IEEE Transactions on Circuits & Systems II Express Briefs*, vol. 66, no. 9, pp. 1552–1556, 2018.
- [21] S. Li, Z. Li, J. Li et al., "Application of event-triggered cubature Kalman filter for remote nonlinear state estimation in wireless sensor network," *IEEE Transactions on Industrial Electronics*, vol. 66, p. 1, 2020.
- [22] M. Ma, J. Gao, J. Wang et al., "Hybrid-driven mechanism based on uncertain network for markov jump system with quantizations and delay," *Complexity*, vol. 66, 2020.
- [23] J. Liu, J. Xia, E. Tian, and S. Fei, "Hybrid-driven-based H_{∞} filter design for neural networks subject to deception attacks," *Applied Mathematics and Computation*, vol. 320, pp. 158–174, 2018.
- [24] J. Yu, C. Yang, X. Tang, and P. Wang, " H_{∞} control for uncertain linear system over networks with Bernoulli data dropout and actuator saturation," *ISA Transactions*, vol. 74, pp. 1–13, 2018.
- [25] J. Hu, Z. Wang, F. E. Alsaadi, and T. Hayat, "Event-based filtering for time-varying nonlinear systems subject to multiple missing measurements with uncertain missing probabilities," *Information Fusion*, vol. 38, pp. 74–83, 2017.
- [26] L. Su and D. Ye, "Observer-based output feedback H_{∞} control for cyber-physical systems under randomly occurring packet dropout and periodic DoS attacks," *ISA Transactions*, vol. 95, pp. 58–67, 2019.
- [27] H. Wang and A. Xue, "Adaptive event-triggered H_{∞} filtering for discrete-time delayed neural networks with randomly occurring missing measurements," *Signal Processing*, vol. 153, pp. 221–230, 2018.
- [28] D. E. Quevedo and D. Nešić, "Robust stability of packetized predictive control of nonlinear systems with disturbances and Markovian packet losses," *Automatica*, vol. 48, no. 8, pp. 1803–1811, 2012.
- [29] X. Tang, L. Deng, J. Yu, and H. Qu, "Output feedback predictive control of interval type-2 T-S fuzzy systems with markovian packet loss," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 4, pp. 2450–2459, 2018.
- [30] A. Elahi and A. Alfi, "Stochastic H_{∞} finite-time control of networked cascade control systems under limited channels, network delays and packet dropouts," *ISA Transactions*, vol. 97, pp. 352–364, 2020.
- [31] Z. Li and J. Zhao, "Resilient adaptive control of switched nonlinear cyber-physical systems under uncertain deception attacks," *Information Sciences*, vol. 543, pp. 398–409, 2021.
- [32] J. Wu, C. Peng, J. Zhang, and B.-L. Zhang, "Event-triggered finite-time H_{∞} filtering for networked systems under deception attacks," *Journal of the Franklin Institute*, vol. 357, no. 6, pp. 3792–3808, 2020.
- [33] H. Yuan and Y. Xia, "Secure filtering for stochastic non-linear systems under multiple missing measurements and deception attacks," *IET Control Theory & Applications*, vol. 12, no. 4, pp. 515–523, 2018.
- [34] Z. Gu, X. Zhou, T. Zhang et al., "Event-triggered filter design for nonlinear cyber-physical systems subject to deception attacks," *ISA Transactions*, vol. 104, pp. 130–137, 2019.
- [35] D. Wang, Z. Wang, B. Shen, and F. E. Alsaadi, "Security-guaranteed filtering for discrete-time stochastic delayed systems with randomly occurring sensor saturations and deception attacks," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 7, pp. 1194–1208, 2017.
- [36] J. Liu, J. Xia, J. Cao, and E. Tian, "Quantized state estimation for neural networks with cyber attacks and hybrid triggered communication scheme," *Neurocomputing*, vol. 291, pp. 35–49, 2018.
- [37] B. Chen, Y. Niu, and Y. Zou, "Security control for Markov jump system with adversarial attacks and unknown transition

- rates via adaptive sliding mode technique,” *Journal of the Franklin Institute*, vol. 356, no. 6, pp. 3333–3352, 2019.
- [38] S. Li, L. Li, Z. Li et al., “Event-trigger heterogeneous nonlinear filter for wide-area measurement systems in power grid,” *IEEE Transactions on Smart Grid*, vol. 10, no. 3, pp. 2752–2764, 2019.
 - [39] X. Liu, L. Li, Z. Li et al., “Event-trigger particle filter for smart grids with limited communication bandwidth infrastructure,” *IEEE Transactions on Smart Grid*, vol. 9, no. 6, pp. 6918–6928, 2018.
 - [40] D. Yue, E. Tian, and Q.-L. Han, “A delay system method for designing event-triggered controllers of networked control systems,” *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 475–481, 2013.
 - [41] N. Li and J. Hu, “Exponential state estimation for delayed recurrent neural networks with sampled-data,” *Nonlinear Dynamics*, vol. 69, no. 1-2, pp. 555–564, 2012.
 - [42] X. Zhu and G. Yang, “Jensen inequality approach to stability analysis of discrete-time systems with time-varying delay,” 2008.
 - [43] J. Xiong and J. Lam, “Stabilization of networked control systems with a logic ZOH,” *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 358–363, 2009.
 - [44] H. Shen, Y. Zhu, L. Zhang, and J. H. Park, “Extended dissipative state estimation for Markov jump neural networks with unreliable links,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 2, pp. 346–358, 2017.
 - [45] W. Xia, S. Xu, J. Lu, Z. Zhang, and Y. Chu, “Reliable filter design for discrete-time neural networks with Markovian jumping parameters and time-varying delay,” *Journal of the Franklin Institute*, vol. 357, no. 5, pp. 2892–2915, 2020.
 - [46] Y. Xu, Z.-G. Wu, and Y.-J. Pan, “Event-based dissipative filtering of Markovian jump neural networks subject to incomplete measurements and stochastic cyber-attacks,” *IEEE Transactions on Cybernetics*, vol. 99, pp. 1–10, 2019.