Research Article

Construction of Nonsinusoidal Oscillation Waveform Function and Technological Parameters for Continuous Casting Mold

Chao Zhou ^{1,2} Xingzhong Zhang ² Fang Wang ³ and Subo Ren²

¹Ocean College, Hebei Agricultural University, Qinhuangdao 066003, China

²National Engineering Research Center for Equipment and Technology of Cold Strip Rolling, Yanshan University, Qinhuangdao 066004, China

³School of Science, Yanshan University, Qinhuangdao 066004, China

Correspondence should be addressed to Xingzhong Zhang; zhangxzh@ysu.edu.cn and Fang Wang; wangfang@ysu.edu.cn

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Nonsinusoidal oscillation techniques can shorten the negative strip time and improve the slab surface quality in the process of steel continuous casting. But the acceleration of nonsinusoidal oscillation is higher than that of the sinusoidal oscillation. So it is easy to result in the impact for the mold movement. To solve this problem, a nonsinusoidal oscillation waveform function of mold was constructed, which had good dynamic characteristics. And the smaller acceleration can reduce the impact of the oscillator. By analyzing the parameters of oscillation technology, the calculation method of each oscillation technological parameter for seven section functions was presented and the multitechnological parameter curves were given. Based on the technological parameters of sinusoidal oscillation, a synchronous control model of nonsinusoidal oscillation was determined. The results show that the parameters can satisfy the limit value in theory, which can enhance the casting speed, surface quality of slab, and provide reference for further practice in industry.

1. Introduction

Mold oscillation plays an important role in continuous casting. It decides whether the continuous casting progress is smooth or not. The velocity rules of mold oscillation went through rectangular, trapezoid, sinusoidal, and nonsinusoidal oscillation. With the demand growing for the surface quality and casting speed of slab, nonsinusoidal oscillation has been one of the key technologies to realize highly efficient continuous casting [1–4].

Nowadays, mold oscillation technologies used in production mainly include sinusoidal and nonsinusoidal oscillations. Sinusoidal oscillation could reduce negative strip time and improve surface quality of slab when high frequency and small amplitude operation are adopted [5, 6]. Lin et al. [7] optimized the sinusoidal oscillation parameters for Nanjing steel, the lubrication between the mold and slab had been improved, the demoulding resistance was reduced, the hook-type oscillation mark was eliminated, and the slab

quality was enhanced. Wang et al. [8] found that the hook-type oscillation marks of slabs could be reduced significantly by increasing frequency and reducing amplitude. Although sinusoidal oscillation can reduce negative strip time, the positive strip time also reduces. In contrast, at the time of reducing negative strip time for nonsinusoidal oscillation, positive strip time can be prolonged which is useful for mold powder consuming. Nonsinusoidal oscillation can obtain reasonable technological parameters and is an ideal oscillation mode for mold [9]. The research of nonsinusoidal oscillation is mainly focused on oscillation waveform, realization way, and parameter control. The waveform functions of nonsinusoidal oscillation mainly contain entire function and piecewise function. An oscillation waveform of entire function was developed by Suzuki et al. [10], which had been used in many iron and steel enterprises. The principle of nonsinusoidal oscillation was studied by means of the rotation vector method, and Li et al. proposed an oscillation waveform which could be realized by using a

couple of elliptical gears [11], which had been applied in many companies and achieved good results. Zhang et al. [12, 13] proposed a nonsinusoidal oscillation waveform composed of two sinusoidal functions, which could be realized by double eccentric shafts. Liu et al. presented the entire function of nonsinusoidal oscillation waveform which could be realized by noncircular gears [14]. Hong et al. [15] proposed an oscillation waveform function composed of Fourier series. Although the entire function is easy to be constructed, the values of basic parameters have a certain limit. An oscillation waveform function consisting of three sections of sine trigonometric functions was proposed. The displacement and velocity curves are continuous, but the acceleration curves are discontinuous. For this oscillation waveform function, the oscillator would not move smoothly and vibratory shock should occur [16]. Meng and Zhu [17] and Li et al. [18] constructed a nonsinusoidal oscillation waveform of piecewise function. The displacement, velocity, and acceleration curves are continuous. However, the maximum acceleration is larger and it has bigger impact for the oscillator when the mold is at the uppermost point and lowest point. Then, an oscillation waveform function was proposed in this paper, which has lower maximum acceleration and good dynamic characteristic.

Compared with sinusoidal oscillation, nonsinusoidal oscillation has the advantages of shorter negative strip time, longer positive strip time, smaller positive strip velocity, and larger negative strip distance. However, the acceleration and

inertia force of nonsinusoidal oscillation are larger, which will result in the impact and make against to the normal operation of oscillator. The inertia force of nonsinusoidal oscillation is mainly decided by waveform function. In order to obtain better dynamic characteristic of oscillation waveform, a nonsinusoidal oscillation waveform composed of seven section functions was proposed, which had lower acceleration and inertia force. Based on nonsinusoidal oscillation waveform of seven section functions, the calculation methods of technological parameters were given, the multitechnological parameter curve was obtained, and the synchronous control model of casting speed and oscillation frequency was determined. The calculation results indicate that the oscillation waveform function proposed in this paper will be useful to further improve the quality of slab and highly efficient continuous casting.

2. Construction of Nonsinusoidal Oscillation Waveform

The lower acceleration for oscillator results in smaller inertia force. The velocity function of nonsinusoidal oscillation waveform is constructed, which is composed of seven sections, shown in Figure 1. From Figure 1, it can be seen that AB , CD , FG , and HK are straight lines and BC , DEF , and GH are parabolas. The functions of oscillation waveform are as follows.

The velocity function is

$$v = \begin{cases} v_B, & 0 \leq t \leq t_B, \\ -k(t - t_B)^2 + v_B, & t_B \leq t \leq t_C, \\ -2k(t_C - t_B)(t - t_C), & t_C \leq t \leq t_D, \\ k\left(t - \frac{1}{2f}\right)^2 - v_B - 2k(t_C - t_B)\left(\frac{1}{2f} + t_B - 2t_C\right), & t_D \leq t \leq t_F, \\ 2k(t_C - t_B)\left(t + t_C - \frac{1}{f}\right), & t_F \leq t \leq t_G, \\ -k\left(t + t_B - \frac{1}{f}\right)^2 + v_B, & t_G \leq t \leq t_H, \\ v_B, & t_H \leq t \leq t_K, \end{cases} \quad (1)$$

where v is the velocity of mold, mm/s, h is the oscillation amplitude, mm, f is the oscillation frequency, Hz, and α is the waveform modification ratio of nonsinusoidal oscillation. t_C , t_D , t_F , and t_G are the time periods of different

positions for mold, s, v_B is the velocity of point B , $t_F = (1/f) - t_D$, $t_G = (1/f) - t_C$, and $t_H = (1/f) - t_B$, $v_B = k(t_C - t_B)^2$, $k = (3h)/[3t_C(t_C - t_B)^2 - (t_C - t_B)^3]$, $t_B = \alpha/[f(1 + \alpha)]$, $t_C = (1 + \alpha)/(4f)$, and $t_D = 1/(2f) + t_B - t_C$.

The displacement function is

$$s = \begin{cases} v_B t, & 0 \leq t \leq t_B, \\ \frac{k}{3}(t - t_B)^3 + v_B t, & t_B \leq t \leq t_C, \\ -k(t_C - t_B)(t - t_C)^2 + c_1, & t_C \leq t \leq t_D, \\ \frac{k}{3}\left(t - \frac{1}{2f}\right)^3 - v_B t - 2kt(t_C - t_B)\left(\frac{1}{2f} + t_B - 2t_C\right) + c_2, & t_D \leq t \leq t_F, \\ k(t_C - t_B)\left[t - \left(\frac{1}{f} - t_C\right)\right]^2 + c_3, & t_F \leq t \leq t_G, \\ -\frac{k}{3}\left[t - \left(\frac{1}{f} - t_B\right)\right]^3 + v_B t + c_4, & t_G \leq t \leq t_H, \\ v_B\left(t - \frac{1}{f}\right), & t_H \leq t \leq t_K, \end{cases} \quad (2)$$

where s is the displacement of mold, mm, $c_1 = v_B t_C - (k/3)(t_C - t_B)^3 = (v_B/3)(t_B + 2t_C)$, $c_2 = (v_B/2f) + (k/f)(t_C - t_B)((1/2f) + t_B - 2t_C)$, $c_3 = -(v_B/3)(t_B + 2t_C)$, and $c_4 = -(v_B/f)$.

The acceleration function is

$$a = \begin{cases} 0, & 0 \leq t \leq t_B, \\ -2k(t - t_B), & t_B \leq t \leq t_C, \\ -2k(t_C - t_B), & t_C \leq t \leq t_D, \\ 2k\left(t - \frac{1}{2f}\right), & t_D \leq t \leq t_F, \\ 2k(t_C - t_B), & t_F \leq t \leq t_G, \\ -2k\left(t + t_B - \frac{1}{f}\right), & t_G \leq t \leq t_H, \\ 0, & t_H \leq t \leq t_K, \end{cases} \quad (3)$$

where a is the acceleration, mm/s².

When h equals 4 mm and f equals 2 Hz, the oscillation waveform curves are shown in Figure 2 with different waveform modification ratios. From Figure 2, it can be found that the velocity curve is smooth and continuous. So there will be no rigid vibration impact occurring. The acceleration curve is continuous without the value sudden change. Thus, there will be no flexible vibration impact occurred. The maximum acceleration is lower compared

with oscillation waveform of five sections with the same basic parameters, shown in Figure 3. Then, the inertia force is smaller. In brief, the oscillation waveform has good dynamic characteristics.

3. Technological Parameters and Synchronous Control Model

Technological parameters decide the production of continuous caster and the surface quality of slab, so it is necessary to analysis them.

3.1. Negative Strip Time Curves. Negative strip time is the period time that the velocity of mold moving downward is faster than casting speed shown in Figure 4. If the negative strip time is longer, the oscillation mark will be deeper:

$$t_N = 2\left(\frac{1}{2f} - t_1\right). \quad (4)$$

For processing,

$$t_1 = \frac{1}{2}\left(\frac{1}{f} - t_N\right). \quad (5)$$

The velocity of mold is v_c at time t_1 . Submitting equation (5) into equation (1) (the third equation),

$$t_N = \frac{1 - \alpha}{2f} - \frac{v_c(1 - \alpha)^2(\alpha^2 + 4\alpha + 1)}{24hf^2(\alpha + 1)^2}, \quad (6)$$

where t_N is the negative strip time, s, and v_c is the casting speed, mm/s.

If $Z = (2h/E)$ (mm · min/m) and $E = (v_c/1000) \times 60$, the relationship between t_N and Z is obtained as

FIGURE 1: Sketch of velocity curve for nonsinusoidal oscillation.

FIGURE 2: Nonsinusoidal oscillation waveform of seven section functions. (a) Displacement curves. (b) Velocity curves. (c) Acceleration curves.

FIGURE 3: Acceleration curves with $h = 4$ mm, $f = 2$ Hz, and $\alpha = 30\%$.

FIGURE 4: Velocity curve for mold oscillation.

$$t_N = \frac{1 - \alpha}{2f} - \frac{25(1 - \alpha)^2(\alpha^2 + 4\alpha + 1)}{18Zf^2(\alpha + 1)^2}, \quad (7)$$

when α equals 20% and Z equals different values; negative strip time curves are shown in Figure 5.

3.2. Negative Strip Ratio. Negative strip ratio is defined as

$$NS = 1 - \frac{4fh}{(1 - \alpha)v_c}. \quad (8)$$

The larger negative strip ratio is helpful for removing the slab from the mold. Submitting (8) into equation (6), the relationship between t_N and NS can be expressed as

FIGURE 5: Negative strip time curves.

FIGURE 6: Multiparameter curves of nonsinusoidal oscillation with $\alpha = 20\%$ and $h = 4$ mm.

$$t_N = \frac{1 - \alpha}{2f} - \frac{(1 - \alpha)(\alpha^2 + 4\alpha + 1)}{6f(\alpha + 1)^2(1 - NS)}. \quad (9)$$

When α equals 20%, NS equals different values, and the change of t_N with f is shown in Figure 5.

3.3. Negative Strip Time Ratio. Negative strip time ratio is the ratio between negative strip time and half an oscillation period:

$$NSR = \frac{t_N}{0.5T} = 2ft_N = 1 - \alpha - \frac{v_c(\alpha - 1)^2(\alpha^2 + 4\alpha + 1)}{12hf(\alpha + 1)^2}, \quad (10)$$

where NSR is the negative strip time ratio, %.

TABLE 1: Caster parameters.

Item	Unit	Value
Slab section size	mm × mm	180 × 1800
Amplitude	mm	±4
Frequency	min ⁻¹	104
Casting speed	m/min	1.15

TABLE 2: Technological parameters of nonsinusoidal oscillation.

v_c (m/min)	f (min ⁻¹)	t_N (s)	NS (%)	NSA (mm)	NSR (%)	t_p (s)	Δv (m/min)
0.3	60	0.357	-300	6.107	71.48	0.643	1.239
0.4	60	0.343	-200	5.523	68.64	0.657	1.339
0.5	66	0.305	-164	5.214	67.09	0.604	1.533
0.6	72	0.274	-140	4.962	65.80	0.559	1.727
0.7	78	0.249	-122.86	4.753	64.71	0.520	1.921
0.8	84	0.228	-110	4.577	63.77	0.487	2.115
0.9	90	0.210	-100	4.426	62.96	0.457	2.309
1.0	96	0.195	-92	4.296	62.25	0.431	2.503
1.1	102	0.181	-85.45	4.182	61.63	0.407	2.697
1.15	105	0.175	-82.61	4.130	61.34	0.396	2.794
1.2	108	0.170	-80	4.081	61.07	0.386	2.890
1.3	114	0.159	-75.38	3.993	60.57	0.367	3.084
1.4	120	0.150	-71.43	3.913	60.12	0.350	3.278
1.5	126	0.142	-68	3.842	59.72	0.334	3.472

Note. $h = 4$ mm; $\alpha = 20\%$.

3.4. Negative Strip Distance. Negative strip distance is the displacement of mold moving downward relative to the slab during the negative strip time of an oscillation period. Then,

NSA (the area, shown in Figure 4) can be calculated as follows:

$$\begin{aligned}
 \text{NSA} &= \int_{t_1}^{t_2} |v_m - v_c| dt = 2 \int_{(T/2)-(T_N/2)}^{T/2} |v_m - v_c| dt \\
 &= 2 \left\{ \int_{(T/2)-(T_N/2)}^{t_D} [2k(t_C - t_B)(t - t_C)] dt + \int_{t_D}^{T/2} \left[v_B + 2k(t_C - t_B) \left(\frac{1}{2f} + t_B - 2t_C \right) - k \left(t - \frac{1}{2f} \right)^2 \right] dt \right\} - v_c t_N \\
 &= 2k(t_C - t_B) \left(\frac{1}{2f} + t_B - 2t_C \right) \left(\frac{1}{2f} - t_B \right) - v_c \left(\frac{1}{2f} - \frac{t_N}{2} - t_C \right) + \frac{4}{3} k(t_C - t_B)^3 - v_c t_N \\
 &= \frac{4h(1-\alpha)^2}{\alpha^2 + 6\alpha + 1} + \frac{4\alpha}{\alpha^2 + 4\alpha + 1} + \frac{v_c(\alpha - 1)}{4f} - \frac{v_c t_N}{2}.
 \end{aligned} \tag{11}$$

where NSA is the negative strip distance, mm.

3.5. Positive Strip Time. Positive strip time t_p is the time of the mold moving upward relative to the slab in an oscillating period. It can be determined as

$$t_p = T - t_N = \frac{1}{f} - t_N = \frac{1 + \alpha}{2f} + \frac{v_c(1 - \alpha)^2(\alpha^2 + 4\alpha + 1)}{24hf^2(\alpha + 1)^2}. \tag{12}$$

where t_p is the positive strip time, s.

3.6. Positive Strip Velocity. The positive strip velocity Δv is the maximum velocity of mold moving upward relative to the slab. That is,

$$\Delta v = |v_{u \max}| + |v_c| = \frac{6fh(1 + \alpha)}{\alpha^2 + 4\alpha + 1} + |v_c|, \tag{13}$$

where $v_{u \max}$ is the maximum velocity of mold moving upward. A group of curves of positive strip velocity are illustrated in Figure 6 with different constants of Δv .

3.7. Curves for Multiparameters. Nonsinusoidal oscillation technological parameters of t_N , NS, t_p , and Δv are illustrated in Figure 6. According to Figure 6, the values of these

TABLE 3: Technological parameters of sinusoidal and nonsinusoidal oscillation.

Oscillation mode	v_c (m/min)	f (min ⁻¹)	t_N (s)	NSA (mm)	t_p (s)	Δv (m/min)
Sinusoidal	1.15	104	0.205	3.259	0.372	3.764
Nonsinusoidal	1.15	105	0.175	4.130	0.396	2.794

technological parameters can be taken into account simultaneously when the relationship between oscillation frequency and casting speed is determined. So by using multiparameter curves of oscillation shown in Figure 6, it is convenient to determine the synchrocontrol model of casting speed and oscillation frequency.

By using equations (6), (8), (12), and (13), a group of curves for negative strip time, negative strip ratio, positive strip time, and positive strip velocity are obtained with different values of t_N , NS, t_p , and Δv , which are shown in Figure 6. When NS equals 57.41%, t_N equals zero. If NS is greater than or equal to 57.41%, there will be no negative strip time which is not permitted for steel continuous casting production. If the oscillation parameters of α and h changed, the multiparameter curves should be calculated again. According to the multiparameter curves, the synchrocontrolling model between casting speed and oscillation frequency determined can be suitable for different kinds of steel continuous casting.

The parameters of sinusoidal oscillation used in a steel plant are shown in Table 1 [19].

To further enhance the slab surface quality, without changing oscillation amplitude of 4 mm and making modification ratio equal to 20%, the synchrocontrol model of nonsinusoidal oscillation between casting speed and oscillation frequency is determined according to Figure 6 as follows:

$$f = \begin{cases} 60, & v_c < 0.4, \\ 60 \cdot v_c + 36, & v_c \geq 0.4. \end{cases} \quad (14)$$

Using the relationship of equation (14), the non-sinusoidal oscillation technological parameters are calculated and illustrated in Table 2.

The comparison of technological parameters between sinusoidal oscillation and nonsinusoidal oscillation is illustrated in Table 3. From Table 3, it can be found that under the same casting speed, the negative strip time and the positive strip velocity of the nonsinusoidal oscillation are smaller than those of sinusoidal oscillation, which would make the depth of the oscillation marks shallower and the tensile friction on the slab reduce. The negative strip distance of the nonsinusoidal oscillation is larger than that of sinusoidal oscillation, which is helpful to remove the slab from mold, weld the cracks in a solidified shell together, and increase casting speed. The positive strip time of non-sinusoidal oscillation is longer than that of sinusoidal oscillation, which will lead to a better lubrication condition with a larger quantity of casting powder consumption, reduce the tensile stress, and minimize cracks of solid shell. Thus, nonsinusoidal oscillation is superior to sinusoidal oscillation for improving surface quality of slab and decreasing the probability of steel breakout.

4. Conclusions

A nonsinusoidal oscillation waveform proposed could control the maximum acceleration, which has good dynamic characteristics due to the continuity of velocity and acceleration. By selecting reasonable basic oscillation parameters and the synchrocontrol model, it could satisfy the continuous casting production of different kinds of steel. The synchrocontrol model of oscillation frequency and casting speed proposed is reasonable and could gain better technological oscillation parameters. Compared with sinusoidal oscillation, the nonsinusoidal oscillation is beneficial to improve the surface quality of the slab and enhance casting speed. In the future, the nonsinusoidal oscillation waveform can be realized by different oscillators, which can reduce the inertia force and prolong the service life of the equipment.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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