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Research Article

Finite-Time Projective Synchronization of Fractional-Order Memristive Neural Networks with Mixed Time-Varying Delays

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This paper is concerned with the finite-time projective synchronization problem of fractional-order memristive neural networks (FMNNs) with mixed time-varying delays. Firstly, under the frame of fractional-order differential inclusion and the set-valued map, several criteria are derived to ensure finite-time projective synchronization of FMNNs. Meanwhile, three properties are established to deal with different forms of the finite-time fractional differential inequation, which greatly extend some results on estimation of settling time of FMNNs. In addition to the traditional Lyapunov function with 1-norm form in Theorem 1, a more general and flexible Lyapunov function based on p-norm is constructed in Theorem 2 to analyze the finite-time projective synchronization problem, and the estimation of settling time has been verified less conservative than previous results. Finally, numerical examples are provided to demonstrate the effectiveness of the derived theoretical results.

1. Introduction

In the past several decades, artificial neural networks have been extensively investigated due to their wide applications in signal processing [1], combinatorial optimization [2], pattern recognition [3], associative memories [4], and so on. Memristor, first proposed by Chua in 1971 [5], was realized by Hewlett-Packard Laboratory in 2008 [6]. Compared with the resistor, the memristor possesses a memory characteristic as the synaptic characteristics of biological neurons, which is distinct from other circuit elements. As a result, by using the memristor to replace other resistors, scholars constructed the memristive neural networks (MNNs) to emulate the human brain. Moreover, as a special kind of neural networks, MNNs are nonlinear systems with state-dependent switching jumps and possess more complicated properties than the traditional neural networks.

It is well known that time delays are unavoidable in neural networks. On the one hand, discrete time delays are ubiquitous because of the finite switching speed of amplifiers and the inherent information exchanging time between different neurons [7]. On the other hand, distributed time delays should also be considered in neural networks because neural networks usually have a spatial nature and the presence of a large number of parallel pathways with multifarious axon sizes and lengths [8]. Hence, these time delays, which may bring about instability, chaos, oscillation, or other poor performances of the system, should be considered when investigating the dynamical behavior for neural networks [9–11].

As a generalization of the ordinary differentiation and integration to arbitrary noninteger order, fractional-order calculus has long history and can be traced back to the 17th century. For a long time, the theory of fractional calculus was once considered as a purely theoretical field of mathematics and developed very slow. However, due to its infinite memory and more degrees of freedom [12], many scholars have found that fractional-order models are more accurate than integer-order ones in describing the memory and hereditary properties. In consequence, fractional-order

models have been incorporated into artificial networks. Subsequently, fractional-order memristive neural networks (FMNNs) have attracted lots of researchers to study their dynamical behaviors [13–15].

Since chaos synchronization was firstly proposed in 1990 [16], synchronization control has become an important tool to control the chaos appearing in the practical applications such as security communications [17], image processing [18], and cryptography [19]. So far, many sorts of synchronization schemes have been presented such as complete synchronization [20], antisynchronization [21], phase synchronization [22], lag synchronization [23], exponential synchronization [24], and projective synchronization.

Overall, the above discussed synchronization schemes can be divided into two categories: finite-time and infinite-time synchronization. In practice, the system always expected to realize synchronization as quickly as possible. It is worth noting that the finite-time control technique can not only accelerate the convergence process but also demonstrate better interference suppression properties and robustness [25]. Consequently, the finite-time synchronization is an optimal synchronization method and has attracted lots of attention in many research studies [26–33]. For example, in [27], by using the Gronwall–Bellman integral inequality and the Volterra integral equation, the authors explored the finite-time projective synchronization problem of memristor-based delay fractional-order neural networks. And Zhang and Deng studied the finite-time projective synchronization of fractional-order complex-valued memristor-based neural networks with time delays in [28]. However, we note that [26–28] pay attention to discussing the synchronization conditions and ignore the computation of settling time. In [29], Zhang et al. deduced the fractional-order derivative of Lyapunov function $_{t_0}^{C}D_t^{\alpha}V(t) \leq -aV(t) - b$ and gave a conservative estimation of settling time with $a = 0$. In [30–33], the authors deduced the fractional-order derivative of Lyapunov function $_{t_0}^{C}D_t^{\alpha}V(t) \leq -aV(t) - bV^{\beta}(t)$ and gave a conservative estimation of settling time with $a = 0$. And we propose a new method to estimate settling time with $a \geq 0$, which is less conservative than previous results.

Among a great variety of synchronization schemes, projective synchronization, first proposed by Mainieri and Rehacek in [34], is characterized by a fact that the drive and response systems could achieve synchronization up to a scaling factor. Particularly, due to the proportional feature, projective synchronization can be used to extend binary digital to M-nary digital communication for achieving fast communication [35]. Furthermore, projective synchronization of FMNNs, which can obtain faster communication with its proportional feature, has been widely concerned and deeply studied by researchers [36–40]. For instance, the global projective synchronization for FMNNs is investigated via combining open-loop control with the time-delayed feedback control in [36]. And in [38], Yang et al. discussed the quasi-projective synchronization of fractional-order complex-valued neural networks, and the error bounds of quasi-projective synchronization are estimated. Based on notoriously Barbalat's lemma and the Razumikhin-type

stability theorem, Zhang et al. investigated the projective synchronization of FMNNs with switching jump mismatch in [40]. However, to the best of our knowledge, there are few research studies about the finite-time projective synchronization of FMNNs with mixed time-varying delays.

Inspired by the above discussion, this paper will study the finite-time projective synchronization of FMNNs with mixed time-varying delays via applying differential inequality of the Caputo derivative and the asymptotic expansion property of Mittag-Leffler function. The main innovations and contents of this paper lie in the following aspects:

(1) For fractional-order neural networks, the fractional-order derivative of Lyapunov function $V(t)$ can usually be reduced to $_{t_0}^{C}D_t^{\alpha}V(t) \leq -aV(t) - b$ or $_{t_0}^{C}D_t^{\alpha}V(t) \leq -aV(t) - bV^{\beta}(t)$. In the existing literature studies on finite-time synchronization of fractional-order neural networks, most pay attention to discuss the synchronization condition $a \geq 0$ and ignore the computation of settling time. And a few literature studies estimate the settling time with $a = 0$, which will increase the conservative of the obtained result. In this paper, via applying the asymptotic expansion property of Mittag-Leffler function and the fractional-order power law inequation, we propose three properties to estimate the settling time with $a \geq 0$. And the comparison result can be seen in Tables 1–3, which demonstrate our new estimation of the settling time is more applicative and accurate.

(2) First, via designing a simple time-delayed feedback controller, some sufficient conditions are derived to guarantee the finite-time projective synchronization of FMNNs. And the results can be easily extended to the complete synchronization and antisynchronization.

(3) Unlike the traditional maximum absolute value-based method to propose the synaptic weights of memristive neural networks, by introducing some transformations, FMNNs are translated to a type of fractional-order systems with uncertain parameters.

(4) In addition to the traditional Lyapunov function with 1-norm form, a more general and flexible Lyapunov function based on p-norm is constructed to analyze the projective finite-time synchronization of FMNNs.

The rest of this paper is organized as follows. Section 2 describes the models and introduces the preliminaries including some necessary definitions and lemmas. In Section 3, we present the main results. Simulation examples are given to confirm the validity of our results in Section 4. And conclusions follow in Section 5.

Notation: throughout this study, $\mathbb{N}, \mathbb{C}, \mathbb{R}$, and $\mathbb{R}^*$ denote the set of all natural numbers, complex numbers, real numbers, and nonnegative real numbers, respectively. $C^n([t_0, \infty), \mathbb{R})$ represents the family of continuous and n-order differentiable function form $[t_0, \infty)$ to $\mathbb{R}$. And $\text{sign}(\cdot)$

TABLE 1: The comparisons among T_1^*, T_2, T_3^*, T_4, and T_5 with the projective coefficient $\lambda = 2$.

	Lyapunov function	Fractional-order differential inequation	Settling time
Theorem 1	$V_1(t) = \sum_{i=1}^n |e_i(t)|$	${}_{t_0}^{C}D_t^\alpha V_1(t) \leq -\varphi V_1(t) - \eta$	$T_1^* = 1.235$
Remark 4		${}_{t_0}^{C}D_t^\alpha V_1(t) \leq -\eta$	$T_2 = 5.5778$
Theorem 2	$V_2(t) = \sum_{i=1}^n |e_i(t)|^p$	${}_{t_0}^{C}D_t^\alpha V_2(t) \leq -\overline{\varphi}V_2(t) - \overline{\eta}V_2^{(p-1)/p}(t)$	$T_3^* = 1.9054$
Remark 6		${}_{t_0}^{C}D_t^\alpha V_2(t) \leq -\overline{\eta}V_2^{(p-1)/p}(t)$	$T_4 = 8.0421$ $T_5 = 6.689$

TABLE 2: The comparisons among T_1^*, T_2, T_3^*, T_4, and T_5 with the projective coefficient $\lambda = 1$.

	Lyapunov function	Fractional-order differential inequation	Settling time
Theorem 1	$V_1(t) = \sum_{i=1}^n |e_i(t)|$	${}_{t_0}^{C}D_t^\alpha V_1(t) \leq -\varphi V_1(t) - \eta$	$T_1^* = 1.0351$
Remark 4		${}_{t_0}^{C}D_t^\alpha V_1(t) \leq -\eta$	$T_2 = 3.5169$
Theorem 2	$V_2(t) = \sum_{i=1}^n |e_i(t)|^p$	${}_{t_0}^{C}D_t^\alpha V_2(t) \leq -\overline{\varphi}V_2(t) - \overline{\eta}V_2^{(p-1)/p}(t)$	$T_3^* = 1.5937$
Remark 6		${}_{t_0}^{C}D_t^\alpha V_2(t) \leq -\overline{\eta}V_2^{(p-1)/p}(t)$	$T_4 = 5.1232$ $T_5 = 4.4965$

TABLE 3: The comparisons among T_1^*, T_2, T_3^*, T_4, and T_5 with the projective coefficient $\lambda = -1$.

	Lyapunov function	Fractional-order differential inequation	Settling time
Theorem 1	$V_1(t) = \sum_{i=1}^n |e_i(t)|$	${}_{t_0}^{C}D_t^\alpha V_1(t) \leq -\varphi V_1(t) - \eta$	$T_1^* = 0.6934$
Remark 4		${}_{t_0}^{C}D_t^\alpha V_1(t) \leq -\eta$	$T_2 = 1.4814$
Theorem 2	$V_2(t) = \sum_{i=1}^n |e_i(t)|^p$	${}_{t_0}^{C}D_t^\alpha V_2(t) \leq -\overline{\varphi}V_2(t) - \overline{\eta}V_2^{(p-1)/p}(t)$	$T_3^* = 1.3539$
Remark 6		${}_{t_0}^{C}D_t^\alpha V_2(t) \leq -\overline{\eta}V_2^{(p-1)/p}(t)$	$T_4 = 2.3539$ $T_5 = 2.1816$

denotes the sign function, where $\text{sign}(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ -1, & \text{if } x < 0 \end{cases}$.

The symbol $*$ involves the convolution operator. And $\Gamma(\alpha)$ is Euler's gamma function which is defined by $\Gamma(\alpha) = \int_0^{+\infty} s^{\alpha-1}e^{-s}\,ds$.

2. Preliminaries and Model Description

2.1. Preliminaries. In this section, we recall some basic definitions with respect to fractional calculus and several important lemmas used in this paper.

Definition 1 (see [41]). The Riemann–Liouville fractional integral with fractional-order α for an integrable function $f(t)$ is defined as

$$
{}_{t_0}I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t-s)^{\alpha-1} f(s)\,ds, \tag{1}
$$

where $t \geq t_0$ and $\alpha > 0$.

Definition 2 (see [42]). The Caputo fractional derivative with fractional-order α for a differentiable function $f(t) \in C^n([t_0, +\infty), \mathbb{R})$ is defined as

$$
{}_{t_0}^{C}D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t \frac{f^{(n)}(s)}{(t-s)^{\alpha-n+1}}\,ds, \tag{2}
$$

where $t \geq t_0$ and n is a positive integral such that $n-1 < \alpha < n$. Especially, when $\alpha \in (0, 1)$, then

$$
{}_{t_0}^{C}D_t^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t \frac{f'(s)}{(t-s)^\alpha}\,ds. \tag{3}
$$

Definition 3 (see [41]). Mittag-–Leffler function $E_{p,q}(\cdot)$ with two parameters is defined as

$$
E_{p,q}(\varsigma) = \sum_{k=0}^{\infty} \frac{\varsigma^k}{\Gamma(pk+q)}, \tag{4}
$$

where $p > 0$, $q > 0$, and $\varsigma \in \mathbb{C}$. And if $q = 1$, the Mittag-Leffler function with one parameter is shown as

$$
E_p(\varsigma) = E_{p,1}(\varsigma) = \sum_{k=0}^{\infty} \frac{\varsigma^k}{\Gamma(pk+1)}. \tag{5}
$$

Definition 4. If there exists a nonzero constant $\lambda \in \mathbb{R}$, for any solutions of the n-dimensional drive system and the response system, $x_i(t)$ and $y_i(t), i = 1, 2, \ldots, n$, with initial values $x_i(t_0)$ and $y_i(t_0)$ are such that

$$
\lim_{t \longrightarrow T^*} [y_i(t) - \lambda x_i(t)] = 0,
$$

$$
y_i(t) - \lambda x_i(t) \equiv 0, \quad t \geq T^*, \tag{6}
$$

where λ represents the projective coefficient. T^* is called the settling time for synchronization. Then, the response system is said to be finite-time projective synchronization to the drive system.

Particularly, the drive-response system can be called finite-time complete synchronization if $\lambda = 1$ and finite-time antisynchronization if $\lambda = -1$.

Lemma 1 (see [41]). *If $f(t)$: $[t_0, +\infty) \longrightarrow \mathbb{R}$, then for all $t \in [t_0, +\infty)$,*

$$_{t_0}I_t^{\alpha}\left(_{t_0}^{C}D_t^{\alpha}f(t)\right) = f(t) - \sum_{i=1}^{n-1}\frac{f^{(i)}(t_0)}{i!}(t-t_0)^i. \qquad (7)$$

In particular, when $0 < \alpha < 1$, it has

$$_{t_0}I_t^{\alpha}\left(_{t_0}^{C}D_t^{\alpha}f(t)\right) = f(t) - f(t_0). \qquad (8)$$

Lemma 2 (see [43]). *For all $0 < \alpha < 2$, $\beta \in \mathbb{C}$, $m \in \mathbb{N}$, and $z \neq 0$, $|\mathrm{arg}z| < \min\{\pi, \alpha\pi\}$, then the following asymptotic formula holds:*

$$E_{\alpha,\beta}(z) = \frac{1}{\alpha}z^{(1-\beta)\alpha}e^{z^{1\alpha}} - \sum_{k=1}^{m}\frac{z^{-k}}{\Gamma(\beta - k\alpha)} + O\left[\frac{1}{z^{m+1}}\right], \quad |z| \longrightarrow \infty. \qquad (9)$$

Lemma 3 (see [44]). *Let $z_1 > 0, z_2 > 0, z_3 > 1, z_4 > 1$, and $z_3^{-1} + z_4^{-1} = 1$; then, the following inequality holds:*

$$z_1 z_2 \leq \varepsilon\frac{z_1^{z_3}}{z_3} + \varepsilon^{-z_3/z_4}\frac{z_2^{z_4}}{z_4}, \qquad (10)$$

where ε is an arbitrary positive constant.

Lemma 4 (see [45]). *Assume that $a_1, a_2, \ldots, a_n$ are nonnegative constants. For two arbitrary constants p and q satisfying $0 < p < q$, then*

$$\left(\sum_{i=1}^{n}a_i^q\right)^{1/q} \leq \left(\sum_{i=1}^{n}a_i^p\right)^{1/p}. \qquad (11)$$

Lemma 5 (see [46]). *Let $f(t)$ be a continuous function on the positive interval $[t_0, +\infty)$ and satisfy*

$$_{t_0}^{C}D_t^{\alpha}f(t) \leq af(t), \qquad (12)$$

where a is a constant and $0 < \alpha < 1$; then,

$$f(t) \leq f(t_0)E_{\alpha}(a(t-t_0)^{\alpha}). \qquad (13)$$

Lemma 6. *Let $f(t)$ be a continuous function on the positive interval $[t_0, +\infty)$ and satisfy*

$$_{t_0}^{C}D_t^{\alpha}f(t) \leq af(t) + b, \qquad (14)$$

where a and b are constants and $0 < \alpha < 1$; then,

$$f(t) \leq \left(f(t_0) + \frac{b}{a}\right)E_{\alpha}(a(t-t_0)^{\alpha}) - \frac{b}{a}. \qquad (15)$$

Proof. Let $h(t) = f(t) + (b/a)$, and we can easily obtain

$$_{t_0}^{C}D_t^{\alpha}h(t) = _{t_0}^{C}D_t^{\alpha}(t) \leq af(t) + b = ah(t). \qquad (16)$$

By applying Lemma 5, we have $h(t) \leq h(t_0)E_{\alpha}(a(t-t_0)^{\alpha})$, which is equivalent to

$$f(t) \leq \left(f(t_0) + \frac{b}{a}\right)E_{\alpha}(a(t-t_0)^{\alpha}) - \frac{b}{a}. \qquad (17)$$

$\square$

Remark 1. For Lemma 6, Yang et al. in [38] obtain a consistent result with $a > 0$ and $b \geq 0$. And we believe that the conditions of $a > 0$ and $b \geq 0$ are conservative. And in Lemma 6, we consider that a and b are constants, which is more general than the results in [38].

Lemma 7 (see [30]). *Suppose $0 < \alpha \leq 1$, $\beta \geq 1$, and $f(t) \in C^1([t_0, +\infty), \mathbb{R})$; then,*

$$_{t_0}^{C}D_t^{\alpha}f^{\beta}(t) = \frac{\Gamma(2-\alpha)\Gamma(1+\beta)}{\Gamma(1+\beta-\alpha)}f^{\beta-1}(t)_{t_0}^{C}D_t^{\alpha}f(t). \qquad (18)$$

Lemma 8 (see [47]). *Let $f(t) \in C^1([t_0, +\infty), \mathbb{R})$ be a continuous and differential function; the following inequation satisfies almost everywhere*

$$_{t_0}^{C}D_t^{\alpha}|f(t)| \leq \mathrm{sign}\,(f(t))_{t_0}^{C}D_t^{\alpha}f(t), \qquad (19)$$

where $0 < \alpha < 1$.

In the following, two new fractional-order inequalities are established to propose the finite-time convergence, which play a critical role to obtain the finite-time synchronization and calculate the settling time. And we can gain three new properties.

Property 1. Let $f(t) \in C^1([t_0, +\infty), \mathbb{R}^*)$ be continuous and nonnegative definite. And the following inequation holds:

$$_{t_0}^{C}D_t^{\alpha}f(t) \leq -af(t) - b, \qquad (20)$$

where $a > 0, b > 0$, and $0 < \alpha < 1$. It can be deduced as follows:

$$f(t) \leq \left[f(t_0) + \frac{b}{a}\right]\frac{1}{\alpha}\exp\left\{-a^{1/\alpha}(t-t_0)\right\} - \frac{b}{a}, \quad t \geq t_0. \qquad (21)$$

Then, we can judge $f(t)$ converges to 0 within the settling time T_1^* which can be estimated as follows:

$$t_1^* \leq t_0 + a^{-1/\alpha}\ln\frac{af(t_0) + b}{b\alpha}. \qquad (22)$$

Proof. By utilizing Lemma 6, we have

$$f(t) \leq \left[f(t_0) + \frac{b}{a}\right]E_{\alpha}(-a(t-t_0)^{\alpha}) - \frac{b}{a}. \qquad (23)$$

From Lemma 2, when $\beta = 1$, we can get

$$E_{\alpha}(z) = \frac{1}{\alpha}e^{z^{1\alpha}} - \sum_{k=1}^{m}\frac{z^{-k}}{\Gamma(1-k\alpha)} + O\left[\frac{1}{z^{m+1}}\right], \quad |z| \longrightarrow \infty. \qquad (24)$$

Based on $\lim_{z\to\infty} O[1/z^{m+1}] = 0$, we can easily gain

$$\lim_{z\to\infty} E_\alpha(z) \longrightarrow \lim_{z\to\infty}\left[\frac{1}{\alpha}e^{z^{1\alpha}} - \sum_{k=1}^{m}\frac{z^{-k}}{\Gamma(1-k\alpha)}\right]. \quad (25)$$

Apparently, $\sum_{k=1}^{m} z^{-k}/\Gamma(1-k\alpha) > 0$; then, we can judge that there exists $z_0 > 0$ such that

$$E_\alpha(z) \le \frac{1}{\alpha}e^{z^{1\alpha}}, \quad z > z_0. \quad (26)$$

Similarly, we have

$$E_\alpha\left(-a(t-t_0)^\alpha\right) \le \frac{1}{\alpha}\exp\left\{-a^{1/\alpha}(t-t_0)\right\}, \quad t \ge t_0. \quad (27)$$

From the above conclusion, inequation (20) can be converted to

$$f(t) \le \left[f(t_0) + \frac{b}{a}\right]\frac{1}{\alpha}\exp\left\{-a^{1/\alpha}(t-t_0)\right\} - \frac{b}{a}, \quad t \ge t_0. \quad (28)$$

In order to better analyze the problem, let $\Omega(t) = [f(t_0) + (b/a)](1/\alpha)e^{-a^{1/\alpha}(t-t_0)} - (b/a)$. We can easily know

that the function $\Omega(t)$ is monotonically decreasing. And $\Omega(t_0) = (1/\alpha)f(t_0) + ((1/\alpha) - 1)(b/a) > (1/\alpha)f(t_0) > 0$ and $\lim_{t\to\infty}\Omega(t) \longrightarrow -(b/a) < 0$. According to the zero-point theorem, there exists $t_1^* \in [t_0, \infty)$ such that $\Omega(t_1^*) = 0$. Then, we can gain $0 \le f(t_1^*) \le \Omega(t_1^*) = 0$, so we have $f(t_1^*) = 0$. And when $t \ge t_1^*$, then $f(t) \equiv 0$.

Based on Definition 4, the function $f(t)$ converges to 0 in finite time. And the settling time t_1^* can be calculated as follows:

$$t_1^* \le t_0 + a^{-1/\alpha}\ln\frac{af(t_0)+b}{b\alpha}. \quad (29)$$

□

Property 2. Assume that function $f(t) \in C([t_0, +\infty), \mathbb{R}^*)$ is continuous and nonnegative definite. If the following inequality holds,

$$\,^C_{t_0}D_t^\alpha f(t) \le -af^\beta(t) - b, \quad (30)$$

where $a > 0$, $b > 0$, $0 < \alpha < 1$, $0 < \beta \le 1$, then it can be deduced as follows:

$$f(t) \le \left[\left(f(t_0)+\left(\frac{b}{a}\right)^{1/\beta}\right)^{1-\beta} - \frac{a\Gamma(1+(1/(1-\beta))-\alpha)(t-t_0)^\alpha}{\Gamma(1+(1/(1-\beta)))\Gamma(2-\alpha)\Gamma(1+\alpha)}\right]^{1/(1-\beta)} - \left(\frac{b}{a}\right)^{1/\beta}. \quad (31)$$

And we can judge $f(t)$ converges to 0 within the settling time T_2^* which can be estimated as follows:

$$t_2^* \le t_0 + \left[f^{1-\beta}(t_0)\frac{\Gamma(1+(1/(1-\beta)))\Gamma(2-\alpha)\Gamma(1+\alpha)}{a\Gamma(1+(1/(1-\beta))-\alpha)}\right]^{1/\alpha}. \quad (32)$$

Proof. Let $g(t) = f(t) + (b/a)^{1/\beta} > 0$; according to Lemma 4, we gain

$$g^\beta(t) = \left(f(t)+\left(\frac{b}{a}\right)^{1/\beta}\right)^\beta \le f^\beta(t) + \frac{b}{a}. \quad (33)$$

Multiplying $-a$ on sides of the above inequation, then we can get

$$-af^\beta(t) - b \le -ag^\beta(t). \quad (34)$$

According to the conditions of Property 2,

$$\,^C_{t_0}D_t^\alpha g(t) = \,^C_{t_0}D_t^\alpha f(t) \le -af^\beta(t) - b \le -ag^\beta(t). \quad (35)$$

Let $w(t) = g^{\beta/m}(t)$, where m is a positive integer. Then, the above inequation can be written as follows:

$$\,^C_{t_0}D_t^\alpha w^{m/\beta}(t) \le -aw^m(t). \quad (36)$$

When $m \ge \beta$, via applying Lemma 7, we can gain

$$\frac{\Gamma(2-\alpha)\Gamma(1+(m/\beta))}{\Gamma(1+(m/\beta)-\alpha)}w^{(m/\beta)-1}(t)D_t^\alpha w(t) \le -aw^m(t). \quad (37)$$

Then,

$$w^{(m/\beta)-m-1}(t)D_t^\alpha w(t) \le \frac{-a\Gamma(1+(m/\beta)-\alpha)}{\Gamma(2-\alpha)\Gamma(1+(m/\beta))}. \quad (38)$$

Considering $m \ge (\beta/(1-\beta))$, thus $(m/\beta) - m - 1 \ge 0$. Once again, we apply Lemma 7 into the above inequation. And we have

$$D_t^\alpha w^{(m/\beta)-m}(t) \le \frac{-a\Gamma(1+(m/\beta)-\alpha)\Gamma(1+(m/\beta)-m)}{\Gamma(1+(m/\beta))\Gamma(1+(m/\beta)-m-\alpha)}. \quad (39)$$

Combining Lemma 1, we take the fractional integral with fractional-order α from t_0 to t on sides of in equation (3); we have

$$w^{(m/\beta)-m}(t) - w^{(m/\beta)-m}(t_0) \le \frac{-a\Gamma(1+(m/\beta)-\alpha)\Gamma(1+(m/\beta)-m)(t-t_0)^\alpha}{\Gamma(1+(m/\beta))\Gamma(1+(m/\beta)-m-\alpha)\Gamma(1+\alpha)}. \quad (40)$$

Substituting $w(t) = [f(t) + (b/a)^{1/\beta}]^{\beta/m}$ into the above inequation, we can obtain

$$f(t) \le \left[\left(f(t_0) + \left(\frac{b}{a}\right)^{1/\beta}\right)^{1-\beta} - \frac{a\Gamma(1 + (m/\beta) - \alpha)\Gamma(1 + (m/\beta) - m)(t - t_0)^{\alpha}}{\Gamma(1 + (m/\beta))\Gamma(1 + (m/\beta) - m - \alpha)\Gamma(1 + \alpha)}\right]^{1/(1-\beta)} - \left(\frac{b}{a}\right)^{1/\beta}. \qquad (41)$$

Apparently, the nonnegative function $f(t)$ is monotonically decreasing. Hence, we can conclude that $f(t)$ converges to 0 within the settling time t_2^*. By simple computation, the estimation of settling time t_2^* is given by

$$t_2^* \le t_0 + \left[\left(\left(f(t_0) + \left(\frac{b}{a}\right)^{(1/\beta)}\right)^{1-\beta} - \left(\frac{b}{a}\right)^{(1-\beta)/\beta}\right)\frac{\Gamma(1 + (m/\beta))\Gamma(1 + (m/\beta) - m - \alpha)\Gamma(1 + \alpha)}{a\Gamma(1 + (m/\beta) - \alpha)\Gamma(1 + (m/\beta) - m)}\right]^{1/\alpha}. \qquad (42)$$

From the above analysis, the estimation of settling time is connected with a positive integer $m \ge (\beta/(1 - \beta))$. Then, we would like to discuss the effect of m for settling time. First, we define a function $H(m)$ as follows:

$$H(m) = \frac{\Gamma(1 + (m/\beta))}{\Gamma(1 + (m/\beta) - m)} \cdot \frac{\Gamma(1 + (m/\beta) - m - \alpha)}{\Gamma(1 + (m/\beta) - \alpha)}. \qquad (43)$$

Based on the property of Euler's gamma function, we have

$$H(m) = \frac{(1 + (m/\beta) - 1)(1 + (m/\beta) - 2) \cdots (1 + (m/\beta) - m)}{(1 + (m/\beta) - \alpha - 1)(1 + (m/\beta) - \alpha - 2) \cdots (1 + (m/\beta) - \alpha - m)} = \prod_{i=1}^{m} A_i, \qquad (44)$$

where $A_i = (1 + (m/\beta) - i)/(1 + (m/\beta) - \alpha - i)$, $i = 1, 2, \ldots, m$. Obviously, $A_i > 1$. On the contrary,

$$\frac{\mathrm{d}A_i}{\mathrm{d}i} = \frac{\alpha}{(1 + (m/\beta)\alpha - i)^2} > 0. \qquad (45)$$

According to the above analysis, it can be obtained that $H(m)$ is monotone increasing about m. Therefore, it can be

deduced that the settling time is monotone increasing about m. In order to obtain more accurate estimation of settling time, we should choose a smaller value of m.

From the above analysis, if we consider $m = \beta/(1 - \beta)$, we can gain a smaller estimation of settling time as follows:

$$t_2^* \le t_0 + \left[\left(\left(f(t_0) + \left(\frac{b}{a}\right)^{1/\beta}\right)^{1-\beta} - \left(\frac{b}{a}\right)^{(1-\beta)/\beta}\right)\frac{\Gamma(1 + (1/(1 - \beta)))\Gamma(1 + (1/(1 - \beta)) - (\beta/(1 - \beta)) - \alpha)\Gamma(1 + \alpha)}{a\Gamma(1 + (1/(1 - \beta)) - \alpha)\Gamma(1 + (1/(1 - \beta)) - (\beta/(1 - \beta)))}\right]^{1/\alpha}$$

$$= t_0 + \left[\left(\left(f(t_0) + \left(\frac{b}{a}\right)^{1/\beta}\right)^{1-\beta} - \left(\frac{b}{a}\right)^{(1-\beta)/\beta}\right)\frac{\Gamma(1 + (1/(1 - \beta)))\Gamma(2 - \alpha)\Gamma(1 + \alpha)}{a\Gamma(1 + (1/(1 - \beta)) - \alpha)}\right]^{1/\alpha}. \qquad (46)$$

For Property 2, considering $b = 0$, we can obtain Corollary 1 as follows.

$\square$

Corollary 1. *Assume that function $f(t) \in C([t_0, +\infty), \mathbb{R}^*)$ is continuous and nonnegative definite. If the following inequality holds,*

$$^C_{t_0}D_t^\alpha f(t) \le -af^\beta(t), \qquad (47)$$

where $a > 0$, $0 < \alpha < 1$, and $0 < \beta \le 1$, then it can be deduced as follows:

$$f(t) \le \left[f^{1-\beta}(t_0) - \frac{a\Gamma(1 + (1/(1 - \beta)) - \alpha)(t - t_0)^\alpha}{\Gamma(1 + (1/(1 - \beta)))\Gamma(2 - \alpha)\Gamma(1 + \alpha)}\right]^{(1/(1-\beta))}. \qquad (48)$$

And we can judge $f(t)$ converges to 0 within the settling time t_3^* which can be estimated as follows:

$$t_3^* \le t_0 + \left[f^{1-\beta}(t_0) \frac{\Gamma(1 + (1/(1-\beta)))\Gamma(2-\alpha)\Gamma(1+\alpha)}{a\Gamma(1 + (1/(1-\beta)) - \alpha)} \right]^{1/\alpha}. \tag{49}$$

Remark 2. Peng et al. and Wu et al. in [31, 32] proposed the estimation of settling time for the differential inequation ${}_{t_0}^{C}D_t^\alpha f(t) \le -af^\beta(t)$. According to the results in [31, 32], we can get another estimation of settling time:

$$t_4^* \le t_0 + \left[f^{\alpha(1+\alpha)-\beta-1}(t_0) \frac{\Gamma(1+\alpha)\Gamma(1 - ((1+\beta)/(1+\alpha)))}{a\Gamma(2+\alpha)\Gamma(1+\alpha - ((1+\beta)/(1+\alpha)))} \right]^{1/\alpha}, \tag{50}$$

where $0 < \beta < \alpha(1+\alpha) - 1$. Apparently, the results in [31, 32] need a limiting condition on α and β. Compared with the above results, the estimation of settling time in Property 2 and Corollary 1 has no limits about α and β when we select a suitable value of m. Hence, the result we obtained is more general and less conservative.

Property 3. Assume that function $f(t) \in C([t_0, +\infty), \mathbb{R}^*)$ is continuous and nonnegative definite. If the following inequality holds,

$$ {}_{t_0}^{C}D_t^\alpha f(t) \le -af(t) - bf^\beta(t), \tag{51}$$

where $a > 0$, $b > 0$, $0 < \alpha < 1$, and $0 < \beta \le 1$, then it can be deduced as follows:

$$f^{1-\beta}(t) \le \left[f^{1-\beta}(t_0) + \frac{b}{a} \right] \frac{1}{\alpha} \exp\left\{ -\left(\frac{a\Gamma(1 + (1/\beta) - \alpha)}{\Gamma(1 + (1/\beta))\Gamma(2-\alpha)} \right)^{1/\alpha} (t - t_0) \right\} - \frac{b}{a}, \qquad t \ge t_0. \tag{52}$$

And we can judge $f(t)$ converges to 0 within the settling time T_2^* which can be estimated as follows:

$$t_5^* \le t_0 + \left[\frac{a\Gamma(1 + (1/\beta) - \alpha)}{\Gamma(1 + (1/\beta))\Gamma(2-\alpha)} \right]^{-1/\alpha} \ln \frac{af^{1-\beta}(t_0) + b}{b\alpha}. \tag{53}$$

Proof. Let $w(t) = f^{\beta/m}(t)$, where m is a positive integer. Then, inequation (51) can be written as follows:

$$ {}_{t_0}^{C}D_t^\alpha w^{m/\beta}(t) \le -aw^{m/\beta}(t) - bw^m(t). \tag{54}$$

When $m \ge \beta$, via applying Lemma 7, we can gain

$$\frac{\Gamma(2-\alpha)\Gamma(1 + (m/\beta))}{\Gamma(1 + (m/\beta) - \alpha)} w^{(m/\beta)-1}(t) D_t^\alpha w(t) \le -aw^{m/\beta}(t) - bw^m(t). \tag{55}$$

Then,

$$\frac{\Gamma(2-\alpha)\Gamma(1 + (m/\beta))}{\Gamma(1 + (m/\beta) - \alpha)} w^{(m/\beta)-m-1}(t) D_t^\alpha w(t) \le -aw^{(m/\beta)-m}(t) - b. \tag{56}$$

Considering $m \ge (\beta/(1-\beta))$, thus $(m/\beta) - m - 1 \ge 0$. Once again, we apply Lemma 7 into the above inequation. And we have

$$D_t^\alpha w^{(m/\beta)-m}(t) \le -\hat{a} w^{(m/\beta)-m}(t) - \hat{b}, \tag{57}$$

where

$$\hat{a} = \frac{\Gamma(1 + (m/\beta) - \alpha)\Gamma(1 + (m/\beta) - m)}{\Gamma(1 + (m/\beta))\Gamma(1 + (m/\beta) - m - \alpha)} a,$$

$$\hat{b} = \frac{\Gamma(1 + (m/\beta) - \alpha)\Gamma(1 + (m/\beta) - m)}{\Gamma(1 + (m/\beta))\Gamma(1 + (m/\beta) - m - \alpha)} b. \tag{58}$$

Combining Property 1, inequation (57) can be written as follows:

$$w^{(m/\beta)-m}(t) \le \left[w^{(m/\beta)-m}(t_0) + \frac{\hat{b}}{\hat{a}} \right] \frac{1}{\alpha} \exp\left(-\hat{a}^{1/\alpha}(t - t_0)\right) - \frac{\hat{b}}{\hat{a}}, \quad t \ge t_0. \tag{59}$$

Substituting $w(t) = f^{\beta/m}(t)$ and $\hat{b}/\hat{a} = b/a$ into the above inequation, we can obtain

$$f^{1-\beta}(t) \le \left[f^{1-\beta}(t_0) + \frac{b}{a} \right] \frac{1}{\alpha} \exp\left(-\hat{a}^{1/\alpha}(t - t_0)\right) - \frac{b}{a}, \quad t \ge t_0. \tag{60}$$

Because $f(t)$ is a nonnegative function, we can gain $f(t) = 0$ if $f^{1-\beta}(t) = 0$. Therefore, based on the analysis in Property 1, we can conclude that $f(t)$ converges to 0 within the settling time t_2^*. By simple computation, the estimation of settling time t_2^* is given by

$$t_5^* \le t_0 + \left[\frac{\Gamma(1 + (m/\beta) - \alpha)\Gamma(1 + (m/\beta) - m)}{(1 + (m/\beta))\Gamma(1 + (m/\beta) - m - \alpha)} a \right]^{-1/\alpha} \ln \frac{af(t_0) + b}{b\alpha}. \tag{61}$$

Similar to Property 1, $H(m) = ((\Gamma(1 + (m/\beta)))/(\Gamma(1 + (m/\beta) - m))) \cdot ((\Gamma(1 + (m/\beta) - m - \alpha))/(\Gamma(1 + (m/\beta) - \alpha)))$ is monotone increasing about m.

$$t_5^* \le t_0 + \left[\frac{a}{H(m)} \right]^{-1/\alpha} \ln \frac{af(t_0) + b}{b\alpha}. \tag{62}$$

Therefore, it can be deduced that the settling time is monotone increasing about m. In order to obtain more accurate estimation of settling time, we should choose a smaller value of $\beta/(1-\beta)$.

$$f^{1-\beta}(t) \leq \left[f^{1-\beta}(t_0) + \frac{b}{a} \right] \frac{1}{\alpha} \exp \left\{ -\left(\frac{a\Gamma(1 + (1/\beta) - \alpha)}{\Gamma(1 + (1/\beta))\Gamma(2 - \alpha)} \right)^{1/\alpha} (t - t_0) \right\} - \frac{b}{a}, \quad t \geq t_0. \tag{63}$$

And we can obtain the settling time t_2^* as follows:

$$t_5^* \leq t_0 + \left[\frac{a\Gamma(1 + (1/\beta) - \alpha)}{\Gamma(1 + (1/\beta))\Gamma(2 - \alpha)} \right]^{-1/\alpha} \ln \frac{a f^{1-\beta}(t_0) + b}{b\alpha}. \tag{64}$$

□

2.2. Model Description. A practical integer-order memristive neural network can usually be implemented by a very large-scale integration (VISI) circuit as shown in Figure 1. According to Kirchhoff's current law, the ith subsystem with mixed delays can be described as follows:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t))$$

$$+ \sum_{j=1}^{n} b_{ij}(x_i(t)) g_j(x_j(t - \tau_j(t)))$$

$$+ \sum_{j=1}^{n} c_{ij}(x_i(t)) \int_{t-\rho_j(t)}^{t} h_j(x_j(s)) \mathrm{d}s + I_i, \tag{65}$$

$$i = 1, 2, \ldots, n, t \geq t_0.$$

In this paper, the above integer-order FNN model is expended to fractional-order FNNs to explore the finite-time projective synchronization. And the drive system can be described by the following differential equation:

$$_{t_0}^{C}D_t^{\alpha} x_i(t) = -d_i x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j(x_j(t))$$

$$+ \sum_{j=1}^{n} b_{ij}(x_i(t)) g_j(x_j(t - \tau_j(t)))$$

$$+ \sum_{j=1}^{n} c_{ij}(x_i(t)) \int_{t-\rho_j(t)}^{t} h_j(x_j(s)) \mathrm{d}s + I_i, \tag{66}$$

$$i = 1, 2, \ldots, n, t \geq t_0.$$

where $_{t_0}^{C}D_t^{\alpha}$ denotes the Caputo fractional derivative of order $0 < \alpha < 1$ from t_0 to t; $d_i \geq 0$ is the self-feedback connection weight of the ith neuron; $x_i(t)$ represents the corresponding state variable of the number of ith neurons at time t; $f_j(\cdot)$, $g_j(\cdot)$, and $h_j(\cdot)$ denote the activation function between the jth-dimension of the memristor and the state variable; $\tau_j(t)$ and $\rho_j(t)$, respectively, describe the discrete and distributed time-varying delays which satisfied $0 \leq \tau_j(t) \leq \tau_j^{\max}$ and

$0 \leq \rho_j(t) \leq \rho_j^{\max}$; and I_i represents the external input or bias, and it is a constant. Furthermore, $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$, and $c_{ij}(x_i(t))$ are memristive synaptic connection weights which can be given by

$$a_{ij}(x_i(t)) = \frac{W_{aij}}{C_i} \times \mathrm{sign}_{ij},$$

$$b_{ij}(x_i(t)) = \frac{W_{bij}}{C_i} \times \mathrm{sign}_{ij}, \tag{67}$$

$$c_{ij}(x_i(t)) = \frac{W_{cij}}{C_i} \times \mathrm{sign}_{ij},$$

where $\mathrm{sign}_{ij} = \begin{cases} 1, & i \neq j \\ -1, & i = j \end{cases}$; C_i represents the capacitors; and W_{aij}, W_{bij}, and W_{cij}, respectively; denote the memductance of memristors by R_{aij}, R_{bij}, and R_{cij}. And R_{aij} describes the memristor between activation function $f_j(x_j(t))$ and $x_i(t)$; R_{bij} denotes the memristor between activation function $g_j(x_j(t - \tau_j))$ and $x_i(t)$; and R_{cij} is the memristor between activation function $\int_{t-\rho_j(t)}^{t} h_j(x_j(s))\mathrm{d}s$ and $x_i(t)$. According to the characteristics of memristors and the current-voltage, we use a simplified memristor model as follows:

$$a_{ij}(x_i(t)) = \begin{cases} a_{ij}^*, & |x_i(t)| \leq \Omega_i, \\ a_{ij}^{**}, & |x_i(t)| > \Omega_i, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} b_{ij}^*, & |x_i(t)| \leq \Omega_i, \\ b_{ij}^{**}, & |x_i(t)| > \Omega_i, \end{cases} \tag{68}$$

$$c_{ij}(x_i(t)) = \begin{cases} c_{ij}^*, & |x_i(t)| \leq \Omega_i, \\ c_{ij}^{**}, & |x_i(t)| > \Omega_i, \end{cases}$$

where the switching jumps $\Omega_i > 0$ and a_{ij}^*, a_{ij}^{**}, b_{ij}^*, b_{ij}^{**}, c_{ij}^*, and c_{ij}^{**} are all constants. And drive system (66) is supplemented with initial conditions as follows:

$$x_i(s) = \Xi_i(s),$$

$$s \in [t_0 - \tau, t_0], \tag{69}$$

$$\tau = \max_{1 \leq j \leq n} \{\tau_j(t), \rho_j(t)\}.$$

In order to express the proof process clearly, we set some mathematical simplified forms. Let $a_{ij}'' = \max\{a_{ij}^*, a_{ij}^{**}\}$, $a_{ij}' = \min\{a_{ij}^*, a_{ij}^{**}\}$, $b_{ij}'' = \max\{b_{ij}^*, b_{ij}^{**}\}$, $b_{ij}' = \min\{b_{ij}^*, b_{ij}^{**}\}$, $c_{ij}'' = \max\{c_{ij}^*, c_{ij}^{**}\}$, and $c_{ij}' = \min\{c_{ij}^*, c_{ij}^{**}\}$. And $\bar{a}_{ij} = (1/2)(a_{ij}'' + a_{ij}')$, $\tilde{a}_{ij} = (1/2)(a_{ij}'' + a_{ij}')$, $\bar{b}_{ij} = (1/2)(b_{ij}'' + b_{ij}')$,
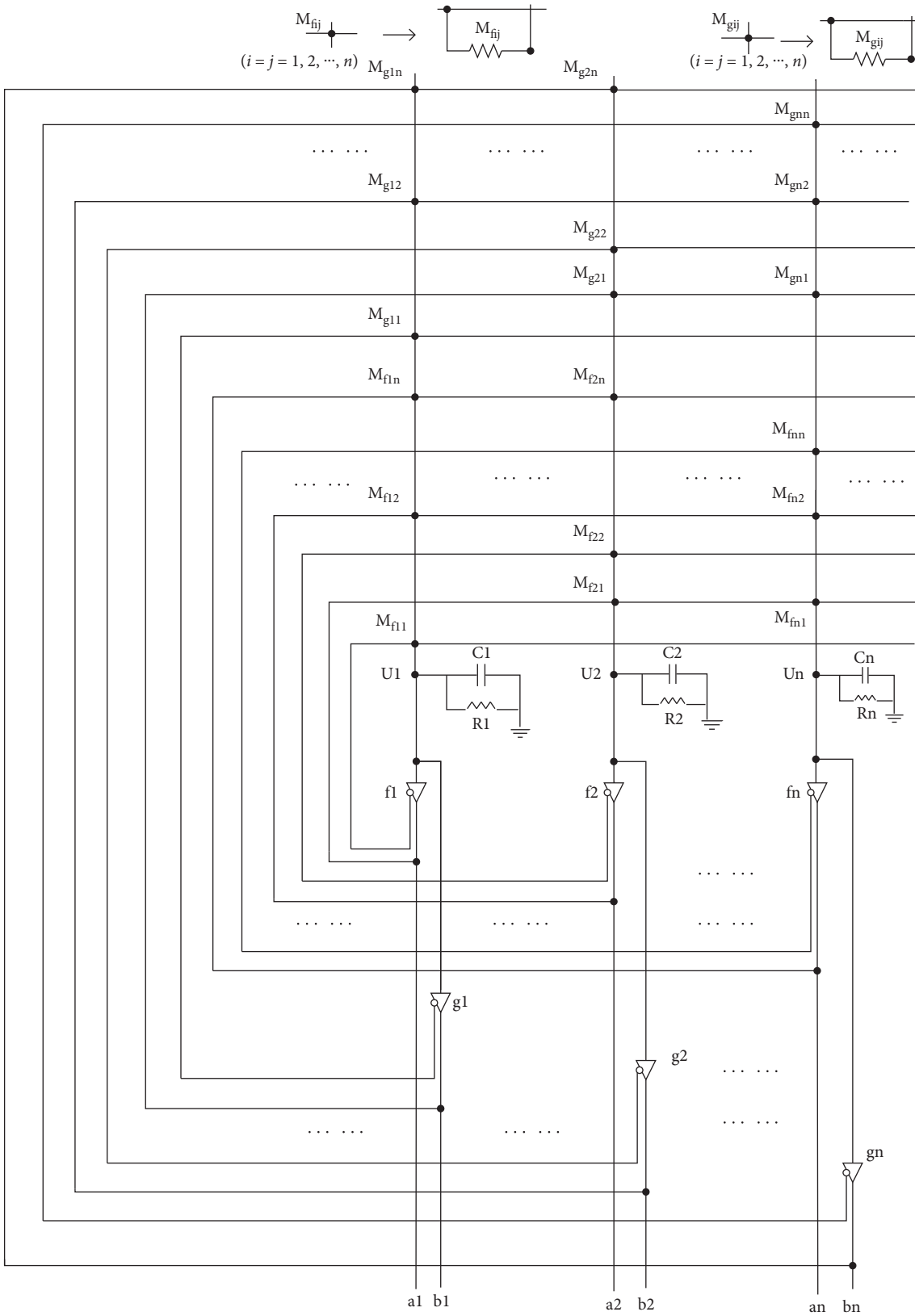
FIGURE 1: The circuit of the memristive neural network.

$\widetilde{b}_{ij} = (1/2)(b_{ij}'' + b_{ij}')$, $\overline{c}_{ij} = (1/2)(c_{ij}'' + c_{ij}')$, and $\widetilde{c}_{ij} = (1/2)$ $(c_{ij}'' + c_{ij}')$. Also, $a_{ij}^+ = \max\{|a_{ij}^*|, |a_{ij}^{**}|\}$, $b_{ij}^+ = \max\{|b_{ij}^*|, |b_{ij}^{**}|\}$, and $c_{ij}^+ = \max\{|c_{ij}^*|, |c_{ij}^{**}|\}$.

From the above description, we note that system (66) can be regarded as a class of the complex switching system with discontinuous right-hand side. In order to discuss the solution of the system, we have to consider them in Filippov's sense. With the theories of the set-valued map and differential inclusions [48, 49], drive system (66) could be converted to the following differential inclusion:

$$
{}_{t_0}^C D_t^\alpha x_i(t) \in -d_i x_i(t) + \sum_{j=1}^n \left(\overline{a}_{ij} + co\left[-\widetilde{a}_{ij}, \widetilde{a}_{ij}\right]\right) f_j\left(x_j(t)\right) + \sum_{j=1}^n \left(\overline{b}_{ij} + co\left[-\widetilde{b}_{ij}, \widetilde{b}_{ij}\right]\right) g_j\left(x_j\left(t - \tau_j(t)\right)\right)
$$
$$
+ \sum_{j=1}^n \left(\overline{c}_{ij} + co\left[-\widetilde{c}_{ij}, \widetilde{c}_{ij}\right]\right) \int_{t-\rho_j(t)}^t h_j\left(x_j(s)\right) ds + I_i, \quad i = 1, 2, \ldots, n, \ t \geq t_0.
$$

(70)

According to the set-valued map and the measurable selection theorem in [50], there exist measurable functions $\delta_{ij}^{(1)}, \delta_{ij}^{(2)}, \delta_{ij}^{(3)} \in co[-1, 1]$ such that

$$
{}_{t_0}^C D_t^\alpha x_i(t) = -d_i x_i(t) + \sum_{j=1}^n \left(\overline{a}_{ij} + \widetilde{a}_{ij}\delta_{ij}^{(1)}\right) f_j\left(x_j(t)\right) + \sum_{j=1}^n \left(\overline{b}_{ij} + \widetilde{b}_{ij}\delta_{ij}^{(2)}\right) g_j\left(x_j\left(t - \tau_j(t)\right)\right)
$$
$$
+ \sum_{j=1}^n \left(\overline{c}_{ij} + \widetilde{c}_{ij}\delta_{ij}^{(3)}\right) \int_{t-\rho_j(t)}^t h_j\left(x_j(s)\right) ds + I_i, \quad i = 1, 2, \ldots, n, \ t \geq t_0.
$$

(71)

In order to investigate the synchronization of FMNNs, system (66) is regarded as the drive system. Similarly, the response system is described as follows:

$$
{}_{t_0}^C D_t^\alpha y_i(t) = -d_i y_i(t) + \sum_{j=1}^n a_{ij}\left(y_i(t)\right) f_j\left(y_j(t)\right)
$$
$$
+ \sum_{j=1}^n b_{ij}\left(y_i(t)\right) g_j\left(y_j\left(t - \tau_j(t)\right)\right)
$$
$$
+ \sum_{j=1}^n c_{ij}\left(y_i(t)\right) \int_{t-\rho_j(t)}^t h_j\left(y_j(s)\right) ds + I_i + u_i(t), \quad i = 1, 2, \ldots, n, \ t \geq t_0.
$$

(72)

where $y_i(t)$ represents the state of the response system and $u_i(t)$ is the controller to be designed. And the initial conditions of response system (72) are given as follows:

$$
y_i(s) = \Psi_i(s),
$$
$$
s \in [t_0 - \tau, t_0],
$$
$$
\tau = \max_{1 \leq j \leq n}\{\tau_j(t), \rho_j(t)\}.
$$

(73)

By using the set-valued map and the measurable selection theorem, there exist measurable functions $\gamma_{ij}^{(1)}, \gamma_{ij}^{(2)}, \gamma_{ij}^{(3)} \in co[-1, 1]$ such that

$$
\begin{aligned}
{}^{C}_{t_0}D^{\alpha}_{t}y_i(t) \;=\; & -d_i y_i(t) + \sum_{j=1}^{n}\left(\overline{a}_{ij} + \widetilde{a}_{ij}\gamma_{ij}^{(1)}\right)f_j\big(y_j(t)\big) + \sum_{j=1}^{n}\left(\overline{b}_{ij} + \widetilde{b}_{ij}\gamma_{ij}^{(2)}\right)g_j\big(y_j(t - \tau_j(t))\big) \\
& + \sum_{j=1}^{n}\left(\overline{c}_{ij} + \widetilde{c}_{ij}\gamma_{ij}^{(3)}\right)\int_{t-\rho_j(t)}^{t} h_j\big(y_j(s)\big)\mathrm{d}s + I_i + u_i(t), \quad i = 1, 2, \ldots, n,\ t \geq t_0.
\end{aligned}
\tag{74}
$$

Remark 3. The drive system and the response system are a distinct network, so $\Delta_{ij}(x_i(t))$ probably is not equal to $\Delta_{ij}(y_i(t))$. Thus, we need to select six different measurable functions $\delta_{ij}^{(1)}, \delta_{ij}^{(2)}, \delta_{ij}^{(3)}, \gamma_{ij}^{(1)}, \gamma_{ij}^{(2)}, \gamma_{ij}^{(3)}$.

Let $e_i(t) = y_i(t) - \lambda x_i(t)$ denote the synchronization error. And λ represents the projective coefficient. Hence, the fractional order of the synchronization error system can be given as follows:

$$
\begin{aligned}
{}^{C}_{t_0}D^{\alpha}_{t}e_i(t) = & -d_i e_i(t) + (1-\lambda)I_i + \sum_{j=1}^{n}\left(\overline{a}_{ij} + \widetilde{a}_{ij}\gamma_{ij}^{(1)}\right)f_j\big(y_j(t)\big) - \lambda\sum_{j=1}^{n}\left(\overline{a}_{ij} + \widetilde{a}_{ij}\delta_{ij}^{(1)}\right)f_j\big(x_j(t)\big) \\
& + \sum_{j=1}^{n}\left(\overline{b}_{ij} + \widetilde{b}_{ij}\gamma_{ij}^{(2)}\right)g_j\big(y_j(t - \tau_j(t))\big) - \lambda\sum_{j=1}^{n}\left(\overline{b}_{ij} + \widetilde{b}_{ij}\delta_{ij}^{(2)}\right)g_j\big(x_j(t - \tau_j(t))\big) \\
& + \sum_{j=1}^{n}\left(\overline{c}_{ij} + \widetilde{c}_{ij}\gamma_{ij}^{(3)}\right)\int_{t-\rho_j(t)}^{t} h_j\big(y_j(s)\big)\mathrm{d}s - \lambda\sum_{j=1}^{n}\left(\overline{c}_{ij} + \widetilde{c}_{ij}\delta_{ij}^{(3)}\right)\int_{t-\rho_j(t)}^{t} h_j\big(x_j(s)\big)\mathrm{d}s + u_i(t).
\end{aligned}
\tag{75}
$$

The initial conditions associated with system (75) are written as follows:

$$
\begin{aligned}
& e_i(s) = \Phi_i(s) = \Xi_i(t) - \lambda\Psi_i(s), \\
& s \in [t_0 - \tau, t_0], \\
& \tau = \max_{1 \leq j \leq n}\{\tau_j(t), \rho_j(t)\}.
\end{aligned}
\tag{76}
$$

To obtain the main results expediently, we set up the following assumptions:

Assumption 1. The activation functions $f_j(\cdot), g_j(\cdot)$, and $h_j(\cdot)$ are Lipschitz continuous. That is, for any $j \in \mathbb{N}$, there exist three constants $L_j^{(1)}, L_j^{(2)}$, and $L_j^{(3)}$ such that

$$
\begin{aligned}
& \left|f_j(y) - f_j(x)\right| \leq L_j^{(1)}|y - x|, \\
& \left|g_j(y) - f_j(x)\right| \leq L_j^{(2)}|y - x|, \\
& \left|h_j(y) - h_j(x)\right| \leq L_j^{(3)}|y - x|,
\end{aligned}
\tag{77}
$$

where $x, y \in \mathbb{R}, x \neq y$.

Assumption 2. The activation functions $f_j(\cdot), g_j(\cdot)$, and $h_j(\cdot)$ are bounded. That is, for any $j \in \mathbb{N}$, there exist three constants $M_j^{(1)}, M_j^{(2)}$, and $M_j^{(3)}$ such that

$$
\begin{aligned}
& \left|f_j(x)\right| \leq M_j^{(1)}, \\
& \left|g_j(x)\right| \leq M_j^{(2)}, \\
& \left|h_j(x)\right| \leq M_j^{(3)},
\end{aligned}
\tag{78}
$$

where $x \in \mathbb{R}$.

It is obvious that error system (75) can be expressed as follows:

$$\begin{aligned}
{}_{t_0}^{C}D_t^{\alpha}e_i(t) &= -d_i e_i(t) + (1-\lambda)I_i + +u_i(t) + \sum_{j=1}^{n}\left(\overline{a}_{ij} + \widetilde{a}_{ij}\gamma_{ij}^{(1)}\right)\left[f_j\left(y_j(t)\right) - f_j\left(\lambda x_j(t)\right)\right] \\
&\quad + \sum_{j=1}^{n}\left(\overline{a}_{ij} + \widetilde{a}_{ij}\gamma_{ij}^{(1)}\right)\left[f_j\left(\lambda x_j(t)\right) - f_j\left(x_j(t)\right)\right] + \sum_{j=1}^{n}\left[(1-\lambda)\overline{a}_{ij} + \left(\gamma_{ij}^{(1)} - \lambda\delta_{ij}^{(1)}\right)\widetilde{a}_{ij}\right]f_j\left(x_j(t)\right) \\
&\quad + \sum_{j=1}^{n}\left(\overline{b}_{ij} + \widetilde{b}_{ij}\gamma_{ij}^{(2)}\right)\left[g_j\left(y_j\left(t-\tau_j(t)\right)\right) - g_j\left(\lambda x_j\left(t-\tau_j(t)\right)\right)\right] \\
&\quad + \sum_{j=1}^{n}\left(\overline{b}_{ij} + \widetilde{b}_{ij}\gamma_{ij}^{(2)}\right)\left[g_j\left(\lambda x_j\left(t-\tau_j(t)\right)\right) - g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right] \\
&\quad + \sum_{j=1}^{n}\left[(1-\lambda)\overline{b}_{ij} + \left(\gamma_{ij}^{(2)} - \lambda\delta_{ij}^{(2)}\right)\widetilde{b}_{ij}\right]g_j\left(x_j\left(t-\tau_j(t)\right)\right) \\
&\quad + \sum_{j=1}^{n}\left(\overline{c}_{ij} + \widetilde{c}_{ij}\gamma_{ij}^{(3)}\right)\int_{t-\rho_j(t)}^{t}h_j\left(y_j(s)\right)\mathrm{d}s - \lambda\sum_{j=1}^{n}\left(\overline{c}_{ij} + \widetilde{c}_{ij}\delta_{ij}^{(3)}\right)\int_{t-\rho_j(t)}^{t}h_j\left(x_j(s)\right)\mathrm{d}s \\
&\leq -d_i e_i(t) + (1-\lambda)I_i + u_i(t) + \sum_{j=1}^{n}a_{ij}^{+}\left|f_j\left(y_j(t)\right) - f_j\left(\lambda x_j(t)\right)\right| + \sum_{j=1}^{n}a_{ij}^{+}\left|f_j\left(\lambda x_j(t)\right) - f_j\left(x_j(t)\right)\right| \\
&\quad + \sum_{j=1}^{n}\left|(1-\lambda)\overline{a}_{ij} + \left(\gamma_{ij}^{(1)} - \lambda\delta_{ij}^{(1)}\right)\widetilde{a}_{ij}\right|\left|f_j\left(x_j(t)\right)\right| + \sum_{j=1}^{n}b_{ij}^{+}\left|tg_jn\left(y_j\left(t-\tau_j(t)\right)\right)q - hg_{j\left(\lambda x_j\left(t-\tau_j(t)\right)\right)}\right| \\
&\quad + \sum_{j=1}^{n}b_{ij}^{+}\left|tg_jn\left(\lambda x_j\left(t-\tau_j(t)\right)\right)q - hg_{j\left(x_j\left(t-\tau_j(t)\right)\right)}\right| + \sum_{j=1}^{n}\left|(1-\lambda)\overline{b}_{ij} + \left(\gamma_{ij}^{(2)} - \lambda\delta_{ij}^{(2)}\right)\widetilde{b}_{ij}\right|\left|g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right| \\
&\quad + \sum_{j=1}^{n}c_{ij}^{+}\int_{t-\rho_j(t)}^{t}\left|h_j\left(y_j(s)\right)\right|\mathrm{d}s + \sum_{j=1}^{n}c_{ij}^{+}|\lambda|\int_{t-\rho_j(t)}^{t}\left|h_j\left(x_j(s)\right)\right|\mathrm{d}s
\end{aligned} \tag{79}$$

According to Assumption 1, we can judge

$$\begin{aligned}
&\sum_{j=1}^{n}a_{ij}^{+}\left|f_j\left(y_j(t)\right) - f_j\left(\lambda x_j(t)\right)\right| \leq \sum_{j=1}^{n}a_{ij}^{+}L_j^{(1)}\left|e_j(t)\right|, \\
&\sum_{j=1}^{n}b_{ij}^{+}\left|g_j\left(y_j\left(t-\tau_j(t)\right)\right) - g_j\left(\lambda x_j\left(t-\tau_j(t)\right)\right)\right| \\
&\leq \sum_{j=1}^{n}b_{ij}^{+}L_j^{(2)}\left|e_j\left(t-\tau_j(t)\right)\right|.
\end{aligned} \tag{80}$$

According to Assumption 2, we have

$$\begin{aligned}
&\sum_{j=1}^{n}a_{ij}^{+}\left|f_j\left(\lambda x_j(t)\right) - f_j\left(x_j(t)\right)\right| \leq \sum_{j=1}^{n}2a_{ij}^{+}M_j^{(1)} \\
&\sum_{j=1}^{n}\left|(1-\lambda)\overline{a}_{ij} + \left(\gamma_{ij}^{(1)} - \lambda\delta_{ij}^{(1)}\right)\widetilde{a}_{ij}\right|\left|f_j\left(x_j(t)\right)\right| \leq \sum_{j=1}^{n}\left|(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}\right|M_j^{(1)} \\
&\sum_{j=1}^{n}b_{ij}^{+}\left|g_j\left(\lambda x_j\left(t-\tau_j(t)\right)\right) - g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right| \leq \sum_{j=1}^{n}2b_{ij}^{+}M_j^{(2)} \\
&\sum_{j=1}^{n}\left|(1-\lambda)\overline{b}_{ij} + \left(\gamma_{ij}^{(2)} - \delta_{ij}^{(2)}\right)\widetilde{b}_{ij}\right|\left|g_j\left(x_j\left(t-\tau_j(t)\right)\right)\right| \leq \sum_{j=1}^{n}\left|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\right|M_j^{(2)} \\
&\sum_{j=1}^{n}c_{ij}^{+}\int_{t-\rho_j(t)}^{t}\left|h_j\left(y_j(s)\right)\right|\mathrm{d}s + \sum_{j=1}^{n}c_{ij}^{+}|\lambda|\int_{t-\rho_j(t)}^{t}\left|h_j\left(x_j(s)\right)\right|\mathrm{d}s \leq \sum_{j=1}^{n}(1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}.
\end{aligned} \tag{81}$$

Substituting inequations (80) and (81) into equation (79), we can obtain

$$
\begin{aligned}
{}_{t_0}^{C}D_t^{\alpha}e_i(t) \leq &-d_i e_i(t) + (1-\lambda)I_i + u_i(t) \\
&+ \sum_{j=1}^{n}\Big[L_j^{(1)}a_{ij}^{+}\big|e_j(t)\big| + L_j^{(2)}b_{ij}^{+}\big|e_j\big(t-\tau_j(t)\big)\big| + \Big(2a_{ij}^{+} + |(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}|\Big)M_j^{(1)} \\
&+ \Big(2b_{ij}^{+} + \big|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\big|\Big)M_j^{(2)} + (1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\Big], \quad i=1,2,\ldots,n,\ t \geq t_0.
\end{aligned}
\tag{82}
$$

3. Main Results

In this section, a feedback controller is designed for actualizing finite-time projective synchronization of fractional-order memristor-based neural networks with mixed time-varying delays. And we choose the following feedback control strategy:

$$
u_i(t) = -k_i e_i(t) - \text{sign}\,(e_i(t))\big(h_i\big|e_i\big(t-\tau_i(t)\big)\big| + q_i\big) - (1-\lambda)I_i,
\tag{83}
$$

where k_i, h_i, and q_i denote the controller parameters.

3.1. The Finite-Time Projective Synchronization of FMNNs with a Lyapunov Function Based on 1-Norm

$$
\varphi_i = d_i + k_i - \sum_{j=1}^{n} L_i^{(1)}a_{ji}^{+},
$$

$$
\delta_i = h_i - \sum_{j=1}^{n} L_i^{(2)}b_{ji}^{+},
$$

$$
\begin{aligned}
\eta_i = q_i - \sum_{j=1}^{n}\Big[&\Big(2a_{ij}^{+} + |(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}|\Big)M_j^{(1)} \\
&+ \Big(2b_{ij}^{+} + \big|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\big|\Big)M_j^{(2)} + (1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\Big].
\end{aligned}
\tag{85}
$$

Moreover, the settling time T_1 is estimated by

$$
T_1^{*} \leq t_0 + \varphi^{-1/\alpha}\ln\frac{\varphi V_1(t_0) + \eta}{\alpha\eta}.
\tag{86}
$$

Proof. Firstly, we consider the following Lyapunov function:

$$
V_1(t) = \sum_{i=1}^{n}\big|e_i(t)\big|.
\tag{87}
$$

Theorem 1. *When Assumptions 1 and 2 hold, drive system (66) and response system (72) achieve the finite-time projective synchronization under state feedback controller (83) if*

$$
\begin{aligned}
\varphi &= \min_{1 \leq i \leq n}\{\varphi_i\} > 0, \\
\delta &= \min_{1 \leq i \leq n}\{\delta_i\} > 0, \\
\eta &= \sum_{i=1}^{n}\eta_i > 0,
\end{aligned}
\tag{84}
$$

where

Based on Lemma 8, the above fractional-order derivative of $V(t)$ can be written as follows:

$$
{}_{t_0}^{C}D_t^{\alpha}V_1(t) \leq \sum_{i=1}^{n}\text{sign}\,(e_i(t))\,{}_{t_0}^{C}D_t^{\alpha}e_i(t).
\tag{88}
$$

From error system (82), we have

$$
{}_{t_0}^{C}D_t^{\alpha}V_1(t) \le \sum_{i=1}^{n}\text{sign}(e_i(t))\,{}_{t_0}^{C}D_t^{\alpha}e_i(t)
$$

$$
\le \sum_{i=1}^{n}\text{sign}(e_i(t))\left\{-d_ie_i(t)+(1-\lambda)I_i+u_i(t)+\sum_{j=1}^{n}\left[L_j^{(1)}a_{ij}^{+}\big|e_j(t)\big|+L_j^{(2)}b_{ij}^{+}\big|e_j(t-\tau_j(t))\big|\right.\right.
$$

$$
\left.\left.+\left(2a_{ij}^{+}+\big|(1-\lambda)\overline{a}_{ij}+(1+|\lambda|)\widetilde{a}_{ij}\big|\right)M_j^{(1)}+\left(2b_{ij}^{+}+\big|(1-\lambda)\overline{b}_{ij}+(1+|\lambda|)\widetilde{b}_{ij}\big|\right)M_j^{(2)}+(1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\right]\right\}
$$

$$
\le \sum_{i=1}^{n}\left\{-d_i\big|e_i(t)\big|+\text{sign}(e_i(t))((1-\lambda)I_i+u_i(t))+\sum_{j=1}^{n}L_i^{(1)}a_{ji}^{+}\big|e_i(t)\big|+L_i^{(2)}b_{ji}^{+}\big|e_i(t-\tau_i(t))\big|\right.
$$

$$
\left.+\left(2a_{ij}^{+}+\big|(1-\lambda)\overline{a}_{ij}+(1+|\lambda|)\widetilde{a}_{ij}\big|\right)M_j^{(1)}+\left(2b_{ij}^{+}+\big|(1-\lambda)\overline{b}_{ij}+(1+|\lambda|)\widetilde{b}_{ij}\big|\right)M_j^{(2)}+(1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\right\}.
$$

$$
\tag{89}
$$

Substituting the controller (83) into inequation (89),

$$
{}_{t_0}^{C}D_t^{\alpha}V_1(t) \le \sum_{i=1}^{n}\left\{-d_i\big|e_i(t)\big|+\sum_{j=1}^{n}\left[L_i^{(1)}a_{ji}^{+}\big|e_i(t)\big|+L_i^{(2)}b_{ji}^{+}\big|e_i(t-\tau_i(t))\big|+\left(2a_{ij}^{+}+\big|(1-\lambda)\overline{a}_{ij}+(1+|\lambda|)\widetilde{a}_{ij}\big|\right)M_j^{(1)}\right.\right.
$$

$$
\left.\left.+\left(2b_{ij}^{+}+\big|(1-\lambda)\overline{b}_{ij}+(1+|\lambda|)\widetilde{b}_{ij}\big|\right)M_j^{(2)}+(1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\right]-k_i\big|e_i(t)\big|-h_i\big|e_i(t-\tau_i(t))\big|-q_i\right\}
$$

$$
= -\sum_{i=1}^{n}\left[d_i+k_i-\sum_{j=1}^{n}L_i^{(1)}a_{ji}^{+}\right]\big|e_i(t)\big|-\sum_{i=1}^{n}\left(h_i-\sum_{j=1}^{n}L_i^{(2)}b_{ji}^{+}\right)\big|e_i(t-\tau_i(t))\big|
$$

$$
-\sum_{i=1}^{n}\left\{q_i-\sum_{j=1}^{n}\left[\left(2a_{ij}^{+}+\big|(1-\lambda)\widetilde{a}_{ij}+(1+|\lambda|)\widetilde{a}_{ij}\big|\right)M_j^{(1)}+\left(2b_{ij}^{+}+\big|(1-\lambda)\overline{b}_{ij}+(1+|\lambda|)\widetilde{b}_{ij}\big|\right)M_j^{(2)}+(1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\right]\right\}.
$$

$$
\tag{90}
$$

Let

$$
\varphi_i = d_i+k_i-\sum_{j=1}^{n}L_i^{(1)}a_{ji}^{+},
$$

$$
\delta_i = h_i-\sum_{j=1}^{n}L_i^{(2)}b_{ji}^{+},
$$

$$
\eta_i = q_i-\sum_{j=1}^{n}\left[\left(2a_{ij}^{+}+\big|(1-\lambda)\overline{a}_{ij}+(1+|\lambda|)\widetilde{a}_{ij}\big|\right)M_j^{(1)}\right.
$$

$$
\left.+\left(2b_{ij}^{+}+\big|(1-\lambda)\overline{b}_{ij}+(1+|\lambda|)\widetilde{b}_{ij}\big|\right)M_j^{(2)}+(1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\right].
$$

$$
\tag{91}
$$

Therefore, in equation (90) can be rewritten as follows:

$$
{}_{t_0}^{C}D_t^{\alpha}V_1(t) \le -\sum_{i=1}^{n}\varphi_i\big|e_i(t)\big|-\sum_{i=1}^{n}\delta_i\big|e_i(t-\tau_i(t))\big|-\sum_{i=1}^{n}\eta_i
$$

$$
\le -\varphi\sum_{i=1}^{n}\big|e_i(t)\big|-\delta\sum_{i=1}^{n}\big|e_i(t-\tau_i(t))\big|-\eta
$$

$$
= -\varphi V_1(t)-\delta V_1(t-\tau_i(t))-\eta,
$$

$$
\tag{92}
$$

where $\varphi = \min_{1\le i\le n}\{\varphi_i\}$, $\delta = \min_{1\le i\le n}\{\delta_i\}$, and $\eta = \sum_{i=1}^{n}\eta_i$.

Based on the condition $\delta > 0$ in Theorem 1, we have

$$
{}_{t_0}^{C}D_t^{\alpha}V_1(t) \le -\varphi V_1(t)-\eta,
$$

$$
\tag{93}
$$

According to the conditions of Theorem 1, we can gain $\varphi > 0, \eta > 0$. Furthermore, by virtue of Property 1, it follows that

$$
V_1(t) \le \left[V_1(t_0)+\frac{\eta}{\varphi}\right]\frac{1}{\alpha}\exp\left\{-\varphi^{1/\alpha}(t-t_0)\right\}-\frac{\eta}{\varphi}, \quad t\ge t_0.
$$

$$
\tag{94}
$$

And we can judge that the Lyapunov function $V_1(t)$ converges to 0, which represents drive system (66) and response system (72) actualizing finite-time projective synchronization under feedback controller (83). And the settling time T_1 which can be estimated is as follows:

$$
T_1^{*} \le t_0+\varphi^{-1/\alpha}\ln\frac{\varphi V_1(t_0)+\eta}{\eta\alpha}.
$$

$$
\tag{95}
$$

This completes the proof.

□

Remark 4. For the differential inequation ${}_{t_0}^{C}D_t^{\alpha}V_1(t) \le -\varphi V_1(t)-\eta$ in fractional-order neural networks, Zhang et al.

[29] ignored the effort of parameter φ when they calculated the settling time.

$$\ _{t_0}^{C}D_t^{\alpha}V_1(t) \leq -\eta. \tag{96}$$

Taking the fractional integral with variable order α on both sides of the above inequation, we can gain

$$V_1(t) - V_1(t_0) \leq \frac{-1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1}\eta \, ds. \tag{97}$$

Through Definition 1, we can calculate that $V(t)$ converges to 0 within the settling time T_2, and T_2 is evaluated by

$$T_2 \leq t_0 + \left[\frac{V_1(t_0)\Gamma(1+\alpha)}{\eta}\right]^{1/\alpha}. \tag{98}$$

Apparently, the authors ignored the impact of parameter η, and let $\eta = 0$ which will increase the conservation of the settling time. Considering the parameter $\eta > 0$; the estimation of settling time T_1^* in Theorem 1 is less conservative than the results in [29]. To verify that the settling time T_1^* is more accurate than T_2, a comparison table has been given in numerical simulation.

Remark 5. For the differential inequation $_{t_0}^{C}D_t^{\alpha}V_1(t) \leq -\varphi V_1(t) - \eta$, when fractional order $\alpha = 1$, the settling time t_1 is estimated by

$$t_1 \leq t_0 + \frac{\ln[\varphi V_1(t_0)] - \ln \eta}{\varphi}. \tag{99}$$

Obviously, the estimate time t_1 is consistent with the result of the integer-order system given in [51]. Thus, it shows the estimation of settling time T_1^* in Theorem 1 is more accurate and general.

3.2. The Finite-Time Projective Synchronization of FMNNs with a Lyapunov Function Based on p-Norm

Theorem 2. *Under the state feedback controller (83) and Assumptions 1 and 2, drive system (66) and response system (72) achieve the finite-time projective synchronization if*

$$\overline{\varphi} = \min_{1 \leq i \leq n}\{\overline{\varphi}_i\} > 0,$$

$$\overline{\delta} = \min_{1 \leq i \leq n}\{\overline{\delta}_i\} > 0,$$

$$\overline{\eta} = \min_{1 \leq i \leq n}\{\overline{\eta}_i\} > 0,$$

$$\overline{\varphi}_i = \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\left[d_i + k_i - \sum_{j=1}^{n}\left(L_j^{(1)}a_{ij}^+ \frac{(p-1)\varepsilon_1}{p} + \frac{L_i^{(1)}a_{ji}^+}{p\varepsilon_1^{p-1}} + L_j^{(2)}b_{ij}^+ \frac{(p-1)\varepsilon_2}{p}\right)\right],$$

$$\overline{\delta}_i = \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\left(h_i - \sum_{j=1}^{n}\frac{L_i^{(2)}b_{ji}^+}{p\varepsilon_2^{p-1}}\right),$$

$$\overline{\eta}_i = \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\left\{q_i - \sum_{j=1}^{n}\left[\left(2a_{ij}^+ + |(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}|\right)M_j^{(1)}\right.\right.$$

$$\left.\left. + \left(2b_{ij}^+ + |(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}|\right)M_j^{(2)} + (1+|\lambda|)c_{ij}^+\rho_j^{\max}M_j^{(3)}\right]\right\},$$

$$\tag{100}$$

where ε_1 and ε_2 are arbitrary positive constants.

Moreover, the settling time T_2 is estimated by

$$T_3^* \leq t_0 + \left[\frac{\Gamma(1+(p/(p-1))-\alpha)\overline{\varphi}}{\Gamma(1+(p/(p-1)))\Gamma(2-\alpha)}\right]^{-1/\alpha} \ln\frac{\overline{\varphi}V_2^{1/p}(t_0) + \overline{\eta}}{\overline{\eta}\alpha}. \tag{101}$$

Proof. Firstly, we consider the following Lyapunov function:

$$V_2(t) = \sum_{i=1}^{n}|e_i(t)|^p, \tag{102}$$

where p is a positive integer with $p > 1$.

Calculating the derivate of $V_2(t)$ and using Lemma 7 and error system (82), one has

$$\ _{t_0}^{C}D_t^{\alpha}V_2(t) = \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\sum_{i=1}^{n}|e_i(t)|^{p-1}\ _{t_0}^{C}D_t^{\alpha}|e_i(t)|. \tag{103}$$

According to Lemma 8, the above fractional-order derivative of $V(t)$ can be written as follows:

$$
{}^{C}_{t_0}D^{\alpha}_{t}V_2(t) \leq \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)} \sum_{i=1}^{n} |e_i(t)|^{p-1} \operatorname{sign}(e_i(t))^{C}_{t_0}D^{\alpha}_{t}e_i(t)
$$

$$
\leq \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)} \sum_{i=1}^{n} \Bigg\{ -d_i|e_i(t)|^p + \operatorname{sign}(e_i(t))\big((1-\lambda)I_i + u_i(t)\big)|e_i(t)|^{p-1}
$$

$$
+ \sum_{j=1}^{n} \Big[L_i^{(1)}a_{ij}^{+}|e_j(t)| + L_i^{(2)}b_{ij}^{+}|e_j(t-\tau_j(t))| + \Big(2a_{ij}^{+} + \big|(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}\big|\Big)M_j^{(1)}
$$

$$
+ \Big(2b_{ij}^{+} + \big|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\big|\Big)M_j^{(2)} + (1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\Big]|e_i(t)|^{p-1}\Bigg\}.
$$

$$(104)$$

Substituting the feedback controller (83) into inequation (104), we can get

$$
{}^{C}_{t_0}D^{\alpha}_{t}V_2(t) \leq \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)} \sum_{i=1}^{n} \Bigg\{ -d_i|e_i(t)|^p + \sum_{j=1}^{n}\Big[L_j^{(1)}a_{ij}^{+}|e_i(t)|^{p-1}|e_j(t)| + L_j^{(2)}b_{ij}^{+}|e_i(t)|^{p-1}|e_j(t-\tau_j(t))|\Big]
$$

$$
+ \sum_{j=1}^{n}\Big[\Big(2a_{ij}^{+} + |(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}|\Big)M_j^{(1)} + \Big(2b_{ij}^{+} + |(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}|\Big)M_j^{(2)}
$$

$$
+ (1+|\lambda|)c_{ij}^{+}\rho_j^{\max}M_j^{(3)}\Big]|e_i(t)|^{p-1} - k_i|e_i(t)|^p - h_i|e_i(t-\tau_i(t))|^p - q_i|e_i(t)|^{p-1}\Bigg\}.
$$

$$(105)$$

By applying Lemma 3, we have

$$
|e_i(t)|^{p-1}|e_j(t)| \leq \frac{(p-1)\varepsilon_1}{p}|e_i(t)|^p + \frac{1}{p\varepsilon_1^{p-1}}|e_j(t)|^p,
$$

$$
|e_i(t)|^{p-1}|e_j(t-\tau_j(t))| \leq \frac{(p-1)\varepsilon_2}{p}|e_i(t)|^p + \frac{1}{p\varepsilon_2^{p-1}}|e_j(t-\tau_j(t))|^p,
$$

$$(106)$$

where ε_1 and ε_2 are arbitrary positive constants.

Combining with inequations (105) and (106), one has

$$
\begin{aligned}
{}^{C}_{t_0}Dt\alpha V_1(t) \le{} & \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\sum_{i=1}^{n}\Bigg\{-(d_i+k_i)\big|e_i(t)\big|^p - h_i\big|e_i(t-\tau_i(t))\big|^p - q_i\big|e_i(t)\big|^{p-1} \\
& + \sum_{j=1}^{n}\left(L_j^{(1)}a_{ij}^+\frac{(p-1)\varepsilon_1}{p} + \frac{L_i^{(1)}a_{ji}^+}{p\varepsilon_1^{p-1}} + L_j^{(2)}b_{ij}^+\frac{(p-1)\varepsilon_2}{p}\right)\big|e_i(t)\big|^p + \sum_{j=1}^{n}\frac{L_i^{(2)}b_{ji}^+}{p\varepsilon_2^{p-1}}\big|e_i(t-\tau_i(t))\big|^p \\
& + \sum_{j=1}^{n}\Bigg[\Big(2a_{ij}^+ + \big|(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}\big|\Big)M_j^{(1)} + \Big(2b_{ij}^+ + \big|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\big|\Big)M_j^{(2)} + (1+|\lambda|)c_{ij}^+\rho_j^{\max}M_j^{(3)}\Bigg]\big|e_i(t)\big|^{p-1}\Bigg\} \\
={} & -\frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\sum_{i=1}^{n}\Bigg[d_i+k_i - \sum_{j=1}^{n}\left(L_j^{(1)}a_{ij}^+\frac{(p-1)\varepsilon_1}{p} + \frac{L_i^{(1)}a_{ji}^+}{p\varepsilon_1^{p-1}} + L_j^{(2)}b_{ij}^+\frac{(p-1)\varepsilon_2}{p}\right)\Bigg]\big|e_i(t)\big|^p \\
& -\frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\sum_{i=1}^{n}\left(h_i - \sum_{j=1}^{n}\frac{L_i^{(2)}b_{ji}^+}{p\varepsilon_2^{p-1}}\right)\big|e_i(t-\tau_i(t))\big|^p \\
& -\frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\sum_{i=1}^{n}\Bigg\{q_i - \sum_{j=1}^{n}\Bigg[\Big(2a_{ij}^+ + \big|(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}\big|\Big)M_j^{(1)} \\
& + \Big(2b_{ij}^+ + \big|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\big|\Big)M_j^{(2)} + (1+|\lambda|)c_{ij}^+\rho_j^{\max}M_j^{(3)}\Bigg]\Bigg\}\big|e_i(t)\big|^{p-1} \\
\le{} & -\sum_{i=1}^{n}\overline{\varphi}_i\big|e_i(t)\big|^p - \sum_{i=1}^{n}\overline{\delta}_i\big|e_i(t-\tau_j(t))\big|^p - \sum_{i=1}^{n}\overline{\eta}_i\big|e_i(t)\big|^{p-1},
\end{aligned}
\tag{107}
$$

where

$$
\begin{aligned}
\overline{\varphi}_i &= \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\Bigg[d_i+k_i - \sum_{j=1}^{n}\left(L_j^{(1)}a_{ij}^+\frac{(p-1)\varepsilon_1}{p} + \frac{L_i^{(1)}a_{ji}^+}{p\varepsilon_1^{p-1}} + L_j^{(2)}b_{ij}^+\frac{(p-1)\varepsilon_2}{p}\right)\Bigg], \\
\overline{\delta}_i &= \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\left(h_i - \sum_{j=1}^{n}\frac{L_i^{(2)}b_{ji}^+}{p\varepsilon_2^{p-1}}\right), \\
\overline{\eta}_i &= \frac{\Gamma(2-\alpha)\Gamma(1+p)}{\Gamma(1+p-\alpha)}\Bigg\{q_i - \sum_{j=1}^{n}\Bigg[\Big(2a_{ij}^+ + \big|(1-\lambda)\overline{a}_{ij} + (1+|\lambda|)\widetilde{a}_{ij}\big|\Big)M_j^{(1)} \\
& \qquad + \Big(2b_{ij}^+ + \big|(1-\lambda)\overline{b}_{ij} + (1+|\lambda|)\widetilde{b}_{ij}\big|\Big)M_j^{(2)} + (1+|\lambda|)c_{ij}^+\rho_j^{\max}M_j^{(3)}\Bigg]\Bigg\}.
\end{aligned}
\tag{108}
$$

Denote $\overline{\varphi} = \min_{1\le i\le n}\{\overline{\varphi}_i\}$, $\overline{\delta} = \min_{1\le i\le n}\{\overline{\delta}_i\}$, and $\overline{\eta} = \min_{1\le i\le n}\{\overline{\eta}_i\}$; one gets

$$
{}^{C}_{t_0}D_t^{\alpha}V_2(t) \le -\overline{\varphi}\sum_{i=1}^{n}\big|e_i(t)\big|^p - \overline{\delta}\sum_{i=1}^{n}\big|e_i(t-\tau_i(t))\big|^p - \overline{\eta}\sum_{i=1}^{n}\big|e_i(t)\big|^{p-1}.
\tag{109}
$$

Applying Lemma 4, we have

$$
\begin{aligned}
{}^{C}_{t_0}D_t^{\alpha}V_2(t) &\le -\overline{\varphi}\sum_{i=1}^{n}\big|e_i(t)\big|^p - \overline{\delta}\sum_{i=1}^{n}\big|e_i(t-\tau_i(t))\big|^p - \overline{\eta}\left(\sum_{i=1}^{n}\big|e_i(t)\big|^p\right)^{(p-1)/p} \\
&= -\overline{\varphi}V_2(t) - \overline{\delta}V_2(t-\tau_i(t)) - \overline{\eta}V_2(t)^{(p-1)/p}.
\end{aligned}
\tag{110}
$$

Based on the conditions of Theorem 2, we know $\overline{\varphi} > 0$, $\overline{\delta} > 0$, and $\overline{\eta} > 0$. Thus,

$$
{}^{C}_{t_0}D_t^{\alpha}V_2(t) \le -\overline{\varphi}V_2(t) - \overline{\eta}V_2^{(p-1)/p}(t).
\tag{111}
$$

Hence, from Property 3, we have

$$V_2^{1/p}(t) \leq \left[V_2^{1/p}(t_0) + \frac{\overline{\eta}}{\overline{\varphi}} \right] \frac{1}{\alpha} \exp\left\{ -\left(\frac{a\Gamma(1 + (p/(p-1)) - \alpha)}{\Gamma(1 + (p/(p-1)))\Gamma(2-\alpha)} \right)^{1/\alpha} (t - t_0) \right\} - \frac{\overline{\eta}}{\overline{\varphi}}. \tag{112}$$

And we can judge that the Lyapunov function $V_2(t)$ converges to 0, which represents drive system (66) and response system (72) achieve finite-time projective synchronization under feedback controller (83). And the settling time T_3^* can be estimated by

$$T_3^* \leq t_0 + \left[\frac{\Gamma(1 + (p/(p-1)) - \alpha)\overline{\varphi}}{\Gamma(1 + (p/(p-1)))\Gamma(2-\alpha)} \right]^{-(1/\alpha)} \ln \frac{\overline{\varphi}V_2^{1/p}(t_0) + \overline{\eta}}{\overline{\eta}\alpha}. \tag{113}$$

The proof is accomplished. $\square$

Remark 6. For the differential inequation ${}_{t_0}^C D_t^{\alpha} V_2(t) \leq -\overline{\varphi}V_2(t) - \overline{\eta}V_2^{(p-1)/p}(t)$, considering the parameter $\overline{\varphi} = 0$, then

$$_{t_0}^C D_t^{\alpha} V_2(t) \leq -\overline{\eta}V_2^{(p-1)/p}(t), \tag{114}$$

where $\overline{\eta} > 0$.

According to Corollary 1, it has

$$V_2(t) \leq \left[V_2^{1/p}(t_0) - \overline{\eta} \cdot \frac{a\Gamma(1 + p - \alpha)(t - t_0)^{\alpha}}{\Gamma(1 + p)\Gamma(2 - \alpha)\Gamma(1 + \alpha)} \right]^{1/(1-\beta)}. \tag{115}$$

And we can judge that the Lyapunov function $V_2(t)$ converges to 0. Furthermore, the settling time

$$T_4 \leq t_0 + \left[V_2^{(1/p)}(t_0) \frac{\Gamma(1 + p)\Gamma(2 - \alpha)\Gamma(1 + \alpha)}{\Gamma(1 + p - \alpha)\overline{\eta}} \right]^{(1/\alpha)}. \tag{116}$$

From Remark 2, if $((p - 1)/p) < \alpha(1 + \alpha) - 1$ holds, we can obtain another estimation of settling time:

$$T_5 \leq t_0 + \left[V_2^{\alpha(1+\alpha)+(1/p)-2}(t_0) \frac{\Gamma(1 + \alpha)\Gamma(1 - ((2p - 1)/p(1 + \alpha)))}{\overline{\eta} \cdot \Gamma(2 + \alpha)\Gamma(1 + \alpha - ((2p - 1)/p(1 + \alpha)))} \right]^{(1/\alpha)}. \tag{117}$$

Obviously, compared with T_3^* in Theorem 2, the estimate times T_4 and T_5 are conservative estimations of settling time. And a comparison table about T_3^*, T_4, and T_5 has been given in numerical simulation.

4. Numerical Simulation

In this section, several numerical examples are given to illustrate the effectiveness of the obtained results. We consider the following two-dimensional fractional-order memristive neural networks with mixed time-varying delays:

$$_{t_0}^C D_t^{0.98} x_i(t) = -d_i x_i(t) + \sum_{j=1}^{2} a_{ij}(x_i(t)) f_j(x_j(t)) + \sum_{j=1}^{2} b_{ij}(x_i(t)) g_j(x_j(t - \tau_j(t))) + \sum_{j=1}^{2} c_{ij}(x_i(t)) \int_{t-\rho_j(t)}^{t} h_j(x_j(s)) \, ds + I_i,$$

$$_{t_0}^C D_t^{0.98} y_i(t) = -d_i y_i(t) + \sum_{j=1}^{2} a_{ij}(y_i(t)) f_j(y_j(t)) + \sum_{j=1}^{2} b_{ij}(y_i(t)) g_j(y_j(t - \tau_j(t))) + \sum_{j=1}^{2} c_{ij}(y_i(t)) \int_{t-\rho_j(t)}^{t} h_j(y_j(s)) \, ds + I_i + u_i(t),$$

$$\tag{118}$$

where $i = 1, 2$. And the values of the memristors are given as follows:

$$a_{11}(x_1(t)) = \begin{cases} 0.3, & |x_1(t)| \le 1, \\ 0.2, & |x_1(t)| > 1, \end{cases}$$

$$a_{12}(x_1(t)) = \begin{cases} -0.4, & |x_1(t)| \le 1, \\ -0.22, & |x_1(t)| > 1, \end{cases}$$

$$a_{21}(x_2(t)) = \begin{cases} -0.9, & |x_2(t)| \le 1, \\ -0.8, & |x_2(t)| > 1, \end{cases}$$

$$a_{22}(x_2(t)) = \begin{cases} 0.5, & |x_2(t)| \le 1, \\ 0.8, & |x_2(t)| > 1, \end{cases}$$

$$b_{11}(x_1(t)) = \begin{cases} -0.6, & |x_1(t)| \le 1, \\ -0.2, & |x_1(t)| > 1, \end{cases}$$

$$b_{12}(x_1(t)) = \begin{cases} 0.5, & |x_1(t)| \le 1, \\ 0.5, & |x_1(t)| > 1, \end{cases} \tag{119}$$

$$b_{21}(x_2(t)) = \begin{cases} 0.1, & |x_2(t)| \le 1, \\ 0.4, & |x_2(t)| > 1, \end{cases}$$

$$b_{22}(x_2(t)) = \begin{cases} -0.4, & |x_2(t)| \le 1, \\ -0.3, & |x_2(t)| > 1, \end{cases}$$

$$c_{11}(x_1(t)) = \begin{cases} -0.1, & |x_1(t)| \le 1, \\ -0.4, & |x_1(t)| > 1, \end{cases}$$

$$c_{12}(x_1(t)) = \begin{cases} 0.7, & |x_1(t)| \le 1, \\ 0.8, & |x_1(t)| > 1, \end{cases}$$

$$c_{21}(x_2(t)) = \begin{cases} 0.4, & |x_2(t)| \le 1, \\ 0.3, & |x_2(t)| > 1, \end{cases}$$

$$c_{22}(x_2(t)) = \begin{cases} -0.85, & |x_2(t)| \le 1, \\ -0.95, & |x_2(t)| > 1. \end{cases}$$

For drive-response system (118), we select the system parameters as follows: $I_1 = I_2 = 0.1$, $d_1 = d_2 = 0.1$, $\tau_1(t) = e^t/(e^t + 1)$, $\tau_2(t) = 0.8 * e^t/(e^t + 1)$, $\rho_1(t) = 0.5\cos(t) + 0.5$, and $\rho_2(t) = 0.4\sin(t) + 0.4$. Hence, we can obtain that $\tau_1^{\max} = 1$, $\tau_2^{\max} = 0.8$, $\rho_1^{\max} = 1$, $\rho_2^{\max} = 0.8$, and $\tau = \max_{j=1,2}\{\tau_j(t), \rho_j(t)\} = 1$. And the initial values $x(s) = [-1.5, 2.5]^T$, $y(s) = [2.5, -0.5]^T$, $\forall s \in [-1, 0]$.

The activations are adopted as follows:

$$f_j(x_j(t)) = g_j(x_j(t)) = h_j(x_j(t)) = \frac{1 - e^{-x_j(t)}}{1 + e^{-x_j(t)}}, \quad j = 1, 2. \tag{120}$$

It is easy to check that Assumption 1 and 2 are satisfied. By simple calculation, $L_j^{(1)} = L_j^{(2)} = L_j^{(3)} = M_j^{(1)} = M_j^{(2)} = M_j^{(3)} = 1$, for $j = 1, 2$.

4.1. Projective Synchronization. First, we choose the projective coefficient $\lambda = 2$. We can get the initial values $\Phi_1(s) = y_1(s) - 2x_1(s) = 5.5$, $\Phi_2(s) = y_2(s) - 2x_2(s) = -5.5$, $V_1(t_0) = 11$, and $V_2(t_0) = 60.5$.

According to Theorem 1, it can be calculated as follows:

$$\varphi_1 = k_1 - 1.1, \varphi_2 = k_2 - 1.1,$$
$$\delta_1 = h_1 - 1, \delta_2 = h_2 - 0.9, \tag{121}$$
$$\eta_1 = q_1 - 8.9, \eta_2 = q_2 - 10.38.$$

And we choose $k_1 = 3.1$, $k_2 = 3.1$, $h_1 = 1$, $h_2 = 0.9$, $q_1 = 9.9$, and $q_2 = 11.38$ which make $\varphi = 2$ and $\eta = 2$.

Then, we have a finite-time fractional-order differential inequation as follows:

$${}_{t_0}^C D_t^\alpha V_1(t) \le -2V_1(t) - 2. \tag{122}$$

Based on the result in Theorem 1, drive-response system (118) could achieve synchronization in finite time. And we can calculate the setting time T_1^* as follows:

$$T_1^* \le t_0 + \varphi^{-1/\alpha} \ln\frac{\varphi V_1(t_0) + \eta}{\alpha\eta} \le 2^{-1/0.98} \ln\left(\frac{2 \times 11 + 2}{0.98 \times 2}\right) = 1.235. \tag{123}$$

To verify Remark 4, we can calculate T_2 to demonstrate that T_1^* is less conservative than T_2. It follows that

$$T_2 \le t_0 + \left[\frac{V_1(t_0)\Gamma(1+\alpha)}{\eta}\right]^{1/\alpha} = \left(\frac{11 \cdot \Gamma(1.98)}{2}\right)^{1/\alpha} = 5.5778. \tag{124}$$

On the contrary, we select $p = 2$ and $\varepsilon_1 = \varepsilon_2 = 1$ for Theorem 2. And it is easy to calculate

$$\overline{\varphi}_1 = 1.9608(k_1 - 1.4), \quad \overline{\varphi}_2 = 1.9608(k_2 - 1.85);$$
$$\overline{\delta}_1 = 1.9608(h_1 - 0.5), \quad \overline{\delta}_2 = 1.9608(h_2 - 0.45);$$
$$\overline{\eta}_1 = 1.9608(q_1 - 8.9), \quad \overline{\eta}_2 = 1.9608(q_2 - 10.38). \tag{125}$$

Under the controller parameter, $k_1 = 3.1$, $k_2 = 3.1$, $h_1 = 1$, $h_2 = 0.9$, $q_1 = 9.9$, and $q_2 = 11.38$ which lead to $\overline{\varphi} = 2.451$ and $\overline{\eta} = 1.9608$.

Hence, we have another finite-time fractional-order differential inequation as follows:

$${}_{t_0}^C D_t^\alpha V_2(t) \le -2.451V_2(t) - 1.9608V_2^{1/2}(t). \tag{126}$$

Based on the result in Theorem 2, we can estimate the setting time T_3 as follows:

$$T_3^* \le t_0 + \left[\frac{\Gamma(1 + (p/(p-1)) - \alpha)\overline{\varphi}}{\Gamma(1 + (p/(p-1)))\Gamma(2-\alpha)}\right]^{-1/\alpha} \ln\frac{\overline{\varphi}V_2^{1/p}(t_0) + \overline{\eta}}{\overline{\eta}\alpha} = \left[\frac{2.451\Gamma(2.02)}{\Gamma(3)\Gamma(1.02)}\right]^{-1/0.98} \ln\frac{2.451 \times \sqrt{60.5} + 1.9608}{1.9608 \times 0.98} = 1.9054. \tag{127}$$

To verify Remark 6, we can calculate T_4 to demonstrate that T_3^* is less conservative than T_4. It follows that

$$T_4 \leq t_0 + \left[V_2^{\frac{1}{p}}(t_0) \frac{\Gamma(1+p)\Gamma(2-\alpha)\Gamma(1+\alpha)}{\Gamma(1+p-\alpha)\overline{\eta}} \right]^{1/\alpha}$$

$$= \left[\sqrt{60.5} \cdot \frac{\Gamma(3)\Gamma(1.02)\Gamma(1.98)}{1.9608\Gamma(2.02)} \right]^{1/0.98} = 8.0421.$$

(128)

$$T_5 \leq t_0 + \left[V_2^{\alpha(1+\alpha)+(1/p)-2}(t_0) \frac{\Gamma(1+\alpha)\Gamma(1-((2p-1)/p(1+\alpha)))}{\overline{\eta} \cdot \Gamma(2+\alpha)\Gamma(1+\alpha-((2p-1)/p(1+\alpha)))} \right]^{1/\alpha} = \left[60.5^{0.4404} \cdot \frac{\Gamma(1.98)\Gamma(0.2424)}{1.9608 \cdot \Gamma(2.98)\Gamma(1.2224)} \right]^{1/0.98} = 6.689.$$

(129)

Therefore, drive-response system (118) can achieve finite-time projective synchronization under controller (83). And the state trajectories and error curves of FMCGNNs are as shown in Figures 2–4, which demonstrate the effectiveness of Theorems 1 and 2.

4.2. Complete Synchronization.
First, we choose the projective coefficient $\lambda = 1$. We can get $\Phi_1(s) = y_1(s) - 2x_1(s) = 4$, $\Phi_2(s) = y_2(s) - 2x_2(s) = -3$, $V_1(t_0) = 7$, and $V_2(t_0) = 25$.

According to Theorem 1, it can be calculated as follows:

$$\begin{aligned}
\varphi_1 &= k_1 - 1.1, \\
\varphi_2 &= k_2 - 1.1; \\
\delta_1 &= h_1 - 1, \\
\delta_2 &= h_2 - 0.9; \\
\eta_1 &= q_1 - 6.36, \\
\eta_2 &= q_2 - 8.12.
\end{aligned}$$

(130)

And we choose $k_1 = 3.1$, $k_2 = 3.1$, $h_1 = 1$, $h_2 = 0.9$, $q_1 = 7.36$, and $q_2 = 9.12$ which make $\varphi = 2$ and $\eta = 2$.

Then, we have a finite-time fractional-order differential inequation as follows:

$$^C_{t_0}D_t^\alpha V_1(t) \leq -2V_1(t) - 2.$$

(131)

Based on the result in Theorem 1, drive-response system (118) could achieve synchronization in finite time. And we can calculate the setting time T_1^* as follows:

Considering $p = 2$ and $\alpha = 0.98$, $((p-1)/p) < \alpha(1+\alpha) - 1$ holds. Thus, we have

$$T_1^* \leq t_0 + \varphi^{-1/\alpha} \ln \frac{\varphi V_1(t_0) + \eta}{\alpha\eta} \leq 2^{-1/0.98} \ln\left(\frac{2 \times 7 + 2}{0.98 \times 2}\right) = 1.0351.$$

(132)

To verify Remark 4, we can calculate T_2 to demonstrate that T_1^* is less conservative than T_2. It follows that

$$T_2 \leq t_0 + \left[\frac{V_1(t_0)\Gamma(1+\alpha)}{\eta} \right]^{1/\alpha} = \left(\frac{7 \cdot \Gamma(1.98)}{2} \right)^{1/\alpha} = 3.5169.$$

(133)

On the contrary, we select $p = 2$ and $\varepsilon_1 = \varepsilon_2 = 1$ for Theorem 2. And it is easy to calculate

$$\begin{aligned}
\overline{\varphi}_1 &= 1.9608(k_1 - 1.4), \\
\overline{\varphi}_2 &= 1.9608(k_2 - 1.85); \\
\overline{\delta}_1 &= 1.9608(h_1 - 0.5), \\
\overline{\delta}_2 &= 1.9608(h_2 - 0.45); \\
\overline{\eta}_1 &= 1.9608(q_1 - 6.36), \\
\overline{\eta}_2 &= 1.9608(q_2 - 8.12).
\end{aligned}$$

(134)

Under the controller parameter, $k_1 = 3.1$, $k_2 = 3.1$, $h_1 = 1$, $h_2 = 0.9$, $q_1 = 7.36$, and $q_2 = 9.12$ which lead to $\overline{\varphi} = 2.451$ and $\overline{\eta} = 1.9608$.

Hence, we have another finite-time fractional-order differential inequation as follows:

$$^C_{t_0}D_t^\alpha V_2(t) \leq -2.451 V_2(t) - 1.9608 V_2^{1/2}(t).$$

(135)

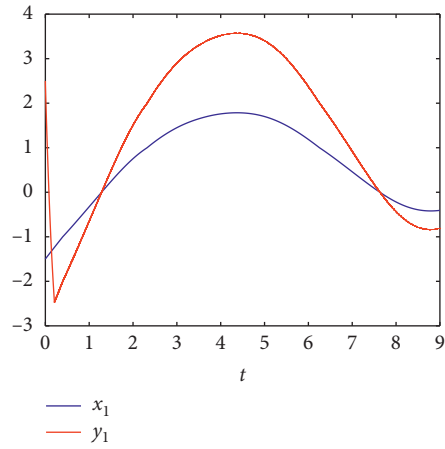Based on the result in Theorem 2, we can estimate the setting time T_3 as follows:

FIGURE 2: The first dimensional state trajectories of the drive system and the response system when the projective coefficient $\lambda = 2$.
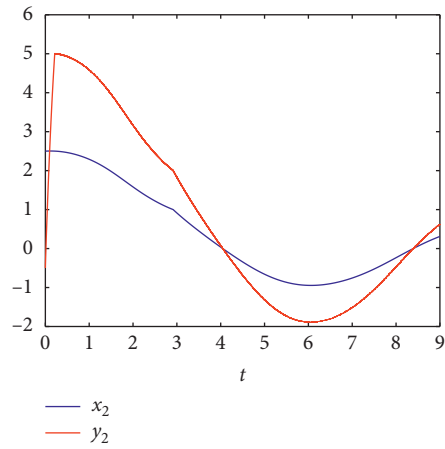

FIGURE 3: The second dimensional state trajectories of the drive system and the response system when the projective coefficient $\lambda = 2$.
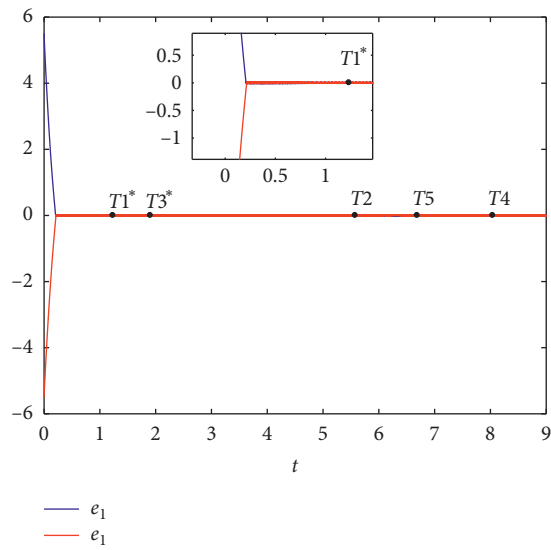

FIGURE 4: The synchronization error evolution when the projective coefficient $\lambda = 2$.

$$T_3^* \leq t_0 + \left[\frac{\Gamma(1 + (p/(p-1)) - \alpha)\overline{\varphi}}{\Gamma(1 + (p/(p-1)))\Gamma(2 - \alpha)} \right]^{-1/\alpha} \ln \frac{\overline{\varphi}V_2^{1/p}(t_0) + \overline{\eta}}{\overline{\eta}\alpha} = \left[\frac{2.451\Gamma(2.02)}{\Gamma(3)\Gamma(1.02)} \right]^{-1/0.98} \ln \frac{2.451 \times \sqrt{25} + 1.9608}{1.9608 \times 0.98} = 1.5937.$$

(136)

To verify Remark 6, we can calculate T_4 to demonstrate that T_3^* is less conservative than T_4. It follows that

$$T_4 \leq t_0 + \left[V_2^{1/p}(t_0) \frac{\Gamma(1 + p)\Gamma(2 - \alpha)\Gamma(1 + \alpha)}{\Gamma(1 + p - \alpha)\overline{\eta}} \right]^{1/\alpha} = \left[\sqrt{25} \cdot \frac{\Gamma(3)\Gamma(1.02)\Gamma(1.98)}{1.9608\Gamma(2.02)} \right]^{1/0.98} = 5.1232.$$

(137)

Considering $p = 2$ and $\alpha = 0.98$, $(p-1)/p < \alpha(1 + \alpha) - 1$ holds. Thus, we have

$$T_5 \leq t_0 + \left[V_2^{\alpha(1+\alpha)+(1/p)-2}(t_0) \frac{\Gamma(1 + \alpha)\Gamma(1 - ((2p-1)/p(1+\alpha)))}{\overline{\eta} \cdot \Gamma(2 + \alpha)\Gamma(1 + \alpha - ((2p-1)/p(1+\alpha)))} \right]^{1/\alpha} = \left[25^{0.4404} \cdot \frac{\Gamma(1.98)\Gamma(0.2424)}{1.9608 \cdot \Gamma(2.98)\Gamma(1.2224)} \right]^{1/0.98} = 4.4965.$$

(138)

Therefore, drive-response system (118) can achieve finite-time complete synchronization under controller (83). And the state trajectories and error curves of FMCGNNs are as shown in Figures 5–7, which demonstrate the effectiveness of Theorems 1 and 2.

4.3. Antisynchronization.
First, we choose the projective coefficient $\lambda = -1$. We can get $\Phi_1(s) = y_1(s) - 2x_1(s) = 1$, $\Phi_2(s) = y_2(s) - 2x_2(s) = 2$, $V_1(t_0) = 3$, and $V_2(t_0) = 5$.

According to Theorem 1, it can be calculated as follows:

$$\begin{aligned}
\varphi_1 &= k_1 - 1.1, \\
\varphi_2 &= k_2 - 1.1; \\
\delta_1 &= h_1 - 1, \\
\delta_2 &= h_2 - 0.9; \\
\eta_1 &= q_1 - 9.08, \\
\eta_2 &= q_2 - 11.12.
\end{aligned}$$

(139)

And we choose $k_1 = 3.1$, $k_2 = 3.1$, $h_1 = 1$, $h_2 = 0.9$, $q_1 = 10.08$, and $q_2 = 12.12$ which make $\varphi = 2$ and $\eta = 2$.

Then, we have a finite-time fractional-order differential inequation as follows:

$${}^{C}_{t_0}D_t^{\alpha}V_1(t) \leq -2V_1(t) - 2.$$

(140)

Based on the result in Theorem 1, drive-response system (118) could achieve synchronization in finite time. And we can calculate the setting time T_1^* as follows:

$$T_1^* \leq t_0 + \varphi^{-1/\alpha} \ln \frac{\varphi V_1(t_0) + \eta}{\alpha\eta} \leq 2^{-1/0.98} \ln \left(\frac{2 \times 3 + 2}{0.98 \times 2} \right) = 0.6934.$$

(141)

To verify Remark 4, we can calculate T_2 to demonstrate that T_1^* is less conservative than T_2. It follows that

$$T_2 \leq t_0 + \left[\frac{V_1(t_0)\Gamma(1 + \alpha)}{\eta} \right]^{1/\alpha} = \left(\frac{3 \cdot \Gamma(1.98)}{2} \right)^{1/\alpha} = 1.4814.$$

(142)

On the contrary, we select $p = 2$ and $\varepsilon_1 = \varepsilon_2 = 1$ for Theorem 2. And it is easy to calculate

$$\begin{aligned}
\overline{\varphi}_1 &= 1.9608(k_1 - 1.4), \overline{\varphi}_2 = 1.9608(k_2 - 1.85); \\
\overline{\delta}_1 &= 1.9608(h_1 - 0.5), \overline{\delta}_2 = 1.9608(h_2 - 0.45); \\
\overline{\eta}_1 &= 1.9608(q_1 - 9.08), \overline{\eta}_2 = 1.9608(q_2 - 11.12).
\end{aligned}$$

(143)

Under the controller parameter, $k_1 = 3.1$, $k_2 = 3.1$, $h_1 = 1$, $h_2 = 0.9$, $q_1 = 10.08$, and $q_2 = 12.12$ which lead to $\overline{\varphi} = 2.451$ and $\overline{\eta} = 1.9608$.

Hence, we have another finite-time fractional-order differential inequation as follows:

$${}^{C}_{t_0}D_t^{\alpha}V_2(t) \leq -2.451V_2(t) - 1.9608V_2^{1/2}(t).$$

(144)

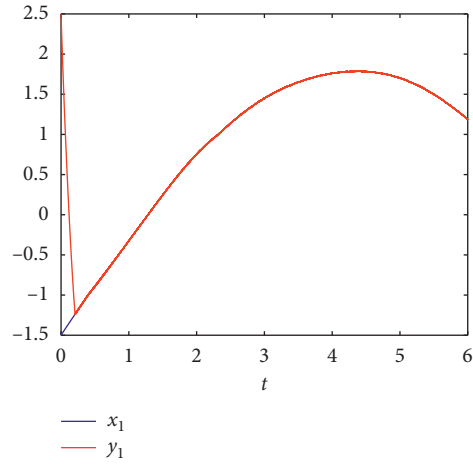Based on the result in Theorem 2, we can estimate the setting time T_3 as follows:

FIGURE 5: The first dimensional state trajectories of the drive system and the response system when the projective coefficient $\lambda = 1$.
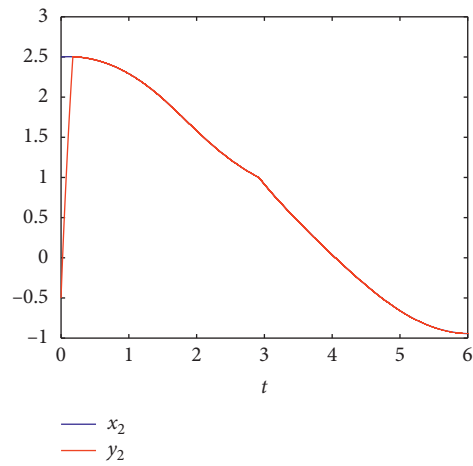

FIGURE 6: The second dimensional state trajectories of the drive system and the response system when the projective coefficient $\lambda = 1$.
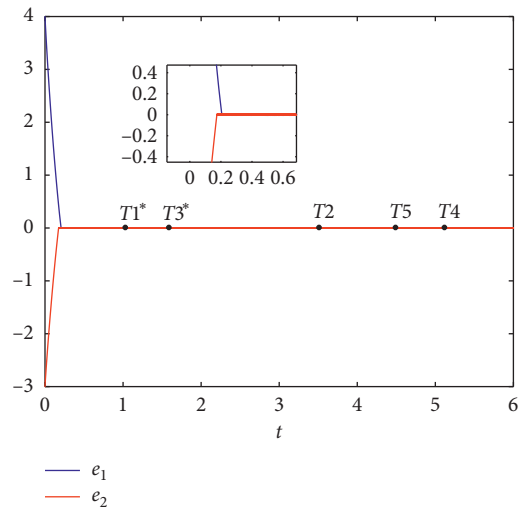

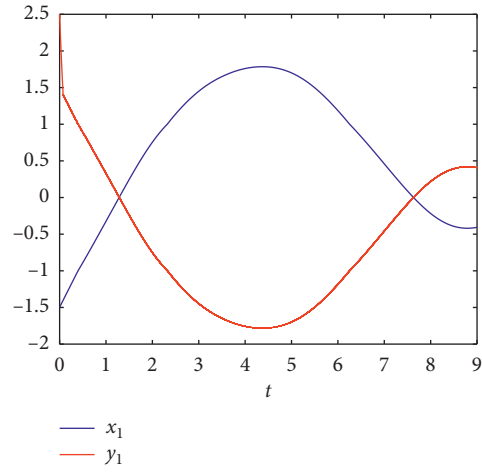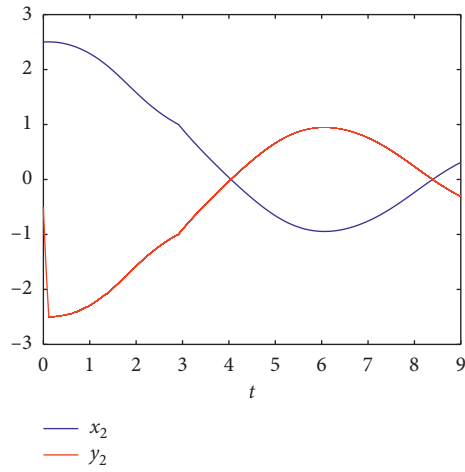FIGURE 7: The synchronization error evolution when the projective coefficient $\lambda = 1$.

FIGURE 8: The first dimensional state trajectories of the drive system and the response system when the projective coefficient $\lambda = -1$.

FIGURE 9: The second dimensional state trajectories of the drive system and the response system when the projective coefficient $\lambda = -1$.
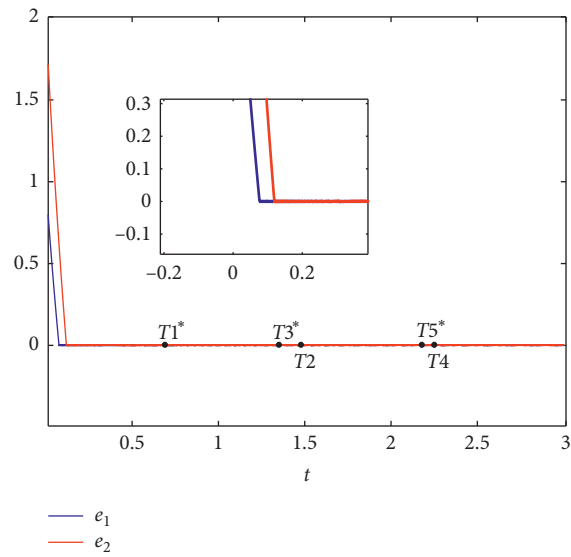

FIGURE 10: The synchronization error evolution when the projective coefficient $\lambda = -1$.

$$T_3^* \leq t_0 + \left[\frac{\Gamma(1 + (p/(p-1)) - \alpha)\overline{\varphi}}{\Gamma(1 + (p/(p-1)))\Gamma(2-\alpha)} \right]^{-1/\alpha} \ln \frac{\overline{\varphi} V_2^{1/p}(t_0) + \overline{\eta}}{\overline{\eta}\alpha} = \left[\frac{2.451\Gamma(2.02)}{\Gamma(3)\Gamma(1.02)} \right]^{-1/0.98} \ln \frac{2.451 \times \sqrt{5} + 1.9608}{1.9608 \times 0.98} = 1.3539.$$

$$(145)$$

To verify Remark 6, we can calculate T_4 to demonstrate that T_3^* is less conservative than T_4. It follows that

$$T_4 \leq t_0 + \left[V_2^{1/p}(t_0) \frac{\Gamma(1+p)\Gamma(2-\alpha)\Gamma(1+\alpha)}{\Gamma(1+p-\alpha)\overline{\eta}} \right]^{1/\alpha}$$

$$= \left[\sqrt{5} \cdot \frac{\Gamma(3)\Gamma(1.02)\Gamma(1.98)}{1.9608\Gamma(2.02)} \right]^{1/0.98} = 2.2539.$$

$$(146)$$

Considering $p = 2$ and $\alpha = 0.98$, $((p-1)/p) < \alpha(1+\alpha) - 1$ holds. Thus, we have

$$T_5 \leq t_0 + \left[V_2^{\alpha(1+\alpha)+(1/p)-2}(t_0) \frac{\Gamma(1+\alpha)\Gamma(1-(2p-1)/p(1+\alpha))}{\overline{\eta} \cdot \Gamma(2+\alpha)\Gamma(1+\alpha-(2p-1)/p(1+\alpha))} \right]^{1/\alpha} = \left[5^{0.4404} \cdot \frac{\Gamma(1.98)\Gamma(0.2424)}{1.9608 \cdot \Gamma(2.98)\Gamma(1.2224)} \right]^{1/0.98} = 2.1816.$$

$$(147)$$

Therefore, drive-response system (118) can achieve finite-time antisynchronization under controller (83). And the state trajectories and error curves of FMCGNNs are as shown in Figures 8–10, which demonstrate the effectiveness of Theorems 1 and 2.

5. Conclusion

In this paper, we have extensively discussed the finite-time projective synchronization for a class of FMNNs which are translated to a type of fractional-order systems with time delays and uncertain parameters. By means of applying the differential inequality of the Caputo derivative and the asymptotic expansion property of Mittag-Leffler function, three useful properties are proposed which play a vital role in calculating the settling time of finite-time synchronization. Besides the time-delayed feedback control utilized in this paper, other control approaches, such as impulsive control, intermittent control, and pinning control, can also be applied in the future work for studying projective synchronization of memristive neural networks with mixed time-varying delays.

Data Availability

All data included in this study are available upon request by contact with the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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