Research Article

Approximation-Based Fixed-Time Adaptive Tracking Control for a Class of Uncertain Nonlinear Pure-Feedback Systems

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This paper examines approximation-based fixed-time adaptive tracking control for a class of uncertain nonlinear pure-feedback systems. Novel virtual and actual controllers are designed that resolve the meaninglessness of virtual and actual controllers at the origin and in the negative domain, and the sufficient condition for the system to have semiglobal fixed-time stability is also provided. Radial basis function neural networks are introduced to approximate unknown functions for solving the fixed-time control problem of unknown nonlinear pure-feedback systems, and the mean value theorem is used to solve the problem of nonaffine structure in nonlinear pure-feedback systems. The controllers designed in this paper ensure that all signals in the closed-loop system are semiglobally uniform and ultimately bounded in a fixed time. Two simulation results show that appropriate design parameters can limit the tracking error within a region of the origin in a fixed time.

1. Introduction

Nonlinear pure-feedback systems [1, 2] are more common than general strict feedback nonlinear systems or nonlinear systems with input affine structure. Such systems have been studied widely in recent years because they can reflect more accurately the working conditions of actual engineering systems owing to the nonaffine structure of their control inputs and state variables.

In 1988, David S. Broomhead proposed the radial basis function (RBF) neural network [3], which is used widely in pattern recognition, signal processing, and control system theory and application because of its simple structure and generalizability. Many scholars have used it to address the uncertainties of nonlinear systems based on its approximation ability, which has been demonstrated by many researchers [4–7]. Kanellakopoulos et al. [8, 9] proposed the backstepping method and the backstepping adaptive control scheme for a class of strict feedback nonlinear systems in 1991. The adaptive control of RBF neural networks has attracted extensive attention [10–18], resulting in the stability analysis method of RBF neural networks based on the Lyapunov method [19–21]. Finite-time stability technology

can be traced back to the 1960s [22], and it developed rapidly in the 1990s, owing primarily to the improvement of finite-time Lyapunov theory [23] and homogeneous systems [24]. Much has been achieved in recent years on finite-time stability [25–30]. A finite-time control system ensures that a nonlinear system converges in a finite time, but the convergence time is related to the initial state of the system. For this reason, fixed-time control systems have been proposed [31–34] that ensure that the upper limit of the fixed convergence time is no longer related to the initial state of the system but only depends on the design parameters. Since Polyakov et al. first proposed fixed-time stability control [33], nonlinear fixed-time control has developed rapidly and has been studied by many scholars. For example, in [35], a fixed-time control with generalized directional topology was proposed for nonlinear multiagent systems; in [36], a prescribed performance fixed-time recurrent neural network control was proposed for a class of uncertain nonlinear systems; in [37], a fast fixed-time nonsingular terminal sliding mode control was proposed to solve the problem of chaos suppression of power systems; and in [38], a fixed-time observer was proposed to detect distributed faults of nonlinear multiagent systems.

In [35–40], strict feedback nonlinear systems are considered primarily, and the fixed-time control problem of more common nonlinear systems is not solved. In this paper, the fixed-time control problem is solved based on nonlinear pure-feedback systems, and the sufficient condition and design steps for semiglobal fixed-time stability are provided.

The principal contributions of this paper are as follows:

- (1) The fixed-time control algorithm proposed in [35–40] does not solve the problem of the nonaffine structure of the control input $u(t)$. This paper applies fixed-time control theory in nonlinear pure-feedback systems to solve this problem.
- (2) The controller designed in [40–43] has a power function similar to z^{2q-1} , $0 < q < 1$. Not selecting q properly results in singularity. A novel fixed-time controller is designed in this paper to solve this problem.
- (3) An RBF neural network control algorithm is introduced to approximate the unknown functions $f_i(\cdot)$ to overcome the difficulty of modeling accurately and solving the problem of interference in nonlinear pure-feedback systems.

The remainder of this paper is organized as follows. Section 2 presents the problem description and preliminaries. Section 3 proposes fixed-time adaptive neural tracking control using backstepping, adaptive neural networks, and Lyapunov functions for a class of unknown nonlinear pure-feedback systems to solve the problem of fixed-time tracking control for nonlinear systems with nonaffine structure. In Section 4, all signals in the closed-loop system are proved to be semiglobally uniform and ultimately bounded. In Section 5, the proposed control scheme is proved to be effective through simulation experiments. Section 6 draws conclusions.

2. Problem Description and Preliminaries

2.1. Problem Description. Consider the following nonlinear pure-feedback system:

$$\begin{cases} \dot{x}_i(t) = f_i(\bar{x}_i(t), x_{i+1}(t)) + d_i(\bar{x}_i, t), \\ \dot{x}_n(t) = f_n(\bar{x}_n(t), u(t))d_n(\bar{x}_n, t), \\ y(t) = x_1(t), \end{cases} \quad (1)$$

where $1 \leq i \leq n-1$, $\bar{x}_i = [x_1(t), \dots, x_i(t)]^T \in R^i$ with $i = 1, \dots, n$, $u(t) \in R$, and $y(t) \in R$ are state variables, system input, and system output, respectively. $f_i(\cdot)$ are unknown but smooth nonaffine functions, and $d_i(\cdot)$ are unknown but bounded disturbances.

According to the mean value theorem [44], $f_i(\bar{x}_i, x_{i+1}) = f_i(\bar{x}_i, x_{i0}) + h_{i\mu_i}(x_{i+1} - x_{i0})$ and $f_n(\bar{x}_n, u) = f_n(\bar{x}_n, x_{n0}) + h_{n\mu_n}(u - x_{n0})$, where $h_{i\mu_i} = h_i(\bar{x}_i, x_{\mu_i}) = \partial f_i(\bar{x}_i, x_{\mu_i}) / \partial x_{\mu_i}$ with $i = 1, \dots, n$, $x_{\mu_i} = \mu_i x_{i+1} + (1 - \mu_i)x_{i0}$ with $0 < \mu_i < 1$, and x_{i0} is a known quantity at the given time t_0 . System (1) can be written as

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i, x_{i0}) + h_{i\mu_i} \cdot (x_{i+1} - x_{i0}) + d_i(\bar{x}_i, t), \\ \dot{x}_n = f_n(\bar{x}_n, x_{n0}) + h_{n\mu_n} \cdot (u - x_{n0}) + d_n(\bar{x}_n, t), \\ y = x_1. \end{cases} \quad (2)$$

Remark 1. It can be seen from system (2) that the mean value theorem separates the nonaffine structure from $f_i(\cdot)$ in system (1).

This paper aims to design a fixed-time controller that can meet the fixed-time control requirements in nonlinear pure-feedback systems, enabling the system output y to track the reference signal y_d in a fixed time. All of the signals of the closed-loop system are semiglobally uniform and ultimately bounded.

Assumption 1 (see [45]). Unknown smooth nonlinear functions $h_i(\cdot)$ are bounded, and there are known positive constants b and c with $0 < b \leq |h_i(\cdot)| < c < \infty$, $\forall (\bar{x}_i, x_{i+1}) \in R^i \times R$. Without loss of generality, we assume that $0 < b \leq h_i(\cdot)$, $i = 1, \dots, n$.

For ease of calculation, vector functions are defined as $\bar{y}_{d,i} = [y_d, y_d^{(1)}, \dots, y_d^{(i)}]^T$, $i = 1, \dots, n$, where $y_d^{(i)}$ is the i^{th} derivative of y_d .

Assumption 2. Reference signal vector functions $\bar{y}_{d,i}$ are known smooth continuous bounded functions. $\bar{y}_{d,i} \in \Omega_{d,i} \subset R^{i+1}$ with $i = 1, \dots, n$, where $\Omega_{d,i}$ are known compact sets and reference signal y_d is an n -order differentiable and bounded function.

2.2. Fixed Time

Definition 1. Consider the following nonlinear system:

$$\dot{x}(t) = f(t, x), x(0) = x_0, \quad (3)$$

where $x \in R^n$ and $f: R_+ \times R^n \rightarrow R^n$, and assume that the origin is an equilibrium point.

Lemma 1 (see [46]). If there exist design constants $\phi_1 > 0$, $\phi_2 > 0$, $\alpha \in (1, +\infty)$, and $\beta \in (0, 1)$ such that

$$\dot{V}(x) \leq -\phi_1 V^\alpha(x) - \phi_2 V^\beta(x), \quad (4)$$

where $V(x)$ is a continuous differentiable positive definite function, then system (3) is global fixed-time stable, and the fixed convergence time satisfies

$$T \leq T_{\max} := \frac{1}{\phi_1(\alpha-1)} + \frac{1}{\phi_2(1-\beta)}. \quad (5)$$

Lemma 2 (see [40]). If there exist design constants $\phi_1 > 0$, $\phi_2 > 0$, $\alpha \in (1, \infty)$, $\beta \in (0, 1)$, $\tau \in (0, \infty)$, and $\omega \in (0, 1)$ such that

$$\dot{V}(x) \leq -\phi_1 V^\alpha(x) - \phi_2 V^\beta(x) + \tau, \quad (6)$$

then the origin of system (3) is practical fixed-time stable and the fixed-time T can be estimated by

$$T \leq T_{\max} := \frac{1}{\phi_1 \omega (\alpha - 1)} + \frac{1}{\phi_2 \omega (1 - \beta)}. \quad (7)$$

The residual set of the solution of system (3) is given by

$$x \in \left\{ V(x) \leq \min \left\{ \left(\frac{\tau}{(1-\omega)\phi_1} \right)^{1/\alpha}, \left(\frac{\tau}{(1-\omega)\phi_2} \right)^{1/\beta} \right\} \right\}. \quad (8)$$

Lemma 3 (see [47]). *Let $x_1, x_2, \dots, x_n \geq 0$. Then,*

$$\begin{aligned} \sum_{i=1}^n x_i^\rho &\geq \left(\sum_{i=1}^n x_i \right)^\rho, & \text{if } 0 < \rho \leq 1, \\ \sum_{i=1}^n x_i^\rho &\geq n^{1-\rho} \left(\sum_{i=1}^n x_i \right)^\rho, & \text{if } 1 < \rho < \infty. \end{aligned} \quad (9)$$

Lemma 4 (see [48]). *For any variable $x \in R$ and any positive constant κ , the following relationship holds:*

$$0 \leq |x| < \kappa + \frac{x^2}{\sqrt{x^2 + \kappa^2}}. \quad (10)$$

Lemma 5. *For $y \geq x > 0$, $x, y \in R$ and any positive constant χ , then the following is satisfied*

$$\frac{y}{\sqrt{y+\chi}} \geq \frac{x}{\sqrt{x+\chi}}. \quad (11)$$

Proof.

$$\begin{aligned} \frac{y}{\sqrt{y+\chi}} - \frac{x}{\sqrt{x+\chi}} &= \frac{y+\chi}{\sqrt{y+\chi}} - \frac{x+\chi}{\sqrt{x+\chi}} - \frac{\chi}{\sqrt{y+\chi}} + \frac{\chi}{\sqrt{x+\chi}} \\ &= (\sqrt{y+\chi} - \sqrt{x+\chi}) + \left(\frac{\chi}{\sqrt{x+\chi}} - \frac{\chi}{\sqrt{y+\chi}} \right) \\ &\geq 0. \end{aligned} \quad (12) \quad \square$$

2.3. Neural Network. An RBF neural network [11, 49] is applied in this paper to approximate arbitrary continuous functions. The mathematical expression of an RBF neural network is as follows:

$$\hat{\varphi} = W^T S(Z), \quad (13)$$

where $W = [w_1, w_2, \dots, w_l]^T \in R^l$ is the weight vector, $l > 1$ is the number of nodes of the neural network, $Z \in \Omega_Z \subset R^q$ is the input of the RBF neural network, q is the input dimension of the RBF neural network, $S(Z) = [s_1(Z), s_2(Z), \dots, s_l(Z)]^T \in R^l$ is the basis vector function, and $s_i(Z)$ is the output of the i^{th} node. A Gaussian function is always chosen as $s_i(Z)$, i.e., $s_i(Z) = \exp[-(Z - \xi_i)^T(Z - \xi_i)/r_i^2]$, $i = 1, \dots, l$, where r_i is the width of the base function and $\xi_i = [\xi_{i1}, \xi_{i2}, \dots, \xi_{iq}]^T$ is

the center of the basis function. With a sufficient number l of nodes selected, an RBF neural network can approximate an arbitrary continuous function $\varphi(Z)$ in a compact set $\Omega_Z \in R^q$ with arbitrary accuracy ε .

$$\varphi(Z) = W^{*T} S(Z) + \delta(Z), \quad \forall Z \in \Omega_Z \in R^q, \quad (14)$$

where $\delta(Z)$ is the approximation error with $|\delta(Z)| \leq \varepsilon$ and W^* is the given ideal constant weight vector, which is defined as

$$W^* = \arg \min_{W \in R^l} \sup_{Z \in \Omega_Z} \{|\varphi(Z) - \hat{\varphi}(Z)|\}. \quad (15)$$

In this paper, let $\theta_i = \max\{\|W_i^*\|^2/b, i = 1, 2, \dots, n\}$ with $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, where $\hat{\theta}_i$ are the estimates of the unknown constants θ_i , W_i^* are the ideal weight vectors of the RBF neural network, b is a positive design parameter, and $\|\cdot\|$ is the norm.

Remark 2. b is related to Assumption 1.

Assumption 3 (see [40]). There are unknown constants Q_i that make $|\tilde{\theta}_i| \leq Q_i < \infty$, $i = 1, 2, \dots, n$.

Lemma 6 (see [50]). *Consider the Gaussian function (13). $\|S(Z)\|$ has an upper bound such that*

$$\|S(Z)\| \leq \sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{(-2p^2 k^2)/r^2} := s, \quad (16)$$

where $p = 1/2(\min_{i \neq j} \|\xi_i - \xi_j\|)$.

Lemma 6 has been proved in [50, 51]. Since $\sum_{k=0}^{\infty} 3q(k+2)^{q-1} e^{(-2p^2 k^2)/r^2}$ is convergent, s is a limited value. In addition, s is independent of the neural network node numbers l and the neural network inputs Z .

3. Design of a Fixed-Time Controller

The backstepping design coordinate transformation is as follows:

$$\begin{aligned} z_1 &= x_1 - y_d, \\ z_i &= x_i - \alpha_{i-1}, \quad i = 2, \dots, n, \end{aligned} \quad (17)$$

where α_i is the virtual controller of the i^{th} subsystem.

An RBF neural network is used in this paper to approximate the unknown functions $\hat{f}_i(Z_i)$:

$$\hat{f}_i(Z_i) = W_i^{*T} S_i(Z_i) + \delta_i(Z_i), \quad (18)$$

and the inequalities involved in the following text are as follows:

$$z_i \delta_i(Z_i) \leq b k_{i3} z_i^2 + \frac{\varepsilon_i^2}{4b k_{i3}}, \quad (19)$$

$$z_i W_i^{*T} S_i(Z_i) \leq \frac{b \theta_i}{2 \eta_i^2} S_i^T(Z_i) S_i(Z_i) z_i^2 + \frac{\eta_i^2}{2}, \quad (20)$$

$$\begin{aligned}
& -bk_{i2}2^{(1+\beta)/2} \left(\left(\frac{1}{2}z_i^2 \right)^{(1+\beta)/2} + \left(\frac{b\tilde{\theta}_i^2}{2\gamma} \right)^{(1+\beta)/2} \right) \\
& \leq -2bk_{i2} \left(\frac{1}{2}z_i^2 + \frac{b\tilde{\theta}_i^2}{2\gamma} \right)^{(1+\beta)/2},
\end{aligned} \quad (21)$$

$$\begin{aligned}
& -bk_{i1}2^{(1+\alpha)/2} \left(\left(\frac{1}{2}z_i^2 \right)^{(1+\alpha)/2} + \left(\frac{b\tilde{\theta}_i^2}{2\gamma} \right)^{(1+\alpha)/2} \right) \\
& \leq -bk_{i1}2^{(1+\alpha)/2} \left(\frac{1}{2}z_i^2 + \frac{b\tilde{\theta}_i^2}{2\gamma} \right)^{(1+\alpha)/2},
\end{aligned} \quad (22)$$

where $Z_1 = [x_1, \hat{\theta}_1, y_d, \dot{y}_d] \in \Omega_{Z_1} \subset R^{3+1}$ and $Z_i = [x_1, x_2, \dots, x_i, \hat{\theta}_1, \hat{\theta}_2, \dots, \hat{\theta}_i, \bar{y}_{di}] \in \Omega_{Z_i} \subset R^{3i+1}$ with $2 \leq i \leq n$ are input vectors and $\eta_i, k_{i1}, k_{i2}, k_{i3}$, and γ are positive design parameters.

Step 1. According to $z_1 = x_1 - y_d$ and (2), we have

$$\dot{z}_1 = f_1(\bar{x}_1, x_{10}) + h_{1\mu_1}(x_2 - x_{10}) + d_1(\bar{x}_1, t) - \dot{y}_d. \quad (23)$$

Construct a Lyapunov function as

$$V_1 = \frac{1}{2}z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma}. \quad (24)$$

Time differentiation of V_1 yields

$$\dot{V}_1 = z_1 f_1(\bar{x}_1, x_{10}) + z_1 h_{1\mu_1}(x_2 - x_{10}) + z_1 d_1 - z_1 \dot{y}_d - \frac{b\tilde{\theta}_1 \dot{\tilde{\theta}}_1}{\gamma}. \quad (25)$$

Substituting $z_2 = x_2 - \alpha_1$ into (25) yields

$$\begin{aligned}
\dot{V}_1 &= z_1 f_1(\bar{x}_1, x_{10}) + z_1 h_{1\mu_1} z_2 + z_1 h_{1\mu_1} \alpha_1 - z_1 h_{1\mu_1} x_{10} \\
&\quad + z_1 d_1 - z_1 \dot{y}_d - \frac{b\tilde{\theta}_1 \dot{\tilde{\theta}}_1}{\gamma}.
\end{aligned} \quad (26)$$

The virtual controller α_1 is defined as

$$\begin{aligned}
\alpha_1 &= -k_{11}S_{z_1,1} - k_{12}S_{z_1,2} - \frac{\hat{\theta}_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1 \\
&\quad - k_{13}z_1 + x_{10},
\end{aligned} \quad (27)$$

where k_{11}, k_{12}, k_{13} , and η_1 are positive design parameters.

In (27), $S_{z_1,1}$ and $S_{z_1,2}$ are defined as

$$\begin{aligned}
S_{z_1,1} &= \begin{cases} \frac{(z_1^2)^{(\alpha+1)/2}}{z_1}, & |z_1| \geq \varepsilon_{10}, \\ \sum_{j=1}^n a_j (z_1^2)^j (\varepsilon_{10}^2)^{-j+(\alpha+1)/2}, & |z_1| < \varepsilon_{10}, \end{cases} \\
S_{z_1,2} &= \begin{cases} \frac{(z_1^2)^{(\beta+1)/2}}{z_1}, & |z_1| \geq \varepsilon_{10}, \\ \sum_{j=1}^n a_j (z_1^2)^j (\varepsilon_{10}^2)^{-j+(\beta+1)/2}, & |z_1| < \varepsilon_{10}, \end{cases}
\end{aligned} \quad (28)$$

where ε_{10} is a positive design parameter and coefficients a_j , $j = 1, 2, \dots, n$, are calculated using the following equation:

$$\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ 1 & 2 & \cdots & n-1 & n \\ 0 & 2 \times 1 & \cdots & (n-1)(n-2) & n(n-1) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \prod_{j=0}^{n-2} (n-1-j) & \prod_{j=0}^{n-2} (n-j) \end{bmatrix} \times \begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_n \end{bmatrix}, \quad (29)$$

where $b_1 = 1$, $b_2 = 3/4$, $b_3 = (3/4)((3/4) - 1)$, ..., and $b_n = \prod_{j=0}^{n-2} ((3/4) - j)$.

Remark 3. One of the main contributions of this paper is to design suitable virtual controllers α_i so that nonlinear pure-feedback systems meet the requirement of fixed-time control. In [40–43], virtual controllers designed exhibit similar power functions $z^{2\alpha-1}$, where $0 < \alpha < 1$. If α is not appropriately selected, it will make z unsolvable at the origin and in the negative domain. As $0 < \alpha < 1$, power exponents $2\alpha - 1 \in (-1, 1)$, which leads to the possibility of negative power exponents. For example, if $2\alpha - 1 = -1/3$, $z^{-1/3}$ is unsolvable at $z = 0$, that is, $(0)^{-1/3}$ does not exist. Suppose $2\alpha - 1 = q_1/q_2 \in (-1, 1)$, where q_2 is an even number; then, z^{q_1/q_2} is unsolvable at the negative domain. For example, if $q_1/q_2 = 1/2$, $z^{1/2}$ is unsolvable at the negative domain, that is, $(-1)^{1/2}$ does not exist. The controller designed in this paper overcomes the aforementioned defect and promotes the application of fixed-time control in more common nonlinear systems.

Substituting α_1 into (26) yields

$$\dot{V}_1 \leq z_1 \hat{f}_1(Z_1) + z_1 h_{1\mu_1} z_2 - \frac{b\tilde{\theta}_1 \dot{\hat{\theta}}_1}{\gamma} - bk_{11} z_1 S_{z_1,1} \quad (30)$$

$$- bk_{12} z_1 S_{z_1,2} - \frac{b\hat{\theta}_1}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 - bk_{13} z_1^2,$$

where $\hat{f}_1(Z_1) = f_1(\bar{x}_1, x_{10}) + d_1 - \dot{y}_d$.

The RBF neural network (18) is utilized to approximate $\hat{f}_1(Z_1)$ and introduce inequalities (19) and (20), following which (30) can be rewritten as

$$\begin{aligned} \dot{V}_1 \leq & \frac{b\tilde{\theta}_1}{\gamma} \left(\frac{\gamma}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 - \dot{\hat{\theta}}_1 \right) \\ & + \sigma_1 + z_1 h_{1\mu_1} z_2 - bk_{11} z_1 S_{z_1,1} - bk_{12} z_1 S_{z_1,2}, \end{aligned} \quad (31)$$

where $\sigma_1 = \eta_1^2/2 + \varepsilon_1^2/4bk_{13}$.

The adaptive law $\hat{\theta}_1$ is then defined as

$$\dot{\hat{\theta}}_1 = \frac{\gamma}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 - \lambda \hat{\theta}_1, \quad (32)$$

where λ is a positive design parameter.

Combining with Assumption 3 and substituting (32) into (31), one can obtain

$$\begin{aligned} \dot{V}_1 \leq & -bk_{11} z_1 S_{z_1,1} - bk_{12} z_1 S_{z_1,2} - bk_{11} 2^{(1+\alpha)/2} \left(\frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\alpha)/2} \\ & - bk_{12} 2^{(1+\beta)/2} \left(\frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\beta)/2} + z_1 h_{1\mu_1} z_2 + C_1, \end{aligned} \quad (33)$$

where

$$\begin{aligned} \beta_1 = & bk_{11} \left(\frac{bQ_1^2}{\gamma} \right)^{(1+\alpha)/2} + bk_{12} \left(\frac{bQ_1^2}{\gamma} \right)^{(1+\beta)/2}, \frac{\lambda b \tilde{\theta}_1 \hat{\theta}_1}{\gamma} \\ \leq & \frac{\lambda b}{\gamma} \left(\frac{\theta_1^2}{2} - \frac{\tilde{\theta}_1^2}{2} \right) \leq \frac{\lambda b}{2\gamma} \theta_1^2, \end{aligned} \quad (34)$$

According to (21), (22), and (28), if $|z_1| \geq \varepsilon_{10}$, then (33) is rewritten as

$$\begin{aligned} \dot{V}_1 \leq & -bk_{11} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\alpha)/2} \\ & - 2bk_{12} \left(\frac{1}{2} z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\beta)/2} + z_1 h_{1\mu_1} z_2 + C_1. \end{aligned} \quad (35)$$

According to (21), (22), and (28), if $|z_1| < \varepsilon_{10}$, then (33) is rewritten as

$$\begin{aligned} \dot{V}_1 \leq & bk_{11} (z_1^2)^{(1+\alpha)/2} - bk_{11} z_1 \sum_{j=1}^n a_j (z_1^2)^j (\varepsilon_{10}^2)^{-j+(1+\alpha)/2} \\ & + bk_{12} (z_1^2)^{(1+\beta)/2} - bk_{12} z_1 \sum_{j=1}^n a_j (z_1^2)^j (\varepsilon_{10}^2)^{-j+(1+\beta)/2} \\ & - bk_{11} 2^{(1+\alpha)/2} \left(\frac{1}{2} z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\alpha)/2} \\ & - 2bk_{12} \left(\frac{1}{2} z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\beta)/2} + z_1 h_{1\mu_1} z_2 + C_1. \end{aligned} \quad (36)$$

Remark 4 (see [52]). Based on (28), when $|z_1| < \varepsilon_{10}$, there is an additional term in (36): $bk_{11} (z_1^2)^{(1+\alpha)/2} - bk_{11} z_1 \sum_{j=1}^n a_j (z_1^2)^j (\varepsilon_{10}^2)^{-j+(1+\alpha)/2} + bk_{12} (z_1^2)^{(1+\beta)/2} - bk_{12} z_1 \sum_{j=1}^n a_j (z_1^2)^j (\varepsilon_{10}^2)^{-j+(1+\beta)/2}$. Note that if $|z_1| < \varepsilon_{10}$, then this additional term is obviously limited by some smaller constant ε_{11} , so the structure of (35) is retained, while the constant term C_1 only slightly increases. Owing to page limitations and to avoid repetitive discussions, we will omit this part in the rest of the analysis.

Step 2. According to $z_2 = x_2 - \alpha_1$, we have

$$\dot{z}_2 = f_2(\bar{x}_2, x_{20}) + h_{2\mu_2}(x_3 - x_{20}) - \dot{\alpha}_1 + d_2(\bar{x}_2, t), \quad (37)$$

where

$$\dot{\alpha}_1 = \frac{\partial \alpha_1}{\partial x_1} \dot{x}_1 + \frac{\partial \alpha_1}{\partial y_d} \dot{y}_d + \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1. \quad (38)$$

Construct a Lyapunov function as

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{b\tilde{\theta}_2^2}{2\gamma}. \quad (39)$$

The derivative of V_2 is written as

$$\begin{aligned}
\dot{V}_2 \leq & -bk_{11}2^{(1+\alpha)/2} \left(\frac{1}{2}z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& - 2bk_{12} \left(\frac{1}{2}z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\beta)/2} + z_2\hat{f}_2(Z_2) + z_2h_{2\mu_2}z_3 \\
& + z_2h_{2\mu_2}\alpha_2 - z_2h_{2\mu_2}x_{20} - z_2\frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 - \frac{b\tilde{\theta}_2\dot{\hat{\theta}}_2}{\gamma} + z_2M_1(Z_2) + C_1,
\end{aligned} \tag{40}$$

where

$$\hat{f}_2(Z_2) = f_2(\bar{x}_2, x_{20}) - \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d + z_1h_{1\mu_1} + d_2 - M_1(Z_2). \tag{41}$$

Remark 5. $M_1(Z_2)$ is a smooth function that is used to overcome the design difficulty of $\hat{\theta}_1\partial\alpha_1/\partial\hat{\theta}_1$.

The virtual controller α_2 is defined as

$$\alpha_2 = -k_{21}S_{z_2,1} - k_{22}S_{z_2,2} - \frac{\tilde{\theta}_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2 - k_{23}z_2 + x_{20}, \tag{42}$$

where k_{21} , k_{22} , k_{23} , and η_2 are positive design parameters.

In (42), $S_{z_2,1}$ and $S_{z_2,2}$ are defined as

$$\begin{aligned}
S_{z_2,1} &= \begin{cases} \frac{(z_2^2)^{(\alpha+1)/2}}{z_2}, & |z_2| \geq \varepsilon_{20}, \\ \sum_{j=1}^n a_j(z_2^2)^j (\varepsilon_{20}^2)^{-j+(\alpha+1)/2}, & |z_2| < \varepsilon_{20}, \end{cases} \\
S_{z_2,2} &= \begin{cases} \frac{(z_2^2)^{(\beta+1)/2}}{z_2}, & |z_2| \geq \varepsilon_{20}, \\ \sum_{j=1}^n a_j(z_2^2)^j (\varepsilon_{20}^2)^{-j+(\beta+1)/2}, & |z_2| < \varepsilon_{20}, \end{cases}
\end{aligned} \tag{43}$$

where ε_{20} is a positive design parameter.

Choose the adaptive law $\hat{\theta}_2$ as

$$\dot{\hat{\theta}}_2 = \frac{\gamma}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2^2 - \lambda\hat{\theta}_2. \tag{44}$$

Combining (18)–(20) and Assumption 3, substituting (42) and (44) into (40), and adopting the same design method as in Step 1 yields (45) as a rewriting of (40).

$$\begin{aligned}
\dot{V}_2 \leq & -bk_{21}z_2S_{z_2,1} - bk_{21}2^{(1+\alpha)/2} \left(\frac{b\tilde{\theta}_2^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& - bk_{22}z_2S_{z_2,2} - bk_{22}2^{(1+\beta)/2} \left(\frac{b\tilde{\theta}_2^2}{2\gamma} \right)^{(1+\beta)/2} \\
& - bk_{11}2^{(1+\alpha)/2} \left(\frac{1}{2}z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& - 2bk_{12} \left(\frac{1}{2}z_1^2 + \frac{b\tilde{\theta}_1^2}{2\gamma} \right)^{(1+\beta)/2} \\
& + \sum_{j=1}^2 C_j + z_2h_{2\mu_2}z_3 + z_2 \left(M_1(Z_2) - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 \right),
\end{aligned} \tag{45}$$

where

$$\begin{aligned}
\frac{\lambda b\tilde{\theta}_2\dot{\hat{\theta}}_2}{\gamma} &\leq \frac{\lambda b}{\gamma} \left(\frac{\theta_2^2}{2} - \frac{\tilde{\theta}_2^2}{2} \right) \leq \frac{\lambda b}{2\gamma}\theta_2^2, \\
\beta_2 &= bk_{21} \left(\frac{bQ_2^2}{\gamma} \right)^{(1+\alpha)/2} + bk_{22} \left(\frac{bQ_2^2}{\gamma} \right)^{(1+\beta)/2}, \\
\sigma_2 &= \frac{\eta_2^2}{2} + \frac{\varepsilon_2^2}{4bk_{23}}, \\
C_2 &= \sigma_2 + \beta_2 + \frac{\lambda b\theta_2^2}{2\gamma}.
\end{aligned} \tag{46}$$

According to (21) and (22) and Remark 4, we consider only $|z_2| \geq \varepsilon_{20}$. Then, (45) can be written as

$$\begin{aligned}
\dot{V}_2 \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^2 k_{j1} \left(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& - 2b \sum_{j=1}^2 k_{j2} \left(\frac{1}{2}z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} \\
& + \sum_{j=1}^2 C_j + z_2h_{2\mu_2}z_3 + z_2 \left(M_1(Z_2) - \frac{\partial\alpha_1}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_1 \right).
\end{aligned} \tag{47}$$

It can be seen from (47) that defining the design smooth function $M_1(Z_2)$ to overcome the design difficulty of $(\partial\alpha_1/\partial\hat{\theta}_1)\hat{\theta}_1$ is one of the difficulties of designing the controllers in this paper.

From Lemmas 4–6 and (32), it follows that

$$\begin{aligned}
-z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 &\leq -z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \left(\frac{\gamma}{2\eta_1^2} S_1^T(Z_1) S_1(Z_1) z_1^2 - \lambda \hat{\theta}_1 \right) \\
&\leq \frac{z_2^2 (\partial \alpha_1 / \partial \hat{\theta}_1)^2 ((\gamma/2\eta_1^2) S_1^T(Z_1) S_1(Z_1) z_1^2)^2}{\sqrt{z_2^2 (\partial \alpha_1 / \partial \hat{\theta}_1)^2 ((\gamma/2\eta_1^2) S_1^T(Z_1) S_1(Z_1) z_1^2)^2 + \varsigma_{2,1}^2}} \\
&\quad + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \lambda \hat{\theta}_1 \\
&\leq \frac{z_2^2 (\partial \alpha_1 / \partial \hat{\theta}_1)^2 ((\gamma/2\eta_1^2) S_1^T(Z_1) S_1(Z_1) z_1^2)^2}{\sqrt{z_2^2 (\partial \alpha_1 / \partial \hat{\theta}_1)^2 ((\gamma/2\eta_1^2) S_1^T(Z_1) S_1(Z_1) z_1^2)^2 + \varsigma_{2,1}^2}} + z_2 \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \lambda \hat{\theta}_1.
\end{aligned} \tag{48}$$

Therefore, $M_1(Z_2)$ can be defined as

$$M_1(Z_2) = -\lambda \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \hat{\theta}_1 - \frac{z_2 (\partial \alpha_1 / \partial \hat{\theta}_1)^2 ((\gamma/2\eta_1^2) S_1^T(Z_1) S_1(Z_1) z_1^2)^2}{\sqrt{z_2^2 (\partial \alpha_1 / \partial \hat{\theta}_1)^2 ((\gamma/2\eta_1^2) S_1^T(Z_1) S_1(Z_1) z_1^2)^2 + \varsigma_{2,1}^2}}, \tag{49}$$

with the result that

$$z_2 \left(M_1(Z_2) - \frac{\partial \alpha_1}{\partial \hat{\theta}_1} \dot{\hat{\theta}}_1 \right) \leq 0. \tag{50}$$

Substituting (50) into (47) yields

$$\begin{aligned}
\dot{V}_2 &\leq -b2^{(1+\alpha)/2} \sum_{j=1}^2 k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
&\quad - 2b \sum_{j=1}^2 k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^2 C_j + z_2 h_{2\mu_2} z_3.
\end{aligned} \tag{51}$$

Step 3. ($3 \leq k \leq n-1$). According to $z_k = x_k - \alpha_{k-1}$, we have

$$\dot{z}_k = f_k(\bar{x}_k, x_{k0}) + h_{k\mu_k}(x_{k+1} - x_{k0}) + d_k(\bar{x}_k, t) - \dot{\alpha}_{k-1}, \tag{52}$$

where

$$\dot{\alpha}_{k-1} = \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_j} \dot{x}_j + \frac{\partial \alpha_{k-1}}{\partial \bar{y}_{d,k-1}} \dot{\bar{y}}_{d,k-1} + \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j. \tag{53}$$

Constructing a Lyapunov function as $V_k = V_{k-1} + (z_k^2/2) + (b\tilde{\theta}_k^2/2\gamma)$ yields

$$\begin{aligned}
\dot{V}_k &\leq -b2^{(1+\alpha)/2} \sum_{j=1}^{k-1} k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
&\quad - 2b \sum_{j=1}^{k-1} k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^{k-1} C_j + z_k \hat{f}_k(Z_k) \\
&\quad + z_k h_{k\mu_k} z_{k+1} + z_k h_{k\mu_k} \alpha_k - z_k h_{k\mu_k} x_{k0} - z_k \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\
&\quad - \frac{b\tilde{\theta}_k \dot{\hat{\theta}}_k}{\gamma} + z_k M_{k-1}(Z_k),
\end{aligned} \tag{54}$$

where

$$\begin{aligned}
\hat{f}_k(Z_k) &= f_k(\bar{x}_k, x_{k0}) - \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial x_j} \dot{x}_j - \frac{\partial \alpha_{k-1}}{\partial \bar{y}_{d,k-1}} \dot{\bar{y}}_{d,k-1} \\
&\quad + z_{k-1} h_{k-1\mu_{k-1}} + d_k(\bar{x}_k, t) - M_{k-1}(Z_k).
\end{aligned} \tag{55}$$

Define virtual controller α_k as

$$\alpha_k = -k_{k1} S_{z_k,1} - k_{k2} S_{z_k,2} - \frac{\hat{\theta}_k}{2\eta_k^2} S_k^T(Z_k) S_k(Z_k) z_k - k_{k3} z_k + x_{k0}, \tag{56}$$

where k_{k1} , k_{k2} , k_{k3} , and η_k are positive design parameters.

In (56), $S_{z_k,1}$ and $S_{z_k,2}$ are defined as

$$\begin{aligned}
S_{z_k,1} &= \begin{cases} \frac{(z_k^2)^{(\alpha+1)/2}}{z_k}, & |z_k| \geq \varepsilon_{k0}, \\ \sum_{j=1}^n a_j (z_k^2)^j (\varepsilon_{k0}^2)^{-j+(\alpha+1)/2}, & |z_k| < \varepsilon_{k0}, \end{cases} \\
S_{z_k,2} &= \begin{cases} \frac{(z_k^2)^{(\beta+1)/2}}{z_k}, & |z_k| \geq \varepsilon_{k0}, \\ \sum_{j=1}^n a_j (z_k^2)^j (\varepsilon_{k0}^2)^{-j+(\beta+1)/2}, & |z_k| < \varepsilon_{k0}, \end{cases}
\end{aligned} \tag{57}$$

where ε_{k0} is a positive design parameter.

Define the adaptive law $\hat{\theta}_k$ as

$$\dot{\hat{\theta}}_k = \frac{\gamma}{2\eta_k^2} S_k^T(Z_k) S_k(Z_k) z_k^2 - \lambda \hat{\theta}_k. \tag{58}$$

Combining (18)–(20) and Assumption 3 and substituting (56) and (58) into (54) enable (54) to be rewritten as

$$\begin{aligned}
\dot{V}_k \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^{k-1} k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& -bk_{k1} z_k S_{z_{k,1}} - bk_{k1} 2^{(1+\alpha)/2} \left(\frac{b\tilde{\theta}_k^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& -2b \sum_{j=1}^{k-1} k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} \\
& -bk_{k2} z_k S_{z_{k,2}} - bk_{k2} 2^{(1+\beta)/2} \left(\frac{b\tilde{\theta}_k^2}{2\gamma} \right)^{(1+\beta)/2} \\
& + \sum_{j=1}^k C_j + z_k h_{k\mu_k} z_{k+1} + z_k \left(M_{k-1}(Z_k) - \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \right),
\end{aligned} \tag{59}$$

where

$$\frac{\lambda b \tilde{\theta}_k \hat{\theta}_k}{\gamma} \leq \frac{\lambda b}{\gamma} \left(\frac{\theta_k^2}{2} - \frac{\tilde{\theta}_k^2}{2} \right) \leq \frac{\lambda b}{2\gamma} \theta_k^2,$$

$$\beta_k = bk_{k1} \left(\frac{bQ_k^2}{\gamma} \right)^{(1+\alpha)/2} + bk_{k2} \left(\frac{bQ_k^2}{\gamma} \right)^{(1+\beta)/2}, \tag{60}$$

$$\sigma_k = \frac{\eta_k^2}{2} + \frac{\varepsilon_k^2}{4bk_{k3}},$$

$$C_k = \sigma_k + \beta_k + \frac{\lambda b \theta_k^2}{2\gamma}.$$

In accordance with (21) and (22) and Remark 4, we consider only $|z_k| \geq \varepsilon_{k0}$. Then, (59) can be written as

$$\begin{aligned}
\dot{V}_k \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^k k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& -2b \sum_{j=1}^k k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^k C_j \\
& + z_k h_{k\mu_k} z_{k+1} + z_k \left(M_{k-1}(Z_k) - \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \right).
\end{aligned} \tag{61}$$

Lemmas 4–6 and (58) yield

$$\begin{aligned}
-z_k \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j & \leq -z_k \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \left(\frac{\gamma}{2\eta_j^2} S_j^T(Z_j) S_j(Z_j) z_j^2 - \lambda \hat{\theta}_j \right) \\
& \leq \sum_{j=1}^{k-1} \frac{z_k^2 (\partial \alpha_{k-1} / \partial \hat{\theta}_j)^2 ((\gamma/2\eta_j^2) s^2 z_j^2)}{\sqrt{z_k^2 (\partial \alpha_{k-1} / \partial \hat{\theta}_j)^2 ((\gamma/2\eta_j^2) s^2 z_j^2)^2 + \varsigma_{k,j}^2}} \\
& \quad + \lambda z_k \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \hat{\theta}_j.
\end{aligned} \tag{62}$$

Therefore, the smooth function $M_{k-1}(Z_k)$ can be defined as

$$\begin{aligned}
M_{k-1}(Z_k) & = - \sum_{j=1}^{k-1} \frac{z_k (\partial \alpha_{k-1} / \partial \hat{\theta}_j)^2 ((\gamma/2\eta_j^2) s^2 z_j^2)^2}{\sqrt{z_k^2 (\partial \alpha_{k-1} / \partial \hat{\theta}_j)^2 ((\gamma/2\eta_j^2) s^2 z_j^2)^2 + \varsigma_{k,j}^2}} \\
& \quad - \lambda \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \hat{\theta}_j,
\end{aligned} \tag{63}$$

with the result that

$$z_k \left(M_{k-1}(Z_k) - \sum_{j=1}^{k-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \right) \leq 0. \tag{64}$$

Substituting (64) into (61) yields

$$\begin{aligned}
\dot{V}_k \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^k k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& -2b \sum_{j=1}^k k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^k C_j + z_k h_{k\mu_k} z_{k+1}.
\end{aligned} \tag{65}$$

Step 4. According to $z_n = x_n - \alpha_{n-1}$, we have

$$\dot{z}_n = f_n(\bar{x}_n, x_{n0}) + h_{n\mu_n}(u - x_{n0}) + d_n(\bar{x}_n, t) - \dot{\alpha}_{n-1}, \tag{66}$$

where

$$\dot{\alpha}_{n-1} = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \dot{x}_j + \frac{\partial \alpha_{n-1}}{\partial \bar{y}_{d,n-1}} \dot{\bar{y}}_{d,n-1} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j. \tag{67}$$

Constructing a Lyapunov function as $V_n = V_{n-1} + z_n^2/2 + b\tilde{\theta}_n^2/2\gamma$ yields

$$\begin{aligned}
\dot{V}_n \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^{n-1} k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& - 2b \sum_{j=1}^{n-1} k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^{n-1} C_j \\
& + z_n \hat{f}_n(Z_n) + z_n h_{n\mu_n} u - z_n h_{n\mu_n} x_{n0} - z_n \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \\
& - \frac{b\tilde{\theta}_n \dot{\hat{\theta}}_n}{\gamma} + z_n M_{n-1}(Z_n),
\end{aligned} \tag{68}$$

where

$$\begin{aligned}
\hat{f}_n(Z_n) = & f_n(\bar{x}_n, x_{n0}) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \dot{x}_j - \frac{\partial \alpha_{n-1}}{\partial y_{d,n-1}} \dot{y}_{d,n-1} \\
& + z_{n-1} h_{n-1\mu_{n-1}} + d_n(\bar{x}_n, t) - M_{n-1}(Z_n).
\end{aligned} \tag{69}$$

Define an actual controller u as

$$\begin{aligned}
u = & -k_{n1} S_{z_{n,1}} - k_{n2} S_{z_{n,2}} - \frac{\hat{\theta}_n}{2\eta_n^2} S_n^T(Z_n) S_n(Z_n) z_n \\
& - k_{n3} z_n + x_{n0},
\end{aligned} \tag{70}$$

where k_{n1} , k_{n2} , k_{n3} , and η_n are positive design parameters.

In (70), $S_{z_{n,1}}$ and $S_{z_{n,2}}$ are defined as

$$\begin{aligned}
S_{z_{n,1}} = & \begin{cases} \frac{(z_n^2)^{(\alpha+1)/2}}{z_n}, & |z_n| \geq \varepsilon_{n0}, \\ \sum_{j=1}^n a_j (z_n^2)^j (\varepsilon_{n0}^2)^{-j+((\alpha+1)/2)}, & |z_n| < \varepsilon_{n0}, \end{cases} \\
S_{z_{n,2}} = & \begin{cases} \frac{(z_n^2)^{(\beta+1)/2}}{z_n}, & |z_n| \geq \varepsilon_{n0}, \\ \sum_{j=1}^n a_j (z_n^2)^j (\varepsilon_{n0}^2)^{-j+((\beta+1)/2)}, & |z_n| < \varepsilon_{n0}, \end{cases}
\end{aligned} \tag{71}$$

where ε_{n0} is a positive design parameter.

The adaptive law $\hat{\theta}_n$ is defined as

$$\dot{\hat{\theta}}_n = \frac{\gamma}{2\eta_n^2} S_n^T(Z_n) S_n(Z_n) z_n^2 - \lambda \hat{\theta}_n. \tag{72}$$

Combining (18)–(20) and Assumption 3 and substituting (70) and (72) into (68) enable (68) to be rewritten as

$$\begin{aligned}
\dot{V}_n \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^{n-1} k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} - b k_{n1} z_n S_{z_{n,1}} \\
& - b k_{n1} 2^{(1+\alpha)/2} \left(\frac{b\tilde{\theta}_n^2}{2\gamma} \right)^{(1+\alpha)/2} - 2b \sum_{j=1}^{n-1} k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} \\
& - b k_{n2} z_n S_{z_{n,2}} - b k_{n2} 2^{(1+\beta)/2} \left(\frac{b\tilde{\theta}_n^2}{2\gamma} \right)^{(1+\beta)/2} \\
& + \sum_{j=1}^n C_j + z_n \left(M_{n-1}(Z_n) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \right),
\end{aligned} \tag{73}$$

where

$$\begin{aligned}
\frac{\lambda b \tilde{\theta}_n \hat{\theta}_n}{\gamma} \leq & \frac{\lambda b}{\gamma} \left(\frac{\theta_n^2}{2} - \frac{\tilde{\theta}_n^2}{2} \right) \leq \frac{\lambda b}{2\gamma} \theta_n^2 \beta_n = b k_{n1} \left(\frac{b Q_n^2}{\gamma} \right)^{(1+\alpha)/2} \\
& + b k_{n2} \left(\frac{b Q_n^2}{\gamma} \right)^{(1+\beta)/2}, \\
\sigma_n = & \frac{\eta_n^2}{2} + \frac{\varepsilon_n^2}{4b k_{n3}}, \\
C_n = & \sigma_n + \beta_n + \frac{\lambda b \theta_n^2}{2\gamma}.
\end{aligned} \tag{74}$$

According to (21) and (22) and Remark 4, we consider only $|z_n| \geq \varepsilon_{n0}$. Then, (73) can be written as

$$\begin{aligned}
\dot{V}_n \leq & -b2^{(1+\alpha)/2} \sum_{j=1}^n k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \\
& - 2b \sum_{j=1}^n k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^n C_j \\
& + z_n \left(M_{n-1}(Z_n) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \right).
\end{aligned} \tag{75}$$

The treatment of $M_{n-1}(Z_n)$ is similar to that of (62) and (63), with $M_{n-1}(Z_n)$ being defined as

$$\begin{aligned}
M_{n-1}(Z_n) = & - \sum_{j=1}^{n-1} \frac{z_n (\partial \alpha_{n-1} / \partial \hat{\theta}_j)^2 ((\gamma/2\eta_j^2) s^2 z_j^2)^2}{\sqrt{z_n^2 (\partial \alpha_{n-1} / \partial \hat{\theta}_j)^2 ((\gamma/2\eta_j^2) s^2 z_j^2)^2 + \zeta_{n,j}^2}} \\
& - \lambda \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \hat{\theta}_j,
\end{aligned} \tag{76}$$

with the result that

$$z_n \left(M_{n-1}(Z_n) - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j \right) \leq 0. \quad (77)$$

Substituting (77) into (75) yields

$$\dot{V}_n \leq -b2^{(1+\alpha)/2} \sum_{j=1}^n k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} - 2b \sum_{j=1}^n k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} + \sum_{j=1}^n C_j, \quad (78)$$

where $\sum_{j=1}^n C_j = \tau$.

Making $\phi_1 = \min 2^{(1+\alpha)/2} (bk_{11}, bk_{21}, \dots, bk_{n1})$ and $\phi_2 = \min 2 (bk_{12}, bk_{22}, \dots, bk_{n2})$, in accordance with Lemma 3, yields

$$-b2^{(1+\alpha)/2} \sum_{j=1}^n k_{j1} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\alpha)/2} \leq -\phi_1 \left(\sum_{j=1}^n \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right) \right)^{(1+\alpha)/2}, \quad (79)$$

$$-2b \sum_{j=1}^n k_{j2} \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right)^{(1+\beta)/2} \leq -n^{(1-\beta)/2} \phi_2 \left(\sum_{j=1}^n \left(\frac{1}{2} z_j^2 + \frac{b\tilde{\theta}_j^2}{2\gamma} \right) \right)^{(1+\beta)/2}, \quad (80)$$

and then substituting (79) and (81) into (78) yields

$$\dot{V}_n \leq -\phi_1 V_n^{(1+\alpha)/2} - n^{(1-\beta)/2} \phi_2 V_n^{(1+\beta)/2} + \tau. \quad (81)$$

Up to this point, the design of the controller is finished.

4. Stability Analysis

Theorem 1. *If system (1) satisfies Assumptions 1–3 and uses the virtual controllers (27), (42), and (56), the actual controller (70), and the adaptive laws (32), (44), (58), and (72), all signals in the closed-loop system are semiglobally uniform and ultimately bounded, and the upper limit of the fixed convergence time is irrelevant to the initial state.*

In accordance with Lemma 2, design proper parameters $k_{j1} > 0, k_{j2} > 0, k_{j3} > 0$, and $C_j > 0, j = 1, \dots, n$, to make (81) satisfy the following situation.

Case 1. If $V_n > (\tau / ((1 - \omega)\phi_1))^{2/(1+\alpha)}$, $\omega \in (0, 1)$, then (81) can be written as

$$\dot{V}_n \leq -\omega\phi_1 V_n^{(1+\alpha)/2} - n^{(1-\beta)/2} \phi_2 V_n^{(1+\beta)/2}, \quad (82)$$

and in that manner, the solution of system (1) converges on the compact set

$$x \in \left\{ V_n(x) \leq \left(\frac{\tau}{(1 - \omega)\phi_1} \right)^{2/(1+\alpha)} \right\}. \quad (83)$$

The fixed convergence time is

$$T \leq T_{\max} := \frac{2}{\omega\phi_1(1 - \alpha)} + \frac{2n^{(\beta-1)/2}}{\phi_2(\beta - 1)}. \quad (84)$$

Case 2. If $V_n \geq (\tau n^{(\beta-1)/2} / ((1 - \omega)\phi_2))^{2/(1+\beta)}$, then (81) can be written as

$$\dot{V}_n \leq -\phi_1 V_n^{(1+\alpha)/2} - \omega n^{(1-\beta)/2} \phi_2 V_n^{(1+\beta)/2}, \quad (85)$$

and in that manner, the solution of system (1) converges on the compact set

$$x \in \left\{ V_n(x) \leq \left(\frac{\tau n^{(\beta-1)/2}}{(1 - \omega)\phi_2} \right)^{2/(1+\beta)} \right\}. \quad (86)$$

The fixed convergence time is

$$T \leq T_{\max} := \frac{2}{\phi_1(1 - \alpha)} + \frac{2n^{(\beta-1)/2}}{\omega\phi_2(\beta - 1)}. \quad (87)$$

When cases 1 and 2 are combined, the solution of system (1) converges on

$$x \in \left\{ V_n(x) \leq \min \left\{ \left(\frac{\tau}{(1 - \omega)\phi_1} \right)^{2/(1+\alpha)}, \left(\frac{\tau n^{(\beta-1)/2}}{(1 - \omega)\phi_2} \right)^{2/(1+\beta)} \right\} \right\}. \quad (88)$$

FIGURE 1: System output y and reference signal y_d . FIGURE 2: State variables x_1 and x_2 .

The fixed convergence time is

$$T_s \leq T_{\max} := \frac{2}{\bar{\omega}\phi_1(1-\alpha)} + \frac{2n^{(\beta-1)/2}}{\bar{\omega}\phi_2(\beta-1)}. \quad (89)$$

It can be seen from (82) and (85) that V_n is bounded, with the result that z_j and $\tilde{\theta}_j$ are bounded. As $\tilde{\theta}_j = \theta_j - \bar{\theta}_j$, $\hat{\theta}_j$ is also bounded, where $j = 1, \dots, n$. Because $z_1 = x_1 - y_d$ and z_1 and y_d are bounded, x_1 is bounded. Because α_1 is a function of z_1 , y_d , $\dot{y}_d$, and $\hat{\theta}_1$, α_1 is bounded. Because $z_2 = x_2 - \alpha_1$, x_2 is bounded. Similarly, α_{j-1} and x_j , $j = 1, \dots, n$, are bounded. Therefore, all signals in system (1) are bounded.

Remark 6. The fixed-time control algorithm in this paper is different from previous control algorithms. The principle differences are as follows:

FIGURE 3: Actual control u . FIGURE 4: Adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.

- (1) The fixed-time control algorithm proposed in [35–40] does not solve the problem of the nonaffine structure of the control input $u(t)$. The fixed-time control algorithm proposed here solves that problem.
- (2) Some systems' $f_i(\cdot)$ structure is complex, which interferes with direct usage of $f_i(\cdot)$ for designing controllers. An RBF neural network is used here to approximate unknown functions $f_i(\cdot)$, thereby obviating the need to know the structure of $f_i(\bar{x}_i, x_{i+1})$ and avoiding the difficult design problem of controllers derived from complex system structures.

5. Simulation Results

In this section, two samples are studied to verify the effectiveness of the controller designed in the paragraphs above.

FIGURE 5: Tracking error $y - y_d$.

5.1. Mathematical Example. Consider the following non-linear pure-feedback system:

$$\begin{cases} \dot{x}_1 = \frac{1 - e^{-x_1}}{1 + e^{-x_2}} + x_2^3 + x_2 e^{-1-x_1^2} + d_1(\bar{x}_1, t), \\ \dot{x}_2 = x_1^2 x_2 + (2u + u^3)(x_1^2 + x_2^2) + d_2(\bar{x}_2, t), \\ y = x_1, \end{cases} \quad (90)$$

where x_1 and x_2 are the system state variables, u is the system control input, y is the system output, $d_1(\bar{x}_1, t) = 0.7x_1^2 \cos(1.5t)$ and $d_2(\bar{x}_2, t) = 0.5(x_1^2 + x_2^2) \sin^3(t)$ are the external disturbance terms, and $y_d = 0.5 \sin(1.5t) + \cos(0.5t)$ is the reference signal. It can be seen from system (90) that the state variables and the control input $(2u + u^3)(x_1^2 + x_2^2)$ have the nonaffine structures. The simulation study aims to design a fixed-time controller based on system (90) that ensures that the output signal y can track the reference signal y_d .

For system (90), the fixed-time controller is defined as follows:

$$\begin{aligned} \alpha_1 &= -k_{11}S_{z_1,1} - k_{12}S_{z_1,2} - \frac{\hat{\theta}_1}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1 - k_{13}z_1 + x_{10}, \\ u &= -k_{21}S_{z_2,1} - k_{22}S_{z_2,2} - \frac{\hat{\theta}_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2 - k_{23}z_2 + x_{20}, \\ \dot{\hat{\theta}}_1 &= \frac{\gamma}{2\eta_1^2}S_1^T(Z_1)S_1(Z_1)z_1^2 - \lambda\hat{\theta}_1, \\ \dot{\hat{\theta}}_2 &= \frac{\gamma}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2^2 - \lambda\hat{\theta}_2, \end{aligned} \quad (91)$$

where $a_1 = 5/4$, $a_2 = -(1/4)$, $z_1 = x_1 - y_d$, $z_2 = x_2 - \alpha_1$, $Z_1 = [x_1, \hat{\theta}_1, y_d, \dot{y}_d]^T$, and $Z_2 = [x_1, x_2, \hat{\theta}_1, \dot{y}_d, \ddot{y}_d]^T$.

Choose initial conditions as $[x_1(0), x_2(0)]^T = [0.5, 0.5]^T$ and $[\hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [0, 0]^T$. The design parameters are chosen as follows: $\alpha = 1/3$, $\beta = 2$, $k_{11} = 0.2$, $k_{12} = 0.2$, $k_{13} = 1$, $k_{21} = 0.2$, $k_{22} = 0.2$, $k_{23} = 1$, $\gamma = 5$, $\eta_1 = 0.25$, $\eta_2 = 0.25$, $\varepsilon_{10} = 0.001$, $\varepsilon_{20} = 0.001$, $\lambda = 0.1$, $x_{10} = 0.2$, and $x_{20} = 0.5$. We set the width of the RBF neural network as four. $W_1S_1(Z_1)$ and $W_2S_2(Z_2)$ contain seven and five nodes, respectively. The center of the Gaussian function is set as

FIGURE 6: Tracking error $y - y_d$. FIGURE 7: System output y and reference signal y_d . FIGURE 8: State variables x_1 , x_2 , and x_3 .

FIGURE 9: Actual control u . FIGURE 11: Tracking error $y - y_d$. FIGURE 10: Adaptive parameters $\hat{\theta}_2$ and $\hat{\theta}_3$.

$$\xi_1 = \begin{bmatrix} -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \\ -2 & -1.5 & -1 & 0 & 1 & 1.5 & 2 \end{bmatrix}, \quad (92)$$

$$\xi_2 = \begin{bmatrix} -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -2 & -1 & 0 & 1 & 2 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \\ -3 & -2 & 0 & 2 & 3 \end{bmatrix}.$$

Figure 1 shows the system output y and the reference signal y_d . It can be seen that y can track reference signals y_d effectively. Figures 2–4 show the state variables x_1 and x_2 , the actual controller u , and the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$.

For a further simulation study, we selected two sets of different data to verify the tracking performance of system (90). It can be seen from Figure 5 that the error between the system output y and the reference signal y_d increases with an increase in the α value. It can be seen from Figure 6 that the error between the system output y and the reference signal y_d does not change significantly with an increase in the β value.

Remark 7. In [40–43], because the controller has the power function $z^{2\alpha-1}$, α can select only specific values; otherwise, the power function $z^{2\alpha-1}$ is unsolvable at the origin and in the negative range. In this paper, α can be any number between zero and one.

It can be seen from Figures 1–6 that the state variables x_1 and x_2 , the actual controller u , and the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are bounded, with the consequence that all signals in the closed-loop system (90) are bounded.

5.2. Physical Example. Consider the following electromechanical system [41]:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = e_{21}x_2x_3^2 + e_{22}\sin x_1 + e_{23}x_2 - e_{23}x_2^2x_3^3, \\ \dot{x}_3 = e_{31}u + e_{32}x_2 + e_{33}x_3^2 - e_{33}x_2^2\sin x_3, \\ y = x_1, \end{cases} \quad (93)$$

where $e_{21} = 1/M$, $e_{22} = -N/M$, $e_{23} = -B/M$, $e_{31} = 1/L$, $e_{32} = -K_B/L$, and $e_{33} = -R/L$ are system parameters. The

FIGURE 12: Tracking error $y - y_d$.

descriptions of M , N , B , L , K_B , and R can be found in [53]. $y_d = \sin(0.5t) + 0.5 \sin(t)$ is the reference signal. The system parameters are chosen as follows: $M = 0.0642$, $N = 1.1408$, $B = 0.0181$, $L = 0.025$, $K_B = 0.9$, and $R = 5.0$. The simulation study aims to design a fixed-time controller based on system (93) to enable the output signal y to track the reference signal y_d .

For system (93), the fixed-time controller is defined as follows:

$$\begin{aligned}
 \alpha_1 &= -k_{11}S_{z_1,1} - k_{12}S_{z_1,2} + \dot{y}_d, \\
 \alpha_2 &= -k_{21}S_{z_2,1} - k_{22}S_{z_2,2} - \frac{\hat{\theta}_2}{2\eta_2^2}S_2^T(Z_2)S_2(Z_2)z_2 - k_{23}z_2 + x_{20}, \\
 u &= -k_{31}S_{z_3,1} - k_{32}S_{z_3,2} - \frac{\hat{\theta}_3}{2\eta_3^2}S_3^T(Z_3)S_3(Z_3)z_3 - k_{33}z_3 + x_{30}, \\
 \dot{\hat{\theta}}_2 &= \frac{\gamma}{2\eta_2^2}S_2^T(Z_2)S_2(Z_1)z_2^2 - \lambda\hat{\theta}_2, \\
 \dot{\hat{\theta}}_3 &= \frac{\gamma}{2\eta_3^2}S_3^T(Z_3)S_3(Z_3)z_3^2 - \lambda\hat{\theta}_3,
 \end{aligned} \tag{94}$$

where $a_1 = 45/32$, $a_2 = -(18/32)$, $a_3 = 5/32$, and $z_i = x_i - \alpha_{i-1}$ with $i = 1, 2, 3$.

To verify the superiority of our designed controller, the tracking performance of this controller is compared with those of a fixed-time controller [40] and a finite-time controller [41]. The same design parameters as [40] are used, as follows: $k_{11} = 10$, $k_{12} = 10$, $k_{21} = 10$, $k_{22} = 10$, $k_{31} = 10$, $k_{32} = 10$, $k_{23} = 10$, $k_{33} = 10$, $\alpha = 97/101$, $\beta = 107/100$, $x_{10} = 0.1$, $x_{20} = 0.1$, $x_{30} = 0.1$, $\varepsilon_{10} = 10^{-5}$, $\varepsilon_{20} = 10^{-5}$, and $\varepsilon_{30} = 10^{-5}$. The initial conditions are selected as $[x_1(0), x_2(0), x_3(0)]^T = [0.15, 0.15, 0.15]^T$ and $[\hat{\theta}_2(0), \hat{\theta}_3(0)] = [0.3, 0.3]^T$. The design parameters of the neural network are the same as [41].

Figure 7 shows the system output y and the reference signal y_d . As can be seen from Figure 7, the system output y

can track reference signals y_d effectively. Figures 8–10 show the state variables, the actual controller, and the adaptive parameters, respectively. Figure 11 shows the tracking error comparison between the fixed-time controller in this paper and the fixed-time controller in [40]. Figure 12 shows the tracking error comparison between the fixed-time controller in this paper and the finite-time controller in [41]. It can be seen from the simulation diagram that the performance of the fixed-time controller proposed in this paper is superior.

It can be seen from Figures 7–12 that the state variables x_1 and x_2 , the actual controller u , and the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are bounded, with the result that all signals in the closed-loop system (93) are bounded.

6. Conclusion

A novel virtual controller and a novel actual controller designed in this paper solve the singularity problem of the virtual controller and the actual controller at the origin and in the negative domain. The fixed-time controller designed is applied to nonlinear pure-feedback systems and solves the problem of nonaffine structure. The controller enables the system output to track the reference signal in a fixed time and also to make the tracking error converge to a region of the origin in a fixed time. The simulation results prove the efficiency of the controller designed in this paper. We intend to apply the approximation-based fixed-time adaptive tracking control for a class of uncertain nonlinear pure-feedback systems in time-delay systems in the future.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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