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Research Article

Orthopedic Boot-Tree 3D Design by means of the Integral Curves of Solutions of Differential Equations

Merab Shalamberidze,[1] Zaza Sokhadze ,[2] and Malvina Tatvidze[3]

[1]*Department of Design and Technology, Akaki Tsereteli State University, Tamar Mepe Str. No. 59, Kutaisi 4600, Georgia*
[2]*Department of Mathematics, Akaki Tsereteli State University, Tamar Mepe Str. No. 59, Kutaisi 4600, Georgia*
[3]*Department of Chemical Technology, Akaki Tsereteli State University, Tamar Mepe Str. No. 59, Kutaisi 4600, Georgia*

Correspondence should be addressed to Zaza Sokhadze; z.soxadze@gmail.com

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For the preparation of orthopedic shoes, it is necessary to design boot-trees, where pathological abnormalities of club feet are taken into account as much as possible. For the normal functioning of the club foot, we have to develop such an internal shape of special-purpose footwear, which is comfortable for the patient. This paper describes the methods and issues of the 3D design for constructing the geometric shapes of the main cross sections of the orthopedic boot-tree. In the research process, the authors' team of this article relied mainly on the patient database, containing the anthropometric, strain-gauge, and pedographic data on club and pathological feet. To construct the shapes of the main cross sections of the orthopedic boot-tree, we have used the integral curves to the suitable second-order differential equations. By means of a computer program, we executed turning and connection of sections of the obtained curves, on the basis of which we have the shapes of transverse-vertical cross sections of the orthopedic boot-tree. This paper also describes the main longitudinal-vertical section and the print of the orthopedic boot-tree in 3D format. By using a program of 3D design (Delcam), a skeleton of the orthopedic boot-tree was constructed in the spatial format.

1. Introduction

In the special-purpose shoe industry, considerable attention is paid to design special-purpose boot-trees. It is well known that geometrically, the orthopedic boot-tree has a complex shape and its description using mathematical investigation methods is a long enough and arduous process. In general, the technical side of the boot-tree design is diverse. In the process of the boot-tree design, it is necessary to take into account the size and shape of the foot.

The algorithm describing a geometrical shape of the surface of the boot-tree was considered in academic writings of different researchers [1–5]. To describe a geometrical shape of the boot-tree, they have used the following methods of mathematical investigation: radius-graphical, biquadratic spline, and bicubic interpolating spline. These methods are quite complex, labor-intensive, and time-consuming in the process of boot-tree design.

The authors' team of this article conducted the anthropometric, strain-gauge, and pedographic investigations of club and pathological feet. A database of patients has been created, where pathological abnormalities of each patient's feet are specified. Based on biomechanics of the movement of foot, it is necessary to transform the obtained parameters and on that basis to determine curvilinear lines describing the surface of the boot-tree. In particular, it is necessary to construct the shapes of transverse-vertical, longitudinal-vertical, and longitudinal-horizontal cross sections of orthopedic shoes. Development of a new algorithm describing a geometric surface of the boot-tree and its use in practice is one of the main and pressing problems in the orthopedic shoe industry.

2. Research Methods

The authors' team of this article decided to describe a geometrical shape of the orthopedic boot-tree by means of the integral curves of solutions of the singular Dirichlet boundary value problem. The issue will be completed in a relatively short time, and the result will be much more accurate.

The goal of our work is to design the appropriate shapes of transverse-vertical cross sections of the orthopedic boot-tree for patients with club and pathological feet. To achieve this goal, it would not be sufficient to use the simple functional relationships. We consider the particular cases of the singular Dirichlet boundary value problem. The multiplicity of solutions of these problems allows us in choosing a shape of orthopedic shoes we are seeking for club and pathological feet.

An important novelty of research consists in obtaining the approximate shapes of transverse-vertical cross sections of the orthopedic boot-tree by means of the integral curves of solutions of the singular Dirichlet boundary value problem.

In the work [6], Rachunkova et al. consider the singular Dirichlet boundary value problem:

$$u''(t) + \frac{a}{t}u'(t) - \frac{a}{t^2}u(t) = f(t, u(t), u'(t)), \tag{1}$$

$$\begin{aligned} u(t) &= 0, \\ u(T) &= 0, \end{aligned} \tag{2}$$

where $a \in (-\infty; 1)$ and f satisfies the local Caratheodory condition for a set $[0, T] \times D$, $D = (0; +\infty) \times R$.

This paper dwells on studying the existence of solution of (1)-(2) problems, and besides, Lemma 3.1 [6] in this article clearly shows the solution of (1)-(2) problems, particularly:

$$u(t) = c_1 t + c_2 t^{-a} + t\int_t^T S^{-a-2}\left(\int_S^T \xi^{a+1} f(t, u(\xi), u'(\xi) \mathrm{d}\xi)\right)\mathrm{d}\xi, \tag{3}$$

where $c_1\, c_2 \in R$, $t \in [0, T]$.

Our goal is to write clearly the solution for the different cases of a function $f(t, u(t), u'(t))$ presented on the right side of the equation (1), and also using the formula (3) for different values of a, and then to construct the integral curves of these solutions that will allow us for obtaining the desirable shapes of transverse-vertical cross sections of the orthopedic boot-tree.

Let us consider the first particular case of (1) and (2) problems:

$$u'' + \frac{2}{t}u' - \frac{2}{t^2}u = t, \tag{4}$$

$$\begin{aligned} u(1) &= 0, \\ u'(1) &= c, \end{aligned} \tag{5}$$

i.e., we put $f(t, u(t), u'(t)) = t$, $a = -2$, and $t \in [0, 1]$, and the solutions of (4) and (5) problem is as follows:

$$u(t) = \frac{t^3}{2} - \frac{1}{3}ct^{-2} - \left(1 - \frac{1}{3}c\right)t + \frac{1}{2}. \tag{6}$$

If $c = 0$, then we have $u(t) = (t^3/2) - t + (1/2)$.

Figure 1 illustrates the appropriate integral curve graphics for obtained solutions. Let us consider the same particular case of (1) and (2) problems:

$$u'' + \frac{a}{t}u' - \frac{a}{t^2}u = t^2, \tag{7}$$

$$\begin{aligned} u(1) &= 0, \\ u'(1) &= c, \end{aligned} \tag{8}$$

viz, with provision $f(t, u(t), u'(t)) = t$, $a = -2$, and $t \in [0, 1]$, the solution of (7) and (8) problems is as follows:

$$u(t) = \left(-\frac{1}{3} - \frac{1}{3}c\right)t + \frac{1}{3}c\frac{1}{t^2} + \frac{2}{3}t - \frac{t^2}{2} + \frac{t^4}{6}. \tag{9}$$

If $c = 0$, then we have $u(t) = (t^4/6) - (t^2/2) + (1/3)t$.

Figure 2 illustrates the appropriate integral curve graphics for obtained solutions.

Thus, by means of the integral curves of the solution to suitable second-order differential equations, it is possible to describe the shapes of the transverse-vertical cross section of the orthopedic boot-tree.

By means of the abovementioned mathematical investigation method and the integral curves of solutions of differential equations with deviating argument, we constructed the shapes (boot-tree print) of transverse-vertical, main longitudinal-vertical, and longitudinal-horizontal cross sections of the orthopedic boot-tree [7–11].

3. Discussion of Research Findings

Based on the 3D scanning results of abnormal and club feet, the right-side terms of equations (4) and (6) were chosen because various parts of the integral curves of the solutions to these equations are as close as possible to the shapes of the foot presented in the patient database.

Using the abovementioned methods of mathematical research, we constructed the shapes of the following transverse-vertical cross sections: $0.4D$, $0.8D$, and $0.9D$ (where D is the length of the foot). The following shapes of transverse-vertical cross section have been constructed for women's orthopedic boot-tree, size 38.

To obtain the shape of the mentioned cross section, we have used mainly the patient database. To construct the transverse-vertical cross section of the orthopedic boot-tree ($0.4D$), it was divided previously into eight parts and each enumerated section was described by means of the integral curves of solutions of differential equations given in the following. From the integral curves, we choose those eight parts that are identical to the geometric shapes of transverse-vertical cross section of the orthopedic boot-tree on $0.4D$, in particular:

(1) AB curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t+$

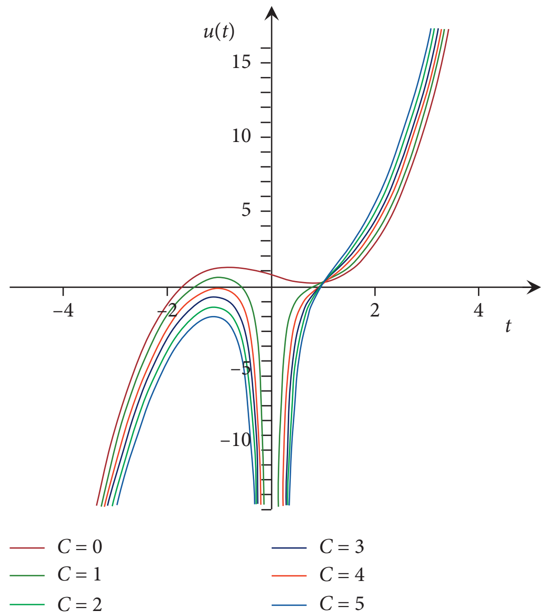

Figure 1: Integral curve graphics for solutions of (4) and (5) problems.

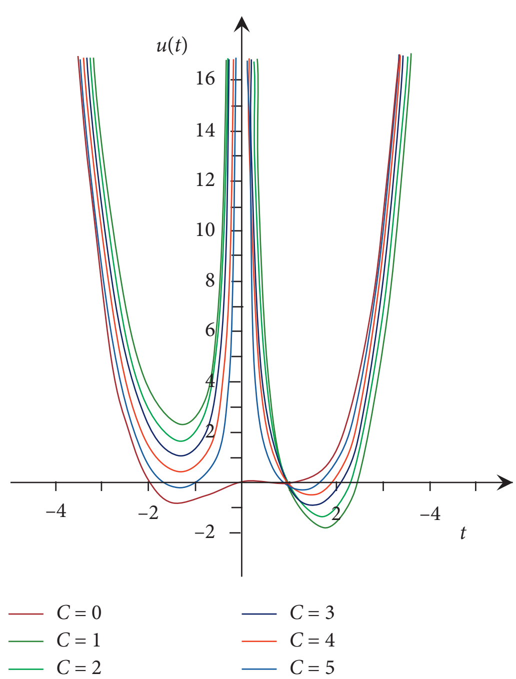

Figure 2: Integral curve graphics for solutions of (4) and (5) problems.

$(1/2)$, for which $c = 0$ and corresponds to a set $[-2{,}2; -2{,}95] \times [-1{,}3; 1]$;

(2) BC curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 5$ and corresponds to a set $[-1{,}95; 1{,}3] \times [-2{,}4; 3{,}1]$;

(3) CD curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 3$ and corresponds to a set $[-0{,}8; 4] \times [-0{,}44; 13{,}2]$;

(4) DE curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 0$ and corresponds to a set $[3; 3{,}8] \times [2{,}6; 2{,}6]$;

(5) EF curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 0$ and corresponds to a set $[-0{,}8; 1] \times [0{,}3; 0{,}2]$;

(6) FN curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 2$ and corresponds to a set $[-2{,}3; -7{,}1] \times [-1{,}4; -2.7]$;

(7) NM curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 5$ and corresponds to a set $[-2{,}9; 2{,}9] \times [-2{,}7; 4{,}1]$;

(8) MA curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 4$ and corresponds to a set $[1{,}55; 1{,}9] \times [3; 11{,}9][3; 9, 11]$.

By means of a computer program, we executed turning and connection of these curves, on the basis of which we obtained a shape of the transverse-vertical cross section of the orthopedic boot-tree on $0.4D$, as shown in Figure 3.

Likewise, we have constructed the shapes of transverse-vertical cross sections of the orthopedic boot-tree on $0.8D$ and $0.9D$.

To construct a shape of transverse-vertical cross sections of the boot-tree on $0.8D$, we have divided it previously into six parts. Each enumerated section was described by means of the integral curves of solutions of differential equations given in the following. From the integral curves, we choose those six parts that are identical to the geometric shapes of transverse-vertical cross section of the orthopedic boot-tree on $0.8D$, in particular:

(1) AB curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 2$ and corresponds to a set $[-2{,}1; -6] \times [-1{,}1; -2{,}1]$;

(2) BC curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 0$ and corresponds to a set $[-1{,}7; -0{,}5] \times [-2.6; 2{,}7]$;

(3) CD curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 0$ and corresponds to a set $[1{,}8; 1] \times [2{,}8; 7{,}4]$;

(4) DE curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 0$ and corresponds to a set $[-1{,}5; 0{,}5] \times [-1{,}9; 1]$;

(5) EF curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)$

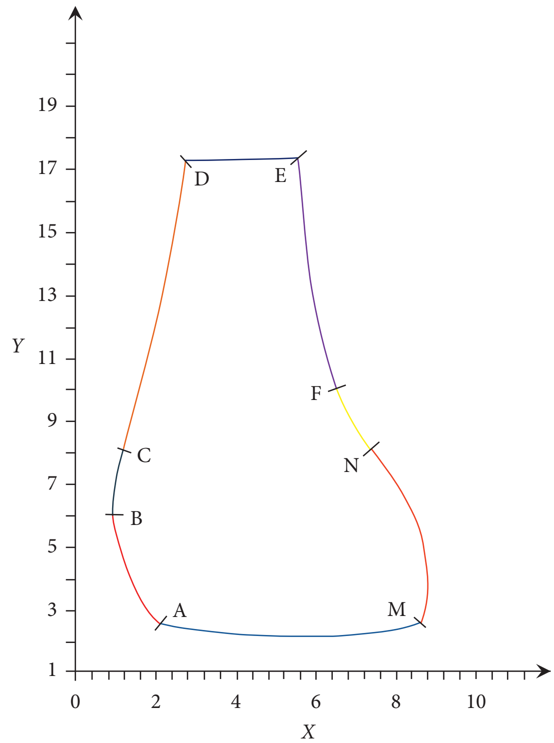

Figure 3: A shape of the transverse-vertical cross section of the orthopedic boot-tree on $0.4D$.

$c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 0$ and corresponds to a set $[1{,}6;\ 0{,}5] \times [\ 2{,}1;\ 3]$;

(6) FA curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)\ c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 1$ and corresponds to a set $[1{,}4;\ 1{,}3] \times [\ 2{,}9;\ 14{,}2]$.

Here, too, by means of a computer program, we constructed a shape of the transverse-vertical cross section of the orthopedic boot-tree on $0.8D$, as shown in Figure 4.

To construct a shape of transverse-vertical cross sections of the boot-tree on $0.9D$, we have divided it previously into four parts. Each enumerated section was described similarly, and it is given as follows:

(1) AB curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)c)t + (1/3)\ c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 0$ and corresponds to a set $[-2{,}2;\ -2{,}1] \times [0{,}4;\ 0{,}3]$;

(2) BC curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 3$ and corresponds to a set $[-2;\ 4] \times [-2{,}6;\ 8]$;

(3) CD curve corresponds that part of the solution of the equation $u(t) = (t^3/2) + (1/3)ct^{-2} - (1 - (1/3)c)t + (1/2)$, for which $c = 0$ and corresponds to a set $[-1{,}6;\ -0{,}8][-2{,}5;\ 2{,}05]$;

(4) DA curve corresponds that part of the solution of the equation $u(t) = (-(1/3) - (1/3)\ c)t + (1/3)c(1/t^2) + (2/3)t + (t^2/2) + (t^4/6)$, for which $c = 1$ and corresponds to a set $[1{,}4;\ 1] \times [\ 2{,}85;\ 13{,}05]$.

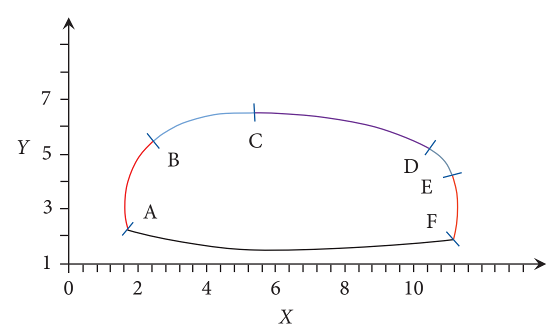

Figure 4: A shape of the transverse-vertical cross section of the orthopedic boot-tree on $0.8D$.

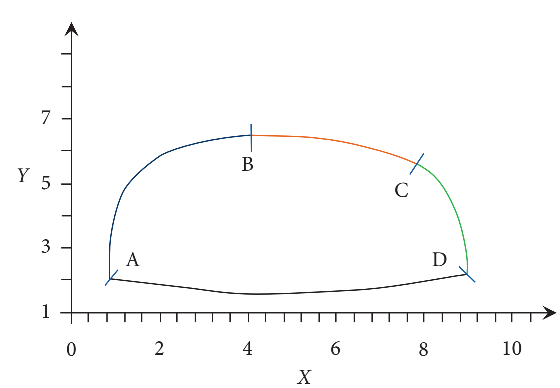

Figure 5: A shape of of the transverse-vertical cross section of the orthopedic boot-tree on $0.9D$.

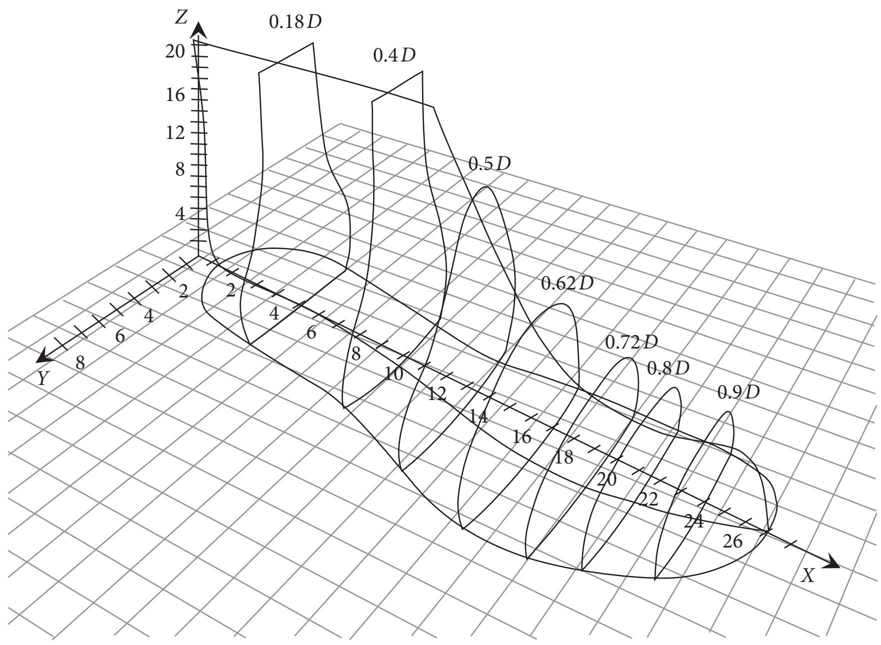

Figure 6: A skeleton of the orthopedic boot-tree in 3D format.

Figure 5 illustrates a shape of the transverse-vertical cross section of the orthopedic boot-tree on $0.9D$.

By using a program of 3D design (Delcam), Figure 6 illustrates the main transverse-vertical, longitudinal-vertical, and longitudinal-horizontal (boot-tree print) cross sections in the spatial format.

4. Conclusions

Thus and so, based on the patient database, the authors' team of this article has constructed the shapes by means of the integral curves of solutions to suitable second-order Euler differential equations. This method allows for describing with high accuracy the shapes of the main cross section of the orthopedic boot-tree. It also enables us to change the shapes of the main transverse-vertical, longitudinal-vertical, and longitudinal-horizontal cross sections of the orthopedic boot-tree an unlimited number of times, while changing the sizes of orthopedic boot-trees. The latter is particularly relevant during the production of orthopedic shoes, when we deal with patients having nonstandard or club and pathological feet.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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