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Research Article

New Periodic Wave, Cross-Kink Wave, Breather, and the Interaction Phenomenon for the (2 + 1)-Dimensional Sharmo–Tasso–Olver Equation

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In this paper, with the aid of symbolic computation, several kinds of exact solutions including periodic waves, cross-kink waves, and breather are proposed by using a trilinear form for the (2 + 1)-dimensional Sharmo–Tasso–Olver equation. Then, by combing the different forms, the interactions between a lump and one-kink soliton and between a lump and periodic waves are generated. Moreover, the dynamic characteristics of interaction solutions are analyzed graphically by selecting suitable parameters with the help of Maple.

1. Introduction

In soliton theory, the study of exact solutions of nonlinear partial differential equations (NLPDES) has attracted more and more attention. Therefore, finding exact solutions of nonlinear partial differential equations is becoming more and more important for the study of oceanographic engineering, atmosphere, chemistry, biology, finance, and social science. With the aid of Maple or Mathematica, one can get the complex solutions including periodic wave, cross-kink wave, and breather of nonlinear partial differential equations. For example, the complex solutions of the Jimbo-Miwa-like equation [1] and the (2 + 1)-dimensional breaking soliton equation [2] have been obtained. Later on, interaction solutions to lump-kink, lump-soliton, and lump periodic waves of many NLPDES have been presented so that the complex solutions can be constructed. To get the interaction between the lump and other nonlinear waves, Ma, Lou, and Yang et al. obtained some new ways by using the bilinear method and CRE method [3–5], which have been applied in many fields [6–23]. All of them are very important and useful. In this article, based on a trilinear form, our main purpose is to study the periodic waves, cross-kink waves, breather, and interaction solutions between lump-soliton and lump periodic waves of the (2 + 1)-dimensional Sharmo–Tasso–Olver equation by combining a positive quadratic function with an exponential function or a trigonometric function. The (2 + 1)-dimensional Sharmo–Tasso–Olver equation is usually written as follows [24]:

$$\left(u_t + \alpha u_x^3 + \frac{3}{2}\alpha u_{x,x}^2 + \alpha u_{x,x,x}\right)_x + \beta u_{y,y} = 0, \tag{1}$$

where $u(x, y, t)$ is an analytic function.

The abovementioned equation is a generalization of the (1 + 1)-dimensional Sharmo–Tasso–Olver equation [25–27]. It can describe the propagation of a nonlinear dispersive wave of inhomogeneous media. The generalized Kaup–Newell-type hierarchy of nonlinear evolution equations is explicitly related to the Sharma-Tasso-Olver equation. By the Hirota direct method and the Bäcklund transformation, the fission and fusion of the solitary waves have been obtained. [28].

In this paper, based on a transformation, we mainly introduce a trilinear form of generalized the (2 + 1)-dimensional Sharmo–Tasso–Olver equation. In Section 3, we present a breather of the (2 + 1)-dimensional Sharmo–Tasso–Olver equation. In Section 4, we study periodic waves and cross-kink waves of the $(2+1)$-dimensional Sharmo–Tasso–Olver equation. In Section 5, with the help of analysis and symbolic computations, by mixing a positive quadratic function with an exponential function or a trigonometric function, the interaction between a lump and one-kink soliton and the interaction between a lump and periodic waves of equation (1) are studied. At the same time, plots are presented to show the change of the equation, and the interactional phenomena are discussed.

2. The Trilinear Equation for the Sharmo–Tasso–Olver Equation

Hirota bilinear forms play an important role in solving the lump solutions. With the widespread use of the bilinear form, trilinear forms and even polylinearity forms gradually appear. By truncated Painleve analysis [29], we can get the solutions of (1) as follows:

$$u = (\ln f)_x, \tag{2}$$

where $f(x, y, t)$ is an unknown real function. Through equation (2), the trilinear equation of (1) can be presented as follows:

$$\begin{aligned} &f^2 f_{x,x,t} + 2f_x^2 f_t - 2f f_x f_{x,t} - f f_{x,x} f_t \\ &\quad + \alpha f^2 f_{x,x,x,x,x} - 2\alpha f f_x f_{x,x,x,x} \\ &\quad - \alpha f f_{x,x} f_{x,x,x} + 2\alpha f_x^2 f_{x,x,x} + \beta f^2 f_{x,y,y} \\ &\quad - 2\beta f f_y f_{x,y} + 2\beta f_x f_y^2 - \beta f f_x f_{y,y} = 0, \end{aligned} \tag{3}$$

where α and β are the arbitrary positive numbers. Next, we will use equation (3) to get a series of solutions.

3. Breather Wave Solutions for the Sharmo–Tasso–Olver Equation

We use the extended homoclinic test method [30–32] to construct $f(x, y, t)$ as a form of solutions, which is

$$f(x, y, t) = k_1 \exp(\xi_1) + k_2 \exp(-\xi_1) + +k_3 \cos(\xi_2), \tag{4}$$

and ξ_1 and ξ_2 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \end{cases} \tag{5}$$

where a_i, $i = 1, \ldots, 8, k_1, k_2, k_3$ are all real numbers. By substituting equation (4) into equation (2), we can get the breather solution of equation (1), which is

$$u(x, y, t) = \frac{k_1 a_1 \exp(\xi_1) - k_2 a_5 \exp(\xi_2) - k_3 a_3 \sin(\xi_3)}{k_1 \xi_1 + k_2 \exp(-\xi_1) + k_3 \cos(\xi_2)}, \tag{6}$$

and ξ_1 and ξ_2 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \end{cases} \tag{7}$$

where a_i, $i = 1, \ldots, 8, k_1, k_2, k_3$, are all real numbers. By substituting (4) into (3), we can get some relations of parameters as follows:

$$a_3 = -a_1^3 \alpha, a_7 = a_5^3 \alpha, \beta = 0, \tag{8}$$

where a_1, a_5, and α are some free real numbers.

Substituting (8) into (6), we can get

$$u(x, y, t) = \frac{k_1 a_1 \exp(\xi_1) - k_2 a_5 \exp(\xi_2) - k_3 a_3 \sin(\xi_3)}{k_1 \xi_1 + k_2 \exp(-\xi_1) + k_3 \cos(\xi_2)}, \tag{9}$$

and ξ_1 and ξ_2 are given by

$$\begin{cases} \xi_1 = a_1 x + a_2 y - a_1^3 \alpha t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_5^3 \alpha t + a_8, \end{cases} \tag{10}$$

where $a_1, a_2, a_4, a_5, a_6, a_8$, and α are some free real numbers.

Therefore, the change of the equation is described in Figure 1.

4. Diversity of Wave Solutions

4.1. The Periodic Cross-Kink Wave Solutions of the $(2+1)$-Dimensional Sharmo–Tasso–Olver Equation. In order to study the periodic cross-kink waves of the $(2+1)$-dimensional Sharmo–Tasso–Olver equation, we assume that the solutions for (1) are determined by

$$\begin{aligned} f(x, y, t) &= \exp(-\xi_1) + k_1 \exp(\xi_1) + k_2 \cos(\xi_2) \\ &\quad + k_3 \cosh(\xi_3) + a_{13}, \end{aligned} \tag{11}$$

and ξ_1 and ξ_2 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \\ \xi_3 = a_9 x + a_{10} y + a_{11} t + a_{12}, \end{cases} \tag{12}$$

where a_i, $i = 1, \ldots, 13, k_1, k_2, k_3$ are all real numbers. Substituting (11) into (2), we can get the solutions of (1):

$$u(x, y, t) = \frac{-a_1 \exp(-\xi_1) + k_1 a_1 \exp(\xi_1) - k_2 a_5 \cos(\xi_2) + k_3 a_9 \sinh(\xi_3)}{\exp(-\xi_1) + k_1 \exp(\xi_1) + k_2 \cos(\xi_2) + k_3 \cosh(\xi_3)}, \tag{13}$$

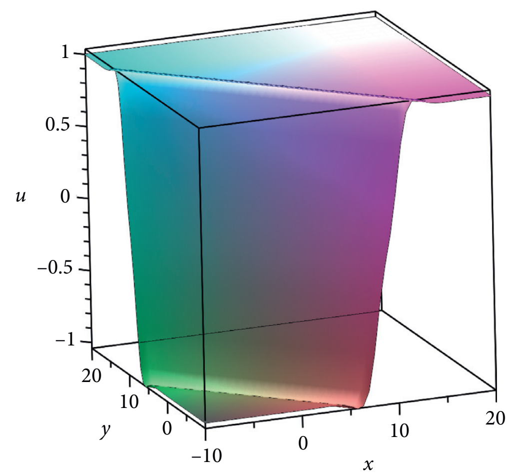


FIGURE 1: Spatiotemporal structure of solution (9) with parameter selections $a_1 = 1$, $a_5 = 1$, and $\alpha = 1$.

and ξ_1, ξ_2, and ξ_3 are given by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \\ \xi_3 = a_9 x + a_{10} y + a_{11} t + a_{12}, \end{cases} \tag{14}$$

where a_i, $i = 1, \ldots, 13, k_1, k_2, k_3$ are all real numbers. Substituting (11) into (3), we can get some relations between parameters.

4.1.1. Case 1

$$\begin{aligned} a_1 &= 0, \\ a_2 &= 0, \\ a_3 &= 0, \\ a_7 &= \frac{a_5^4\alpha - a_6^2\beta}{a_5}, \\ a_{10} &= \frac{a_6 a_9}{a_5}, \\ a_{11} &= -\frac{a_9\left(a_5^2 a_9^2\alpha + a_6^2\beta\right)}{a_5^2}, \\ a_{13} &= 0, \end{aligned} \tag{15}$$

where a_5, a_6, a_9, α, and β are free real numbers. Taking (15) into (3), we can get

$$u(x,y,t) = \frac{-a_1\exp(-\xi_1) + k_1 a_1 \exp(\xi_1) - k_2 a_5 \cos(\xi_2) + k_3 a_9 \sinh(\xi_3)}{\exp(-\xi_1) + k_1\exp(\xi_1) + k_2\cos(\xi_2) + k_3\cosh(\xi_3)}, \tag{16}$$

and ξ_1, ξ_2, and ξ_3 are determined by

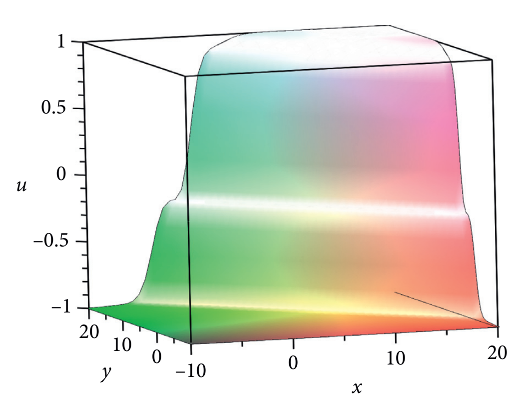


FIGURE 2: Spatiotemporal structure of solution (16) with parameter selections: $a_5 = 1$, $a_6 = 1$, $a_9 = 1$, $\alpha = 1$, and $\beta = 1$.

$$\begin{cases} \xi_1 = a_4, \\ \xi_2 = a_5 x + a_6 y + \dfrac{a_5^4\alpha - a_6^2\beta}{a_5} t + a_8, \\ \xi_3 = a_9 x + \dfrac{a_6 a_9}{a_5} y - \dfrac{a_9\left(a_5^2 a_9^2 \alpha + a_6^2\beta\right)}{a_5^2} t + a_{12}, \end{cases} \tag{17}$$

where a_4, a_5, a_6, a_8, a_9, a_{12}, α, and β are free real numbers.

The value of (16) will change when coefficients of the equation are replaced by suitable values. The three-dimensional dynamic Figure 2 is plotted as follows:

4.1.2. Case 2

$$\begin{aligned} a_1 &= 0, \\ a_3 &= 0, \\ a_7 &= a_5^3\alpha, \\ a_{11} &= -a_9^3\alpha, \\ a_{13} &= 0, \\ \beta &= 0, \end{aligned} \tag{18}$$

where a_5 and α are free real numbers. Substituting (18) into (13), we can get

$$u(x,y,t) = \frac{-a_1\exp(-\xi_1) + k_1 a_1 \exp(\xi_1) - k_2 a_5 \cos(\xi_2) + k_3 a_9 \sinh(\xi_3)}{\exp(-\xi_1) + k_1\exp(\xi_1) + k_2\cos(\xi_2) + k_3\cosh(\xi_3)}, \tag{19}$$

and ξ_1, ξ_2, and ξ_3 are given by

$$\begin{cases} \xi_1 = a_2 y + a_4, \\ \xi_2 = a_5 x + a_6 y + a_5^3\alpha t + a_8, \\ \xi_3 = a_9 x + a_{10} y - a_9^3\alpha t + a_{12}, \end{cases} \tag{20}$$

where a_2, a_4, a_5, a_6, a_8, a_9, a_{10}, a_{12}, and α are free real numbers.

So, we can draw Figure 3 to describe change of the equation.

4.2. The Periodic Wave Solutions of the Sharmo–Tasso–Olver Equation. For purpose of getting the periodic wave solutions of the $(2+1)$-dimensional Sharmo–Tasso–Olver equation, we assume that the solutions for (1) are determined by

$$f(x,y,t) = k_1\cosh(\xi_1) + k_2\cos(\xi_2) + k_3\cosh(\xi_3) + a_{13}, \tag{21}$$

where three linear wave variables are defined by

$$\begin{cases} \xi_1 = a_1x + a_2y + a_3t + a_4, \\ \xi_2 = a_5x + a_6y + a_7t + a_8, \\ \xi_3 = a_9x + a_{10}y + a_{11}t + a_{12}, \end{cases} \tag{22}$$

where a_i, $i = 1,\ldots,13, k_1, k_2, k_3$ are all real numbers. Substituting (21) into (2), we can get the solutions of (1):

$$u(x,y,t) = \frac{k_1a_1\sinh(\xi_1) - k_2a_5\sin(\xi_2) + k_3a_9\sinh(\xi_3)}{k_1\cosh(\xi_1) + k_2\cos(\xi_2) + k_3\cosh(\xi_3) + a_{13}}, \tag{23}$$

where three linear wave variables are defined by

$$\begin{cases} \xi_1 = a_1x + a_2y + a_3t + a_4, \\ \xi_2 = a_5x + a_6y + a_7t + a_8, \\ \xi_3 = a_9x + a_{10}y + a_{11}t + a_{12}, \end{cases} \tag{24}$$

where a_i, $i = 1,\ldots,13, k_1, k_2, k_3$ are real numbers. Substituting (21) into (3), we can get the relations of parameters as follows:

4.2.1. Case 1

$$\begin{aligned} a_1 &= 0, \\ a_2 &= 0, \\ a_3 &= 0, \\ a_7 &= \frac{a_5^4\alpha - a_6^2\beta}{a_5}, \\ a_{10} &= \frac{a_6a_9}{a_5}, \\ a_{11} &= -\frac{a_9\left(a_5^2a_9^2\alpha + a_6^2\beta\right)}{a_5^2}, \\ a_{13} &= 0, \end{aligned} \tag{25}$$

where a_5, a_6, a_9, α, and β are free real numbers. Substituting (25) into (23), we can get

$$u(x,y,t) = \frac{k_1a_1\sinh(\xi_1) - k_2a_5\sin(\xi_2) + k_3a_9\sinh(\xi_3)}{k_1\cosh(\xi_1) + k_2\cos(\xi_2) + k_3\cosh(\xi_3)}, \tag{26}$$

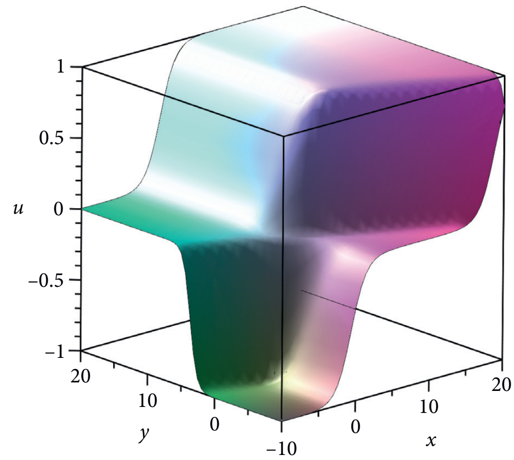


Figure 3: Spatiotemporal structure of solution (19) with parameter selections: $a_5 = 1$, $a_9 = 1$, and $\alpha = 1$.

where three linear wave variables are determined by

$$\begin{cases} \xi_1 = a_4, \\ \xi_2 = a_5x + a_6y + \dfrac{a_5^4\alpha - a_6^2\beta}{a_5}t + a_8, \\ \xi_3 = a_9x + \dfrac{a_6a_9}{a_5}y + -\dfrac{a_9\left(a_5^2a_9^2\alpha + a_6^2\beta\right)}{a_5^2}t + a_{12}, \end{cases} \tag{27}$$

where a_4, a_5, a_6, a_8, a_9, a_{12}, α, and β are free real numbers.

So, we can draw Figure 4 to describe change of the equation.

4.2.2. Case 2

$$\begin{aligned} a_1 &= 0, \\ a_3 &= 0, \\ a_7 &= a_5^3\alpha, \\ a_{11} &= -a_9^3\alpha, \\ a_{13} &= 0, \\ \beta &= 0, \end{aligned} \tag{28}$$

where a_5, a_9, and α are some free real numbers. Substituting (18) into (23), we get

$$u(x,y,t) = \frac{k_1a_1\sinh(\xi_1) - k_2a_5\sin(\xi_2) + k_3a_9\sinh(\xi_3)}{k_1\cosh(\xi_1) + k_2\cos(\xi_2) + k_3\cosh(\xi_3)}, \tag{29}$$

where three linear wave variables are given by

$$\begin{cases} \xi_1 = a_2y + a_4, \\ \xi_2 = a_5x + a_6y + a_5^3\alpha t + a_8, \\ \xi_3 = a_9x + a_{10}y - a_9^3\alpha t + a_{12}, \end{cases} \tag{30}$$

where a_2, a_4, a_5, a_6, a_8, a_{10}, a_{12}, and α are free real numbers.

So, we can draw Figure 5 to describe change of the equation.

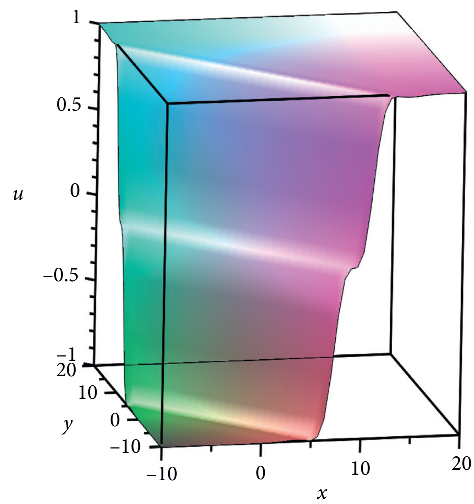


FIGURE 4: Spatiotemporal structure of solution (23) with parameter selections: $a_5 = 1$, $a_6 = 1$, $a_9 = 1$, $\alpha = 1$, and $\beta = 1$.

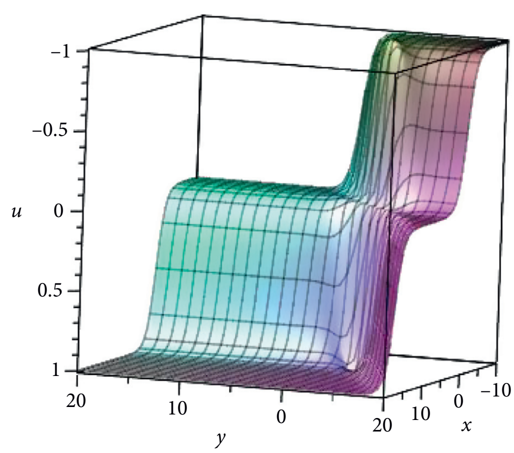


FIGURE 5: Spatiotemporal structure of solution (28) with the parameter selections: $a_5 = 1$, $a_9 = 1$, and $\alpha = 1$.

5. Interaction Solutions of the Sharmo–Tasso–Olver Equation

5.1. Interaction between a Lump Wave and One-Kink Soliton. In this section, in order to get the interaction between a lump wave and one-kink soliton of (1), we assume $f(x, y, t)$ as a combination of a positive quadratic function and an exponential function:

$$f(x, y, t) = \xi_1^2 + \xi_2^2 + \exp(\xi_3) + a_9, \tag{31}$$

and ξ_1, ξ_2, and ξ_3 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \\ \xi_3 = k_1 x + p_1 y + q_1 t + r_1, \end{cases} \tag{32}$$

where a_i, $i = 1, \ldots, 9, p_1, k_1, r_1, q_1$ are real numbers to be determined. Substituting (31) into (2), we can get the interaction solutions of (1):

$$u(x, y, t) = \frac{2a_1\xi_1 + 2a_5\xi_2 + b_1 k_1 \exp(\xi_3)}{\xi_1^2 + \xi_2^2 + \exp(\xi_3) + a_9}, \tag{33}$$

and ξ_1, ξ_2, and ξ_3 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \\ \xi_3 = k_1 x + p_1 y + q_1 t + r_1, \end{cases} \tag{34}$$

where a_i, $i = 1, \ldots, 9, p_1, k_1, r_1, q_1$ are real numbers. Substituting (31) into (3), we can get relations between the parameters as follows.

5.1.1. Case 1

$$\begin{aligned} a_2 &= \frac{a_1 p_1}{k_1}, \\ a_3 &= -\frac{a_1 \beta p_1^2}{k_1^2}, \\ a_6 &= \frac{a_5 p_1}{k_1}, \\ a_7 &= -\frac{a_5 \beta p_1^2}{k_1^2}, \\ q_1 &= \frac{-k_1^4 \alpha + \beta p_1^2}{k_1}, \end{aligned} \tag{35}$$

where a_1, a_5, p_1, k_1, α, and β are free real numbers.

5.1.2. Case 2

$$\begin{aligned} a_3 &= -\frac{\beta a_2^2}{a_1}, \\ a_6 &= \frac{a_2 a_5}{a_1}, \\ a_7 &= -\frac{a_5 \beta a_2^2}{a_1^2}, \\ k_1 &= 0, \\ p_1 &= 0, \\ q_1 &= 0, \end{aligned} \tag{36}$$

where a_1, a_2, a_5, and β are free real numbers.

In order to analyze the dynamics properties concisely, we choose case 1 to analyze. Substituting (35) into (33), we can get

$$u(x, y, t) = \frac{2a_1\xi_1 + 2a_5\xi_2 + b_1 k_1 \exp(\xi_3)}{\xi_1^2 + \xi_2^2 + \exp(\xi_3) + a_9}, \tag{37}$$

and ξ_1, ξ_2, and ξ_3 are given by

$$\begin{cases} \xi_1 = a_1 x + \dfrac{a_1 p_1}{k_1} y - \dfrac{a_1 \beta p_1^2}{k_1^2} t + a_4, \\ \xi_2 = a_5 x + \dfrac{a_5 p_1}{k_1} y - \dfrac{a_5 \beta p_1^2}{k_1^2} t + a_8, \\ \xi_3 = k_1 x + p_1 y + \dfrac{-k_1^4 \alpha + \beta p_1^2}{k_1} t + r_1, \end{cases} \tag{38}$$

where a_1, a_4, a_5, a_8, p_1, k_1, r_1, α, and β are real numbers.

The three-dimensional dynamic figures of the waves are shown in Figure 6. We can find that the lump waves and the exponential function waves will interact with each other and keep moving forward.

5.2. Interaction between a Lump Wave and Periodic Waves. In the previous section, we have obtained interaction solutions between a lump and one-kink soliton of the $(2+1)$-dimensional Sharmo–Tasso–Olver equation. In this part, we will discuss the interaction between a lump wave and periodic waves by combining a positive function with a hyperbolic cosine function. We assume that the solutions for (1) is determined by

$$f(x, y, t) = \xi_1^2 + \xi_2^2 + b_1 \cos(\xi_3) + a_9, \tag{39}$$

and ξ_1, ξ_2, and ξ_3 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \\ \xi_3 = k_1 x + p_1 y + q_1 t + r_1, \end{cases} \tag{40}$$

where a_i, $i = 1, \ldots, 9, p_1, k_1, r_1, q_1$ are real numbers. Substituting (39) into (2), we can get the interaction solutions of (1):

$$u(x, y, t) = \frac{2a_1\xi_1 + 2a_5\xi_2 - b_1 k_1 \sin(\xi_3)}{\xi_1^2 + \xi_2^2 + b_1 \cos(\xi_3) + a_9}, \tag{41}$$

and ξ_1, ξ_2, and ξ_3 are defined by

$$\begin{cases} \xi_1 = a_1 x + a_2 y + a_3 t + a_4, \\ \xi_2 = a_5 x + a_6 y + a_7 t + a_8, \\ \xi_3 = k_1 x + p_1 y + q_1 t + r_1, \end{cases} \tag{42}$$

where a_i, $i = 1, \ldots, 9, p_1, k_1, r_1, q_1$ are real numbers. Substituting (39) into (3), we can obtain the following relations between parameters.

5.2.1. Case 1

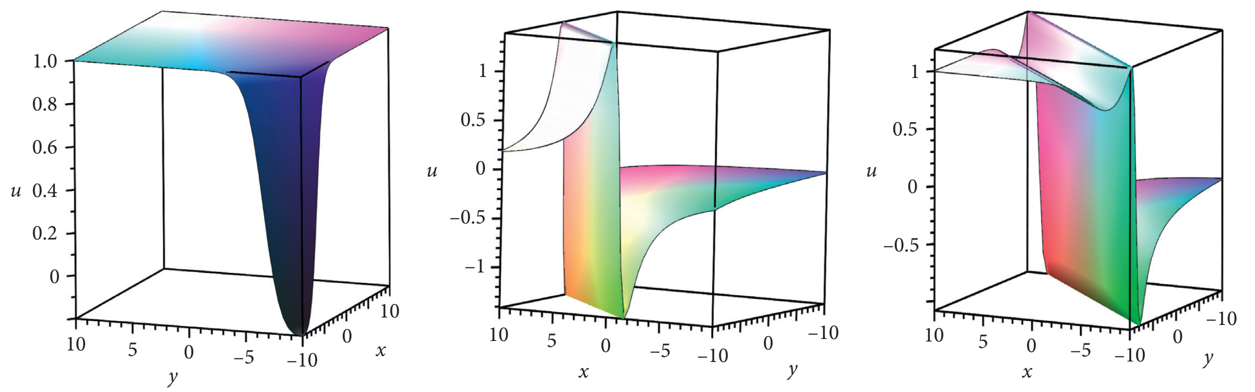
$$\begin{aligned} a_1 &= \frac{a_5 a_4}{a_8}, \\ a_2 &= \frac{a_6 a_4}{a_8}, \\ a_3 &= -\frac{a_6^2 \beta a_4}{a_5 a_8}, \\ a_7 &= -\frac{a_6^2 \beta}{a_5}, \\ p_1 &= \frac{a_6 k_1}{a_5}, \\ q_1 &= \frac{\alpha k_1^3 a_5^2 - \beta a_6^2 k_1}{a_5^2}, \end{aligned} \tag{43}$$

where a_4, a_5, a_6, a_8, k_1, α, and β are some free real numbers.

5.2.2. Case 2

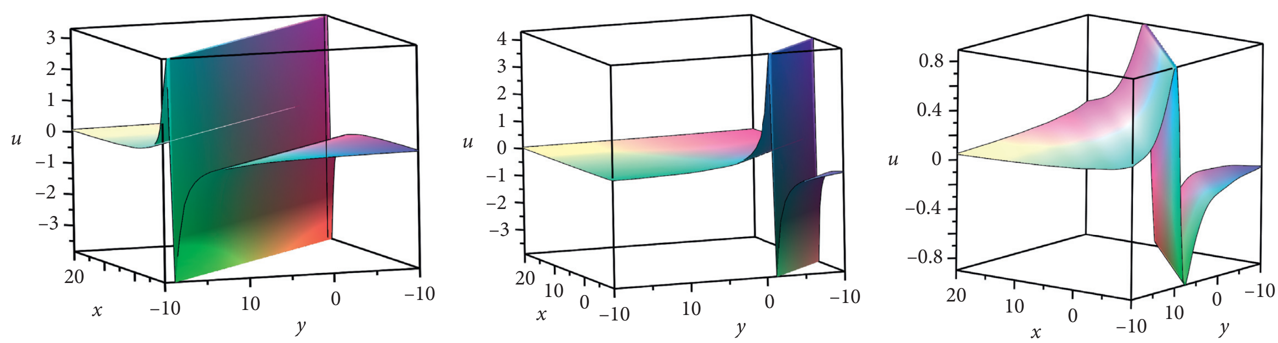
$$\begin{aligned} a_2 &= 0, \\ a_3 &= 0, \\ a_6 &= 0, \\ a_7 &= 0, \\ q_1 &= k_1^3 \alpha, \\ p_1 &= 0, \end{aligned} \tag{44}$$

where k_1 and α are some free real numbers.



(a) (b) (c)

Figure 6: Spatiotemporal structure of solution (37) with parameter selections: a: $t = -10$, $a_1 = 1$, $p_1 = 1$, $k_1 = 1$, $a_5 = 1$, $\alpha = 1$, $\beta = 1$; b: $t = 0$, $a_1 = 1$, $p_1 = 1$, $k_1 = 1$, $a_5 = 1$, $\alpha = 1$, $\beta = 1$; c: $t = 10$, $a_1 = 1$, $p_1 = 1$, $k_1 = 1$, $a_5 = 1$, $\alpha = 1$, and $\beta = 1$.



(a) (b) (c)

Figure 7: Spatiotemporal structure of solution (45) with the parameter selections: (a) $t = -10$, $a_1 = 1$, $a_5 = 1$, $p_1 = 1$, $k_1 = 1$, $\alpha = 1$, $\beta = 1$; (b) $t = 0$, $a_1 = 1$, $a_5 = 1$, $p_1 = 1$, $k_1 = 1$, $\alpha = 1$, $\beta = 1$; (c) $t = 10$, $a_1 = 1$, $a_5 = 1$, $p_1 = 1$, $k_1 = 1$, $\alpha = 1$, and $\beta = 1$.

In order to analyze the dynamic properties concisely, we choose case 1 to analyze. Sustituting (43) into (41), we can get

$$u(x, y, t) = \frac{2a_1\xi_1 + 2a_5\xi_2 - b_1k_1 \sin(\xi_3)}{\xi_1^2 + \xi_2^2 + b_1 \cos(\xi_3) + a_9}, \tag{45}$$

and ξ_1, ξ_2, and ξ_3 are given by

$$\begin{cases} \xi_1 = \dfrac{a_5a_4}{a_8}x + \dfrac{a_6a_4}{a_8}y - \dfrac{a_6^2\beta a_4}{a_5a_8}t + a_4, \\[2ex] \xi_2 = a_5x + a_6y - \dfrac{a_6^2\beta}{a_5}t + a_8, \\[2ex] \xi_3 = k_1x + \dfrac{a_6k_1}{a_5}y + \dfrac{\alpha k_1^3a_5^2 - \beta a_6^2k_1}{a_5^2}t + r_1, \end{cases} \tag{46}$$

where a_4, a_5, a_6, a_8, k_1, r_1, α, and β are real numbers.

The three-dimensional dynamic figures of the waves are shown in Figure 7. We can find that the lump waves and the periodic waves will interact with each other and keep moving forward.

6. Conclusions

In this paper, we derived the periodic wave, cross-kink wave, breather, and the interaction solutions, such as the interaction between a lump and one-kink soliton and the interaction between a lump and periodic waves of the $(2+1)$-dimension Sharmo–Tasso–Olver equation. By using the trilinear form, some interaction solutions of the $(2+1)$-dimension Sharmo–Tasso–Olver equation have been obtained with symbolic computations. The graphs of their interaction evolution processes over time are presented, and their dynamic characteristics are analysed.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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