Research Article

Availability Equivalence Analysis for the Simulation of Repairable Bridge Network System

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The performance of a repairable bridge network system is improved by using the availability equivalence factors. All components for the bridge system have constant failure and repair rates. The system is improved through the use of five methods: reduction, increase, hot duplication, warm duplication, and cold duplication methods. The availability of the original and improved systems is derived. Two types of availability equivalence factors of the system are obtained to compare different system designs. Numerical example to interpret how to utilize the obtained results is provided.

1. Introduction

In reliability analysis, there are two main methods to improve a nonrepairable system design. These two methods are (i) the reduction method which assumes that the system can be improved by reducing the failure rates of a set of components by a factor ρ , $0 < \rho < 1$, and (ii) the redundancy method which in actuality is divided into more than one redundancy method such as hot, warm, cold, and cold with imperfect switch redundancy [1]. The redundancy and reduction methods can be used to improve the repairable systems as well. In addition, the repairable system can be improved by increasing the rate of repair of some system components by a factor σ , $\sigma > 1$ [2].

The use of the redundancy method may not be a practical solution for a system in which the minimum size and weight are excessive [3]. Therefore, the concept of reliability/availability equivalence takes place. In such a concept, the design of the improved system according to the reduction or increase method must be equivalent to the design of the improved system in accordance with one of the redundancy methods specified. That is, using this concept, one can say that system performance can be improved through an alternative design [4]. In this case, different system designs

must be compared based on performance characteristics such as (i) the reliability function or mean time to failure for nonrepairable systems [5–21] or (ii) the availability in the case of repairable systems [2, 22–24].

The reliability systems in life can be divided into the following types:

- (1) The failed component is not repaired or replaced, and the operation of the complete system once one or more of its components fail depends on its structure.
- (2) The failed component can be replaced by a new component, and the failure rate for the new component may be different from the failed item.
- (3) The failed component can be repaired and restored to the system after repair in order for the system to operate again.

The nonrepairable systems can sometimes be considered a special case of repairable systems.

In this paper, the main objective is to derive the availability equivalence factors of a repairable bridge system with independent and identical components. Recent research has been done on the reliability equivalence of nonrepairable bridge systems. Sarhan [14], Mustafa et al. [17], and Mustafa

[18] derived the reliability equivalence factors of the bridge system in the case of no repairs. In [14, 17, 18], the reliability function is computed based on the assumption that the system components are nonrepairable. In addition, the reliability function was computed to obtain the reliability equivalence factors. Four methods are used to improve the reliability of the structure, namely, reduction, hot duplication, perfect switch, and imperfect switch methods. However, this paper assumes that the system components are repairable and could be repaired to function again in the system. Therefore, the goal is to calculate the availability function in order to obtain the availability equivalence factors. In this paper, five different methods were used to improve the system availability, namely, reduction, increase, hot duplication, warm duplication, and cold duplication methods.

This paper is organized as follows. In Section 2, we introduce the original system and present its availability function. The system availability is improved by five different methods, which are introduced in Section 3. The availability equivalence factors of the system are derived in Section 4. Numerical results and conclusions are discussed in Section 5.

2. Materials and Methods

The following notations and abbreviations are used throughout this paper:

- λ , failure rate
- μ , repair rate
- $\mathcal{A}_i$, availability for the component i
- A_{sys} , availability for the system
- $A_{R,\rho}$, availability for the improving system by using the reduction method
- $A_{I,\sigma}$, availability for the improving system by using the increasing method
- $\mathcal{A}_B^{\mathcal{H}}$, availability for the improving system by using the hot method
- $\mathcal{A}_B^{\mathcal{W}}$, availability for the improving system by using the warm method
- $\mathcal{A}_B^{\mathcal{C}}$, availability for the improving system by using the cold method
- N , natural numbers
- ρ , reduction factor
- σ , increasing factor
- AEF, availability equivalence factors
- RAEF, reducing availability equivalence factor
- IAEF, increasing availability equivalence factor
- $\mathcal{D}, \mathcal{H}, \mathcal{W}, \mathcal{C}$, duplication, hot, warm, cold

2.1. Repairable Bridge System. The bridge system has five components connected as shown in Figure 1.

The component i , $i = 1, 2, \dots, 5$, has exponential lifetime distribution with failure rate λ . Also, the repair time is exponential with repair rate μ . The system components are

independent and identical with availability $\mathcal{A}_i$, $i = 1, 2, \dots, 5$. $\mathcal{A}_i$ is calculated using the following equation.

$$\mathcal{A}_i = \frac{\mu}{\mu + \lambda} = \frac{1}{1 + \eta}, \quad (1)$$

where $\eta = \lambda/\mu$.

The availability for the bridge network system can be evaluated by using some techniques such as Tie sets [25]. A tie set is a minimal path of the system. The minimal path sets for the bridge system are 14, 25, 135, and 234.

The component for each path must be connected in series. All the tie sets must fail in order for the system to fail; therefore, the tie sets are connected in parallel mode as shown in Figure 2.

Let A be the availability of the bridge system, then A can be obtained as

$$\begin{aligned} A_{\text{sys}} = P(\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4) = & A_1 A_4 + A_2 A_5 + A_1 A_3 A_5 \\ & + A_2 A_3 A_4 - \sum_{i=1}^5 \prod_{j \in N_i} A_i + 2 \prod_{j \in N} A_i, \end{aligned} \quad (2)$$

where $N = \{1, 2, \dots, 5\}$, $N_i = N \setminus \{i\}$, $i \in N$. Assuming that the components are identical in equations (1) and (2), we obtain

$$A_{\text{sys}} = \frac{2\eta^3 + 8\eta^2 + 5\eta + 1}{(1 + \eta)^5}. \quad (3)$$

3. The Improving Methods

The system availability can be improved by using one of the following methods:

- (1) Reduction method
- (2) Increase method
- (3) Hot duplication method
- (4) Warm duplication method
- (5) Cold duplication method

3.1. The Reduction Method. Assume that the system will improve by reducing the failure rates of the elements in the set R using a factor ρ , $0 < \rho < 1$ and $|R| = r$, $0 \leq r \leq |N|$. Let $A_{R,\rho}$ be the availability function of the improved system in accordance with the reduction method. For a component $i \in R$, the availability A_ρ is given by

$$A_\rho = \frac{\mu}{\mu + \rho \cdot \lambda} = \frac{1}{1 + \rho \cdot \eta}, \quad (4)$$

where $\eta = \lambda/\mu$. The availability $A_{R,\rho}$ can be obtained for some different values of $r \leq 2$ as follows:

- (1) $R \in S_1 = \{\{3\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{2\eta^3 \rho + 4\eta^2 (1 + \rho) + \eta (4 + \rho) + 1}{(1 + \eta)^4 (1 + \eta \rho)}. \quad (5)$$

- (2) $R \in S_2 = \{\{1\}, \{2\}, \{4\}, \{5\}\}$:

FIGURE 1: A bridge structure.

FIGURE 2: Tie sets for the bridge system.

$$\mathcal{A}_{R,\rho} = \frac{\eta^3(1+\rho) + \eta^2(5+3\rho) + \eta(4+\rho) + 1}{(1+\eta)^4(1+\eta\rho)}. \quad (6)$$

(3) $R \in S_3 = \{1, 3, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{3, 5\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{\eta^3\rho(1+\rho) + \eta^2(2+5\rho+\rho^2) + \eta(3+2\rho) + 1}{(1+\eta)^3(1+\eta\rho)^2}. \quad (7)$$

(4) $R \in S_4 = \{\{1, 2\}, \{4, 5\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{2\eta^3\rho + \eta^2(2+6\rho) + \eta(3+2\rho) + 1}{(1+\eta)^3(1+\eta\rho)^2}. \quad (8)$$

(5) $R \in S_5 = \{\{1, 4\}, \{2, 5\}\}$:

$$\mathcal{A}_{R,\rho} = \frac{\eta^3(1+\rho^2) + \eta^2(3+4\rho+\rho^2) + \eta(3+2\rho) + 1}{(1+\eta)^3(1+\eta\rho)^2}. \quad (9)$$

3.2. The Increase Method. Suppose that $\mathcal{A}_{I,\sigma}$ refers to the availability function of the improved system through increasing the repair rates of some system components belonging to the set I by a factor σ , $\sigma > 1$ and $|I| = \ell$, $0 \leq \ell \leq N$. For component $i \in I$, the availability after increasing its repair rate can be given as follows:

$$\mathcal{A}_{i,\sigma} = \frac{\sigma\mu}{\sigma\mu + \lambda} = \frac{\sigma}{\sigma + \eta}, \quad (10)$$

where $\eta = \lambda/\mu$. The function $\mathcal{A}_{I,\sigma}$ for some different I , $\ell \leq 2$, is obtained as

(1) $I \in S_1 = \{\{3\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{2\eta^3 + 4\eta^2(1+\sigma) + \eta(4\sigma+1) + \sigma}{(1+\eta)^4(\sigma+\eta)}. \quad (11)$$

(2) $I \in S_2 = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\eta^3(1+\sigma) + \eta^2(3+5\sigma) + \eta(1+4\sigma) + \sigma}{(1+\eta)^4(\sigma+\eta)}. \quad (12)$$

(3) $I \in S_3 = \{\{1, 3\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{3, 5\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\eta^3(1+\sigma) + \eta^2(1+5\sigma+2\sigma^2) + \eta\sigma(2+3\sigma) + \sigma^2}{(1+\eta)^3(\sigma+\eta)^2}. \quad (13)$$

(4) $I \in S_4 = \{\{1, 2\}, \{4, 5\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\sigma(2\eta^3 + 2\eta^2(3+\sigma) + \eta(2+3\sigma) + \sigma)}{(1+\eta)^3(\sigma+\eta)^2}. \quad (14)$$

(5) $I \in S_5 = \{\{1, 4\}, \{2, 5\}\}$:

$$\mathcal{A}_{I,\sigma} = \frac{\eta^3(1+\sigma^2) + \eta^2(1+4\sigma+3\sigma^2) + \eta\sigma(2+3\sigma) + \sigma^2}{(1+\eta)^3(\sigma+\eta)^2}. \quad (15)$$

3.3. The Hot Duplication Method. Suppose that $\mathcal{A}_B^{\mathcal{H}}$ denotes to the availability of the improved system by hot duplication for the components belonging to the set B and $|B| = m$, $0 \leq m \leq N$. The component $i \in B$ has the availability, say $\mathcal{A}_i^{\mathcal{H}}$, as follows:

$$\mathcal{A}_i^{\mathcal{H}} = 1 - (1 - \mathcal{A}_i)^2 = 1 - \left(\frac{\eta}{1+\eta}\right)^2. \quad (16)$$

Thus, the system availability $\mathcal{A}_B^{\mathcal{H}}$ can be derived as ($m \leq 2$)

(1) $B \in S_1 = \{\{3\}\}$:

$$\mathcal{A}_B^{\mathcal{H}} = \frac{2\eta^4 + 12\eta^3 + 13\eta^2 + 6\eta + 1}{(1+\eta)^6}. \quad (17)$$

(2) $B \in S_2 = \{\{1\}, \{2\}, \{4\}, \{5\}\}$:

$$\mathcal{A}_B^{\mathcal{H}} = \frac{3\eta^4 + 14\eta^3 + 14\eta^2 + 6\eta + 1}{(1+\eta)^6}. \quad (18)$$

(3) $B \in S_3 = \{\{1, 3\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{3, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{H}} = \frac{3\eta^5 + 20\eta^4 + 29\eta^3 + 20\eta^2 + 7\eta + 1}{(1+\eta)^7}. \quad (19)$$

(4) $B \in S_4 = \{\{1, 2\}, \{4, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{H}} = \frac{(1+2\eta)(2\eta^4 + 10\eta^3 + 10\eta^2 + 5\eta + 1)}{(1+\eta)^7}. \quad (20)$$

(5) $B \in S_5 = \{\{1, 4\}, \{2, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{H}} = \frac{5\eta^5 + 25\eta^4 + 33\eta^3 + 21\eta^2 + 7\eta + 1}{(1+\eta)^7}. \quad (21)$$

3.4. The Warm Duplication Method. Suppose that, in this method, each component belonging to the set B is connected with a warm standby component (with constant failure rate ν) via a perfect switch. Let $\mathcal{A}_i^{\mathcal{W}}$ be the availability of the component $i \in B$, then $\mathcal{A}_i^{\mathcal{W}}$ can be obtained as follows [26]:

$$\begin{aligned} \mathcal{A}_i^{\mathcal{W}} &= \frac{\mu^2 + \lambda\mu + \nu\mu}{\mu^2 + \lambda\mu + \nu\mu + (1/2)\lambda^2 + (1/2)\lambda\nu} \\ &= \frac{1 + \eta + \xi}{1 + \eta + \xi + (1/2)\eta^2 + (1/2)\eta\xi}, \end{aligned} \quad (22)$$

where $\eta = \lambda/\mu$ and $\xi = \nu/\mu$. Thus, the availability of the improved system by warm duplication method, $\mathcal{A}_B^{\mathcal{W}}$, is given by

(1) $B \in S_1 = \{3\}$:

$$\mathcal{A}_B^{\mathcal{W}} = \frac{2\eta^4 + 2\eta^3(6 + \xi) + \eta^2(17 + 12\xi) + \eta(10 + 9\xi) + 2(1 + \xi)}{(1 + \eta)^4[\eta^2 + \eta(2 + \xi) + 2(1 + \xi)]}. \quad (23)$$

(2) $B \in S_2 = \{\{1\}, \{2\}, \{4\}, \{5\}\}$:

$$\mathcal{A}_B^{\mathcal{W}} = \frac{3\eta^4 + 3\eta^3(5 + \xi) + \eta^2(19 + 13\xi) + \eta(10 + 9\xi) + 2(1 + \xi)}{(1 + \eta)^4[\eta^2 + \eta(2 + \xi) + 2(1 + \xi)]}. \quad (24)$$

(3) $B \in S_3 = \{\{1, 3\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{3, 5\}\}$:

$$\begin{aligned} \mathcal{A}_B^{\mathcal{W}} &= \frac{1}{(1 + \eta)^3[\eta^2 + \eta(2 + \xi) + 2(1 + \xi)]} \\ &\quad \cdot [3\eta^5 + 3\eta^4(7 + 2\xi) + \eta^3(42 + 40\xi + 3\xi^2) \\ &\quad + \eta^2(40 + 58\xi + 19\xi^2) + 4\eta(5 + 9\xi + 4\xi^2) + 4(1 + \xi)^2]. \end{aligned} \quad (25)$$

(4) $B \in S_4 = \{\{1, 2\}, \{4, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{W}} = \frac{4(1 + \eta + \xi)[\eta^4 + \eta^3(5 + \xi) + \eta^2(6 + 5\xi) + 4\eta(1 + \xi) + \xi + 1]}{(1 + \eta)^3[\eta^2 + \eta(2 + \xi) + 2(1 + \xi)]^2}. \quad (26)$$

(5) $B \in S_5 = \{\{1, 4\}, \{2, 5\}\}$:

$$\begin{aligned} \mathcal{A}_B^{\mathcal{W}} &= \frac{1}{(1 + \eta)^3[\eta^2 + \eta(2 + \xi) + 2(1 + \xi)]^2} \\ &\quad \cdot [5\eta^5 + \eta^4(29 + 10\xi) + \eta^3(52 + 50\xi + 5\xi^2) \\ &\quad + \eta^2(44 + 64\xi + 21\xi^2) + 4\eta(5 + 9\xi + 4\xi^2) + 4(1 + \xi)^2]. \end{aligned} \quad (27)$$

3.5. The Cold Duplication Method. The cold duplication method assumes that each component belonging to the set B is connected with an identical component via a perfect

switch. The availability of the component $i \in B$, $\mathcal{A}_i^{\mathcal{C}}$, can be calculated by [27]

$$\mathcal{A}_i^{\mathcal{C}} = \frac{\mu^2 + \lambda\mu}{\mu^2 + \lambda\mu + (1/2)\lambda^2} = \frac{1 + \eta}{1 + \eta + (1/2)\eta^2}, \quad (28)$$

where $\eta = \lambda/\mu$. In what follows, we present the availability, $\mathcal{A}_B^{\mathcal{C}}$, of the improved system in accordance with the cold duplication method for the components belonging to B for some different values of B as

(1) $B \in S_1 = \{\{3\}\}$:

$$\mathcal{A}_B^{\mathcal{C}} = \frac{2\eta^4 + 12\eta^3 + 17\eta^2 + 10\eta + 2}{(1 + \eta)^4(\eta^2 + 2\eta + 2)}. \quad (29)$$

(2) $B \in S_2 = \{\{1\}, \{2\}, \{4\}, \{5\}\}$:

$$\mathcal{A}_B^{\mathcal{C}} = \frac{3\eta^4 + 15\eta^3 + 19\eta^2 + 10\eta + 2}{(1 + \eta)^4(\eta^2 + 2\eta + 2)}. \quad (30)$$

(3) $B \in S_3 = \{\{1, 3\}, \{1, 5\}, \{2, 3\}, \{2, 4\}, \{3, 4\}, \{3, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{C}} = \frac{3\eta^4 + 18\eta^3 + 24\eta^2 + 16\eta + 4}{(1 + \eta)^2(\eta^2 + 2\eta + 2)^2}. \quad (31)$$

(4) $B \in S_4 = \{\{1, 2\}, \{4, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{C}} = \frac{4(\eta^4 + 5\eta^3 + 6\eta^2 + 4\eta + 1)}{(1 + \eta)^2(\eta^2 + 2\eta + 2)^2}. \quad (32)$$

(5) $B \in S_5 = \{\{1, 4\}, \{2, 5\}\}$:

$$\mathcal{A}_B^{\mathcal{C}} = \frac{5\eta^4 + 24\eta^3 + 28\eta^2 + 16\eta + 4}{(1 + \eta)^2(\eta^2 + 2\eta + 2)^2}. \quad (33)$$

4. Availability Equivalence Factors

In this section, we will derive two different types of availability equivalence factors (AEF), reducing availability equivalence factor (RAEF) and increasing availability equivalence factor (IAEF).

Definition 1. Availability equivalence factor (AEF) is defined as the factor by which the failure rates (repair rates) of some of the system's components should be reduced (increased) in order to reach equality of the availability of a better system.

4.1. The RAEF. The reducing availability equivalence factor can be obtained by solving the following equation with respect to $\rho = \rho_{R,B}^{\mathcal{D}}$.

$$\mathcal{A}_{R,\rho} = \mathcal{A}_B^{\mathcal{D}}, \quad \mathcal{D} = \mathcal{H}, \mathcal{W}, \mathcal{C}. \quad (34)$$

For a specific set R of the system components, we present the different forms of the RAEF of the bridge system that can be derived from equation (34) as follows:

(1) When $R \in S_1$:

$$\rho_{R,B}^{\mathcal{D}} = \frac{(1+\eta)^4 \mathcal{A}_B^{\mathcal{D}} - (2\eta+1)^2}{\eta[2\eta^2 + 4\eta + 1 - (1+\eta)^4 \mathcal{A}_B^{\mathcal{D}}]}. \quad (35)$$

(2) When $R \in S_2$:

$$\rho_{R,B}^{\mathcal{D}} = \frac{(1+\eta)^4 \mathcal{A}_B^{\mathcal{D}} - (\eta^3 + 5\eta^2 + 4\eta + 1)}{\eta[\eta^2 + 3\eta + 1 - (1+\eta)^4 \mathcal{A}_B^{\mathcal{D}}]}. \quad (36)$$

(3) When $R \in S_3$:

$$\rho_{R,B}^{\mathcal{D}} = \frac{-b_1 \pm \sqrt{b_1^2 - 4a_1c_1}}{2a_1}, \quad (37)$$

where $a_1 = \eta^2(\eta+1)[1 - (1+\eta)^2 \mathcal{A}_B^{\mathcal{D}}]$, $b_1 = \eta[\eta^2 + 5\eta + 2 - 2(1+\eta)^3 \mathcal{A}_B^{\mathcal{D}}]$, and $c_1 = (1+\eta)[2\eta + 1 - (1+\eta) \mathcal{A}_B^{\mathcal{D}}]$.

(4) When $R \in S_4$:

$$\rho_{R,B}^{\mathcal{D}} = \frac{-b_2 \pm \sqrt{b_2^2 - 4a_2c_2}}{2a_2}, \quad (38)$$

where $a_2 = (1+\eta)^3 \eta^2 \mathcal{A}_B^{\mathcal{D}}$, $b_2 = 2\eta[(1+\eta)^3 \mathcal{A}_B^{\mathcal{D}} - (\eta^2 + 3\eta + 1)]$, $b_2 = 2\eta[(1+\eta)^3 \mathcal{A}_B^{\mathcal{D}} - (\eta^2 + 3\eta + 1)]$, and $c_2 = (1+\eta)[(1+\eta)^2 \mathcal{A}_B^{\mathcal{D}} - (2\eta + 1)]$.

(5) When $R \in S_5$:

$$\rho_{R,B}^{\mathcal{D}} = \frac{-b_3 \pm \sqrt{b_3^2 - 4a_3c_3}}{2a_3}, \quad (39)$$

where $a_3 = \eta^2(1+\eta)[1 - (1+\eta)^2 \mathcal{A}_B^{\mathcal{D}}]$, $b_3 = 2\eta[(2\eta+1) - (1+\eta)^3 \mathcal{A}_B^{\mathcal{D}}]$, and $c_3 = (\eta+1)^3(1 - \mathcal{A}_B^{\mathcal{D}})$.

For the values of η, ξ and $\mathcal{A}_B^{\mathcal{D}}$ for different $B, \mathcal{D}$, the values for RAEFs, $\rho_{R,B}^{\mathcal{D}}$, can be calculated from (35)–(39).

4.2. The IAEF. The increasing availability equivalence factor, $\sigma = \sigma_{I,B}^{\mathcal{D}}$, can be calculated by solving the following equation:

$$\mathcal{A}_{I,\sigma} = \mathcal{A}_B^{\mathcal{D}}, \quad \mathcal{D} = \mathcal{H}, \mathcal{W}, \mathcal{C}, \quad (40)$$

with respect to σ .

Different forms for IAEF can be calculated from equation (40) for a specific set I as follows:

(1) When $I \in S_1$:

$$\sigma_{I,B}^{\mathcal{D}} = \frac{\eta[(1+\eta)^4 \mathcal{A}_B^{\mathcal{D}} - (2\eta^2 + 4\eta + 1)]}{(2\eta+1)^2 - (1+\eta)^4 \mathcal{A}_B^{\mathcal{D}}}. \quad (41)$$

(2) When $I \in S_2$:

$$\sigma_{I,B}^{\mathcal{D}} = \frac{\eta[(1+\eta)^4 \mathcal{A}_B^{\mathcal{D}} - (\eta^2 + 3\eta + 1)]}{\eta^3 + 5\eta^2 + 4\eta + 1 - (1+\eta)^4 \mathcal{A}_B^{\mathcal{D}}}. \quad (42)$$

(3) When $I \in S_3$:

$$\sigma_{I,B}^{\mathcal{D}} = \frac{-e_1 \pm \sqrt{e_1^2 - 4d_1f_1}}{2d_1}, \quad (43)$$

where $d_1 = (\eta+1)[(2\eta+1) - (1+\eta)^2 \mathcal{A}_B^{\mathcal{D}}]$, $e_1 = \eta[(\eta^2 + 5\eta + 2) - 2(1+\eta)^3 \mathcal{A}_B^{\mathcal{D}}]$, and $f_1 = \eta^2(1+\eta)[1 - (1+\eta)^2 \mathcal{A}_B^{\mathcal{D}}]$.

(4) When $I \in S_4$:

$$\sigma_{I,B}^{\mathcal{D}} = \frac{-e_2 \pm \sqrt{e_2^2 - 4d_2f_2}}{2d_2}, \quad (44)$$

where $d_2 = (\eta+1)[(2\eta+1) - (1+\eta)^2 \mathcal{A}_B^{\mathcal{D}}]$, $e_2 = 2\eta[\eta^2 + 3\eta + 1 - (1+\eta)^3 \mathcal{A}_B^{\mathcal{D}}]$, and $f_2 = -\eta^2(1+\eta)^3 \mathcal{A}_B^{\mathcal{D}}$.

(5) When $I \in S_5$:

$$\sigma_{I,B}^{\mathcal{D}} = \frac{-e_3 \pm \sqrt{e_3^2 - 4d_3f_3}}{2d_3}, \quad (45)$$

where $d_3 = (\eta+1)^3(1 - \mathcal{A}_B^{\mathcal{D}})$, $e_3 = 2\eta[2\eta + 1 - (1+\eta)^3 \mathcal{A}_B^{\mathcal{D}}]$, and $f_3 = \eta^2(\eta+1)[1 - (1+\eta)^2 \mathcal{A}_B^{\mathcal{D}}]$.

From (41)–(45), the values for IAEFs, $\sigma_{I,B}^{\mathcal{D}}$, can be calculated through specified values of η, ξ and $\mathcal{A}_B^{\mathcal{D}}$ for different B and $\mathcal{D} = \mathcal{H}(\mathcal{W}, \mathcal{C})$.

5. Numerical Results and Conclusions

In this section, a numerical example is given to illustrate the theoretical results obtained in the previous sections. We assume $\lambda = 0.24, \mu = 1.2$, and $\nu = 0.12$. In this case, we have $\eta = \lambda/\mu = 0.2$ and $\xi = \nu/\mu = 0.1$.

Figures 3 and 4 show the availability of an improved system through improving different sets of the components in accordance with the reduction (increase) method by the factor ρ , $0 < \rho < 1$, and $(\sigma, \sigma > 1)$, respectively.

According to Figures 3 and 4, we can conclude that

- (1) $\mathcal{A}_{R,\rho}$, decreases with increasing ρ for all possible sets R .
- (2) when σ increases, the $\mathcal{A}_{I,\sigma}$ increases as well for all possible sets I .
- (3) $\mathcal{A} < \mathcal{A}_{S_1,\rho} < \mathcal{A}_{S_2,\rho} < \mathcal{A}_{S_3,\rho} < \mathcal{A}_{S_4,\rho} < \mathcal{A}_{S_5,\rho}$, for all $\rho \in (0, 1)$.
- (4) $\mathcal{A} < \mathcal{A}_{S_1,\sigma} < \mathcal{A}_{S_2,\sigma} < \mathcal{A}_{S_3,\sigma} < \mathcal{A}_{S_4,\sigma} < \mathcal{A}_{S_5,\sigma}$, for all $\sigma > 1$.

The availability of the original system is $\mathcal{A}_{\text{sys}} = 0.951342$ and $\mathcal{A}_B^{\mathcal{D}}$ for the improved system for all possible sets B are presented in Table 1.

From the numerical results in Table 1, the following can be concluded:

- (1) $\mathcal{A}_{\text{sys}} < \mathcal{A}_B^{\mathcal{H}} < \mathcal{A}_B^{\mathcal{W}} < \mathcal{A}_B^{\mathcal{C}}$ for all $B \in S_i$;
- (2) $\mathcal{A}_{\text{sys}} < \mathcal{A}_{S_1}^{\mathcal{D}} < \mathcal{A}_{S_2}^{\mathcal{D}} < \mathcal{A}_{S_3}^{\mathcal{D}} < \mathcal{A}_{S_4}^{\mathcal{D}} < \mathcal{A}_{S_5}^{\mathcal{D}}$ for all $\mathcal{D} = \mathcal{H}(\mathcal{W}, \mathcal{C})$.

FIGURE 3: The $\mathcal{A}_{R,\rho}$ for the reduction method, $R \in S_i, i = 1, 2, \dots, 5$.

FIGURE 4: The $\mathcal{A}_{I,\sigma}$ for the increase method, $I \in S_i, i = 1, 2, \dots, 5$.

TABLE 1: The $\mathcal{A}_B^{\mathcal{D}}$, $\mathcal{D} = \mathcal{H}(\mathcal{W}, \mathcal{C})$ and $B \in S_i, i = 1, 2, \dots, 5$.

$B \in S_i$	$\mathcal{A}_B^{\mathcal{H}}$	$\mathcal{A}_B^{\mathcal{W}}$	$\mathcal{A}_B^{\mathcal{C}}$
S_1	0.955415	0.955492	0.955715
S_2	0.971482	0.971862	0.972962
S_3	0.974125	0.974527	0.975687
S_4	0.976518	0.976702	0.977177
S_5	0.992585	0.99338	0.995692

Finally, the AEFs, $\rho_{R,B}^{\mathcal{D}}$, and $x\rho_{R,B}^{\mathcal{D}}$ for a different $\mathcal{D}, R, I$, and B are calculated according to the previous theoretical formulas. Tables 2–6 contain the values of AEFs.

6. Discussion

According to the numerical results in Tables 1–6, the following can be observed:

- (1) Improving the system by hot duplication method for the components belonging to the set $B \in S_1$ improves the system availability from 0.951342 to 0.955415

TABLE 2: The AEFs for a different set B and $R, I \in S_1$.

$B \in S_i$	$\rho_{R,B}^{\mathcal{D}}$			$\sigma_{I,B}^{\mathcal{D}}$		
	Hot	Warm	Cold	Hot	Warm	Cold
S_1	0.12963	0.115385	0.0744681	7.71429	8.66667	13.4286
S_2	—	—	—	—	—	—
S_3	—	—	—	—	—	—
S_4	—	—	—	—	—	—
S_5	—	—	—	—	—	—

(see Table 1). The system with $\mathcal{A}_B^{\mathcal{H}} = 0.955415$ can be obtained by using one of the following:

- (a) The failure rate of the components belonging to the set (i) $R \in S_1$ is reduced by the factor $\rho^{\mathcal{H}} = 0.12963$; (ii) $R \in S_2$ is reduced by the factor $\rho^{\mathcal{H}} = 0.803674$; (iii) $R \in S_3$ is reduced by the factor $\rho^{\mathcal{H}} = 0.834270$; (iv) $R \in S_4$ is reduced by the factor $\rho^{\mathcal{H}} = 0.896350$; (v) $R \in S_5$ is reduced by the factor $\rho^{\mathcal{H}} = 0.900617$ (see Tables 2–6).
- (b) Increasing the repair rate for the components in the set (i) $I \in S_1$ by the factor $\sigma^{\mathcal{H}} = 7.71429$; (ii) $I \in S_2$ by the factor $\sigma^{\mathcal{H}} = 1.24429$; (iii) $I \in S_3$ by the factor $\sigma^{\mathcal{H}} = 1.19865$; (iv) $I \in S_4$ by the factor $\sigma^{\mathcal{H}} = 1.11564$; (v) $I \in S_5$ by the factor $\sigma^{\mathcal{H}} = 1.11035$ (see Tables 2–6).
- (2) The system availability increases from 0.951342 to 0.955492 (see Table 1), by improving the components belonging to the set $B \in S_1$ using the warm duplication method. Through one of the following, we can obtain the system with availability, $\mathcal{A}_B^{\mathcal{W}} = 0.955492$:
 - (a) Reducing the failure rate of the components in set (i) $R \in S_1$ by the factor $\rho^{\mathcal{W}} = 0.115385$; (ii) $R \in S_2$ by the factor $\rho^{\mathcal{W}} = 0.800084$; (iii) $R \in S_3$ by the factor $\rho^{\mathcal{W}} = 0.831193$; (iv) $R \in S_4$ by the factor $\rho^{\mathcal{W}} = 0.894343$; (v) $R \in S_5$ by the factor $\rho^{\mathcal{W}} = 0.898776$ (see Tables 2–6).
 - (b) Increasing the repair rate for the components belonging to the set (i) $I \in S_1$ by the factor $\sigma^{\mathcal{W}} = 8.66667$; (ii) $I \in S_2$ by the factor $\sigma^{\mathcal{W}} = 1.24987$; (iii) $I \in S_3$ by the factor $\sigma^{\mathcal{W}} = 1.20309$; (iv) $I \in S_4$ by the factor $\sigma^{\mathcal{W}} = 1.11814$; (v) $I \in S_5$ by the factor $\sigma^{\mathcal{W}} = 1.11262$ (see Tables 2–6).
- (3) Cold duplication of the system components belonging to the set $B \in S_1$ increases the availability from 0.951342 to 0.955715 (see Table 1). The design with $\mathcal{A}_B^{\mathcal{C}} = 0.955715$ can be obtained through one of the following:
 - (a) Reducing the failure rate of the system components in (i) $R \in S_1$ by the factor $\rho^{\mathcal{C}} = 0.0744681$; (ii) $R \in S_2$ by the factor $\rho^{\mathcal{C}} = 0.789690$; (iii) $R \in S_3$ by the factor $\rho^{\mathcal{C}} = 0.822279$; (iv) $R \in S_4$ by the factor $\rho^{\mathcal{C}} = 0.888510$; (v) $R \in S_5$ by the factor $\rho^{\mathcal{C}} = 0.8934460$ (see Tables 2–6).
 - (a) Increasing the repair rate of the components in (i) $I \in S_1$ by the factor $\sigma^{\mathcal{C}} = 13.4286$; (ii) $I \in S_2$ by

TABLE 3: The AEFs for a different set B and $R, I \in S_2$.

$B \in S_i$	$\rho_{R,B}^{\mathcal{D}}$			$\sigma_{I,B}^{\mathcal{D}}$		
	Hot	Warm	Cold	Hot	Warm	Cold
S_1	0.803674	0.800084	0.789690	1.24429	1.24987	1.26631
S_2	0.129630	0.115385	0.074468	7.71429	8.66667	13.4286
S_3	0.031872	0.017290	—	31.3751	57.8362	—
S_4	—	—	—	—	—	—
S_5	—	—	—	—	—	—

TABLE 4: The AEFs for a different set B and $R, I \in S_3$.

$B \in S_i$	$\rho_{R,B}^{\mathcal{D}}$			$\sigma_{I,B}^{\mathcal{D}}$		
	Hot	Warm	Cold	Hot	Warm	Cold
S_1	0.834270	0.831193	0.822279	1.19865	1.20309	1.21613
S_2	0.224141	0.210474	0.171020	4.46149	4.75119	5.84728
S_3	0.129630	0.115385	0.074468	7.71429	8.66667	13.4286
S_4	0.045323	0.038871	0.022315	22.0639	25.7262	44.813
S_5	—	—	—	—	—	—

TABLE 5: The AEFs for a different set B and $R, I \in S_4$.

$B \in S_i$	$\rho_{R,B}^{\mathcal{D}}$			$\sigma_{I,B}^{\mathcal{D}}$		
	Hot	Warm	Cold	Hot	Warm	Cold
S_1	0.896350	0.894343	0.888510	1.11564	1.11814	1.12548
S_2	0.393569	0.377786	0.329639	2.54085	2.64700	3.03363
S_3	0.273861	0.253018	0.186415	3.65149	3.95229	5.36438
S_4	0.129630	0.115385	0.074468	7.71429	8.66667	13.4286
S_5	—	—	—	—	—	—

TABLE 6: The AEFs for a different set B and $R, I \in S_5$.

$B \in S_i$	$\rho_{R,B}^{\mathcal{D}}$			$\sigma_{I,B}^{\mathcal{D}}$		
	Hot	Warm	Cold	Hot	Warm	Cold
S_1	0.900617	0.898776	0.8934460	1.11035	1.11262	1.11926
S_2	0.539715	0.531738	0.5087430	1.85283	1.88062	1.96563
S_3	0.484669	0.476396	0.4526690	2.06326	2.09909	2.20912
S_4	0.435802	0.432071	0.4225060	2.29462	2.31443	2.36683
S_5	0.129630	0.115385	0.0744681	7.71429	8.66667	13.4286

the factor $\sigma^{\mathcal{C}} = 1.26631$; (iii) $I \in S_3$ by the factor $\sigma^{\mathcal{C}} = 1.21613$; (iv) $I \in S_4$ by the factor $\sigma^{\mathcal{C}} = 1.12548$; (v) $I \in S_5$ by the factor $\sigma^{\mathcal{C}} = 1.11926$ (see Tables 2–6).

7. Conclusions

In this paper, the equivalence of different designs of a repairable bridge system based on the system availability was derived. The failure rates and repair rates of the system's components were assumed constant. The system availability was improved using five different methods. The availability of the original and the improved systems and the availability equivalence factors of the systems were derived. Numerical results were presented to illustrate how one can utilize the

theoretical results obtained in this work and to compare the different availability factors of the system. Future work includes extending the current paper to include other cases such as systems with nonidentical components, limited repair teams which are available for each component in the system, and failure (repair) rates of the components which are not constant.

Data Availability

No data was used in the study.

Conflicts of Interest

The authors declare no conflicts of interest.

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