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Research Article

Group Controllability of Discrete-Time Time-Delayed Multiagent Systems with Multiple Leaders

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The group controllability is a unique distinct perspective and a further generalization of the controllability problem of discrete-time time-delayed multiagent systems (MASs) with multiple leaders. The group controllability concept of discrete-time time-delayed MASs with multiple leaders is proposed, its equivalent augmented system without time delay is reformulated, and the group controllability criteria are obtained in this paper. Numerical simulations are presented finally.

1. Introduction

Controllability is a novel and significant topic in the field of coordination control for MASs [1–10]. The evolution behavior of MASs is influenced by the agent's dynamics, the information communication links among the agents, and the control protocols. The concept of group controllability is the generalization of controllability of MASs, which follows with interest in the controllability of the entire group consisting of multiple subgroups or multiple clusters to further enrich and improve the controllability theory and provide more and better theoretical basis for other research fields of MASs.

In 2004, Tanner [11] first put forward the concept of controllability of MASs and built a first-order continuous-time dynamic model with a simple linear protocol and an intelligent agent selected as a virtual leader (an external control input or input signal) to control the whole system to obtain the required spatial state from any initial state. According to the agent's time state, MASs can be described by the single integrator [12], double integrator [13], high integrator [14] to generic linear dynamics [15] of continuous-time/ discrete-time models. In particular, Wang et al. [14] showed the controllability of second-order MASs/ high-order MASs/generic linear MASs is equivalent to that of first-order MASs by designing appropriate control protocols. Moreover, Guan and Wang [16] considered the structural controllability of high-order integrator MASs under absolute and relative protocols, respectively, and gave the structured controllable conditions of MASs from the perspective of communication topology. In [17, 18], the first-order controllability of discrete-time MASs was studied through linear algebraic theory and the PBH criterion under different topological structures. The second-order controllability of MASs was discussed in [19], and some graph theory criteria were obtained [20–31]. In [32], the second-order controllability of MASs with sampling data was investigated, and the controllability criteria related to the sampling period were obtained. Guan et al. [33] considered the controllability of heterogeneous MASs composed of (continuous and discrete) agents with the single integrator and double integrator on directed weighted topology, obtained some graphical and algebraic conditions for determining controllability of heterogeneous MASs, and analyzed the controllability relationship between heterogeneous MASs and homogeneous MASs.

However, in reality, the group cooperation and control of MASs with multiple complex subgroups or intelligent communities can be of more practical significance. In the existing literature studies, there are few research studies on the group controllability of MASs. At present, the authors discussed the group controllability for discrete-time MASs in [34] and continuous-time MASs in [35], respectively. Long et al. [13] further studied the second-order group controllability of discrete-time and continuous-time MASs with two-time-scale. In most existing literature studies, researchers rarely consider both the influences of leaders and time delay on the group controllability for MASs. Different from the group controllability problem of discrete-time time-delayed MASs under the leaderless framework studied in [34], the current work has considered the group controllability of discrete-time time-delayed MASs with multiple leaders, which can be expressed by different system matrices, respectively. It is obvious that different models can lead to completely different features for MASs with leaders. The study of this work is not a direct extension of the counterpart in the cited paper [34]. The main difference between this paper and [34] is threefold: (1) the mathematical models are different, (2) the mathematical definitions of the group controllability are different, and (3) the system external control inputs are completely different, where [34] considered the group controllability of discrete-time time-delayed MASs under the leaderless framework, while this paper has considered the group controllability of discrete-time time-delayed MASs under the leader-follower framework, which makes the system control input more complex than that of [34]. We will focus on the group controllability of discrete-time MASs with multiple leaders (external control inputs) as well as communication restrictions and obtain algebraic criteria on the group controllability for MASs in this paper. The main contributions of this paper include the following:

(1) The concept of the group controllability of discrete-time time-delayed MASs with multiple leaders is put forward on fixed topology

(2) The equivalent augmented system without time delay is reformulated for discrete-time time-delayed MASs

(3) Some group controllable features of discrete-time time-delayed MASs with multiple leaders are reestablished based on the group agreement protocol

(4) The influence of leaders on the group controllability is investigated

The rest of the work is arranged as follows. Section 2 states the mathematical preliminaries and model. Section 3 represents the main results. Simulations are given in Section 4. Section 5 summarises the conclusion.

2. Preliminaries and Model

At present, most of the studies for MASs are for a single system. However, in real practice and daily life, MASs can be made up of many clusters or subgroups, which make the research on the controllability of MASs be more important and necessary. This work aims at the controllability of MASs which are divided into different subgroups based on the group consensus protocol. Here, we need to explain that each subgroup, without setting the external control inputs, can be controlled by its leaders and the other subgroups so that we can redivide the agents in the whole group. This is of great significance in social practice.

A discrete-time MAS $(\mathcal{G}, x)$ consists of $m+n+l+p$ dynamical agents for m, n, l, $p>1$, partitioned into two different subgroups $(\mathcal{G}_1, x)$ and $(\mathcal{G}_2, x)$, without loss of generality, as shown in Figure 1, where subgroup $(\mathcal{G}_1, x)$ contains m followers and l leaders, and subgroup $(\mathcal{G}_2, x)$ contains n followers and p leaders, respectively.

Let $\ell_1 = \{1, \ldots, m\}$, $\ell_2 = \{m+1, \ldots, m+n\}$, $\ell_{1_l} = \{m+n+1, \ldots, m+n+l\}$, $\ell_{2_l} = \{m+n+l+1, \ldots, m+n+l+p\}$, $\mathcal{V}_1 = \{v_1, \ldots, v_m\}$, and $\mathcal{V}_2 = \{v_{m+1}, \ldots, v_{m+n}\}$, $\ell = \ell_1 \cup \ell_2$, $\ell_l = \ell_{1_l} \cup \ell_{2_l}$, and $\mathcal{V} = \mathcal{V}_1 \cup \mathcal{V}_2$. $\mathcal{N}_i$ denotes the neighboring set of the i-th agent, $\mathcal{N}_{1i} = \{v_j \in \mathcal{V}_1\colon (v_j, v_i) \in \mathcal{E}\}$, and $\mathcal{N}_{2i} = \{v_j \in \mathcal{V}_2\colon (v_j, v_i) \in \mathcal{E}\}$ with $\mathcal{N}_i = \mathcal{N}_{1i} \cup \mathcal{N}_{2i}$ and $\mathcal{N}_{1i} \cap \mathcal{N}_{2i} = \varnothing$; $\mathcal{N}_{1q}$ and $\mathcal{N}_{2q}$ are the neighbor sets of subgroup 1 and subgroup 2, respectively. Now, let us start from the following discrete-time MAS:

$$x_i(k+1) = \begin{cases} x_i(k) + \sum\limits_{j\in\mathcal{N}_{1i}} a_{ij}\big(x_j(k-h) - x_i(k-h)\big) + \sum\limits_{j\in\mathcal{N}_{2i}} a_{ij}x_j(k) + \sum\limits_{q\in\mathcal{N}_{1q}} b_{iq}\big(y_q(k) - x_i(k)\big), & i \in \ell_1, \\ x_i(k) + \sum\limits_{j\in\mathcal{N}_{2i}} a_{ij}\big(x_j(k-h) - x_i(k-h)\big) + \sum\limits_{j\in\mathcal{N}_{1i}} a_{ij}x_j(k) + \sum\limits_{q\in\mathcal{N}_{2q}} b_{iq}\big(y_q(k) - x_i(k)\big), & i \in \ell_2, \end{cases} \tag{1}$$

in which $x_i \in \mathfrak{R}$ is the i-th agent's state; $a_{ij} \geq 0$, $\forall i, j \in \ell_1, \ell_2$; otherwise, $a_{ij} \in \mathfrak{R}$; $b_{iq} \geq 0$; and time delay $h > 0$ is the integer.

Remark 1. The weighted factors $a_{ij} \in \mathfrak{R}$ are allowed to be negative, making more difficult and more complex to solve the group controllability of the MAS.

Let

$$\begin{aligned} x^1 &\triangleq (x_1, \ldots, x_m)^T, \\ x^2 &\triangleq (x_{m+1}, \ldots, x_{m+n})^T, \\ y^1 &= (y_1, y_2, \ldots, y_l)^T, \\ y^2 &= (y_{l+1}, y_{l+2}, \ldots, y_{l+p})^T. \end{aligned} \tag{2}$$

Then, the followers' dynamics of MAS (1) is redescribed by

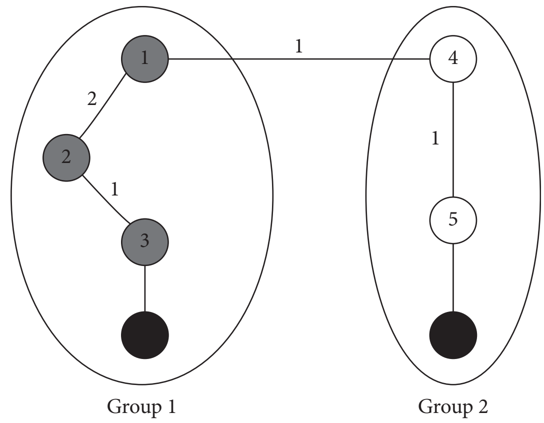

Figure 1: Topology $\mathscr{G}$ with two subgroups.

$$\begin{cases} x^1(k+1) = F_1 x^1(k) - L_{11} x^1(k-h) + \begin{bmatrix} -L_{12} & P_1 \end{bmatrix} \begin{bmatrix} x^2(k) \\ y^1(k) \end{bmatrix}, \\ x^2(k+1) = F_2 x^2(k) - L_{22} x^2(k-h) + \begin{bmatrix} -L_{21} & P_2 \end{bmatrix} \begin{bmatrix} x^1(k) \\ y^2(k) \end{bmatrix}, \end{cases} \tag{3}$$

where $F_1 = E_1 - R_1$ and $F_2 = E_2 - R_2$, with $E_1 \in \mathfrak{R}^{m\times m}$ and $E_2 \in \mathfrak{R}^{n\times n}$ being identity matrices,

$$\begin{aligned} R_1 &= \operatorname{diag}\left(\sum_{q=m+1}^{m+l} b_{1q}, \ldots, \sum_{q=m+1}^{m+l} b_{mq}\right) \in \mathfrak{R}^{m\times m}, \\ R_2 &= \operatorname{diag}\left(\sum_{q=n+1}^{n+p} b_{1q}, \ldots, \sum_{q=n+1}^{n+p} b_{mq}\right) \in \mathfrak{R}^{n\times n}, \\ L &= \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}, \\ \text{with } L_{11} &= \left[l_{ij}\right] \in \mathfrak{R}^{m\times m}, \\ L_{22} &= \left[l_{ij}\right] \in \mathfrak{R}^{n\times n}, \\ L_{12} &= \left[l_{ij}\right] \in \mathfrak{R}^{m\times n}, \\ L_{21} &= \left[l_{ij}\right] \in \mathfrak{R}^{n\times m}, \\ P_1 &= \begin{bmatrix} b_{1(m+n+1)} & \cdots & b_{1(m+n+l)} \\ b_{2(m+n+1)} & \cdots & b_{2(m+n+l)} \\ \vdots & \vdots & \vdots \\ b_{m(m+n+1)} & \cdots & b_{m(m+n+l)} \end{bmatrix} \in \mathfrak{R}^{m\times l}, \\ P_2 &= \begin{bmatrix} b_{(m+1)(m+n+l+1)} & \cdots & b_{(m+1)(m+n+l+p)} \\ b_{(m+2)(m+n+l+1)} & \cdots & b_{(m+2)(m+n+l+p)} \\ \vdots & \vdots & \vdots \\ b_{(m+n)(m+n+l+1)} & \cdots & b_{(m+n)(m+n+l+p)} \end{bmatrix} \in \mathfrak{R}^{n\times p}. \end{aligned} \tag{4}$$

Remark 2. Note that L_{11} and L_{22} are both Laplacian matrices, but $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$ is not a Laplacian matrix since the subgroup-to-subgroup weighted factors $a_{ij} \in \mathfrak{R}$ are allowed to be negative so that the eigenvalues and eigenvectors of matrix L have no same properties with the Laplacian matrix, which is completely different from the typical MASs with the consensus protocol, which makes the theoretical analysis for the group controllability more difficult.

In order to transform system (3) into a classical linear discrete-time control system, we will introduce its equivalent augmented form:

$$\begin{aligned} &\begin{cases} x^1(k+1) = F_1 x^1(k) - L_{11} x^1(k-h) + \begin{bmatrix} -L_{12} & P_1 \end{bmatrix} \begin{bmatrix} x^2(k) \\ y^1(k) \end{bmatrix}, \\ x^1(k) = x^1(k), \\ \qquad\vdots \\ x^1(k-h+1) = x^1(k-h+1), \end{cases} \\ &\begin{cases} x^2(k+1) = F_2 x^2(k) - L_{22} x^2(k-h) + \begin{bmatrix} -L_{21} & P_2 \end{bmatrix} \begin{bmatrix} x^1(k) \\ y^2(k) \end{bmatrix}, \\ x^2(k) = x^2(k), \\ \qquad\vdots \\ x^2(k-h+1) = x^2(k-h+1). \end{cases} \end{aligned} \tag{5}$$

In the following, we take new stacked vectors as $\mathscr{X}^1(k) \triangleq (x^1(k)^T, \ldots, x^1(k-h)^T)^T$ as well as $\mathscr{X}^2(k) \triangleq (x^2(k)^T, \ldots, x^2(k-h)^T)^T$; then, system (3) can be changed into

$$\begin{cases} \mathscr{X}^1(k+1) = \mathscr{F}_1 \mathscr{X}^1(k) + \mathscr{P}_1 \begin{bmatrix} \mathscr{X}^2(k) \\ y^1(k) \end{bmatrix}, \\ \mathscr{X}^2(k+1) = \mathscr{F}_2 \mathscr{X}^2(k) + \mathscr{P}_2 \begin{bmatrix} \mathscr{X}^1(k) \\ y^2(k) \end{bmatrix}, \end{cases} \tag{6}$$

where

$$\mathscr{F}_1 = \begin{bmatrix} F_1 & 0 & \cdots & 0 & -L_{11} \\ E & 0 & \cdots & 0 & 0 \\ 0 & E & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & E & 0 \end{bmatrix}_{(h+1)m\times(h+1)m},$$
$$\mathscr{P}_1 = \begin{bmatrix} -L_{12} & 0 & \cdots & 0 & P_1 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{(h+1)m\times(l+(h+1)m)},$$
$$\mathscr{F}_2 = \begin{bmatrix} F_2 & 0 & \cdots & 0 & -L_{22} \\ E & 0 & \cdots & 0 & 0 \\ 0 & E & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & E & 0 \end{bmatrix}_{(h+1)n\times(h+1)n},$$
$$\mathscr{P}_2 = \begin{bmatrix} -L_{21} & 0 & \cdots & 0 & P_2 \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}_{(h+1)n\times(p+(h+1)n)}. \tag{7}$$

Remark 3. For subgroup $(\mathscr{G}_1, x)$, $\mathscr{X}^2$ and y^1 are its external inputs, and for subgroup $(\mathscr{G}_2, x)$, $\mathscr{X}^1$ and y^2 are its external inputs, respectively. Notice that system (3) with time delay can be turned into system (6) without time delay; therefore, their controllability is equivalent.

Remark 4. On the contrary, this work uses the similar equivalent augmented conversion with [34] and obtains some similar results, but they cannot be obtained straightforwardly from results in [34] because discrete-time time-delayed MASs with multiple leaders rely on leader-to-follower interactions (i.e., matrices P_1, P_2, R_1, and R_2) and the subgroup-to-subgroup interactions (i.e., matrices L_{11}, L_{22}, L_{12}, and L_{21}) regardless of the internal interactions among subgroups (i.e., matrices L_{11}, L_{22}, L_{12}, and L_{21}), which make the subsystem matrices $(\mathscr{F}_1, \mathscr{P}_1)$ and $(\mathscr{F}_2, \mathscr{P}_2)$ more complex than those of [34]. That is, the influence of leaders on the group controllability must be considered, which makes the system matrices more complex and difficult to discuss.

3. Theoretical Analysis

Definition 1. A state $\mathscr{X}(\neq 0)$ of system (6) attains group controllability if

(1) There are a time instant $T \in J$ and the control input $\begin{bmatrix} \mathscr{X}^2 \\ y^1 \end{bmatrix}$ such that $\mathscr{X}^1(0) = \mathscr{X}^1 \& \mathscr{X}^1(T) = 0$ as well as

(2) There are a time instant $T \in J$ and the control input $\begin{bmatrix} \mathscr{X}^1 \\ y^2 \end{bmatrix}$ such that $\mathscr{X}^2(0) = \mathscr{X}^2 \& \mathscr{X}^2(T) = 0$, where J is a time set

Theorem 1. *System (6) attains group controllability iff*

$$\begin{aligned} \operatorname{rank}(Q_1) &= (h+1)m, \\ \operatorname{rank}(Q_2) &= (h+1)n, \end{aligned} \tag{8}$$

where $Q_1 = [\mathscr{P}_1, \mathscr{F}_1\mathscr{P}_1, \ldots, \mathscr{F}_1^{m-1}\mathscr{P}_1]$ *and* $Q_2 = [\mathscr{P}_2, \mathscr{F}_2\mathscr{P}_2, \ldots, \mathscr{F}_2^{n-1}\mathscr{P}_2]$.

The proof is obvious, hence omitted here. It is also fussy to test this result. In order to facilitate calculation, the group controllability of such MAS can be judged by the following PBH.

Theorem 2. *System (6) attains group controllability iff*

(1) rank$(sI - \mathscr{F}_1, \mathscr{P}_1) = (h+1)m$ and rank$(tI - \mathscr{F}_2, \mathscr{P}_2) = (h+1)n, \forall s, t \in \mathscr{C}$ ($\mathscr{C}$ represents the set of complex numbers), or

(2) rank$(\lambda_i I - \mathscr{F}_1, \mathscr{P}_1) = (h+1)m$ and rank$(\mu_i I - \mathscr{F}_2, \mathscr{P}_2) = (h+1)n$, for λ_i $(\forall i = 1, \ldots, m)$ being $\mathscr{F}_1$'s eigenvalues and μ_i $(\forall i = 1, \ldots, n)$ being $\mathscr{F}_2$'s eigenvalues.

Proof. Clearly, condition (2) is absolutely true if condition (1) is true. Therefore, only condition (1) needs to be shown.

Necessity 1. By contradiction, assumed that $\exists \lambda_1 \in \mathscr{C}$, we have

$$\operatorname{rank}(\lambda_1 I - \mathscr{F}_1, \mathscr{P}_1) < (h+1)m. \tag{9}$$

We can know that the rows of $[\lambda_1 I - \mathscr{F}_1, \mathscr{P}_1]$ are all linear-dependent; thus, $\exists \alpha(\neq 0)$ such that $\alpha'[\lambda_1 I - \mathscr{F}_1, \mathscr{P}_1] = 0$. Then,

$$\begin{aligned} \lambda_1 \alpha' &= \alpha' \mathscr{F}_1, \\ \alpha' \mathscr{P}_1 &= 0. \end{aligned} \tag{10}$$

Moreover,

$$\begin{aligned} \alpha' \mathscr{P}_1 &= 0, \\ \alpha' \mathscr{F}_1 \mathscr{P}_1 = 0, \ldots, \alpha' F_1^{m-1} \mathscr{P}_1 &= 0. \end{aligned} \tag{11}$$

By the PBH rank test, we can obtain

$$\alpha' \left[\mathscr{P}_1, \mathscr{F}_1\mathscr{P}_1, \ldots, \mathscr{F}_1^{m-1}\mathscr{P}_1\right] = 0. \tag{12}$$

Since $\alpha \neq 0$, then there must be

$$\operatorname{rank}\left(\left[\mathscr{P}_1, \mathscr{F}_1\mathscr{P}_1, \ldots, \mathscr{F}_1^{m-1}\mathscr{P}_1\right]\right) < (h+1)m. \tag{13}$$

Therefore, we can have the conclusion that system (6) must be uncontrollable, which is at odds with the truth that system (6) is controllable. The necessity of condition (1) is proved.

Sufficiency 1. By contradiction, suppose that system (6) is uncontrollable; then,

$$\operatorname{rank}\left(\left[\mathscr{P}_1, \mathscr{F}_1\mathscr{P}_1, \ldots, \mathscr{F}_1^{m-1}\mathscr{P}_1\right]\right) < (h+1)m. \tag{14}$$

Therefore, $\exists\lambda_2 \in \mathscr{C}$ belongs to $\mathscr{F}_1$ corresponding to eigenvector $\beta \in \mathscr{C}$. Thus,

$$\beta'\left[\mathscr{P}_1, \mathscr{F}_1\mathscr{P}_1, \ldots, \mathscr{F}_1^{m-1}\mathscr{P}_1\right] = 0. \tag{15}$$

Moreover,

$$\begin{gathered} \beta\prime\mathscr{P}_1 = 0, \\ \beta'\mathscr{F}_1\mathscr{P}_1 = 0 = \lambda_2\beta'\mathscr{P}_1, \ldots, \beta'\mathscr{F}_1^{m-1}\mathscr{P}_1 = 0. \end{gathered} \tag{16}$$

We can immediately know that $\beta'[\lambda_2 I - \mathscr{F}_1, \mathscr{P}_1] = 0$ for $\beta \neq 0$ and $\lambda_2 \in \mathscr{C}$; then, $\operatorname{rank}(\lambda_2 I - \mathscr{F}_1, \mathscr{P}_1) < (h+1)m$, which clashes with the fact that $\operatorname{rank}(sI - \mathscr{F}_1, \mathscr{P}_1) = (h+1)m$ for $\forall s \in \mathscr{C}$. The sufficiency of condition (1) is proved.

Notice that it is also fussy to compute $(\mathscr{F}_1, \mathscr{P}_1)$ and $(\mathscr{F}_2, \mathscr{P}_2)$ with higher dimensions. According to the previous analysis, it can be seen that when the dimension of the multiagent system is too high, it is inconvenient to adopt the classical method in control theory to study the system. Moreover, the system matrices are not Laplacian matrices; hence, it is complex and difficult to investigate the group controllability of discrete-time time-delayed MASs. In the following, we will give more simple and easier methods to judge the group controllability of such MAS.

Theorem 3. *System (6) attains group controllability iff* $Y_i = [L_{ii} + \lambda^h(\lambda E - F_i), -L_{ij}, P_i]$ *has full row rank at each root of* $\det(L_{ii} + \lambda^h(\lambda E - F_i)) = 0$ *for* $i, j = 1, 2$, *with* $i \neq j$.

proof. Based on Theorem 2, we can see that system (6) attains group controllability iff

$$[\lambda E - \mathscr{F}_1, \mathscr{P}_1] = \begin{bmatrix} \lambda E - F_1 & 0 & 0 & \cdots & 0 & L_{11} & -L_{12} & \cdots & P_1 \\ -E & \lambda E & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & -E & \lambda E & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & \\ 0 & 0 & 0 & \ddots & \lambda E & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & -E & \lambda E & 0 & \cdots & 0 \end{bmatrix}, \tag{17}$$

has full row rank for each $\lambda \in \mathscr{C}$. Performing elementary transformation on rows of matrix (17), then

$$\begin{bmatrix} -E & \lambda E & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & -E & \lambda E & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & -E & \ddots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & -E & \lambda E & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & L_{11} + \lambda^h(\lambda E - F_1) & -L_{12} & \cdots & P_1 \end{bmatrix}. \tag{18}$$

Clearly, matrices (17) and (18) have the same rank. If matrix (18) has full row rank, then $Y_1 = [L_{11} + \lambda^h(\lambda E - F_1), -L_{12}, P_1]$ also has full row rank, i.e., $\operatorname{rank}(Y_1) = m$ for $\lambda \in \mathscr{C}$. Similarly, we can prove case $i = 2$.

Remark 5. Theorems 1 and 2 provide traditionally important and simple methods to check the group controllability of discrete-time time-delayed MASs with multiple leaders by rank test and PBH test, respectively. However, for system (6), since the group controllability is determined by the system matrix pairs $(\mathscr{F}_1, \mathscr{P}_1)$ and $(\mathscr{F}_2, \mathscr{P}_2)$, it is also fussy to check the group controllability due to the complexity and high dimension of the system matrix pairs $(\mathscr{F}_1, \mathscr{P}_1)$ and $(\mathscr{F}_2, \mathscr{P}_2)$ from Theorem 1 or Theorem 2. Nevertheless, through the analytical process of Theorem 2, we can find that the group controllability of the system matrix pairs $(\mathscr{F}_1, \mathscr{P}_1)$ and $(\mathscr{F}_2, \mathscr{P}_2)$ is completely relied on matrix $Y_i = [L_{ii} + \lambda^h(\lambda E - F_i), -L_{ij}, P_i]$ for $i, j = 1, 2$, with $i \neq j$. Corollaries 1–4, respectively, discussed some special cases to satisfy the condition that matrix $Y_i = [L_{ii} + \lambda^h(\lambda E - F_i), -L_{ij}, P_i]$ has full row rank in the following, which makes it be more concise and straightforward in checking and testing the group controllability of such MAS, which only involves system structure itself regardless of the time-delay value and system dimension.

Corollary 1. *System (6) attains group controllability if* $\det(L_{ii} + \lambda^h(\lambda E - F_i)) \neq 0$ *for* $i = 1, 2$.

Corollary 2. *System (6) attains group controllability if* L_{ij} *has full row rank for* $i, j = 1, 2$, *with* $i \neq j$.

Corollary 3. *System (6) attains group controllability if* P_i $(i = 1, 2)$ *has full row rank.*

Corollary 4. *The roots of* $\det([-L_i + \lambda^h F_i - \lambda^{h+1}E]) = 0$ *are some of the eigenvalues of* $\mathscr{F}_i$ *for* $i = 1, 2$.

4. Example and Simulations

In this work, a discrete-time time-delayed model with multiple leaders and several subgroups is established, the group controllability is analyzed theoretically, and group controllable conditions are given. Numerical examples and simulations are used to verify the correctness of the theoretical analysis. Therefore, in numerical examples and simulations, as long as the topology structure and edge weights satisfy the group controllable conditions, there is no need to consider too much data selection. In the following, we will give an example and some simulations.

For a seven-agent network with two subgroups and $h = 1$, it is shown as Figure 1. For Figure 1, system (6) is defined by

$$F_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$L_{11} = \begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & -1 \\ 0 & -1 & 1 \end{bmatrix},$$
$$L_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$P_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{19}$$
$$F_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},$$
$$L_{22} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix},$$
$$L_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

From Theorem 3, we can have $\det(L_{11} + \lambda(\lambda E - F_1)) = \lambda^6 - 2\lambda^5 + 7\lambda^4 - 7\lambda^3 + 7\lambda^2 - 4\lambda = 0$ and $\det(L_{22} + \lambda(\lambda E - F_2)) = \lambda^4 - \lambda^3 + 2\lambda^2 - \lambda = 0$, whose roots are, respectively, $\{0, 0.4786 + 2.1100i, 0.4786–2.1100i, 0.1366 + 1.0448i, 0.1366–1.0448i, 0.7695\}$ and $\{0, 0.2151 + 1.3071i, 0.2151–1.3071i, 0.5698\}$.

For $\lambda_1 = 0$,

$$Y_{1(\lambda_1)} = [L_{11} + \lambda_1(\lambda_1 E - F_1), -L_{12}, P_1] = \begin{bmatrix} 2 & -2 & 0 & -1 & 0 & 0 \\ -2 & 3 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{bmatrix}, \tag{20}$$

and by calculating, rank $(Y_{1(\lambda_1)}) = 3$.

Analogously, for $\lambda_2 = 0.4786 + 2.1100i$,

$$Y_{1(\lambda_2)} = [L_{11} + \lambda_2(\lambda_2 E - F_1), -L_{12}, P_1] = \begin{bmatrix} -2.7016 - 0.0903i & -2 & 0 & -1 & 0 & 0 \\ -2 & -1.7016 - 0.0903i & -1 & 0 & 0 & 0 \\ 0 & -1 & -3.2230 + 2.0197i & 0 & 0 & 1 \end{bmatrix}, \tag{21}$$

and by calculating, rank $(Y_{1(\lambda_2)}) = 3$.

For $\lambda_3 = 0.4786 - 2.1100i$,

$$Y_{1(\lambda_3)} = [L_{11} + \lambda_3(\lambda_3 E - F_1), -L_{12}, P_1] = \begin{bmatrix} -2.7016 + 0.0903i & -2 & 0 & -1 & 0 & 0 \\ -2 & -1.7016 + 0.0903i & -1 & 0 & 0 & 0 \\ 0 & -1 & -3.2230 - 2.0197i & 0 & 0 & 1 \end{bmatrix}, \tag{22}$$

and by calculating, rank $(Y_{(1\lambda_3)}) = 3$.

For $\lambda_4 = 0.1366 + 1.0448i$,

$$Y_{1(\lambda_4)} = [L_{11} + \lambda_4(\lambda_4 E - F_1), -L_{12}, P_1] = \begin{bmatrix} 0.7905 - 0.7594i & -2 & 0 & -1 & 0 & 0 \\ -2 & 1.7905 - 0.7594i & -1 & 0 & 0 & 0 \\ 0 & -1 & -0.0729 + 0.2854i & 0 & 0 & 1 \end{bmatrix}, \tag{23}$$

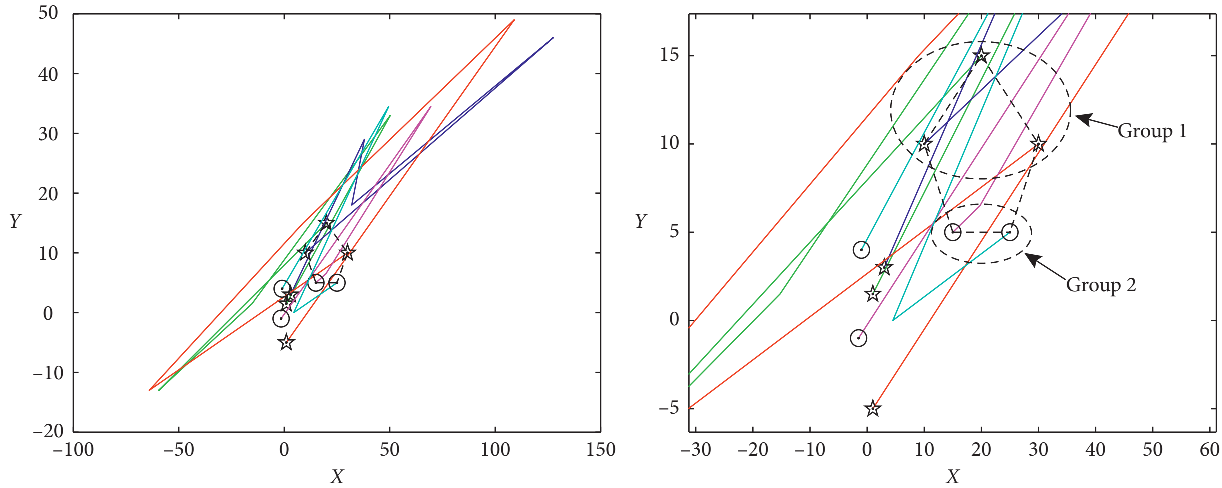

FIGURE 2: A regular pentagon array and its local magnification portion.

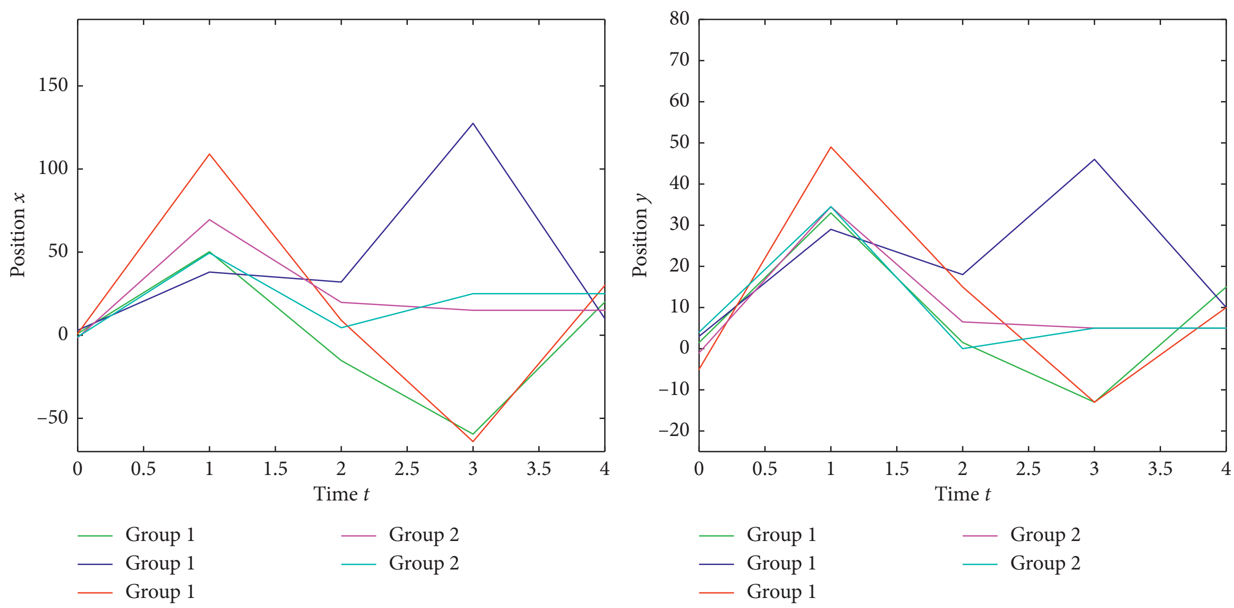

FIGURE 3: The time-evolution trajectories for the x−axis and y−axis corresponding to Figure 2.

and by calculating, rank $(Y_{1(\lambda_4)}) = 3$.

For $\lambda_5 = 0.1366 - 1.0448i$,

$$\begin{aligned} Y_{1(\lambda_5)} &= [L_{11} + \lambda_5 (\lambda_5 E - F_1), -L_{12}, P_1] \\ &= \begin{bmatrix} 0.7905 + 0.7594i & -2 & 0 & -1 & 0 & 0 \\ -2 & 1.7905 + 0.7594i & -1 & 0 & 0 & 0 \\ 0 & -1 & -0.0729 - 0.2854i & 0 & 0 & 1 \end{bmatrix}, \end{aligned} \tag{24}$$

and by calculating, rank $(Y_{1(\lambda_5)}) = 3$.

For $\lambda_6 = 0.7695$,

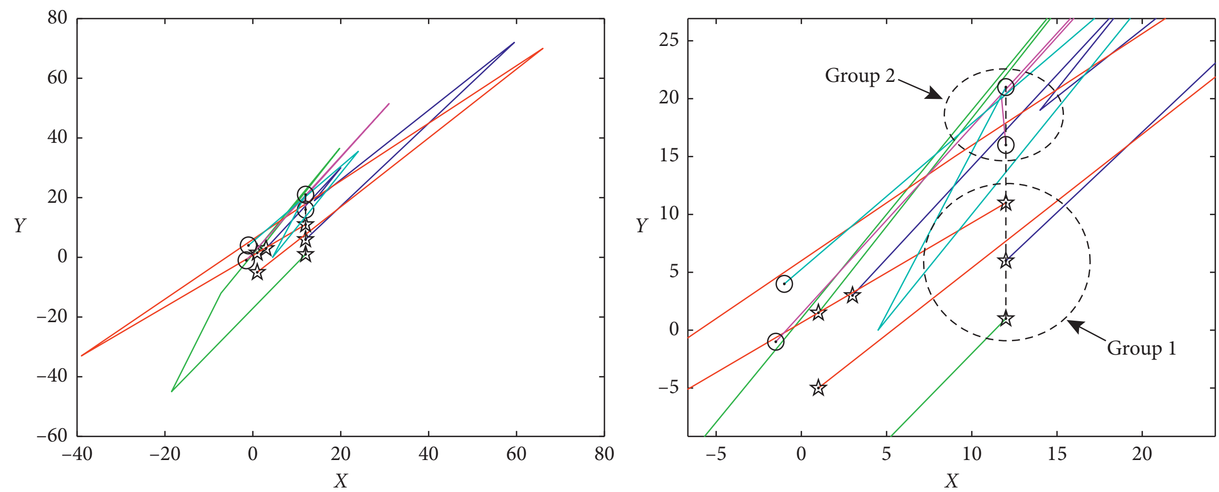

Figure 4: A straight line array and its local magnification portion.

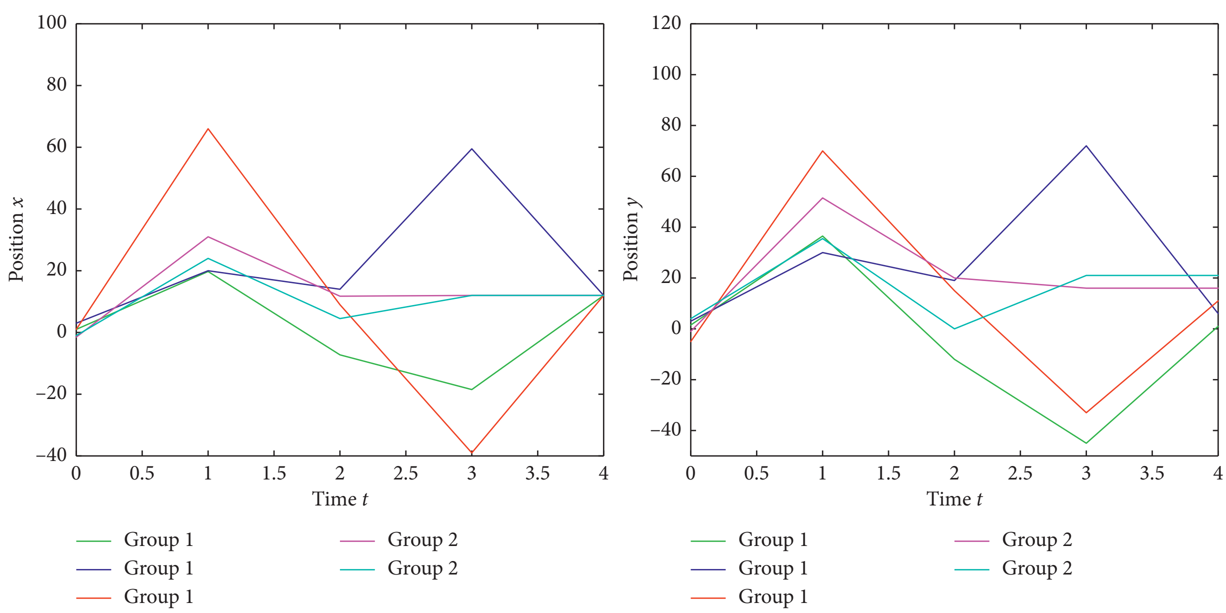

Figure 5: The time-evolution trajectories for the x−axis and y−axis corresponding to Figure 4.

$$Y_{1(\lambda_6)} = [L_{11} + \lambda_6(\lambda_6 E - F_1), -L_{12}, P_1] = \begin{bmatrix} 1.8226 & -2 & 0 & -1 & 0 & 0 \\ -2.0000 & 2.8226 & -1 & 0 & 0 & 0 \\ 0 & -1 & 1.5921 & 0 & 0 & 1 \end{bmatrix}, \quad (25)$$

and by calculating, rank $(Y_{1(\lambda_6)}) = 3$.

For $\delta_1 = 0$,

$$Y_{2(\delta_1)} = [L_{22} + \lambda_7(\lambda_7 E - F_2), -L_{21}, P_2] = \begin{bmatrix} 1 & -1 & -1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (26)$$

and by calculating, rank $(Y_{2(\delta_1)}) = 2$.

For $\delta_2 = 0.2151 + 1.3071i$,

$$Y_{2(\delta_2)} = [L_{22} + \lambda_8(\lambda_8 E - F_2), -L_{21}, P_2] = \begin{bmatrix} -0.8773 - 0.7448i & -1 & -1 & 0 & 0 & 0 \\ -1 & -0.6622 + 0.5623i & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (27)$$

and by calculating, rank $(Y_{(2\delta_2)}) = 2$; for $\delta_3 = 0.2151 - 1.3071i$,

$$Y_{2(\delta_3)} = [L_{22} + \lambda_9(\delta_3 E - F_2), -L_{21}, P_2] = \begin{bmatrix} -0.8773 + 0.7448i & -1 & -1 & 0 & 0 & 0 \\ -1 & -0.6622 - 0.5623i & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (28)$$

and by calculating, rank $(Y_{2(\delta_3)}) = 2$; and for $\delta_4 = 0.5698$,

$$Y_{2(\delta_4)} = [L_{22} + \lambda_{10}(\delta_4 E - F_2), -L_{21}, P_2] = \begin{bmatrix} 0.7549 & -1 & -1 & 0 & 0 & 0 \\ -1 & 1.3247 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad (29)$$

and by calculating, rank $(Y_{2(\delta_4)}) = 2$.

Obviously, $Y_i = [L_{ii} + \lambda^{(\lambda E - F_i)}, -L_{ij}, P_i]$ has full row rank at each root of $\det(L_{ii} + \lambda(\lambda E - F_i)) = 0$ for $i, j = 1, 2$, with $i \neq j$; therefore, system (6) can attain group controllability according to Theorem 3.

Figure 2 shows the followers' movement trajectories from any initial state to a regular pentagon array (corresponding to its local magnification portion), where red, green, and blue stars are the followers in group 1 and pink and light blue circles are the followers in group 2, respectively. Figure 3 shows the followers' time-evolution trajectories for the x−axis and y−axis corresponding to Figure 2, respectively. Figure 4 shows the followers' movement trajectories from any initial state to a straight line array (corresponding to its local magnification portion), where red, green, and blue stars are the followers in group 1 and pink and light blue circles are the followers in group 2, respectively. Figure 5 shows the followers' time-evolution trajectories for the x−axis and y−axis corresponding to Figure 4, respectively.

5. Conclusion

Inspired by leaderless MASs, the model of a discrete-time time-delayed MAS with multiple leaders has been established, the concept of the group controllability has been proposed, and the criteria of the group controllability for such MAS have been obtained. This work has studied the group controllability problem of discrete-time time-delayed multiagent systems with multiple leaders, which can be expanded to more complex cases, such as MASs with the two-time-scale feature, saturation constraints, or signed networks. These directions will be the key and difficult problems in future research.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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