Event-Based Consensus for General Linear Multiagent Systems under Switching Topologies

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Received 14 December 2019; Revised 21 January 2020; Accepted 3 February 2020; Published 28 March 2020

1. Introduction

In recent years, multi-agent systems have great potential of application in the fields of biology, engineering, physics, society and so on. Also it distributed cooperative control [1, 2] has attracted more and more researchers’ attention. For example, the controllability [3–11] and the consensus problem [12–19] are widely studied in multi-agent systems. Among them, as the basic problem of multi-agent cooperative control, consensus is widely used in formation control [20], cluster control, sensor network and other aspects, which is a research hot issue in the control discipline at present.

In practical applications, the information needed for cooperative control among agents is transmitted through the network. It is necessary to design a reasonable controller to ensure the control performance of the system due to the finite energy of the agent and the limited network bandwidth. It is well known that periodic sampling control [21–23] can save resources, but when the system runs in an ideal environment or the system state tends to be consensus gradually, it will cause unnecessary resource waste if the control task is executed periodically. In order to reduce this unnecessary waste of resources, a new simple event-triggered control strategy based on feedback mechanism was proposed in [24]. In short, the event-triggered control strategy means that the control task is executed as required. On the premise of ensuring the closed-loop system has certain performance, only can the task be executed once when a specific event occurs (such as the state error exceeds the preset threshold value). The advantage of event-triggered control strategies is that it can not only guarantee the performance of the system but also save the network and computing resources. At present, event-triggered mechanism has been applied to the research of consensus for multiagent systems effectively. For example, Dimarogonas and Frazoli and Dimarogonas et al. [25, 26] studied the consensus of a first-order multiagent system in undirected topology, and Yan et al. [27] investigated the consensus of a second-order multiagent system based on event-triggered mechanism in directed topology. Event-triggering conditions based on composite measurements were
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2. Preliminaries

2.1. Theory of Graph. For a multiagent system composed of a leader and follower agents, its communication topology can be represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{0, 1, \ldots, N\}$. $0$ denotes the leader, and $1, \ldots, N$ denote the followers. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the edges set. The connection matrix between the follower agent $i(i = 1, \ldots, N)$ and leader 0 is $D = \text{diag}(d_{10}, \ldots, d_{N0})$, where $d_{i0}$ is the connection weight between the leader 0 and follower $i$. If $a_{i0} > 0$, the follower agent $i$ can receive state information of leader 0; otherwise, $a_{i0} = 0$.

The communication network among the followers is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$, where $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ are obtained from $\mathcal{G}$ by removing all edges among the leader 0 and followers in $\mathcal{G}$, and $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ is the weighted adjacency matrix of graph $\mathcal{G}$, where $a_{ij} > 0$ for $(i, j) \in \mathcal{E}$ if agent $i$ obtains information from agent $j$. We assume that $(i, i) \notin \mathcal{E}$, and hence $a_{ii} = 0$. For a given graph $\mathcal{G}$ with the adjacency matrix $A$, the Laplacian matrix used in this paper is $L = D - A$, where $D$ is a diagonal matrix, its diagonal elements are $d_{ii} = \sum_{j \neq i} a_{ij}$, and define $N_i$ as the neighbor set of agent $i$ in $\mathcal{G}$. Therefore, the elements of $L$ are

$$L_{ik} = \begin{cases} \sum_{j \in N_i} a_{ij}, & k = i, \\ -a_{ik}, & k \neq i. \end{cases}$$

A path from the vertex $i$ to vertex $k$ is a sequence of adjacent edges in the form $(i, i + 1), (i + 1, i + 2), \ldots, (k - 1, k)$. The undirected graph is said to be connected if there exists a path between any two distinct vertices.

2.2. System Model. Consider a multiagent system composed of the leader 0 and $N$ followers. The dynamics of leader 0 is

$$x_0(t) = Ax_0(t),$$

where $x_0(t) \in \mathbb{R}^n$ is the state and $A \in \mathbb{R}^{n \times n}$ is constant matrix.

Accordingly, each follower has the following linear dynamic equation:

$$x_i(t) = Ax_i(t) + Bu_i(t), \quad i = 1, \ldots, N,$$

where $x_i(t) \in \mathbb{R}^n$ and $u_i(t) \in \mathbb{R}^p$ are the state and input of the $i$th follower agent, respectively. $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times p}$ are constant matrices. Denote the initial state of the $i$th follower as $x_i(0)$.

**Definition 1.** If there is a control input $u_i(t)$, the leader 0 and follower $i$ for any initial state satisfy the following conditions:

$$\lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0, \quad i = 1, 2, \ldots, N.$$

Then, the leader (2) is said to be successfully tracked by follower (3).

**Assumption 1.** The communication network topology $\mathcal{G}$ among followers is connected.
Assumption 2. The pair $(A, B)$ is stabilizable.

Based on Assumption 2, there is a symmetric positive definite matrix $P$ that satisfies the following algebraic Riccati and Lyapunov inequality with $\beta > 0$:

$$A^T P + PA - 2\beta PBB^T P + \beta I < 0,$$

$$A^T P + PA < 0.$$  (5)  (6)

Lemma 1 (see [37]). For an undirected and connected graph $\mathcal{G}$, the eigenvalues of $L$ are real and can be labelled as

$$0 = \lambda_1(L) < \lambda_2(L) \leq \cdots \leq \lambda_N(L).$$  (7)

Lemma 2 (see [40]). For any $x, y \in \mathbb{R}$ and $\beta > 0$, one has the following property:

$$xy \leq \frac{\beta}{2}x^2 + \frac{1}{2\beta}y^2.$$  (8)

Lemma 3 (see [41]). (Comparison Principle). Consider a differential equation $\dot{x}(t) = f(t, x)$, where $t > 0$, $f(t, x)$ is continuous and satisfies the local Lipschitz condition in $t$. Let $[t_0, T]$ be the maximum existence interval of the solution $x$, where $T$ can be infinite. If, for any $t \in [t_0, T)$ satisfies

$$\frac{dv}{dt} \leq f(t, v),$$

$$v(t_0) \leq u_0,$$  (9)

then $v(t) \leq u(t), t \in [t_0, T]$.

3. Leader-Following Control of Multiagent Systems under Fixed Topology

In this part, we consider the leader-following control of multiagent systems (2) and (3) under the event-triggered strategy. Based on the general event-triggered control law, we put forward two kinds of piecewise continuous control mechanisms, which are centralized event-triggered mechanism and decentralized event-triggered mechanism with state estimation in order to minimize the frequency of controller updating. The analysis shows that under the two control mechanisms, multiagent system (3) can track the system (2) successfully with appropriate event-triggering function. The minimum interval between any two consecutive event-triggering instants under the two control mechanisms is greater than 0, and Zeno behavior can be excluded.

3.1. Centralized Event-Triggered Control Strategy. Under the centralized event-triggered strategy, all agents $i$ in system (3) are triggered synchronously at the time $t_k$ ($k = 0, 1, \ldots$). At the triggering instants, all agents send their states information to neighbours and update the control law with the received state information. Compared with the control protocol in continuous time, each agent $i$ only updates the control input at the event instants under the event-triggered mechanism. So, $u_i$ is a piecewise continuous function, and the updating frequency can be reduced.

We consider the following control input for the $i$th follower:

$$u_i(t) = -K \sum_{j \in N(i)} a_{ij}(t)(x_i(t_k) - x_j(t_k))$$

$$- Ka_{i0}(t)(x_i(t_k) - x_0(t_k)),$$  (10)

where $t \in [t_k, t_{k+1})$, $K \in \mathbb{R}^{p\times n}$ is the control gain matrix to be designed, and $x_i(t_k)$ is the sampling state of agent $i$ at the $k$th triggering instant. Since there does not exist control input for leader 0, we take $x_0(t_k) = x_0(t), t \in [t_k, t_{k+1})$. For convenience, we make $t_0 = 0$.

The event-triggering time sequence $\{t_k\}$ is determined by the following triggering functions:

$$f(t) = -K a_{0i0} \lambda_{\min}(W) - \alpha \lambda_{\max}(P) + \lambda_{\max}(D) \lambda_{\max}(W) \sum_{i=1}^N x_i^T \bar{x}_i$$

$$+ \sum_{i=1}^N e_i^T e_i \geq 0.$$  (11)

That is, $t_{k+1} = \inf\{t > t_k \mid f(t) \geq 0\}$, where $0 < k < 1, 0 < \alpha < (a_{0i0} \lambda_{\min}(W)/\lambda_{\max}(P)), W = PBB^T P$.

The state error between the follower and leader is defined as $x_i(t) = x_i(t) - x_0(t), \bar{x}_i = \bar{x}_i = [\bar{x}_i^T, \ldots, \bar{x}_i^T]^T$. For agent $i$, the measurement error is defined as $e_i(t) = x_i(t_k) - x_i(t), t \in [t_k, t_{k+1})$. Then, formula (10) is converted to

$$u_i(t) = -K \sum_{j=0}^N a_{ij}(t) (\bar{x}_i(t) - \bar{x}_j(t)) + e_i(t) - e_j(t)$$

$$- Ka_{i0}(t)(\bar{x}_i(t) + e_i(t)).$$  (12)

Combining (2), (3), and (12), we get

$$\dot{x}(t) = (I_N \otimes A) \bar{x}(t) - (I_N \otimes BK)((L + D) \otimes I_n)(\bar{x}(t) + e(t))$$

$$= (I_N \otimes A) \bar{x}(t) - ((L + D) \otimes BK)(\bar{x}(t) + e(t)).$$  (13)

Remark 1. Through the model transformation, the leader-following control problem between systems (2) and (3) can be interpreted by the stability problem of system (13).

Next, we will give the following consensus conditions under the centralized event-triggering protocol (10).

Theorem 1. Under Assumptions 1 and 2, centralized event-triggered control strategy (10) can make multiagent system (3) track system (2) successfully under event-triggering condition (4), where feedback gain matrix $K$ satisfies $K = B^T P$ and $W = PBK$. 

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Proof. We consider a candidate Lyapunov function, as follows:

\[ V_1 = e^{at} \sum_{i=1}^{N} \bar{x}_i^T P \bar{x}_i. \]  

(14)

Along with the trajectories of the state as described in (8), the time derivative of Lyapunov function is

\[
\begin{align*}
\dot{V}_1 &= ae^{at} \sum_{i=1}^{N} \bar{x}_i^T P \bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T P \dot{\bar{x}}_i \\
&= e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP \bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T P (A \bar{x}_i + B u_i) \\
&= e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP \bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T P A \bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T P Bu_i, \\
\end{align*}
\]

(15)

where

\[
\begin{align*}
e^{at} \sum_{i=1}^{N} \bar{x}_i^T P Bu_i
&= -e^{at} \sum_{i=1}^{N} \bar{x}_i^T P BK \left( a_{i0} (\bar{x}_i + e_i) + \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j + e_j - e_j) \right) \\
&= -e^{at} \sum_{i=1}^{N} \bar{x}_i^T W a_{i0} (\bar{x}_i + e_i) - e^{at} \sum_{i=1}^{N} \bar{x}_i^T W \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j) \\
&\quad - e^{at} \sum_{i=1}^{N} \bar{x}_i^T W \sum_{j=1}^{N} a_{ij} (e_j - e_j),
\end{align*}
\]

(16)

where \( W = PBK \).

According to the property of \( L = L^T \) in undirected graph \( \mathcal{G} \), we can deduce

\[
\begin{align*}
e^{at} \sum_{i=1}^{N} \bar{x}_i^T W \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (\bar{x}_i - \bar{x}_j) \\
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (\bar{x}_j - \bar{x}_i) \\
&= -e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (\bar{x}_i - \bar{x}_j) \\
&= -e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (\bar{x}_i - \bar{x}_j).
\end{align*}
\]

(17)

Similarly,

\[
\begin{align*}
e^{at} \sum_{i=1}^{N} \bar{x}_i^T W \sum_{j=1}^{N} a_{ij} (e_j - e_j)
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (e_j - e_j) \\
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (e_j - e_j) \\
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (\bar{x}_j - \bar{x}_i) \\
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (\bar{x}_i - \bar{x}_j) \\
&= e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (\bar{x}_i - \bar{x}_j).
\end{align*}
\]

Hence,

\[
\begin{align*}
e^{at} \sum_{i=1}^{N} \bar{x}_i^T P Bu_i
&= -e^{at} \sum_{i=1}^{N} \bar{x}_i^T W a_{i0} (\bar{x}_i + e_i) - e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (\bar{x}_i - \bar{x}_j) \\
&\quad - e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (\bar{x}_i - \bar{x}_j) \\
&\quad - e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (e_j - e_j). \\
\end{align*}
\]

(19)

Combining equality (15) yields

\[
\begin{align*}
\dot{V}_1 &= e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP \bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T P A \bar{x}_i \\
&\quad - 2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W (\bar{x}_i - \bar{x}_j) \\
&\quad - e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (e_j - e_j) \\
&\quad - 2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} \bar{x}_i^T W a_{i0} (\bar{x}_i + e_i).
\end{align*}
\]

(20)

In the light of Lemma 2, we have

\[
\begin{align*}
&\quad - e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (e_i - e_j) \\
&\leq 1/2 e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\bar{x}_i - \bar{x}_j)^T W (\bar{x}_i - \bar{x}_j) + 1/2 e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (e_i - e_j)^T W (e_i - e_j).
\end{align*}
\]

(21)

By substituting the abovementioned formula into equation (20), we obtain
\[
\dot{V}_1 \leq e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP\bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T PA\bar{x}_i \\
-2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\bar{x}_i^T W(\bar{x}_i - \bar{x}_j) \\
+\frac{e^{at}}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(\bar{x}_i - \bar{x}_j)^T W(\bar{x}_i - \bar{x}_j) \\
+\frac{e^{at}}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(e_i - e_j)^T W(e_i - e_j) \\
-2e^{at} \sum_{i=1}^{N} \bar{x}_i^T W a_{i0}(\bar{x}_i + e_i) \\
\leq e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP\bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T PA\bar{x}_i \\
-2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\bar{x}_i^T W(\bar{x}_i - \bar{x}_j) \\
+\frac{e^{at}}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(e_i - e_j)^T W(e_i - e_j) \\
-2e^{at} \sum_{i=1}^{N} \bar{x}_i^T W a_{i0}(\bar{x}_i + e_i). 
\]

From Lemma 2, we have
\[
\frac{e^{at}}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}(e_i - e_j)^T W(e_i - e_j) \leq 2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T W e_i. 
\]

Together with (22), we can get that
\[
\dot{V}_1 \leq e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP\bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T PA\bar{x}_i \\
- e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\bar{x}_i^T W(\bar{x}_i - \bar{x}_j) \\
+ 2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T W e_i. 
\]

Combining (24) and (25), we arrive at
\[
\dot{V}_1 \leq e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP\bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T PA\bar{x}_i \\
- e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\bar{x}_i^T W(\bar{x}_i - \bar{x}_j) \\
- e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\bar{x}_i^T W(\bar{x}_i - \bar{x}_j) \\
+ 2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T W e_i. 
\]

Under Assumption 1, by using Lemma 1, \( \bar{x}^T (L \otimes W) \bar{x} \geq \lambda_2 (L) \bar{x} \), \( \bar{x}^T (I_N \otimes aP) \bar{x} \), \( \bar{x}^T (I_N \otimes (PA + AT P)) \bar{x} \), \( \bar{x}^T (L \otimes W) \bar{x} \), \( \bar{x}^T (D \otimes W) \bar{x} \), \( \bar{x}^T (D \otimes W) e \), \( 2e^{at} e^T (D \otimes W) e \).

Using inequality (5) and event-triggering condition (11), we claim that the following inequality holds:
\[
\dot{V}_1 \leq e^{at} \sum_{i=1}^{N} \bar{x}_i^T aP\bar{x}_i + 2e^{at} \sum_{i=1}^{N} \bar{x}_i^T PA\bar{x}_i \\
- e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij}\bar{x}_i^T W(\bar{x}_i - \bar{x}_j) \\
+ 2e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T W e_i. 
\]

Using inequality (5) and event-triggering condition (11), we claim that the following inequality holds:
\[ V_1 \leq (\kappa - 1)e^{at}(a_{\text{max}}\lambda_{\text{min}}(W) - \alpha\lambda_{\text{max}}(P))\|\bar{x}\|^2 \]

\[ -e^{at}\frac{\lambda_2(L)}{2}\bar{x}^T\bar{x} \leq -e^{at}\frac{\lambda_2(L)}{2} \bar{x}. \]  

(28)

It can be seen from (28) that \( V_1 \) is not increasing; therefore,

\[ V_1(0) \geq V_1(t) = e^{at}\sum_{i=1}^{N} \bar{x}_i(t)^T P \bar{x}_i(i) \geq e^{at}\lambda_{\text{min}}(P)\|\bar{x}(t)\|^2. \]

(29)

That is to say, \( \|\bar{x}(t)\| \leq \sqrt{(V_1(0)/\lambda_{\text{min}}(P))e^{-at})} \), i.e.,

\[ \lim_{t \to \infty} \|\bar{x}(t)\| = 0 \]  

is equivalent to \( \lim_{t \to \infty} \|\bar{x}(t)\| = 0 \), which means

\[ \lim_{t \to \infty} \|\bar{x}(t) - x_0(t)\| = 0, i = 1, 2, \ldots, N \]

holds.

\[ \square \]

**Theorem 2.** Under the conditions of Theorem 1, system (13) does not exhibit Zeno behavior. The interval between any two consecutive event-triggering instants of the system is not less than

\[ \sqrt{(e^{at}a_{\text{max}}\lambda_{\text{min}}(W) - \alpha\lambda_{\text{max}}(P))/(\alpha a_{\text{max}}(W) + 2\lambda_{\text{max}}(D)\lambda_{\text{max}}(W))}. \]

The time derivative of \( \|e(t)\|/\|\bar{x}(t)\| \) has

\[ \begin{align*}
\frac{d}{dt} \frac{\|e(t)\|}{\|\bar{x}(t)\|} &= \frac{d}{dt} \frac{(e(t)^T e(t))^{1/2}}{(\bar{x}(t)^T \bar{x}(t))^{1/2}} \\
&= \frac{(e(t)^T e(t))^{1/2} (\|\bar{x}(t)\| - (e(t)^T e(t))^{1/2} (\bar{x}(t)^T \bar{x}(t))^{1/2})}{\|\bar{x}(t)\|^2} \\
&= \frac{e(t)^T \dot{e}(t)}{\|\bar{x}(t)\| e(t)} - \frac{e(t)^T \ddot{x}(t)}{\|\bar{x}(t)\|^2} \\
&= \frac{-e(t)^T \dot{\bar{x}}(t)}{\|\bar{x}(t)\| e(t)} - \frac{e(t)^T \ddot{\bar{x}}(t)}{\|\bar{x}(t)\|^2} \\
&\leq \frac{\|\ddot{\bar{x}}(t)\|}{\|\bar{x}(t)\|^2} + \frac{\|\dot{\bar{x}}(t)\|}{\|\bar{x}(t)\|^2} = \|\ddot{\bar{x}}(t)\| \left( 1 + \frac{\|e(t)\|}{\|\bar{x}(t)\|} \right) \\
&\leq \left( \|I_N \otimes A\| + \|(L + D) \otimes BK\| \right) \left( 1 + \frac{\|e(t)\|}{\|\bar{x}(t)\|} \right) \\
&+ \left( \|(L + D) \otimes BK\| \|e(t)\| \right) \left( 1 + \frac{\|e(t)\|}{\|\bar{x}(t)\|} \right) \\
&\leq \left( \|I_N \otimes A\| + \|(L + D) \otimes BK\| \right) \left( 1 + \frac{\|e(t)\|}{\|\bar{x}(t)\|} \right) \\
&\leq \left( \|I_N \otimes A\| + \|(L + D) \otimes BK\| \right) \left( 1 + \frac{\|e(t)\|}{\|\bar{x}(t)\|} \right)^2.
\end{align*} \]
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Denote $z = (||e(t)||/\|\tilde{x}(t)||)$, then
\[
\dot{z} \leq \left( ||I_N \otimes A|| + ||(L + \mathcal{D}) \otimes BK|| \right) (1 + z)^2. \tag{32}
\]

Consider that a nonnegative function $\psi(t, \psi_0)$ satisfies $\psi = (||I_N \otimes A|| + ||(L + \mathcal{D}) \otimes BK||) (1 + \psi)^2$, and $\psi_0 = 0$. Then, from Lemma 3, $z \leq \psi(t, 0)$. It can be seen from (11) that
\[
\tau = \left( \frac{||I_N \otimes A|| + ||(L + \mathcal{D}) \otimes BK||}{3} \right) \left( (1 + \psi(t))^3 - 1 \right)
\]
\[
= \left( \frac{||I_N \otimes A|| + ||(L + \mathcal{D}) \otimes BK||}{3} \right) \left( 1 + \frac{\kappa(\lambda_{\min}(W) - \lambda_{\min}(P))}{\lambda_{\max}(W) + 2\lambda_{\max}(D)\lambda_{\max}(W)} \right)^3 - 1 \right). \tag{33}
\]

Obviously, $\tau > 0$.

It is assumed that the Zeno behavior occurs, which means that there exists a positive constant $t^*$ such that $\lim_{\tau \to \infty} t_k = t^*$. Let $\epsilon_0 = (1/2)\tau$. There exists a positive integer $N_0$ such that $t^* - \epsilon_0 \leq t_k \leq t^* + \epsilon_0$ for the abovementioned $\epsilon_0 > 0$ according to the definition of sequence limit, where $k \geq N_0$. Therefore, $t^* + \epsilon_0 \leq t_k + 2\epsilon_0 \leq t_{k+1}$ holds when $k \geq N_0$. This contradicts with $t^* \geq t_{k+1}$ for $k \geq N_0$. Thus, Zeno behavior is strictly excluded. \qed

3.2. Decentralized Event-Triggered Control Strategy. The centralized event-triggered mechanism given in the previous section sets a global state error threshold for all agents. Once the system error reaches the threshold, all agents in the system perform control tasks at the same time. In this section, an error threshold based on the state of its neighbor node is set for each agent. When the state error of the agent reaches the set threshold, the agent triggers the event independently and executes the control task.

The triggering time of the $k$th event of the $i$ agent is defined as $t_{k}^i = (k = 0, 1, \ldots)$. In the design of this section, it should be noted that the agent triggers asynchronously, that is, each agent has its own event-triggering sequence. The measurement error of agent $i$ is defined as $e_i(t) = x_i(t) - x_i(t)$, $t \in [t_k^i, t_{k+1}^i]$. It is clear that $e_i(t_k^i) = 0$ when $t = t_k^i$.

For a multiagent system composed of (2) and (3), we consider the following decentralized event-triggered control protocol:
\[
u_i(t) = -K \sum_{j \in N_i(t)} a_{ij} \left( x_j(t_{k}^j) - x_i(t_{k}^j) \right) - Ka_{0i} \left( x_i(t_{k}^i) - x_0(t) \right), \tag{35}\]
where $t \in [t_k^i, t_{k+1}^i]$, $t_k^i = \arg \min_{t \in \{t_k^i, \ldots, t_{k+1}^i\}} \{t - t_k^i\}$ represents the latest event-triggering time before $t$ for agent $j$. According to (35), agent $i$ will update control input $u_i$ at both its triggering instants ($t_{k}^i, t_{k+1}^i, \ldots$) and neighbor agent $j$ event instants ($t_{k}^j, t_{k+1}^j, \ldots$). The event-triggering instant sequence $\{t_k^i\}$ for agent $i$ is determined by the following decentralized event-triggering function:
\[
\psi(t, 0) = \sqrt{\frac{\kappa(\lambda_{\min}(W) - \lambda_{\min}(P))}{\lambda_{\max}(W) + 2\lambda_{\max}(D)\lambda_{\max}(W)}}. \tag{33}\]

Therefore,
\[
\psi(t, 0) = \frac{\kappa(\lambda_{\min}(W) - \lambda_{\min}(P))}{\lambda_{\max}(W) + 2\lambda_{\max}(D)\lambda_{\max}(W)}.
\]

Theorem 3. Under Assumption 1, the multiagent systems (3) with protocol (35) can track system (2) successfully under the event-triggering condition (36), where $K = B^T P$ and $W = PBB^T P$. 

\[\begin{align*}
\tilde{x}(t) &= (I_N \otimes A)\tilde{x}(t) - ((L + \mathcal{D}) \otimes BK)(\tilde{x}(t) + e(t)). \tag{38}
\end{align*}\n\]
Proof. Define the Lyapunov function

$$V_2 = \frac{1}{2} \sum_{i=1}^{N} \tilde{x}_i^T PA \tilde{x}_i.$$  \hfill (39)

Following the same proof as that of Theorem 1, the time derivation of $V_2$ along the trajectory of system (38) is obtained:

$$\dot{V}_2 \leq \sum_{i=1}^{N} \tilde{x}_i^T PA \tilde{x}_i - \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\tilde{x}_i - \tilde{x}_j)^T W (\tilde{x}_i - \tilde{x}_j)$$

$$- \frac{1}{2} \sum_{i=1}^{N} \tilde{x}_i^T W a_{ii} \tilde{x}_i + \frac{1}{2} \sum_{i=1}^{N} e_i^T W a_{ii} e_i$$

$$+ \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} e_i^T W e_i$$

$$\leq \tilde{x}^T (I_N \otimes PA) \tilde{x} - \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\tilde{x}_i - \tilde{x}_j)^T W (\tilde{x}_i - \tilde{x}_j)$$

$$- \frac{1}{2} \sum_{i=1}^{N} \tilde{x}_i^T W a_{ii} \tilde{x}_i + \frac{1}{2} \sum_{i=1}^{N} e_i^T W a_{ii} e_i$$

$$\leq \tilde{x}^T (I_N \otimes PA) \tilde{x} + \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1}^{N} e_i^T (4a_{ij} + 2a_{ii}) W e_i$$

$$- \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\tilde{x}_i - \tilde{x}_j)^T W (\tilde{x}_i - \tilde{x}_j)$$

$$\leq \tilde{x}^T (I_N \otimes (PA + A^T P)) \tilde{x}$$

$$+ \frac{1}{4} \sum_{i=1}^{N} \lambda_{\max} (W) \|e_i\|^2 \sum_{j=1}^{N} (4a_{ij} + 2a_{ii})$$

$$- \frac{1}{4} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} (\tilde{x}_i - \tilde{x}_j)^T W (\tilde{x}_i - \tilde{x}_j).$$  \hfill (40)

According to (6) and event-triggering condition (36), we can find that

$$\dot{V}_2 \leq \frac{1}{2} \tilde{x}^T (I_N \otimes (PA + A^T P)) \tilde{x}$$

$$- \frac{\kappa}{4} \sum_{j=1}^{N} a_{ij} (\tilde{x}_i - \tilde{x}_j)^T W (\tilde{x}_i - \tilde{x}_j)$$

$$\leq \frac{\kappa}{2} \tilde{x}^T (L \otimes W) \tilde{x}$$

$$\leq \frac{\kappa}{2} \lambda_{\max} (W) \|\tilde{x}\|^2$$

$$\leq 0.$$  \hfill (41)

It can be seen from the abovementioned formula that $V_2$ is not increasing; therefore,

$$V_2 (0) \geq V_2 (t) = \frac{1}{2} \sum_{i=1}^{N} \tilde{x}_i (t)^T P \tilde{x}_i (t) \geq \frac{1}{2} \lambda_{\min} (P) \|\tilde{x} (t)\|^2.$$  \hfill (42)

That is to say, \(\|\tilde{x} (t)\| \leq \sqrt{2} \lambda_{\min} (P) = 0\). According to LaSalle’s invariance principle, we can obtain that system (38) can achieve consensus, that is, \(\lim_{t \to \infty} \tilde{x}_i = 0\), which is equivalent to \(\lim_{t \to \infty} \|x_i (t) - x_0 (t)\| = 0\), \(i = 1, 2, \ldots, N\). The proof is completed. \(\square\)

**Theorem 4.** Under the conditions of Theorem 3, system (38) does not exhibit Zeno behavior. The interval between any two consecutive event-triggering instants of the system is not less than

$$\frac{\|A\| + \|BK ((L_i + a_{ii}) \otimes I_n)\|}{3} \times \left( 1 + \left( 1 - \kappa \right) \frac{a_{ii} \lambda_{\min} (W)}{(2d_i + a_{ii}) \lambda_{\max} (W)} \right)^{1/2} - 1 \right).$$  \hfill (43)

Proof. It is similar to the proof of Theorem 2. The event interval between $t_k$ and $t_{k+1}$ is \(\|e_i (t)\| / \|\tilde{x}_i (t)\|\) which grows from 0 to \((1 - \kappa) (a_{ii} \lambda_{\min} (W) / (2d_i + a_{ii}) \lambda_{\max} (W)))^{1/2}\). The time derivative of \(\|e_i (t)\| / \|\tilde{x}_i (t)\|\) is
\[ \frac{d}{dt} \| e_i \| \leq (\| A \| + \| BK ((L_i + a_{i0}) \otimes I_N) \|) \left( 1 + \frac{\| e_i(t) \|}{\| x_i(t) \|} \right) \]

\[ + \frac{\| BK ((L_i + a_{i0}) \otimes I_N) \||e_i|}{\| x_i(t) \|} \left( 1 + \frac{\| e_i(t) \|}{\| x_i(t) \|} \right) \]

\[ \leq (\| A \| + \| BK ((L_i + a_{i0}) \otimes I_N) \|) \left( 1 + \frac{\| e_i(t) \|}{\| x_i(t) \|} \right) \]

\[ + \frac{\| BK ((L_i + a_{i0}) \otimes I_N) \||e_i|}{\| x_i(t) \|^2} \left( 1 + \frac{\| e_i(t) \|}{\| x_i(t) \|} \right) \]

\[ = (\| A \| + \| BK ((L_i + a_{i0}) \otimes I_N) \|) \left( 1 + \frac{\| e_i(t) \|}{\| x_i(t) \|} \right)^2, \]  

(44)

where \( L_i \) is the row \( i \) of the Laplace matrix \( L \).

Let \( \dot{z}_i = (\| e_i(t) \|/\| x_i(t) \|) \), then

\[ \frac{d}{dt} \dot{z}_i \leq (\| A \| + \| BK ((L_i + a_{i0}) \otimes I_N) \|) \dot{z}_i. \]  

(45)

Consider that a nonnegative function \( \psi(t, \psi) \) satisfies

\[ \dot{\psi} = (\| A \| + \| BK ((L_i + a_{i0}) \otimes I_N) \|)(1 + \psi)^2 \quad \text{and} \quad \dot{\psi}_0 = 0, \]

according to Lemma 3, \( \dot{z}_i \leq \psi(t, 0) \). It can be seen from (36),

\[ \psi(t, 0) = \left( 1 - \kappa \right) \frac{a_{i0} \lambda_{\min}(W)}{2d_{\max} + a_{i0} \lambda_{\max}(W)} \right)^{1/2}. \]  

(46)

Hence,

\[ \frac{d}{dt} x_i \left[ t_k \right] = \frac{(\| A \| + \| BK ((L_i + a_{i0}) \otimes I_N) \|)}{3} \left( 1 + \psi(t) \right)^3 - 1, \]

(47)

Similar to Theorem 2, that Zeno behavior that does not occur can be proved by contradiction, which is omitted here.

4. Leader-Following Control of Multiagent Systems under Switching Topologies

In this part, we consider the extended case that the interconnection network switches according to signal \( \sigma(t) \) and is not connected all the time. It is worth noting that, unlike the fixed topology, the controller updates only when the event is triggered. In the switching topologies, the controller updates in the following two cases: (1) event-triggering instant. (2) Communication topology switching instant.

The control input of the \( i \)-th agent is defined as follows:

\[ u_i(t) = -K \sum_{j \in N_i(t)} a_{ij}(t)(x_i(t) - x_j(t)), \]  

(48)

where \( t \in [t_k, t_{k+1}) \). Different from control protocols (10) and (35), \( N_i(t) \) and \( a_{ij}(t) \) in (48) are changed under the switching topologies. Matrices \( \mathcal{T}_{\sigma(t)} \) and \( \mathcal{D}_{\sigma(t)} \) in \( \mathcal{S}_{\sigma(t)} \) represent Laplacian matrix and connection matrix between leader and agent, respectively. Switching signal \( \sigma(t) : [0, \infty) \rightarrow \mathcal{P} \) is a piecewise continuous constant function, which is used to describe the switching law of communication topology. Also, \( \mathcal{F}_{\sigma(t)} : p \in \mathcal{P} \) is a set of graphs that are switched within a finite set \( \mathcal{P} = \{ 1, 2, \ldots \} \) in any finite time interval. Consider a nonempty and continuous infinite sequence \( \{ t_s, t_{s+1} \} \), where \( k = 0, 1, \ldots, \) and \( t_0 = 0 \). Suppose that \( \mathcal{F}_{\sigma(t)} \) is switched only at and remains unchanged in \( t \in [t_s, t_{s+1}) \).

Remark 2. It should be noted that graph \( \mathcal{F}_{\sigma(t)} \) may be connected or unconnected in interval \( [t_s, t_{s+1}) \).

By replacing the similar variables in Section 3.2, we can derive that

\[ \dot{x}(t) = (I_N \otimes A) \dot{x}(t) - (I_{\sigma(t)} \otimes \mathcal{D}_{\sigma(t)} \otimes BK)(\dot{x}(t) + e(t)). \]  

(49)

Theorem 5. Under Assumptions 1 and 2, if feedback gain matrix \( K \) satisfies \( K = B^TP + W = PBK \), then the protocol (48) still makes the multiagent system with (3) track the system (2) successfully if the event-triggering condition satisfies

\[ f_i(t) = -\kappa \left( a_{i0} \lambda_{\min}(W) - a \lambda_{\max}(P) \right) \sum_{i=1}^{N} x_i^T x_i \]

\[ + \sum_{i=1}^{N} e_i^T e_i \geq 0, \]

where \( 0 < \kappa < 1 \), \( 0 < a < a_{i0} \lambda_{\min}(W)/\lambda_{\max}(P) \).

Proof. Construct the Lyapunov function for system (49) as follows:

\[ V_3 = e^{\alpha} \sum_{i=1}^{N} x_i^T P x_i. \]  

(51)

Similar to Section 3.2, taking the derivative of \( V_3 \) along the trajectory of system (49) yields
\[ V_3 \leq e^{at} \sum_{i=1}^{N} \sum_{j=1}^{N} a_{ij} x_i^T W x_j + e^{at} \sum_{i=1}^{N} a_{i0} x_i + e^{at} \sum_{i=1}^{N} a_{ij} x_i \]

\[ + e^{at} \sum_{i=1}^{N} a_{i0} x_i - e^{at} \sum_{i=1}^{N} x_i^T W x_i \]

\[ \leq e^{at} \bar{x}^T \left( I_n \otimes \left( PA + A^T P \right) \right) \bar{x} - e^{at} \bar{x}^T \left( L_{\alpha}(t) \otimes W \right) \bar{x} \]

\[ + e^{at} \sum_{i=1}^{N} \bar{x}_i^T \left( a_{i0} - a_{0i} \right) W x_i \]

\[ + e^{at} \sum_{i=1}^{N} \lambda_{\max}(W) \left( a_{ij} \lambda_{\max}(W) + 2d_{ij} \lambda_{\max}(W) \right) \]

\[ \leq 0. \]  

\[ (52) \]

(i) If the graph \( \mathcal{G}_p \) is not connected during \( t \in [t_s, t_{s+1}] \), according to the event-triggering condition (50) and equation (6), one has

\[ V_3 \leq e^{at} \bar{x}^T \left( I_n \otimes \left( PA + A^T P \right) \right) \bar{x} \]

\[ + e^{at} \sum_{i=1}^{N} \left( a_{i0} - a_{0i} \right) \lambda_{\min}(W) \left\| \bar{x}_i \right\|^2 \]

\[ + e^{at} \sum_{i=1}^{N} \lambda_{\max}(W) \left( a_{ij} + 2d_{ij} \lambda_{\max}(W) \right) \left\| \bar{x}_i \right\|^2 \]

\[ \leq 0. \]  

\[ (53) \]

It can be seen from the abovementioned formula that \( V_3 \) is not increasing; hence,

\[ V_3(t) \geq V_3(t_{s+1}) = e^{at_{s+1}} \sum_{i=1}^{N} x_i(t_{s+1})^T P x_i(t_{s+1}) \]

\[ \geq e^{at_{s+1}} \lambda_{\min}(P) \left\| \bar{x}(t_{s+1}) \right\|^2, \]

i.e.,

\[ \sqrt{\left( V_3(0) / \lambda_{\min}(P) \right) e^{-\left( a/2 \right) t_{s+1}}} \leq \sqrt{\left( V_3(t) / \lambda_{\min}(P) \right) e^{-\left( a/2 \right) t_{s+1}}}. \]

(ii) If the graph \( \mathcal{G}_p \) is connected during \( t \in [t_s, t_{s+1}] \), then

\[ V_3 \leq e^{at} \bar{x}^T \left( I_n \otimes \left( PA + A^T P - \lambda_2 \left( L_{\alpha}(t) \right) W \right) \right) \bar{x} \]

\[ + e^{at} \sum_{i=1}^{N} \left( a_{i0} - a_{0i} \right) \lambda_{\min}(W) \left\| \bar{x}_i \right\|^2 \]

\[ + e^{at} \sum_{i=1}^{N} \lambda_{\max}(W) \left( a_{ij} + 2d_{ij} \lambda_{\max}(W) \right) \left\| \bar{x}_i \right\|^2. \]

\[ (55) \]

According to event-triggering condition (50) and equation (5),

\[ V_3(t) \geq V_3(t_{s+1}) = e^{at_{s+1}} \sum_{i=1}^{N} x_i(t_{s+1})^T P x_i(t_{s+1}) \]

\[ \geq e^{at_{s+1}} \lambda_{\min}(P) \left\| \bar{x}(t_{s+1}) \right\|^2, \]

i.e.,

\[ \sqrt{\left( V_3(0) / \lambda_{\min}(P) \right) e^{-\left( a/2 \right) t_{s+1}}} \leq \sqrt{\left( V_3(t) / \lambda_{\min}(P) \right) e^{-\left( a/2 \right) t_{s+1}}}. \]
In summary, \[ \|\hat{x}(t_{en})\| \leq \sqrt{(V_3(t_{en+1})/\lambda_{\min}(P))} e^{-(\alpha/2)t_{en}} \leq \cdots \leq \sqrt{(V_3(0)/\lambda_{\min}(P))} e^{-(\alpha/2)t_{en}}, \] i.e., \[ \|\hat{x}(t)\| \leq \sqrt{(V_3(t)/\lambda_{\min}(P))} e^{-(\alpha/2)t} \leq \cdots \leq \sqrt{(V_3(0)/\lambda_{\min}(P))} e^{-(\alpha/2)t}, \] so \( \lim_{t \to \infty} \|\hat{x}(t)\| = 0 \) is equivalent to \( \lim_{t \to \infty} \|\hat{x}_i(t)\| = 0 \), and accordingly \( \lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0 \), \( i = 1, 2, \ldots, N \) is established.

Remark 3. Index \((\alpha/2)\) can be approximated as the convergence rate of multiagent system (49), and the convergence rate can be changed by adjusting \(\alpha\).

**Theorem 6.** Under the conditions of Theorem 5, system (49) does not have Zeno behavior. The interval between any two consecutive event-triggering instants of the system is not less than

\[
\left( \frac{\|I_N \otimes A\| + \|L + \bar{D}\otimes BK\|}{3} \right) \times \left( 1 + \frac{\kappa}{a_{\min}\lambda_{\min}(W) - a\lambda_{\max}(P)} \right)^{1/2} - 1.
\]

\[ (58) \]

**Proof.** The proof is similar to that of Theorem 2. \(\square\)
5. Simulation

In this part, we consider the trajectories of the state errors between the follower and leader under the fixed topology and the switching topology, respectively, where the dynamic equations of the leader and the follower are given by (2) and (3), respectively, and the communication network topology among agents is shown in Figures 1 and 2. Assume that $x_i = [x_{i1}, x_{i2}]^T$, and $A$ and $B$ are chosen as follows:

$$A = \begin{bmatrix} 0 & 0.5 \\ -4.8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ -0.5 \end{bmatrix},$$

it is easy to prove that the Assumption 2 is satisfied. By solving Riccati equation by MATLAB, we know that feedback gain matrix $K = B^T P = [-0.4995, -1.1343]^T$. Let the leader’s initial state be $x_0 (0) = [2, 3]^T$ and the follower’s initial state be $x_1 (0) = [-1, 1]^T$, $x_2 (0) = [-2, -3]^T$, $x_3 (0) = [5, -6]^T$, $x_4 (0) = [4, 2]^T$.

**Example 1.** Under the centralized event-triggering protocol (10), the leader-following consensus of the multi-agent system composed of (2) and (3) is considered. The communication network among agents is shown in Figure 1, and the corresponding weights are all 1. It can be seen from Figures 3 and 4 that followers can successfully follow the leader. Figure 5 shows the event instants of each follower with the centralized event-triggering protocol (10). It can be seen that protocol (10) can effectively reduce the number of communications among agents, thus reducing the waste of resources. Also, there is no Zeno behavior.

**Example 2.** In this example, we illustrate the leader-following consensus of the multiagent system under the distributed event-triggering protocol (35). The communication network among agents is shown in Figure 1. It can be seen from Figures 6 and 7 that followers can successfully follow the leader. Figure 8 shows the event triggering time of each follower under the decentralized event triggering protocol (35), and Zeno behavior is excluded. The simulation results verify Theorems 3 and 4.

**Example 3.** Finally, the leader-following consensus of the multiagent system under the control protocol (48) is considered. The communication network among agents will randomly switch between $\mathcal{G}_1$ and $\mathcal{G}_2$, as shown in Figure 2, where $\mathcal{G}_1$ is a connected graph and $\mathcal{G}_2$ is an unconnected graph. The state errors between the follower agent $i$ and leader 0 are shown in Figures 9 and 10, respectively. It indicates that all followers can successfully follow the leader. Figure 11 shows the event-triggering instants of each follower under (48), and there is no Zeno behavior.
6. Conclusions and Future Work

In this paper, the leader-following control of general linear multiagent systems based on event-triggering mechanism under both fixed topology and switching topologies have been studied. Under the fixed topology, two different control protocols are designed in order to reduce waste of resources. Based on these two control protocols, we propose two different triggering functions, i.e., centralized event-triggering function and decentralized event-triggering function with state error between the follower and leader. When the triggering function exceeds 0, the agent will update the control input at the triggering instants. Through theoretical analysis, the sufficient conditions are derived for the system to achieve leader-following consensus under two control protocols and event-triggering conditions. The conditions obtained under fixed topology are extended to switching topologies (different from the fixed topology, the controller update at the triggering time, and also the switching time). The results show that the conditions to achieve leader-following are also valid under switching topologies. Finally, it is proved that the system can realize leader-following control without Zeno behavior. The simulation results verify the effectiveness of the theoretical analysis. In the future, we will further study the leader-following control of the linear multiagent system with interference, delay, and other factors.

Data Availability
No data were used to support this study.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant.no. 61873136, 61903210, 61374062, and 61603288); Science Foundation of Shandong Province for Distinguished Young Scholars (Grant.no. JQ201419); and Shandong Provincial Natural Science Foundation, China (Grant.no. ZR201709260010).

References

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