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Research Article

A Research on Nonendangered Population Protection Facing Biological Invasion

Meng Zhang[1], **Yi Zhao**[1], **Mingjing Shi**[1] **and Xin Zhao**[2]

[1]*School of Science, Beijing University of Civil Engineering and Architecture, Beijing, 100044, China*
[2]*Canvard College, Beijing Technology and Business University, Beijing, 101118, China*

Correspondence should be addressed to Xin Zhao; lettertozhaoxin@163.com

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In nature, a biological invasion is a common phenomenon that often threatens the existence of local species. Getting rid of the invasive species is hard to achieve after its survival and reproduction. At present, killing some invasive ones and putting artificial local species are usual methods to prevent local species from extinction. An ODE model is constructed to simulate the invasive procedure, and the protection policy is depicted as a series of impulses depending on the state of the variables. Both the ODEs and the impulses form a state feedback impulsive model which describes the invasion and protection together. The existence of homoclinic cycle and bifurcation of order-1 periodic solution of the impulsive model are discussed, and the orbitally asymptotical stability of the order-1 periodic solution is certificated with a novel method. Finally, the numerical simulation result is listed to confirm the theoretical work.

1. Introduction

Biological invasion is the introduction and establishment of a species beyond its natural range where it may proliferate and spread dramatically [1]. As being predicted, the rate of biological invasion is expected to increase with the acceleration of worldwide movement of people and goods [2, 3]. Biological invasions constitute a major environmental change driver, affecting conservation, agriculture, and human health [4, 5]. Inevitably, biological invasions result in species interactions. When local community members are challenged by biological invasions, they may face novel antagonists such as predators or competitors, or they may benefit from new prey, new and underutilized host plants, or even new mutualists [6]. It is well known that some invasion cases such as the diffusion of alien invasive plants in Albania and the Mediterranean [7] and the rampant diffusion of Asian carp in America have caused great disaster in nature niche. Since biological invasions contribute a lot to rapid environmental change [8], biodiversity loss, degradation of ecosystem structure, impairment of ecosystem services [9], and significant impacts on both natural and agricultural ecosystems [10], ecological invasion and subsequent influence are receiving increasing attention from scientists and governors. To prevent some species from extinction and destruction of niche caused by the ecological invasion, ecologists carry out some strategies such as the introduction of natural enemies, substitution of invasion species, physical prevention, and chemical prevention. These artificial controlling methods are executed periodically instead of being performed continuously. And the time point to carry out the artificial assistance is decided by the density of the species, but not the fixed period.

Recently, some mathematicians improved a lot in the state feedback impulsive dynamics. They not only set up the basic framework but also proposed and proved some useful theorems. Chen revised and improved some basic frame definitions and proposed a series of theorems which established the foundation of this area and had been widely applied in dealing with practical problems [11]. Zhang et al. improved the theory about order-1 periodic solution in the model of the Internet worm control [12]. Wei et al. dedicated a lot in the area of homoclinic cycle and heteroclinic orbit with respect to some kinds of dynamic model [13–15].

Inspired by the new results in state feedback impulsive model [16–26], and classical work in limit cycle [26] the authors propose a state feedback impulsive model, which is more suitable than the ODE model, to describe the procedure of ecological invasion and artificial control.

This paper is organized as follows. Section 2 proposes a state feedback impulsive model which describes the artificial auxiliary as an impulse relying on the state of variables and introduces some preliminaries of state feedback impulsive system. The existence of homoclinic cycle and order-1 periodic solution is proved in Section 3, and its stability is proved in Section 4. Some numerical simulations will be exhibited in Section 5.

2. State Feedback Impulsive Artificial Assistance Model and Its Preliminaries

We focus on the artificial control by releasing some artificial breeding ones and getting rid of some invasive ones to help the local species to survive from competition with invasive ones.

2.1. Free Developing Model and Analysis. We consider the model

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x(a - cy), \\ \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = y(d - ex) \end{cases} \tag{1}$$

to describe the competition between the two species, where x and y are densities of local and invasive species separately, and these two species compete for the same natural resource. Considering the practical significance, we only discuss model (1) in the first quadrant. It is easy to find that model (1) has two equilibriums $O(0,0)$ and $E(d/e, a/c)$, where $O(0,0)$ is an unstable node while $E(d/e, a/c)$ is a saddle. The dynamical features of model (1) are shown in Figure 1. Since there is only one saddle in the first quadrant, we claim that there exists no limit cycle in the area $\{(x, y) \mid x > 0, y > 0\}$.

In Figure 1, unstable and stable manifolds of saddle E, shown in heavy lines, are denoted with L_1 and L_2. Then, the first quadrant is separated into four parts G_1, G_2, G_3, and G_4. The trajectories initiated from G_3 and G_4 will tend to $(\infty, 0)$ which means the local species can survive the invasion without artificial assistance. On the contrary, the trend of trajectories starting from G_1 and G_2 implies that the local species will experience a critical shrink after a certain period, and the invasion will lead to the extinction of local species sooner or later. The endangered species should have already been protected in special ways, and in this paper, we focus on the common protection of nonendangered local species during the invasion, so we only study the trajectories in G_1.

The artificial control is designed as follows. When the density of invasive species reaches a certain level which is marked with h, manual intervention will be implemented. Through some technical skills, such as spraying biocide,

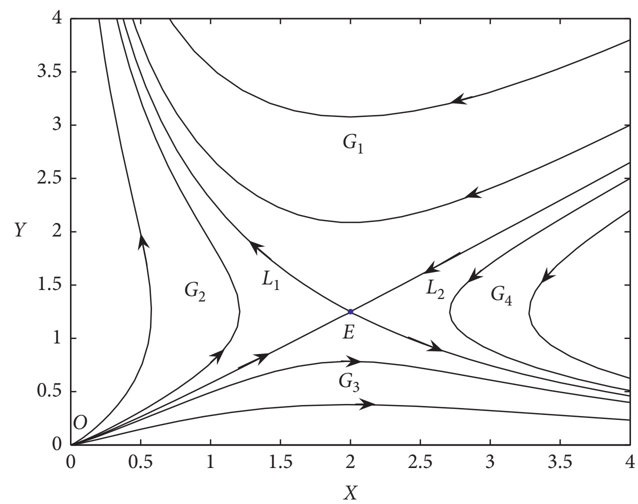

Figure 1: Trajectories of model (1) with parameter values: $a = 5, c = 4, d = 6$, and $e = 3$.

artificial killing, and releasing natural enemies, the invasive species will be partly removed. At the same time, a certain number of local species are put into circulation to replenish its density. Since the perishing of invasive species and the replenishing of local species are carried out according to the actual state of the species and all the artificial auxiliary can be finished in a relatively short period compared with the long-term struggle between the two species, the invasion and protection procedure should be described with a state feedback impulsive model. Without loss of generality, we assume the density of invasive species before mankind help is bigger than the ordinate of equilibrium E, i.e., $h > a/c$. If the mankind help is strong enough to maintain the state of the species in areas G_3 and G_4, the local system will persist. However, the power of mankind is always limited, which means the invasive species cannot be eliminated thoroughly and the local species can only be complemented with limited amount. So, we also assume that the state of two species is located in area G_1 and its boundaries after impulse.

2.2. Construction of the State Feedback Impulsive Model. The state feedback impulse control works in the following procedure. Once the density of invasive species y rises to the threshold value h, people will cull some of them at the rate of β and put in some artificial local ones at the same time. The number of artificial local ones put in the circumstance is infected by density of remainder local species. In circumstantial depiction, we put in more artificial ones to keep the local species permanence if the density of local species is lower, i.e., it is inversely proportional to the density of local species x before the impulse. To construct the model, we denote the intersection of unstable manifold L_1 and the thresh hold line $y = h$ as $A(x_A, h)$, and point $A^+(x_{A^+}, (1 - \beta)h)$ denotes the intersection of stable manifold L_2; the line consisted of the points after impulse. We also denote the intersection of thresh hold line $y = h$ and the horizontal isocline $\mathrm{d}y/\mathrm{d}t = 0$ as $P(x_P, y_P)$ (see Figure 2). Then, the model can be described as

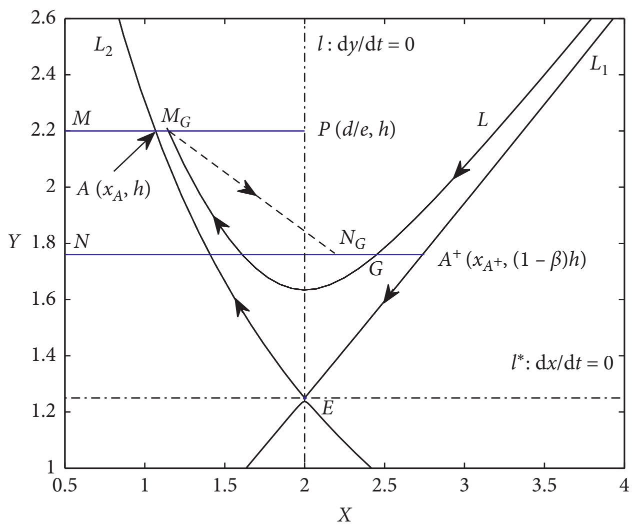

Figure 2: Diagram of state feedback impulsive system with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2$, and $\beta = 0.2$.

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = x(a - cy), \\ & y \neq h \text{ or } y = h,\ x > x_P, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = y(d - ex), \\ \Delta x = \alpha\left(x_{A^+} - x\right), \\ & y = h,\ x_A \le x \le x_P. \\ \Delta y = -\beta y, \end{cases} \tag{2}$$

From the description above, we assume that $\alpha \le 1$ and all the parameters in model (2) are positive. In model (2), the investment of artificial local species is monotonically decreasing with the left ones, which is in accordance with the actual situation. The less the local species left, the more the artificial ones invested.

2.3. Preliminaries. In the following part of this section, some preliminaries about the state feedback impulsive system will be introduced.

Definition 1. A state feedback impulsive system is defined as

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = P(x, y), \\ & (x, y) \notin M\{x, y\}, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = Q(x, y), \\ \Delta x = A(x, y), \\ & (x, y) \in M\{x, y\}. \\ \Delta y = B(x, y), \end{cases} \tag{3}$$

The dynamic system defined with (3) is a kind of semicontinuous dynamic system. In the system, when the variables (x, y) reach set $M\{x, y\}$, the impulse will be carried out according φ: $(x, y) \longrightarrow (x + \Delta x, y + \Delta y)$. Here, $M\{x, y\}$ is called impulse set and ϕ is impulsive function. Without loss of generality, the initial point P_0 of system (3) should be restricted not in impulse set $M\{x, y\}$, i.e., $P \in \Omega = R^2 \backslash M\{x, y\}$. Define $N = \varphi(M)$, and we call it phase set. The state feedback impulsive system can be denoted with (Ω, f, φ, M), where $f(x, y) = (P(x, y), Q(x, y))$, $(x, y) \notin M\{x, y\}$. And the mapping of the state feedback impulsive system is expressed with $f \otimes \varphi(\bullet, t)$.

It is obvious that line segment $\overline{AP}$ is impulsive set of model (2), while a certain part of line $y = (1 - \beta)h$ is the corresponding phase set. While the trajectory L reaches the impulsive set AP at point M_G, the impulsive function φ maps it to the point N_G, i.e. $f \otimes \varphi(G, t) = N_G$.

Definition 2 (see [11]). Since $f \otimes \varphi(\bullet, t)$ is a mapping on itself, there exists a point G^* in phase set N and a corresponding moment t^* satisfying $f(G^*, t^*) = M_G^* \in M$; moreover, $\varphi(M_G^*) = \varphi(f(G^*, t^*)) = G^* \in N$, i.e., $f \otimes \varphi(G^*, t^*) = G^*$. Then, $f \otimes \varphi(G^*, t^*)$ is an order-1 periodic solution of model (3). And the trajectory from G^* to M_G^* controlled by f and the mapping segment line from M_G^* to G^* controlled by φ make up an order-1 limited cycle. Furthermore, if there is a saddle on the order-1 limited cycle, then they form an order-1 homoclinic cycle.

Definition 3 (see [11]). Suppose impulse set M and phase set N are straight lines (see Figure 2). To any point $G \in N$, define the absolutely value of its abscissas as its coordinate. The trajectory initiating from G intersects impulse set M at M_G; then, the impulse function φ maps M_G to N_G in phase set N, and N_G is the subsequent point of G. Then, we define the successor function of G as $F(G) = |x_{N_G}| - |x_G|$.

Remark 1. The sufficient and necessary condition of $F(G) = 0$ is that the solution from G point is an order-1 periodic solution of system (3).

Lemma 1 (see [11]). *Successor function $F(A)$ is continuous.*

Lemma 2 (see [11]). *In semicontinuous dynamic system (Ω, f, φ, M), there exist two points G_1 and G_2 in phase set N; if $F(G_1) \cdot (G_2) < 0$, then there must exist a point G^* between G_1 and G_2 in phase set N satisfying $F(G^*) = 0$, i.e., $f \otimes \varphi(G^*, t)$ is the order-1 periodic solution.*

Lemma 3. *If the impulsive condition is expressed with $\phi(x, y) = 0$, then system (3) can be rewritten as*

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}t} = P(x, y), \\ & \phi(x, y) \neq 0, \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} = Q(x, y), \\ \Delta x = A(x, y), \\ & \phi(x, y) = 0. \\ \Delta y = B(x, y), \end{cases} \tag{4}$$

Assume it has an order-q periodic solution $\Gamma(t): = (\xi(t), \eta(t))$, and the period is T. Then, the order-1 periodic solution is orbitally asymptotically stable if the factor μ_2 satisfies $|\mu_2| < 1$, where

$$\mu_2 = \prod_{k=1}^{q} \Delta_k \exp\left[\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial y}(\xi(t), \eta(t))\right) \mathrm{d}t\right],$$
$$\Delta_k = \frac{P_+ ((\partial B/\partial y)(\partial \phi/\partial x) - (\partial B/\partial x)(\partial \phi/\partial y) + (\partial \phi/\partial x)) + Q_+ ((\partial A/\partial x)(\partial \phi/\partial y) - (\partial A/\partial y)(\partial \phi/\partial x) + (\partial \phi/\partial y))}{P(\partial \phi/\partial x) + Q(\partial \phi/\partial y)}. \tag{5}$$

Here, P, Q, $\partial A/\partial x, \partial A/\partial y, \partial B/\partial x, \partial B/\partial y, \partial \phi/\partial x$, and $\partial \phi/\partial y$ are the corresponding values at point $(\xi(\tau_k), \eta(\tau_k))$, and P_+ and Q_+ are calculated at $(\xi(\tau_k^+), \eta(\tau_k^+))$.

These definitions and lemmas about the state feedback impulsive system are of great significance in the following part of this paper.

3. Homoclinic Cycle and Homoclinic Bifurcation

In order to discuss the existence of homoclinic cycle and order-1 periodic solution of model (2), we choose α as a key parameter. For a certain α, the homoclinic cycle exists and then it will disappear and bifurcate an order-1 periodic solution with the changing of α.

3.1. Homoclinic Cycle of Model (2) about Parameter α. The horizontal isocline l: $\mathrm{d}y/\mathrm{d}t = 0$ of model (2) expressed as dot line in Figure 3 crosses impulsive set M: $y = h$ and phase set N: $y = (1-\beta)h$ at P and Q (see Figure 3), and the vertical isocline l^*: $\mathrm{d}x/\mathrm{d}t = 0$ is a line passing saddle E and paralleling with x axis. From the trajectory properties of model (2), we know that the intersection point A of unstable manifold L_2 and the line $y = h$ must locate on the left side of horizontal isocline l and on the upside of vertical isocline l^*, while the intersection point A^+ of stable manifold L_1 and line $y = (1-\beta)h$ must be on the upper right side.

According to the control of model (2), L_2, unstable manifold of saddle, E, comes out from E and reaches impulsive set M at point A. Restricting $\alpha = 1$, A will be mapped by the impulsive mapping φ of model (2), i.e.,

$$\varphi(x_A, h) = (x_{A^+}, (1-\beta)h), \quad \varphi\colon A \longrightarrow A^+. \tag{6}$$

After the impulsive mapping, the trajectory moves along L_1, the stable manifold of E. So, trajectory EA, impulsive line AA^+, and trajectory A^+E form a closed cycle, and saddle E locates on it. Then, the homoclinic cycle forms (see Figure 3).

Theorem 1. *If $\alpha = 1$, then there exists an order-1 homoclinic cycle in model (2).*

3.2. Homoclinic Bifurcation about Parameter α. $\alpha > 1$ indicates that putting abundant artificial local ones can maintain the permanence of local species. But in practice, the massive release cannot be satisfied every time for the deficiency of artificial breeding capability. Congruous with the above, we only consider the case that the phase point locates in G_1 in this paper.

In this section, we assume $\alpha < 1$, so the phase point of A which is denoted as N_A locates on the left side of A^+. Take another point $B^+(x_{B^+}, (1-\beta)h)$ on the left side of A^+ and assume $x_{B^+} = x_{A^+} - \varepsilon$, where $\varepsilon > 0$ is a small enough number, which means B^+ is sufficiently close to A^+. Following the fundamental theories of ordinary differential equation, there exists a unique trajectory L passing through B^+ and intersecting with the impulse set M at point $B(x_B, h)$. According to the continuous property of solution with respect to the initial value, point B locates on the right side of A and is close enough to A. Furthermore, based on the continuous characteristic of impulse function φ with respect to the independent variable, the phase point $N_B = \varphi(B)$ is also close enough to N_A, i.e., N_B is on the left side of B^+, $F(B^+) < 0$ (see Figure 4).

Then, we consider the trajectory passing Q. Following the properties of trajectories, there exists only one trajectory passing through Q denoted as L^*; it arrives the imagine set M: $y = h$ at D, then the impulsive function φ maps D to N_D. The following discussion is basedfsection, we will discuss the orbital stabi on the location of $N_D(x_{N_D}, (1-\beta)h)$.

Case 1. $x_{N_D} = x_Q = d/e$. This condition ensures the phase points N_D and Q coincide and the successor function of Q is $F(Q) = 0$, i.e., the trajectory $\widehat{QD}$ and the impulsive mapping $\overline{DN_D}$ make up an order-1 limit cycle (see Figure 5). We denote the value of α under this situation as α^*.

Case 2. $x_{N_D} > x_Q = d/e$, i.e., $\alpha^* < \alpha < 1$. With this condition, we have that the successor function of Q is $F(Q) = x_{N_D} - x_Q > 0$. According to Lemma 2, there exists a point $C \in N$ (between Q and B^+) satisfying $F(C) = 0$. That means there exists an order-1 periodic solution of system (2) (see Figure 6).

Case 3. $x_{N_D} < x_Q = d/e$, i.e., $\alpha < \alpha^*$. Here, we can get a contrary successor function of Q, i.e., $F(Q) = x_{N_D} - x_Q < 0$. Denote the intersection of L and phase set N (L moves upside) as B^*; then, $F(B^*) > 0$. Also, from Lemma 2, there exists a point $C^* \in N$ (between B^* and Q) satisfying $F(C^*) = 0$. That means there exists an order-1 periodic solution of system (2) (see Figure 7).

From the discussion of those three cases, we have the following theorem.

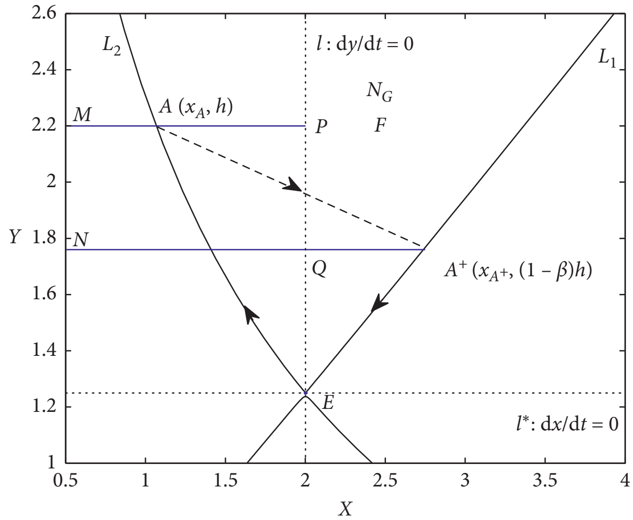

Figure 3: Homoclinic cycle of model (2) with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2, \beta = 0.2$, and $\alpha = 1$.

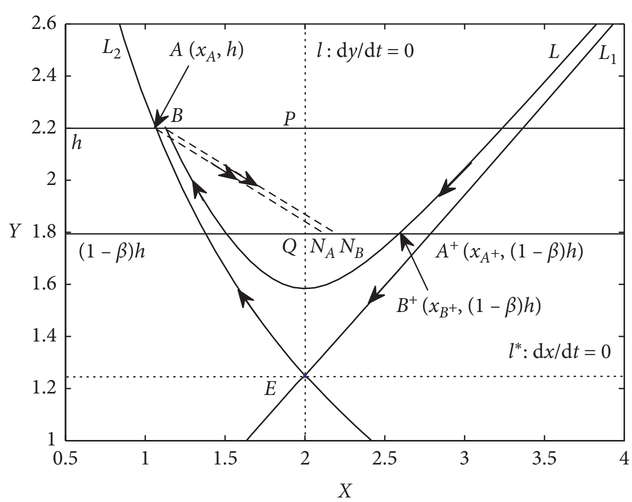

Figure 4: Trajectory L passing B^+ and impulse mapping of B with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2$, and $\beta = 0.2$.

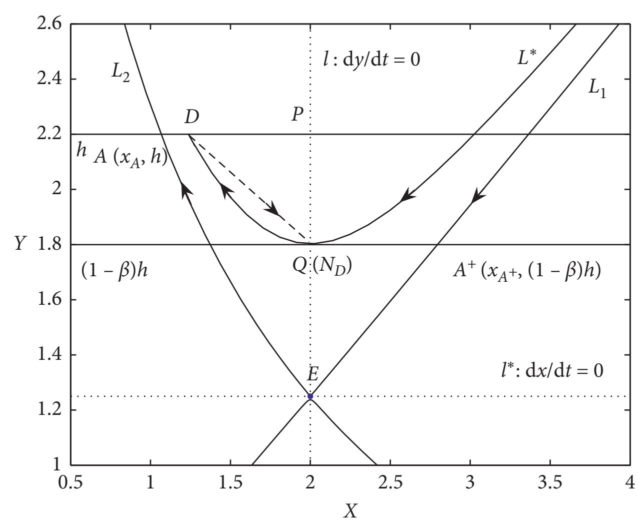

Figure 5: Trajectory L^* passing Q and impulse mapping D of case I with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2$, and $\beta = 0.2$.

Theorem 2. *If $\underline{\alpha} < \alpha < 1$ ($\underline{\alpha}$ is the threshold to make sure the phase point is located in G_1) holds, then there exists an order-1 periodic solution to system (2).*

Remark 2. From Theorems 1 and 2, we know that system (2) has an order-1 homoclinic cycle when $\alpha = 1$. For any $\underline{\alpha} < \alpha < 1$, the order-1 homoclinic cycle breaks and bifurcates an order-1 periodic solution. Then, $\alpha = 1$ is a bifurcation point for system (2).

4. Stability of the Order-1 Periodic Solution

In this section, we will discuss the orbital stability of the order-1 periodic solution.

Theorem 3. *Suppose $(\xi(t), \eta(t))$ is an order-1 periodic solution of model (2) which initiates from $(\xi_0, \eta_0) \in N$ and it arrives the impulsive set at $(\xi_1, \eta_1) \in M$. The order-1 periodic solution $(\xi(t), \eta(t))$ is orbitally asymptotically stable if*

$$(H), \xi_0 < \frac{d}{e} \tag{7}$$

is satisfied.

Proof 1. Assume $\Gamma(t) = (\xi(t), \eta(t))$ is an order-1 periodic solution of system (2) and the period is T. Denote

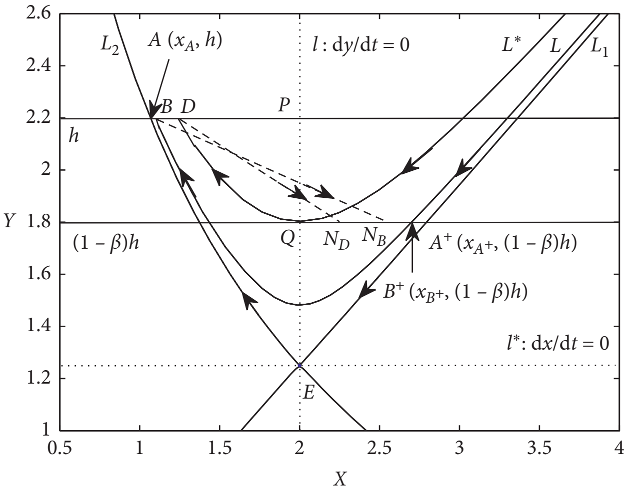

Figure 6: Trajectory L^* passing Q and impulse mapping D of case II with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2$, and $\beta = 0.2$.

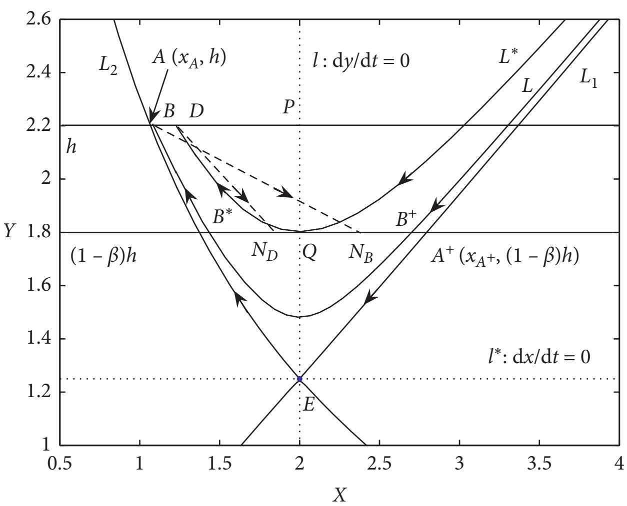

Figure 7: Trajectory L^* passing Q and impulse mapping D of case III with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2$, and $\beta = 0.2$.

$$\begin{aligned} \xi_0 &= \xi(0), \\ \xi_1 &= \xi(T), \\ \eta_0 &= \eta(0) = (1-\beta)h, \\ \eta_1 &= \eta(T) = h, \end{aligned} \tag{8}$$

$$\begin{aligned} \xi_1^+ &= \xi(T^+) = \xi_1 + \alpha(x_{A^+} - \xi_1) = \xi_0, \\ \eta_1^+ &= \eta(T^+) = \eta_0 = (1-\beta)h. \end{aligned} \tag{9}$$

Following the expression of system (2), we have

$$\begin{aligned} P(x,y) &= x(a - cy), \\ Q(x,y) &= y(d - ex), \\ A(x,y) &= \alpha(x_{A^+} - x), \\ B(x,y) &= -\beta y, \\ \phi(x,y) &= y - h. \end{aligned} \tag{10}$$

Then, their values can be calculated as

$$\begin{aligned} \frac{\partial P}{\partial x} &= a - cy, \\ \frac{\partial P}{\partial y} &= -cx, \\ \frac{\partial Q}{\partial x} &= -ey, \\ \frac{\partial Q}{\partial y} &= d - ex, \\ \frac{\partial A}{\partial x} &= -\alpha, \\ \frac{\partial A}{\partial y} &= 0, \\ \frac{\partial B}{\partial x} &= 0, \\ \frac{\partial B}{\partial y} &= -\beta, \\ \frac{\partial \phi}{\partial x} &= 0, \\ \frac{\partial \phi}{\partial y} &= 1. \end{aligned} \tag{11}$$

We also have

$$\begin{aligned} \Delta_1 &= \frac{(1-\alpha)Q_+}{Q} = \frac{(1-\alpha)(d - e\xi_0)\eta_0}{(d - e\xi_1)\eta_1} \\ &= (1-\alpha)(1-\beta)\frac{d - e\xi_0}{d - e\xi_1}, \end{aligned} \tag{12}$$

$$\begin{aligned} &\int_0^T \left(\frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial y}(\xi(t), \eta(t)) \right) \mathrm{d}t \\ &= \int_0^T \left(\frac{\dot{x}}{x} + \frac{\dot{y}}{y} \right) \mathrm{d}t \\ &= \ln \frac{\xi_1}{\xi_0} \cdot \frac{\eta_1}{\eta_0}. \end{aligned} \tag{13}$$

Then,

$$\begin{aligned} \mu_2 &= \Delta_1 \cdot \exp \int_0^T \left(\frac{\partial P}{\partial x}(\xi(t), \eta(t)) + \frac{\partial Q}{\partial y}(\xi(t), \eta(t)) \right) \mathrm{d}t \\ &= (1-\alpha) \cdot \frac{d - e\xi_0}{d - e\xi_1} \cdot \frac{\xi_1}{\xi_0}. \end{aligned} \tag{14}$$

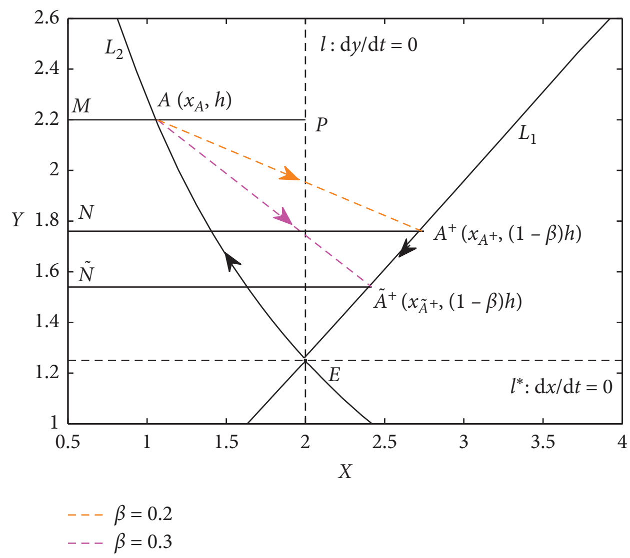

FIGURE 8: Homoclinic cycle of model (2) when $\alpha = 1$ with parameter values: $a = 5, c = 4, d = 6, e = 3$, and $h = 2.2$. The case of $\beta = 0.2$ is shown in hazel line while that of $\beta = 0.3$ is exhibited in pink line.

In area G_1, variable x shows a continuous decrease when it is controlled by the first two equations of model (2). (ξ_0, η_0) and (ξ_1, η_1) are coordinates of the intersections of the order-1 periodic solution with phase set and impulsive set. It is obvious that the trajectories move from the intersection on phase set to the one on impulsive set, so we can draw the conclusion that $\xi_1 < \xi_0$. With these conditions, if

$$(H), \xi_0 < \frac{d}{e}, \tag{15}$$

also holds, i.e., the intersection of the order-1 periodic solution and the phase set locates on the left side of the horizontal isocline, we have $0 < (d - e\xi_0/d - e\xi_1) < 1$. Then, $|\mu_2| < 1$ is satisfied. Following Lemma 3, we can draw the conclusion that the order-1 periodic solution is orbitally asymptotically stable. This completes the proof. □

5. Numerical Simulation and Discussion

System (1) is a state feedback impulsive dynamic system whose corresponding system without impulse is model (2). In Section 3 and Section 4, we have proved that system (2) has either a homoclinic cycle or an order-1 periodic solution according to different parameter values. To verify the result, we show some results which display the conclusion more intuitively.

In the following numerical simulation of model (2), we assume $a = 5, c = 4, d = 6, e = 3$, and $h = 2.2$ while the parameters α and β which control the intensity of impulse take different values.

Figure 8 shows that homoclinic cycles exist when $\alpha = 1$ while the parameter β takes different values. It means the existence of homoclinic cycle is not influenced by the value of parameter β as long as the impulsive set is located above the saddle E.

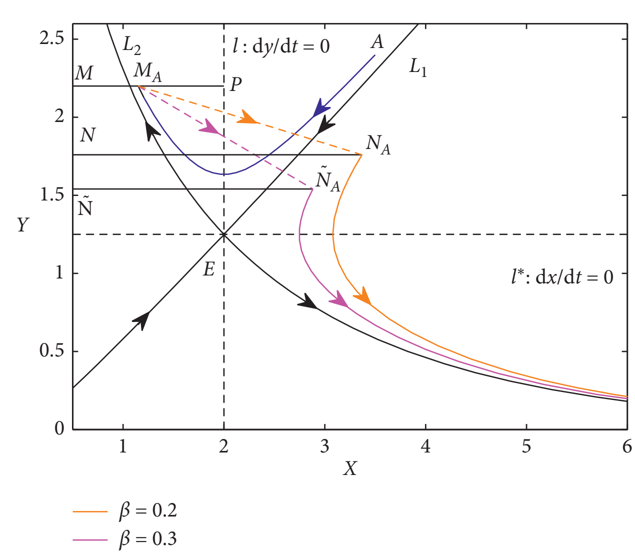

FIGURE 9: Situations of impulsive model (2) when $\alpha = 1.4$ with parameter values: $a = 5, c = 4, d = 6, e = 3$, and $h = 2.2$ and initial values (3.5, 2.4). The case of $\beta = 0.2$ is shown in hazel line while that of $\beta = 0.3$ is exhibited in pink line.

Figure 9 exhibits the situation when $\alpha > 1$. When $\alpha > 1$, the phase point will drop in area G_4 which means the trajectory definitely runs to $(\infty, 0)$. In reality, if there are plenty of artificial local ones can be invested in the competition together with getting rid of a certain amount of invasive ones, then the local species will success and sustain. If artificial breeding local species is easy, impulsive replenish of enough amount of artificial local ones can help the local species hold advantage in the competition with invasive ones. So, strengthening the local species with supplying plentiful amount in single impulse is the best strategy when dealing with ecological invasion.

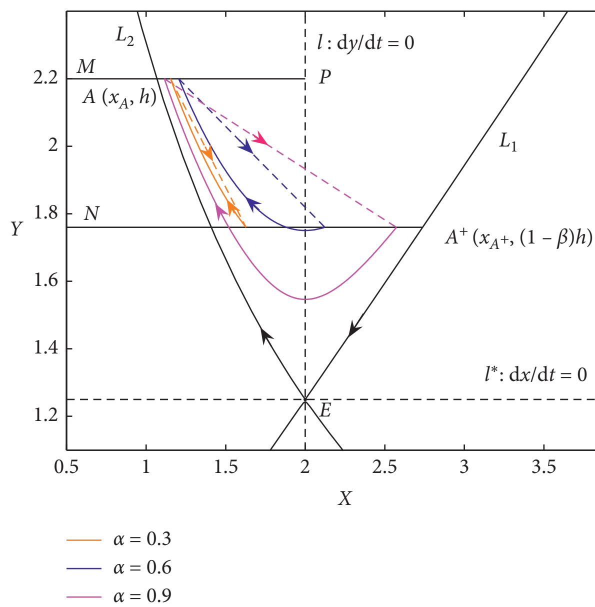

Figure 10: Existence of order-1 periodic solution when $\alpha < 1$ with parameter values: $a = 5, c = 4, d = 6, e = 3, h = 2.2,$ and $\beta = 0.2$ and initial values $(3.5, 2.4)$. The cases of $\alpha = 0.3$, $\alpha = 0.6$, and $\alpha = 0.9$ are shown with hazel, blue, and pink lines coordinately.

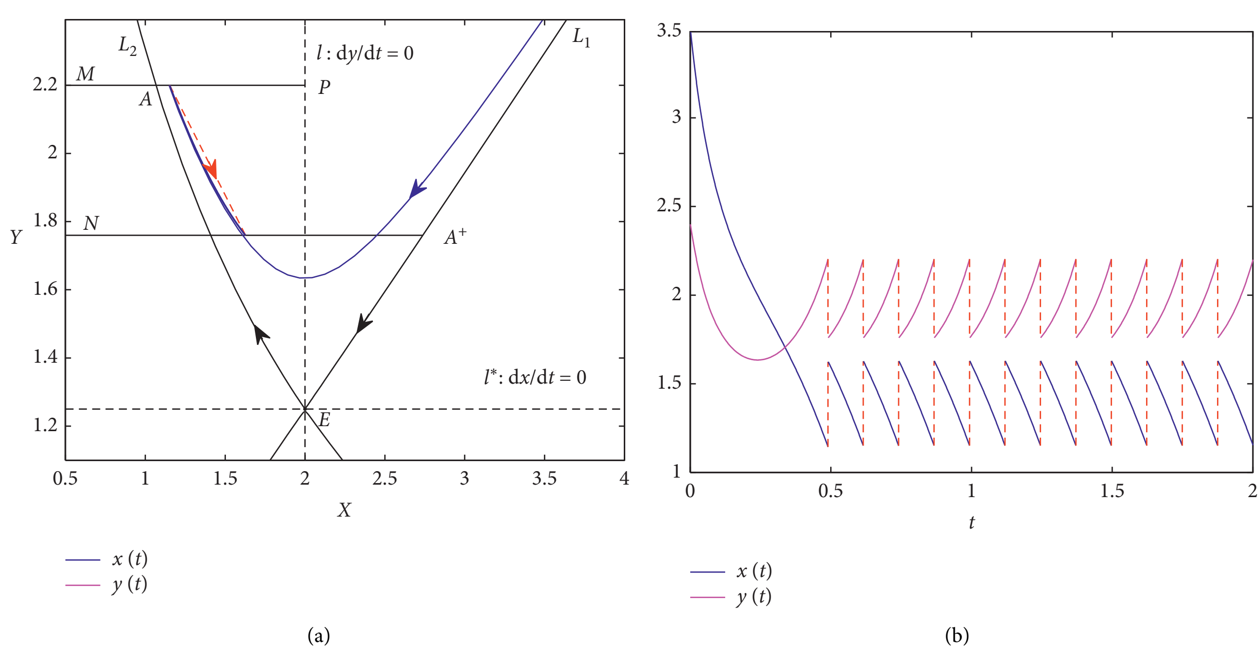

Figure 11: Continued.

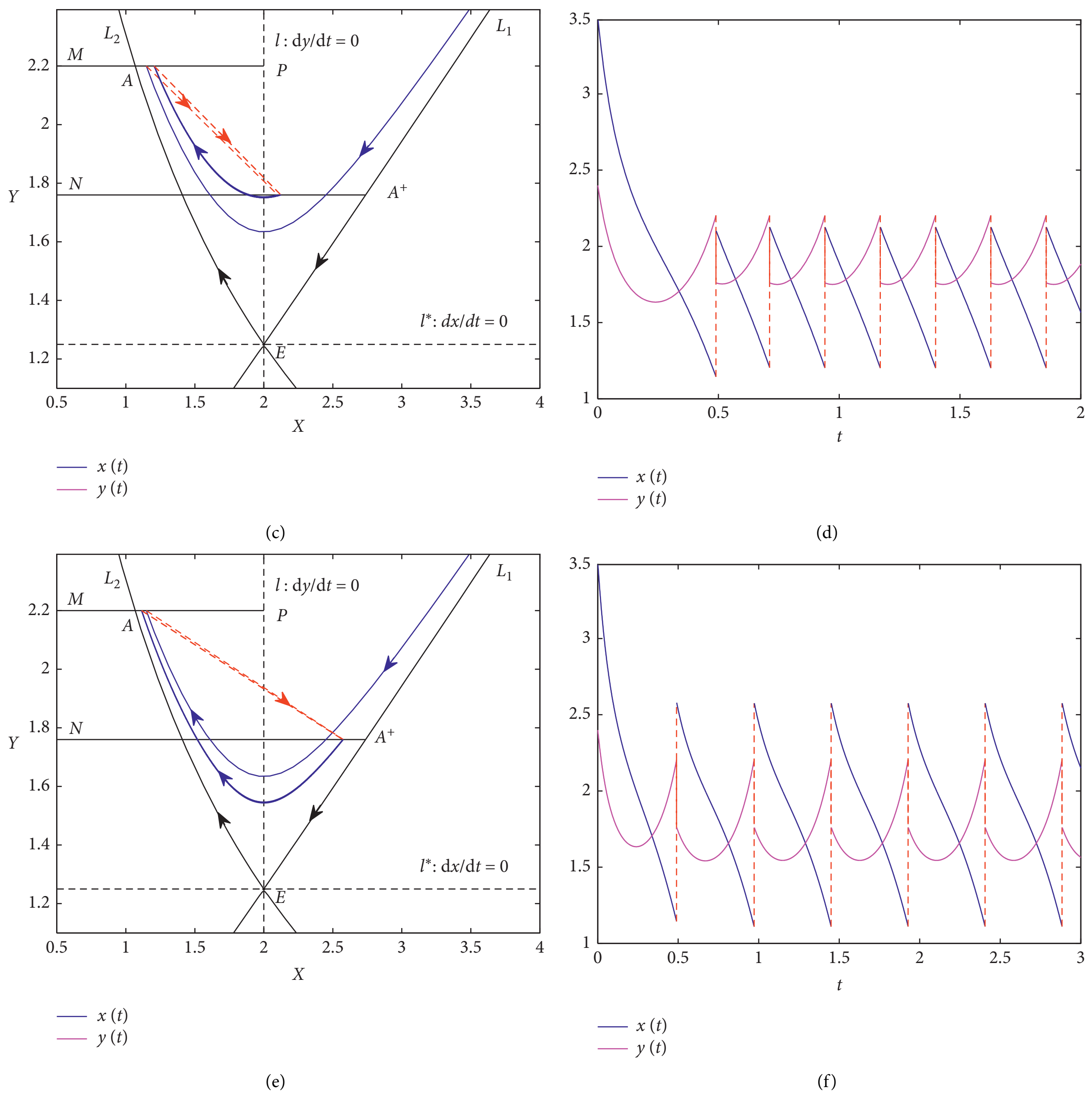

Figure 11: The order-1 periodic solution of model (2) and time series of x and y with parameters $a = 5, c = 4, d = 6, e = 3, h = 2.2$, and $\beta = 0.2$ and initial values $(3.5, 2.4)$. (a, b) $\alpha = 0.3$. (c, d) $\alpha = 0.6$. (e, f) $\alpha = 0.9$. (a), (c), and (e) show the order-1 periodic solution, while (b), (d), and (f) exhibit the time series of variables x and y.

When $\overline{\alpha} < \alpha < 1$, there exists order-1 periodic solution in model (2), and Figure 10 shows the specific situation. It means that removing some invasive species and supplying a certain amount of local species can keep the local species and prevent it from extinction. In reality, when the local species is not quite easy to breed, proper amount investment of local species and removal of invasive ones will keep the two species into periodic circumstances. Both local species and invasive species will be kept, and their amounts appear to have a complex relation with each other. The single situation and the time series of local species x and invasive species y are shown in Figure 11. It is easy to find that both time period and amplitude increase with the growing of α based on the fixed β. This means that more supplement of artificial breeding of local species can make the local species steady when facing disturbance.

To different β, model (2) has an order-1 periodic solution for $\overline{\alpha} < \alpha < 1$. Figure 12 shows the order-1 periodic solutions of model (2) when $\beta = 0.3$.

Based on the theoretical and numerical results, we can conclude that killing invasive species and investing artificial breeding local species into circumstances can help the local ones resist the ecological invasion. Investing plenty of local species in one time can even support the local species win the

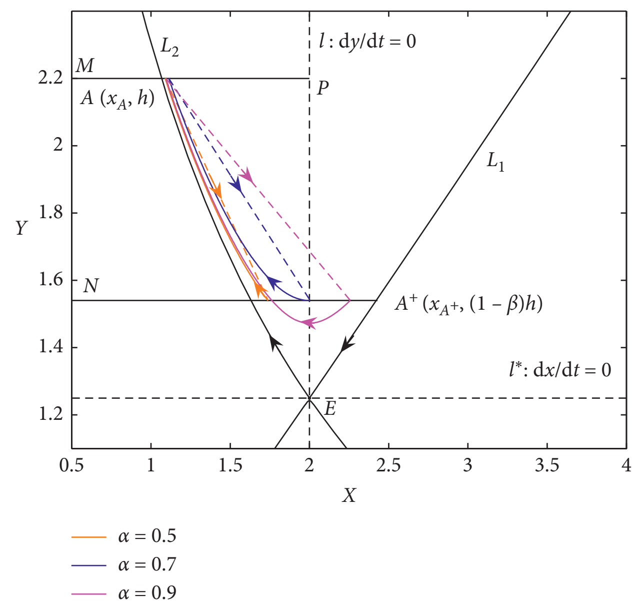

Figure 12: Existence of order-1 periodic solution when $\beta = 0.3$ with parameter values: $a = 5, c = 4, d = 6, e = 3$, and $h = 2.2$ and initial values (3.5, 2.4). The cases of $\alpha = 0.5$, $\alpha = 0.7$, and $\alpha = 0.9$ are shown with hazel, blue, and pink lines coordinately.

competition. If the supplement of local species is not enough, we should also replenish it as abundant as we can, since the more we invest, the stronger the local species will be.

Data Availability

No data were used to support the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] D. Simberloff, *Invasive Species: What Everyone Needs to Know*, Oxford University Press, Oxford, UK, 2013.

[2] F. Essl, S. Dullinger, W. Rabitsch et al., “Socioeconomic legacy yields an invasion debt,” *Proceedings of the National Academy of Sciences*, vol. 108, no. 1, pp. 203–207, 2011.

[3] H. Seebens, M. T. Gastner, and B. Blasius, “The risk of marine bioinvasion caused by global shipping,” *Ecology Letters*, vol. 16, no. 6, pp. 782–790, 2013.

[4] S. L. Chown, K. A. Hodgins, P. C. Griffin, J. G. Oakeshott, M. Byrne, and A. A. Hoffmann, “Biological invasions, climate change and genomics,” *Evolutionary Applications*, vol. 8, no. 1, pp. 23–46, 2015.

[5] P. Peter and D. M. Richardson, “Invasive species, environmental change and management, and ecosystem health,” *Annual Review of Environment and Resources*, vol. 35, pp. 25–55, 2010.

[6] A. I. Queiroz and P. Simon, *Histories of Bioinvasions in the Mediterranean (Environmental history Series*, Springer, Cham, Switzerland, 2018.

[7] D. Simberloff, J.-L. Martin, P. Genovesi et al., “Impacts of biological invasions: what’s what and the way forward,” *Trends in Ecology & Evolution*, vol. 28, no. 1, pp. 58–66, 2013.

[8] N.-M. Gabriela and S. Fei, “Understanding macroscale patterns and processes of invasion. theses and dissertations available from ProQuest,” 2018.

[9] C. Hui and D. M. Richardson, *Invasion dynamics*, Oxford University Press, Oxford, UK, 2017.

[10] J. A. Lau and C. P. Terhorst, “Causes and consequences of failed adaptation to biological invasions: the role of ecological constraints,” *Molecular Ecology*, vol. 24, no. 9, pp. 1987–1998, 2015.

[11] L. Chen, X. Liang, and Y. Pei, “The periodic solutions of the impulsive state feedback dynamical system,” *Communications in Mathematical Biology and Neuroscience*, vol. 2018, Article ID 14, 2018.

[12] M. Zhang, G. Song, and L. Chen, “A state feedback impulse model for computer worm control,” *Nonlinear Dynamics*, vol. 85, no. 3, pp. 1561–1569, 2016.

[13] C. Wei and L. Chen, “Heteroclinic bifurcations of a prey-predator fishery model with impulsive harvesting,” *International Journal of Biomathematics*, vol. 06, no. 05, Article ID 1350031, 2013.

[14] C. Wei and L. Chen, “Dynamic analysis of mathematical model of ethanol fermentation with gas stripping,” *Nonlinear Dynamics*, vol. 57, no. 1-2, pp. 13–23, 2009.

[15] C. Wei and L. Chen, “Periodic solution of prey-predator model with beddington-DeAngelis functional response and impulsive state feedback control,” *Journal of Applied Mathematics*, vol. 2012, Article ID 607105, 2012.

[16] T. Zhang, N. Gao, J. Wang et al., “Dynamic system of microbial culture described bu impulsive differential equations,” *Mathematical Modelling and Its Application*, vol. 8, no. 01, pp. 1–13, 2019.

[17] S. Huang, “Modeling and solution to the problem of optimal epidemic control: a review,” *Mathematical Modelling and Its Application*, vol. 8, no. 03, pp. 1–36, 2019.

[18] C. Wei and L. Chen, “Periodic solution and heteroclinic bifurcation in a predator-prey system with allee effect and impulsive harvesting,” *Nonlinear Dynamics*, vol. 76, no. 2, pp. 1109–1117, 2014.

[19] H. Gritli and S. Belghith, “Walking dynamics of the passive compass-gait model under OGY-based state-feedback control: analysis of local bifurcations via the hybrid poincaré map,” *Chaos, Solitons & Fractals*, vol. 98, pp. 72–87, 2017.

[20] H. Gritli and S. Belghith, “Diversity in the nonlinear dynamic behavior of a one-degree-of-freedom impact mechanical oscillator under OGY-based state-feedback control law: order, chaos and exhibition of the border-collision bifurcation,” *Mechanism and Machine Theory*, vol. 124, pp. 1–41, 2018.

[21] G. Pang and L. Chen, “Periodic solution of the system with impulsive state feedback control,” *Nonlinear Dynamics*, vol. 78, no. 1, pp. 743–753, 2014.

[22] M. Sun, Y. Liu, S. Liu, Z. Hu, and L. Chen, “A novel method for analyzing the stability of periodic solution of impulsive state feedback model,” *Applied Mathematics and Computation*, vol. 273, pp. 425–434, 2016.

[23] H. Guo, L. Chen, and X. Song, "Qualitative analysis of impulsive state feedback control to an algae-fish system with bistable property," *Applied Mathematics and Computation*, vol. 271, pp. 905–922, 2015.

[24] B. Liu, Y. Tian, and B. Kang, "Dynamics on a holling ii predator-prey model with state-dependent impulsive control," *International Journal of Biomathematics*, vol. 05, no. 03, Article ID 1260006, 2012.

[25] Y. Tian, K. Sun, and L. Chen, "Geometric approach to the stability analysis of the periodic solution in a semi-continuous dynamic system," *International Journal of Biomathematics*, vol. 07, no. 02, Article ID 1450018, 2014.

[26] C. Wei and L. Chen, "Homoclinic bifurcation of prey-predator model with impulsive state feedback control," *Applied Mathematics and Computation*, vol. 237, pp. 282–292, 2014.

[27] Y. Ye, *Limit Cycle theory*, Shanghai Science and Technology Press, Shanghai, China, 1984, in Chinese.