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Research Article

The Diffusive Model for West Nile Virus on a Periodically Evolving Domain

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In this paper, we investigate the impact of a periodically evolving domain on the dynamics of the diffusive West Nile virus. A reaction-diffusion model on a periodically and isotropically evolving domain which describes the transmission of the West Nile virus is proposed. In addition to the classical basic reproduction number, the spatial-temporal basic reproduction number depending on the periodic evolution rate is introduced and its properties are discussed. Under some conditions, we explore the long-time behavior of the virus. The virus will go extinct if the spatial-temporal basic reproduction number is less than or equal to one. The persistence of the virus happens if the spatial-temporal basic reproduction number is greater than one. We consider special case when the periodic evolution rate is equivalent to one to better understand the impact of the periodic evolution rate on the persistence or extinction of the virus. Some numerical simulations are performed in order to illustrate our analytical results. Our theoretical analysis and numerical simulations show that the periodic change of the habitat range plays an important role in the West Nile virus transmission, in particular, the increase of periodic evolution rate has positive effect on the spread of the virus.

1. Introduction

West Nile virus (WNv) is an arbovirus with natural transmission cycle between mosquitoes and birds. When infected mosquitoes bite birds or other animals including humans, they transmit the virus [1]. WNv is different from other arbovirus since it involves a cross infection between birds (hosts) and mosquitoes (vector) and those birds might travel with spatial boundaries. Also, WNv can be passed via vertical transmission from mosquito to its offspring which increases the survival of the virus [2]. WNv was first isolated and identified in 1937 from the blood of a febrile woman in the West Nile province of Uganda during research on yellow fever virus [3]. It is worth mentioning that WNv is endemic in some temperate and tropical regions such as Africa and the Middle East; it has now spread to North America; the first epidemic case was introduced in New York City in 1999 and then propagated across the USA [4–6]. The USA had experienced one of its worst epidemics in 2012; there were 5387 cases of infections in humans [7]. As we know, there are no indications that the spread of the virus has stopped. Consequently, it is very necessary to acquire some insights into the propagation of WNv in the mosquito-bird population.

Mathematical nonspatial models have been proposed and analyzed in an attempt to study the transmission dynamics of WNv, in order to elucidate control strategies [2, 6, 8, 9]. It is essential to study and understand its temporal and spatial spread, but most of the models are focused on the nonspatial transmission dynamics of the virus between birds and mosquitoes.

With respect to spatial models of WNv, Lewis et al. [4] studied the spatial spread of WNv to describe the movement of birds and mosquitoes, established the existence of travelling waves, and calculated the spatial spreading rate of the infection. The effects of vertical transmission in the spatial dynamics of the virus for different bird species were proposed by Maidana and Yang in [10], and they studied the travelling wave solutions of the model to determine the speed of virus dissemination. Liu et al. [11] presented the directional dispersal of birds and impact on spatial spreading of WNv. Likewise, Lin and Zhu studied spatial spreading model and dynamics of WNv in birds and mosquitoes with free boundary [12].

To investigate the existence of travelling wave and calculate the spatial spread rate of infection, Lewis et al. in [4] proposed the following simplified WNv model:

$$\begin{cases} \dfrac{\partial I_b}{\partial t} = D_1 \Delta I_b + \alpha_b \beta_b \dfrac{(N_b - I_b)}{N_b} I_m - \gamma_b I_b, & (x,t) \in \Omega \times (0,+\infty), \\ \dfrac{\partial I_m}{\partial t} = D_2 \Delta I_m + \alpha_m \beta_b \dfrac{(A_m - I_m)}{N_b} I_b - d_m I_m, & (x,t) \in \Omega \times (0,+\infty), \end{cases} \tag{1}$$

where the positive constants N_b and A_m denote the total population of birds and adult mosquitoes; $I_b(x,t)$ and $I_m(x,t)$ represent the populations of infected birds and mosquitoes at the location x in the habitat $\Omega \subset \mathbb{R}^N$ and at time $t \geq 0$, respectively, and $I_b(x,0) + I_m(x,0) > 0$. The parameters in the above system are defined as follows:

(i) α_m, α_b: WNv transmission probability per bite to mosquitoes and birds, respectively

(ii) β_b: biting rate of mosquitoes on birds

(iii) d_m: adult mosquito death rate

(iv) γ_b: bird recovery rate from WNv

(v) D_1, D_2: diffusion coefficients for birds and mosquitoes, respectively

As in [13], throughout this paper, we assume that the mosquitoes' population does not diffuse ($D_2 = 0$).

For the corresponding spatially independent model of (1), the basic reproduction number is

$$R_0 = \sqrt{\frac{\alpha_m \alpha_b \beta_b^2 A_m}{d_m \gamma_b N_b}}, \tag{2}$$

such that for $0 < R_0 < 1$, the virus always vanishes, while for $R_0 > 1$, a nontrivial epidemic level appears, which is globally asymptotically stable in the positive quadrant [4]. As pointed out in [14], the basic reproduction number R_0 is a very important concept in epidemiology and it defined as an expected number of secondary cases produced by a typical infected individual during its entire period of infectiousness in a completely susceptible population, and mathematically it is introduced as the dominant eigenvalue of a positive linear operator. It is important to mention that usually the basic reproduction numbers for the nonspatial models are calculated by the next generation matrix method [15], while for the spatially dependent systems, the numbers could expressed in terms of the principal eigenvalue of related eigenvalue problem [16] or the spectral radius of next infection operator [17].

The dynamics of the spatial dependence model (1) has been studied. The existence and nonexistence of the coexistence states in a heterogeneous environment have been investigated in [18]. The impact of the environmental heterogeneity and seasonal periodicity on the transmission of WNv was considered in [19].

In recent years, the impact of change of the habitat range on biological population has attracted much attention. We know there are two aspects: one is the domain changing with unknown boundary, which describes the domain change induced by the activity of population itself, and the other is the domain changing with known boundary, which characterizes the domain change induced by objective environments. For the domain changing with unknown boundary, many researchers have proposed and considered the free boundary problem, for example, [20–23] for the persistence of invasive species and [24, 25] for the transmission of diseases. In addition, Tarboush et al. [13] discussed the corresponding free boundary problem to model (1). Wang et al. investigated the spreading speed for a WNv model with free boundary in a homogeneous environment [26]. Their results indicated that the asymptotic spreading speed of the WNv model with free boundary is strictly less than that of the corresponding model in Lewis et al. [4]. For the domain changing with known boundary, there are also some papers, for instance, [27–30] for a growing domain and [31–34] for a periodically evolving domain. In [31], the authors introduced the periodically evolving domain into a single-species diffusion logistic model and studied the influence of periodic evolution on the survival and extinction of species. Recently, Zhang and Lin considered the diffusive model for *Aedes aegypti* mosquito on a periodically evolving domain in order to explore the diffusive dynamics of *Aedes aegypti* mosquito [32]. Their results indicated that the periodic domain evolution has a significant impact on the dispersal of *Aedes aegypti* mosquito. To investigate the impact of periodically evolving domain on the mutualism interaction of two species, Adam et al. [33] studied a mutualistic model on the periodic evolving domain. They suggested that the periodic evolution of domain places significant influence on the interaction of two species. Zhu et al. [34] used a periodic evolving domain to investigate the gradual transmission of a dengue fever model. They found that the periodic domain evolution has a significant effect on the transmission of dengue.

In this paper, we will consider the impact of the periodic evolving domain on the dynamics of a diffusive WNv model corresponding to system (1). We followed the methods of Adam et al. [33], Zhang and Lin [32], and Zhu et al. [34].

The rest of this paper is organized as follows. We will present the formulation of our problem in Section 2. In Section 3, we introduce the spatial-temporal basic reproduction number and present its properties. The existence and nonexistence of the periodic solutions on a periodically evolving domain Ω_t are discussed in Section 4. Section 5 is devoted to the attractivity of periodic solutions on a periodically evolving domain Ω_t. In Section 6, we deal with the existence, nonexistence, and attractivity of the periodic solutions on a fixed domain Ω_0. Some numerical simulations are given in Section 7. Section 8 provides some conclusions.

2. Model Formulation

Motivated by [27], we let $\Omega_t \subset \mathscr{R}^n\,(n \geq 1)$ be a periodically evolving domain and $\partial\Omega_t$ be the evolving boundary. For any point, $x(t) \in \Omega_t$ satisfies $x(t+T) = x(t)$ for some positive constant T. Also, we assume that the domain Ω_t grows uniformly and isotropically, that is,

$$x(t) = \rho(t), \quad \text{for all } x(t) \in \Omega_t \text{ and } (y,t) \in \Omega_0 \times [0,T], \tag{3}$$

where $\rho(t) \in C^1[[0,T];(0,\infty)]$ and y represents the spatial coordinates of the initial domain Ω_0. Moreover, $\rho(t)$ is T-periodic in time, i.e., $\rho(t+T) = \rho(t)$, $\rho(0) = 1$ and $\dot\rho(t) \geq 0$ for $t > 0$.

According to the principle of mass conservation and Reynolds transport theorem [35], in this paper, we will focus on the following problem:

$$\begin{cases} \dfrac{\partial I_b}{\partial t} + \mathbf{a}\cdot\nabla I_b + I_b(\nabla\cdot\mathbf{a}) = D_1\Delta I_b + \alpha_b(x(t),t)\beta_b(x(t),t)\dfrac{(N_b - I_b)}{N_b}I_m - \gamma_b(x(t),t)I_b, & \text{in } \Omega_t, \\ \dfrac{\partial I_m}{\partial t} + \mathbf{a}\cdot\nabla I_m + I_m(\nabla\cdot\mathbf{a}) = \alpha_m(x(t),t)\beta_b(x(t),t)\dfrac{(A_m - I_m)}{N_b}I_b - d_m(x(t),t)I_m, & \text{in } \Omega_t, \\ I_b(x(t),t) = I_m(x(t),t) = 0, & \text{on } \partial\Omega_t, \\ I_b = I_{b,0}(x),\ I_m = I_{m,0}(x), & \text{in } \Omega_0, \end{cases} \tag{4}$$

where $I_b(x(t),t)$ and $I_m(x(t),t)$ represent the densities of infected birds and mosquitoes at position $x(t) \in \Omega_t$ and time t, respectively, and $I_{b,0}(x)$ and $I_{m,0}(x)$ are positive smooth functions in Ω_0. The functions $\alpha_b(x(t),t)$, $\beta_b(x(t),t)$, $\gamma_b(x(t),t)$, $\alpha_m(x(t),t)$, and $d_m(x(t),t)$ are all sufficiently smooth, T-periodic, and strictly positive when $t \geq 0$. According to [36, 37], the evolution of domain Ω_t generates a flow velocity $\mathbf{a}(x(t),t)$. In addition, the evolving domain Ω_t represents two kinds of extra terms into the problem, one of which is the advection terms $\nabla I_b\cdot\mathbf{a}$ and $\nabla I_m\cdot\mathbf{a}$ representing the transport of material around Ω_t at a rate determined by the flow $\mathbf{a}$, and the other is the dilution terms $(\nabla\cdot\mathbf{a})I_b$ and $(\nabla\cdot\mathbf{a})I_m$ due to local volume expansion.

Since problem (4) is involving the terms of advection and dilution, it is not easy to study the long-time behavior of its solutions; therefore, we will transform the continuously deforming domain in problem (4) to a fixed domain by using Lagrangian transformations [29, 36].

We suppose that $\mathbf{a}(x(t),t) = \dot x(t)$ is the flow velocity, which is identical to the domain velocity.

This means that $\mathbf{a} = \dot x(t) = \dot\rho(t)y = (\dot\rho(t)/\rho(t))x(t)$.

Define

$$\begin{aligned} I_b(x(t),t) &= u(y,t), \\ I_m(x(t),t) &= v(y,t), \end{aligned} \tag{5}$$

and assume

$$\begin{aligned} &\alpha_b(x(t),t) \equiv \alpha_b(y,t), \alpha_m[x(t),t] \equiv \alpha_m(y,t), \\ &\beta_b(x(t),t) \equiv \beta_b(y,t), \\ &\gamma_b(x(t),t) \equiv \gamma_b(y,t), d_m(x(t),t) \equiv d_m(y,t). \end{aligned} \tag{6}$$

Therefore, problem (4) is transformed to

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{D_1(y,t)}{\rho^2(t)}\Delta u + \dfrac{n\dot\rho(t)}{\rho(t)}u = \alpha_b(y,t)\beta_b(y,t)\dfrac{(N_b - u)}{N_b}v - \gamma_b(y,t)u, & y \in \Omega_0, t > 0, \\ \dfrac{\partial v}{\partial t} + \dfrac{n\dot\rho(t)}{\rho(t)}v = \alpha_m(y,t)\beta_b(y,t)\dfrac{(A_m - v)}{N_b}u - d_m(y,t)v, & y \in \Omega_0, t > 0, \\ u(y,t) = v(y,t) = 0, & y \in \partial\Omega_0, t > 0, \end{cases} \tag{7}$$

with the periodic condition

$$\begin{aligned} u(y,0) &= u(y,T), \\ v(y,0) &= v(y,T), \\ & \qquad y \in \Omega_0, \end{aligned} \tag{8}$$

and under the initial condition

$$\begin{aligned} u(y,0) &= \eta_1(y) = I_{b,0}(y), \\ v(y,0) &= \eta_2(y) = I_{m,0}(y), \\ & \qquad y \in \Omega_0. \end{aligned} \tag{9}$$

Moreover, we assume that the functions $\alpha_b(y,t)$, $\alpha_m(y,t)$, $\beta_b(y,t)$, $\gamma_b(y,t)$, and $d_m(y,t) \in C^{\alpha,\alpha/2}(\overline{\Omega_0} \times [0,\infty))$ for some $\alpha \in (0,1)$; all are positive bounded in the sense that there exist constants $\alpha_b^*, \alpha_{b*}, \alpha_m^*, \alpha_{m*}, \beta_b^*, \beta_{b*}, \gamma_b^*, \gamma_{b*}, d_m^*$, and d_{m*} such that $\alpha_{b*} \le \alpha_b(y,t) \le \alpha_b^*$, $\alpha_{m*} \le \alpha_m(y,t) \le \alpha_m^*$, $\beta_{b*} \le \beta_b(y,t) \le \beta_b^*$, $\gamma_{b*} \le \gamma_b(y,t) \le \gamma_b^*$, and $d_{m*} \le d_m(y,t) \le d_m^*$. Furthermore, $\alpha_b(y,t) = \alpha_b(y,t+T)$, $\alpha_m(y,t) = \alpha_m(y,t+T)$, $\beta_b(y,t) = \beta_b(y,t+T)$, $\gamma_b(y,t) = \gamma_b(y,t+T)$, and $d_m(y,t) = d_m(y,t+T)$ for all $t > 0$.

3. Spatial-Temporal Basic Reproduction Number

In this section, we will introduce the spatial-temporal basic reproduction number $R_0(\rho)$ and exhibit its properties. To address this, we consider the following linearized periodic reaction-diffusion problem:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta u = \alpha_b(y,t)\beta_b(y,t)v - \left(\gamma_b(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)u, & y \in \Omega_0, t > 0, \\ \dfrac{\partial v}{\partial t} = \alpha_m(y,t)\beta_b(y,t)\dfrac{A_m}{N_b}u - \left(d_m(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)v, & y \in \Omega_0, t > 0, \\ u(y,t) = v(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ u(y,0) = u(y,T), v(y,0) = v(y,T), & y \in \Omega_0. \end{cases} \tag{10}$$

Employing the ideas go back to [17, 32, 38], and we let C_T be the ordered Banach space consisting of all T-periodic and continuous functions from R to $C(\overline{\Omega}_0, R)\|\cdot\|$ with the maximum norm $C_T^+ = \{\eta \in C_T : \eta(t)y \ge 0, \text{ for all } t \in R, y \in \overline{\Omega}_0\}$ and the positive cone $\eta \in C_T$. For any given $\eta(y,t) = \eta(t)y$, we have $m = (m_1, m_2) \in C_T \times C_T$. Next, we suppose that is the density distribution at the spatial location $y \in \Omega_0$ and time s and let $\{\Phi(t,s), t \ge s\}$ be the evolution family determined by

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta u = -\left(\gamma_b(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)u, & y \in \Omega_0, t > 0, \\ \dfrac{\partial v}{\partial t} = -\left(d_m(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)v, & y \in \Omega_0, t > 0, \\ u(y,t) = v(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ u(y,0) = u(y,T), \ v(y,0) = v(y,T), & y \in \Omega_0. \end{cases} \tag{11}$$

Define the operator $G(t)$ by

$$G(t)\phi = [G_1(t)\phi_2, G_2(t)\phi_1], \quad \text{for all } \phi \in C_T \times C_T, \ t > 0, \tag{12}$$

where

$$\begin{aligned} G_1(t)\phi_2 &= \alpha_b(\cdot,t)\beta_b(\cdot,t)\phi_2, \\ G_2(t)\phi_1 &= \alpha_m(\cdot,t)\beta_b(\cdot,t)\frac{A_m}{N_b}\phi_1. \end{aligned} \tag{13}$$

Now under the same boundary conditions in problem (11), we let $\{\Phi_1(t,s), t \ge s\}$ and $\{\Phi_2(t,s), t \ge s\}$ be the evolution families determined by the first equation and second equation in problem (11), respectively. Moreover, let A and B be two bounded linear operator defined by

$$\begin{aligned} Am &= (A_1 m_1, A_2 m_2), \\ Bm &= (B_1 m_2, B_2 m_1), \end{aligned} \tag{14}$$

for $m \in C_T \times C_T$, where $[A_1 m_1](t) = \int_0^\infty \Phi_1(t,t-s)m_1(t-s)ds$ and $[A_2 m_2](t) = \int_0^\infty \Phi_2(t,t-s)m_2(t-s)ds$, $[B_1 m_2] = G_1(t)m_2$, $[B_2 m_1] = G_2(t)m_1$, and define

$$Lm = ABm = (A_1 B_1 m_2, A_2 B_2 m_1). \tag{15}$$

Consequently, we define the spatial-temporal basic reproduction number of system (10), that is,

$$R_0(\rho) = r(L), \tag{16}$$

where $r(L)$ is spectral radius of the operator L.

With the above discussion, we have the following result (see [19, 32] for more details).

Lemma 1. *$sign[1 - R_0(\rho)] = sign(\lambda_0)$, where $R_0(\rho) = \mu_0$ is the principal eigenvalue of the eigenvalue problem*

$$\begin{cases} \dfrac{\partial\phi}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta\phi = \dfrac{\alpha_b(y,t)\beta_b(y,t)}{\mu}\psi - \left(\gamma_b(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)\phi, & y \in \Omega_0, t > 0, \\ \dfrac{\partial\psi}{\partial t} = \alpha_m(y,t)\beta_b(y,t)\dfrac{A_m}{N_b\mu}\phi - \left(d_m(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)\psi, & y \in \Omega_0, t > 0, \\ \phi(y,t) = \psi(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ \phi(y,0) = \phi(y,T),\ \psi(y,0) = \psi(y,T), & y \in \Omega_0, \end{cases} \tag{17}$$

and λ_0 is the principal eigenvalue of the eigenvalue problem

$$\begin{cases} \dfrac{\partial\phi}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta\phi = \alpha_b(y,t)\beta_b(y,t)\psi - \left[\gamma_b(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right]\phi + \lambda\phi, & y \in \Omega_0, t > 0, \\ \dfrac{\partial\psi}{\partial t} = \alpha_m(y,t)\beta_b(y,t)\dfrac{A_m}{N_b}\phi - \left(d_m(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right)\psi + \lambda\psi, & y \in \Omega_0, t > 0, \\ \phi(y,t) = \psi(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ \phi(y,0) = \phi(y,T), \psi(y,0) = \psi(y,T), & y \in \Omega_0. \end{cases} \tag{18}$$

In what follows, we will present the properties of the spatial-temporal basic reproduction number $R_0(\rho)$, that is, μ_0. Note that problem (17) is degenerate, so we will not able to derive the existence of the principal eigenvalue by using Krein–Rutman theorem [39] because of the lack of compactness for the solution semigroup. Therefore, we first transform the equations of (17) to one equation. To achieve this, let

$$\begin{aligned} g_1(y,t) &= \gamma_b(y,t) + \frac{n\dot{\rho}(t)}{\rho(t)}, \\ g_2(y,t) &= d_m(y,t) + \frac{n\dot{\rho}(t)}{\rho(t)}, \end{aligned} \tag{19}$$

and then the second equation of (17) can be written as

$$\frac{\partial\psi}{\partial t} = \alpha_m\beta_b(y,t)\frac{A_m}{N_b\mu}\phi - g_2(y,t)\psi, \tag{20}$$

which gives rise to

$$\left(e^{\int_0^t g_2(y,s)\mathrm{d}s}\psi\right)_t = \alpha_m\beta_b(y,t)\frac{A_m}{N_b\mu}\phi e^{\int_0^t g_2(y,s)\mathrm{d}s}. \tag{21}$$

Together with the periodic condition $\psi(y,0) = \psi(y,T)$, direct computations yield

$$\begin{aligned} \psi(y,t) &= \frac{e^{-\int_0^t g_2(y,s)\mathrm{d}s}}{1 - e^{-\int_0^t g_2(y,s)\mathrm{d}s}}\int_0^T \alpha_m\beta_b(y,\tau)\frac{A_m}{N_b\mu}\phi(y,\tau)e^{-\int_T^\tau g_2(y,s)\mathrm{d}s} \\ &\quad + \int_0^t \alpha_m\beta_b(y,\tau)\frac{A_m}{N_b\mu}\phi(y,\tau)e^{-\int_t^\tau g_2(y,s)\mathrm{d}s} \\ &:= \frac{1}{\mu}G[\phi(y,t)]. \end{aligned} \tag{22}$$

Here, one can easily see that the function $G(\phi(y,t))$ is monotonically nondecreasing with respect to $\beta_b(y,t)$ and decreasing with respect to $d_m(y,t)$. Hence, problem (17) reduces to the following form:

$$\begin{cases} \dfrac{\partial\phi}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta\phi = \dfrac{\alpha_b(y,t)\beta_b(y,t)}{\mu^2}G[\phi(y,t)] - \left[\gamma_b(y,t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right]\phi, & y \in \Omega_0, t > 0, \\ \phi(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ \phi(y,0) = \phi(y,T), & y \in \Omega_0. \end{cases} \tag{23}$$

By employing the integration by parts to problem (23) and denoting the principal eigenvalue of (17) as μ_0, we obtain

$$\mu_0 \leq \sup_{\phi \in G_1, \phi \neq 0} \left\{ \sqrt{\frac{\int_0^T \int_{\Omega_0} \alpha_m(y,t)\beta_b(y,t) A_m/N_b G(\phi) \mathrm{d}y\mathrm{d}t}{\int_0^T \int_{\Omega_0} D_1/\rho^2(t)|\nabla\phi|^2 \mathrm{d}y\mathrm{d}t + \int_0^T \int_{\Omega_0} g_1(y,t)\phi^2 \mathrm{d}y\mathrm{d}t}} \right\}, \tag{24}$$

where

$$G_1 = \left\{\phi \in \mathscr{C}^{2+\alpha,1+(\alpha/2)}\left(\overline{\Omega_0} \times [0,+\infty)\right): \ \phi(y,t) = 0, \text{ for } y \in \partial\Omega_0, \ t \in [0,+\infty), \ \phi \text{ is T} - \text{periodicin } t\right\}. \tag{25}$$

For formula (24), in some special cases, we can replace the sign of inequality ($\leq$) by equality sign by using the variational methods [40–42].

In order to give the relationship between the spatial-temporal basic reproduction number $R_0(\rho)$ and the periodic evolution rate $\rho(t)$, we adopt the notation $\overline{\rho^{-2}} = 1/T \int_0^T (1/\rho^2(t))\mathrm{d}t$. λ^* is the principal eigenvalue of the following eigenvalue problem:

$$\begin{cases} -\Delta\varphi = \lambda^*\varphi, & y \in \Omega_0, \\ \varphi = 0, & y \in \partial\Omega_0. \end{cases} \tag{26}$$

Consequently, we have the following result.

Theorem 1. *The following assertions are valid:*

(a) *If $\alpha_b(y,t) = \alpha_b(t)$, $\alpha_m(y,t) = \alpha_m(t)$, $\beta_b(y,t) = \beta_b(t)$, $\gamma_b(y,t) = \gamma_b(t)$, and $d_m(y,t) = d_m(t)$, then the principal eigenvalue $R_0(\rho)$ for (17) is expressed by*

$$R_0(\rho) \geq \sqrt{\frac{\left[1/T \int_0^T \sqrt{\alpha_b(t)\alpha_m(t)\beta_b^2(t)(A_m/N_b)\mathrm{d}t}\right]^2}{1/T \int_0^T d_m(t)\mathrm{d}t \left[1/T \int_0^T \gamma_b(t)\mathrm{d}t + \lambda^* D_1 \overline{\rho^{-2}}\right]}}. \tag{27}$$

(b) *Moreover, if $\alpha_b(y,t) = \alpha_b^*$, $\alpha_m(y,t) = \alpha_m^*$, $\beta_b(y,t) = \beta_b^*$, $\gamma_b(y,t) = \gamma_b^*$, and $d_m(y,t) = d_m^*$, then we have*

$$R_0(\rho) \geq \sqrt{\frac{A_m \alpha_b^* (\beta_b^*)^2 \alpha_m^*}{N_b d_m^* \left[D_1 \lambda^* \overline{\rho^{-2}} + \gamma_b^*\right]}}. \tag{28}$$

Furthermore,

$$R_0(1) = \sqrt{\frac{A_m \alpha_b^* (\beta_b^*)^2 \alpha_m^*}{N_b d_m^* \left[D_1 \lambda^* + \gamma_b^*\right]}}, \tag{29}$$

in the sense that $\rho(t) = 1$.

Proof. Let

$$\begin{aligned} \phi(y,t) &= p(t)\varphi(y), \\ \psi(y,t) &= q(t)\varphi(y), \\ &(y,t) \in \Omega_0 \times (0,\infty), \end{aligned} \tag{30}$$

where $p(t)$ and $q(t)$ are functions to be determined later and $[\lambda^*, \varphi(y)]$ is the principal eigenpair of the eigenvalue problem

$$\begin{cases} -\Delta\varphi = \lambda^*\varphi, & y \in \Omega_0, \\ \varphi = 0, & y \in \partial\Omega_0. \end{cases} \tag{31}$$

Together with (17), we obtain

$$\begin{cases} \dfrac{\mathrm{d}p(t)}{\mathrm{d}t} = \dfrac{\alpha_b(t)\beta_b(t)}{R_0(\rho)} q(t) - \left(\dfrac{D_1\lambda^*}{\rho^2(t)} + \gamma_b(t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right) p(t), \\ \dfrac{\mathrm{d}q(t)}{\mathrm{d}t} = \alpha_m(t)\beta_b(t) \dfrac{A_m}{N_b R_0(\rho)} p(t) - \left(d_m(t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right) q(t), \end{cases} \tag{32}$$

where

$$\begin{aligned} p(t) &= p(t+T), \\ q(t) &= q(t \\ t &= [0,\infty), \end{aligned} \tag{33}$$

and $(R_0(\rho); p(t)\varphi(y), q(t)\varphi(y))$ is the unique principal eigenpair of problem (17).

Rewriting (32) as

$$\begin{cases} \dfrac{1}{p(t)} \dfrac{\mathrm{d}p(t)}{\mathrm{d}t} = \dfrac{\alpha_b(t)\beta_b(t)}{R_0(\rho)} \dfrac{q(t)}{p(t)} - \left(\dfrac{D_1\lambda^*}{\rho^2(t)} + \gamma_b(t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right), \\ \dfrac{1}{q(t)} \dfrac{\mathrm{d}q(t)}{\mathrm{d}t} = \alpha_m(t)\beta_b(t) \dfrac{A_m}{N_b R_0(\rho)} \dfrac{p(t)}{q(t)} - \left(d_m(t) + \dfrac{n\dot{\rho}(t)}{\rho(t)}\right), \end{cases} \tag{34}$$

and integrating from 0 to T yield

$$\begin{cases} \dfrac{1}{R_0(\rho)} \displaystyle\int_0^T \alpha_b(t)\beta_b(t)\frac{q(t)}{p(t)}\mathrm{d}t = \int_0^T \left(\frac{D_1\lambda^*}{\rho^2(t)} + \gamma_b(t) + \frac{n\dot{\rho}(t)}{\rho(t)}\right)\mathrm{d}t, \\ \dfrac{A_m}{N_b R_0(\rho)} \displaystyle\int_0^T \alpha_m(t)\beta_b(t)\frac{p(t)}{q(t)}\mathrm{d}t = \int_0^T \left(d_m(t) + \frac{n\dot{\rho}(t)}{\rho(t)}\right)\mathrm{d}t. \end{cases} \tag{35}$$

By using Hölder inequality, one can easily obtain that

$$R_0(\rho)^2 \geq \frac{\left((1/T)\int_0^T \sqrt{\alpha_b(t)\alpha_m(t)\beta_b^2(t)(A_m/N_b)}\,\mathrm{d}t\right)^2}{(1/T)\int_0^T d_m(t)\mathrm{d}t\left((1/T)\int_0^T \gamma_b(t)\mathrm{d}t + \lambda^* D_1\overline{\rho^{-2}}\right)}. \tag{36}$$

The proof of assertion (a) is completed.

For assertion (b), since we assumed that all coefficients are constants, we can get

$$R_0(\rho) \geq \sqrt{\frac{A_m\alpha_b^*(\beta_b^*)^2\alpha_m^*}{N_b d_m^*\left[D_1\lambda^*\overline{\rho^{-2}} + \gamma_b^*\right]}}, \tag{37}$$

directly from (27).

To prove the rest of assertion (b), let $\rho(t) \equiv 1$, that is, $\Omega_t = \Omega_0$ is a fixed domain and rewrite (32) as

$$\begin{cases} \dfrac{\mathrm{d}p(t)}{\mathrm{d}t} = \dfrac{\alpha_b^*\beta_b^*}{R_0(1)}q(t) - (D_1\lambda^* + \gamma_b^*)p(t), \\ \dfrac{\mathrm{d}q(t)}{\mathrm{d}t} = \alpha_m^*\beta_b^*\dfrac{A_m}{N_b R_0(1)}p(t) - d_m^* q(t), \end{cases} \tag{38}$$

which are explicitly given by

$$\begin{cases} p(t) = e^{\left(-D_1\lambda^* - \gamma_b^* + \left(C\alpha_b^*\beta_b^*/R_0(1)\right)\right)t}, \\ q(t) = Ce^{\left(-D_1\lambda^* - \gamma_b^* + \left(C\alpha_b^*\beta_b^*/R_0(1)\right)\right)t}, \end{cases} \tag{39}$$

with

$$C = \frac{-d_m^* + D_1\lambda^* + \gamma_b^* + \sqrt{\left(d_m^* - D_1\lambda^* - \gamma_b^*\right)^2 + 4\left(A_m\alpha_b^*(\beta_b^*)^2\alpha_m^*/N_b R_0^2(1)\right)}}{2\left(\alpha_b^*\beta_b^*/R_0(1)\right)}. \tag{40}$$

According to (38), direct computations yield

$$R_0(1) = \sqrt{\frac{A_m\alpha_b^*(\beta_b^*)^2\alpha_m^*}{N_b d_m^*\left[D_1\lambda^* + \gamma_b^*\right]}}, \tag{41}$$

which is consistent with the result given from the variational method. □

4. Periodic Solutions on Evolving Domain

In this section, we discuss the existence and nonexistence of T-periodic solutions. To begin, we first consider the T-periodic boundary problem corresponding to (7) and (8):

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta u = f_1(t,u,v), & y \in \Omega_0, t > 0, \\ \dfrac{\partial v}{\partial t} = f_2(t,u,v), & y \in \Omega_0, t > 0, \\ u(y,t) = v(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ u(y,0) = u(y,T),\ v(y,0) = v(y,T), & y \in \overline{\Omega}_0, \end{cases} \tag{42}$$

where

$$\begin{aligned} f_1(t,u,v) &= \alpha_b(y,t)\beta_b(y,t)\frac{(N_b - u)}{N_b}v - \left(\gamma_b(y,t) + \frac{n\dot{\rho}(t)}{\rho(t)}\right)u, \\ f_2(t,u,v) &= \alpha_m(y,t)\beta_b(y,t)\frac{(A_m - v)}{N_b}u - \left(d_m(y,t) + \frac{n\dot{\rho}(t)}{\rho(t)}\right)v. \end{aligned} \tag{43}$$

For later analysis, we give the following definition of upper and lower solutions.

Definition 1. A pair of functions $(\tilde{u},\tilde{v})$, $(\hat{u},\hat{v})$ in $\mathscr{C}^{2,1}[\Omega_0\times(0,\infty)\cap\mathscr{C}(\overline{\Omega}_0\times[0,\infty))]$ is called ordered upper and lower solutions of problem (42), if $(0,0)\le(\hat{u},\hat{v})\le(\tilde{u},\tilde{v})\le(N_b,A_m)$ and

$$\begin{cases} \dfrac{\partial\tilde{u}}{\partial t}-\dfrac{D_1}{\rho^2(t)}\Delta\tilde{u}\ge f_1(t,\tilde{u},\tilde{v}), & y\in\Omega_0, t>0,\\ \dfrac{\partial\tilde{v}}{\partial t}\ge f_2(t,\tilde{u},\tilde{v}), & y\in\Omega_0, t>0,\\ \dfrac{\partial\hat{u}}{\partial t}-\dfrac{D_1}{\rho^2(t)}\Delta\hat{u}\le f_1(t,\hat{u},\hat{v}), & y\in\Omega_0, t>0,\\ \dfrac{\partial\hat{v}}{\partial t}\le f_2(t,\hat{u},\hat{v}), & y\in\Omega_0, t>0,\\ \tilde{u}(y,t)\ge0\ge\hat{u}(y,t),\ \tilde{v}(y,t)\ge0\ge\hat{v}(y,t), & y\in\partial\Omega_0, t>0,\\ \tilde{u}(y,0)\ge\tilde{u}(y,T),\ \hat{u}(y,0)\le\hat{u}(y,T), & y\in\Omega_0,\\ \tilde{v}(y,0)\ge\tilde{v}(y,T),\ \hat{v}(y,0)\le\hat{v}(y,T), & y\in\Omega_0. \end{cases} \tag{44}$$

Now we are in a position to state the existence and nonexistence of T-periodic solutions to problem (42) as well as problems (7) and (8). To begin with, in the following result we give the existence of T-periodic solution.

Theorem 2. *If $R_0(\rho)>1$, then problem (42) admits at least one positive T-periodic solution $(u(y,t),v(y,t))$.*

Proof. Since $R_0(\rho)>1$, one can easily verify that $(\tilde{u},\tilde{v})=(N_b,A_m)$ and $(\hat{u},\hat{v})=(\delta\phi,\delta\psi)$ are ordered upper and lower solutions of problem (42), where δ is positive constant and small enough, $(\phi,\psi)\equiv(\phi(y,t),\psi(y,t))$ is (normalized) positive eigenfunction corresponding to λ_0, and λ_0 is the principal eigenvalue of periodic-parabolic eigenvalue problem (11) (for more details, see [19]).

To establish the nonexistence of a T-periodic solution to problem (42), we have the following result. □

Theorem 3. *If $R_0(\rho)\le1$, then problem (42) has no positive T-periodic solution.*

Proof. Suppose that $(u^*(y,t),v^*(y,t))$ is a positive T-periodic solution of problem (42), that is, $(u^*(y,t),v^*(y,t))>(0,0)$ in $\Omega_0\times(0,\infty)$ and satisfies

$$\begin{cases} \dfrac{\partial u^*}{\partial t}-\dfrac{D_1}{\rho^2(t)}\Delta u^*+\dfrac{n\dot{\rho}(t)}{\rho(t)}u^*=\alpha_b(y,t)\beta_b(y,t)\dfrac{(N_b-u^*)}{N_b}v^*-\gamma_b(y,t)u^*, & y\in\Omega_0, t>0,\\ \dfrac{\partial v^*}{\partial t}+\dfrac{n\dot{\rho}(t)}{\rho(t)}v^*=\alpha_m(y,t)\beta_b(y,t)\dfrac{(A_m-v^*)}{N_b}u^*-d_m(y,t)v^*, & y\in\Omega_0, t>0,\\ u^*(y,t)=v^*(y,t)=0, & y\in\partial\Omega_0, t>0,\\ u^*(y,0)=u^*(y,T),\ v^*(y,0)=v^*(y,T), & y\in\Omega_0. \end{cases} \tag{45}$$

From the above equations, we have

$$\begin{cases} \dfrac{\partial u^*}{\partial t}-\dfrac{D_1}{\rho^2(t)}\Delta u^*<\alpha_b(y,t)\beta_b(y,t)v^*-\left[\gamma_b(y,t)+\dfrac{n\dot{\rho}(t)}{\rho(t)}\right]u^*, & y\in\Omega_0, t>0,\\ \dfrac{\partial v^*}{\partial t}<\alpha_m(y,t)\beta_b(y,t)\dfrac{A_m}{N_b}u^*-\left(d_m(y,t)+\dfrac{n\dot{\rho}(t)}{\rho(t)}\right)v^*, & y\in\Omega_0, t>0,\\ u^*(y,t)=v^*(y,t)=0, & y\in\partial\Omega_0, t>0,\\ u^*(y,0)=u^*(y,T),\ v^*(y,0)=v^*(y,T), & y\in\Omega_0. \end{cases} \tag{46}$$

Recalling (17), one can easily deduce from the monotonicity of the principal eigenvalue $R_0(\rho)$ that $R_0(\rho) > 1$ by comparing (17) and (46), which contradicts the fact $R_0(\rho) \leq 1$. □

5. Attractivity of Periodic Solutions

In this section, we first construct the true solutions of problem (42) and then present the attractivity of T-periodic solutions to problems (8) and (9) in relation to the minimal and maximal T-periodic solution of problems (8) and (9). In what follows, we construct the true solutions of problem (42) by using the monotone iterative scheme. Let

$$
\begin{aligned}
k_1 &= \gamma_b^M + \alpha_b^M \beta_b^M \frac{A_m}{N_b} + n\left(\frac{\dot{\rho}(t)}{\rho(t)}\right)^M, \\
k_2 &= d_m^M + \alpha_m^M \beta_b^M + n\left(\frac{\dot{\rho}(t)}{\rho(t)}\right)^M, \\
F_1 &= k_1 u + f_1(t,u,v), \\
F_2 &= k_2 v + f_2(t,u,v),
\end{aligned}
\tag{47}
$$

where $f^m = \min_{(-\infty,\infty)\times[0,T]} f(t)$ and $f^M = \max_{(-\infty,\infty)\times[0,T]} f(t)$ for any given continuous T-periodic function f. It is easy to verify that both F_1 and F_2 are nondecreasing with respect to u and v. Then, problem (42) is equivalent to

$$
\begin{cases}
\dfrac{\partial u}{\partial t} - \dfrac{D_1}{\rho^2(t)}\Delta u + k_1 u = F_1(t,u,v), & y \in \Omega_0, t > 0, \\
\dfrac{\partial v}{\partial t} + k_2 v = F_2(t,u,v)v, & y \in \Omega_0, t > 0, \\
u(y,t) = v(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\
u(y,0) = u(y,T), v(y,0) = v(y,T), & y \in \Omega_0.
\end{cases}
\tag{48}
$$

Using $(\overline{u}^{(0)}, \overline{v}^{(0)}) = (N_b, A_m)$ and $(\underline{u}^{(0)}, \underline{v}^{(0)}) = (\delta\phi, \delta\psi)$ as an initial iteration, one can construct a sequence $\{(u^{(i)}, v^{(i)})\}$ from the iteration process

$$
\begin{cases}
\overline{u}_t^{(i)} - \dfrac{D_1}{\rho^2(t)}\Delta\overline{u}^{(i)} + k_1\overline{u}^{(i)} = F_1\left(t, \overline{u}^{(i-1)}, \overline{v}^{(i-1)}\right), & y \in \Omega_0, t > 0, \\
\overline{v}_t^{(i)} + k_2\overline{v}^{(i)} = F_2\left(t, \overline{u}^{(i-1)}, \overline{v}^{(i-1)}\right), & y \in \Omega_0, t > 0, \\
\underline{u}_t^{(i)} - \dfrac{D_1}{\rho^2(t)}\Delta\underline{u}^{(i)} + k_1\underline{u}^{(i)} = F_1\left(t, \underline{u}^{(i-1)}, \underline{v}^{(i-1)}\right), & y \in \Omega_0, t > 0, \\
\underline{v}_t^{(i)} + k_2\underline{v}^{(i)} = F_2\left(t, \underline{u}^{(i-1)}, \underline{v}^{(i-1)}\right), & y \in \Omega_0, t > 0, \\
\overline{u}^{(i)}(y,t) = \underline{u}^{(i)}(y,t) = \underline{v}^{(i)}(y,t) = \underline{v}^{(i)}(y,t) = 0, & y \in \partial\Omega_0, t > 0,
\end{cases}
\tag{49}
$$

with the periodic condition

$$
\begin{cases}
\overline{u}^{(i)}(y,0) = \overline{u}^{(i-1)}(y,T), \overline{v}^{(i)}(y,0) = \overline{v}^{(i-1)}(y,T), & y \in \Omega_0, \\
\underline{u}^{(i)}(y,0) = \underline{u}^{(i-1)}(y,T), \underline{v}^{(i)}(y,0) = \underline{v}^{(i-1)}(y,T), & y \in \Omega_0,
\end{cases}
\tag{50}
$$

where $i = 1, 2, \ldots$.

Under condition $R_0(\rho) > 1$, we know that (N_b, A_m) and $(\delta\phi, \delta\psi)$ are ordered upper and lower solution of problem (42). Taking (N_b, A_m) and $(\delta\phi, \delta\psi)$ as initial iteration and employing ideas of [43] with the monotonicity of f_1 and f_2, it follows that the well-defined sequences governed by (49) and (50) possess the monotone property

$$
\begin{aligned}
(\hat{u}, \hat{v}) &\leq \left(\underline{u}^{(i-1)}, \underline{v}^{(i-1)}\right) \leq \left(\underline{u}^{(i)}, \underline{v}^{(i)}\right) \leq \left(\overline{u}^{(i)}, \overline{v}^{(i)}\right) \\
&\leq \left(\overline{u}^{(i-1)}, \overline{v}^{(i-1)}\right) \leq (\tilde{u}, \tilde{v}).
\end{aligned}
\tag{51}
$$

Therefore, the pointwise limits

$$
\begin{aligned}
\lim_{i\longrightarrow\infty}\left(\overline{u}^{(i)}, \overline{v}^{(i)}\right) &= (\overline{u}, \overline{v}), \\
\lim_{i\longrightarrow\infty}\left(\underline{u}^{(i)}, \overline{v}^{(i)}\right) &= (\underline{u}, \underline{v}),
\end{aligned}
\tag{52}
$$

exist and their limits possess the relation

$$
(\hat{u}, \hat{v}) \leq \left(\underline{u}^{(i)}, \underline{v}^{(i)}\right) \leq (\underline{u}, \underline{v}) \leq (\overline{u}, \underline{v}) \leq \left(\overline{u}^{(i)}, \overline{v}^{(i)}\right) \leq (\tilde{u}, \tilde{v}).
\tag{53}
$$

Therefore, $(\overline{u}, \overline{v})$ and $(\underline{u}, \underline{v})$ are the true positive T-periodic solutions of problem (42). Moreover, $(\overline{u}, \overline{v})$ and $(\underline{u}, \underline{v})$ in respect are the maximal and minimal solutions in the sense that (u, v) is any other solution of (42) in $\langle(\hat{u}, \hat{v}), (\tilde{u}, \tilde{v})\rangle$, and then $(\underline{u}, \underline{v}) \leq (u, v) \leq (\overline{u}, \overline{v})$. Furthermore, if $\overline{u}(y,0) = \underline{u}(y,0)$ or $\overline{v}(y,0) = \underline{v}(y,0)$, then $(\overline{u}, \overline{v}) =$

$(\underline{u}, \underline{v}) := (u^*, v^*)$ and (u^*, v^*) is the unique solution of (42) in $\overline{\Omega}_0$.

From the above conclusions, we have the following result.

Theorem 4. *Let $(\tilde{u}, \tilde{v})$ and $(\hat{u}, \hat{v})$ be a pair of ordered upper and lower solutions of (42), respectively, and then the sequences $\{(\overline{u}^{(i)}, \overline{v}^{(i)})\}$ and $\{(\underline{u}^{(i)}, \underline{v}^{(i)})\}$ provided from (49), (50) converge monotonically from above to a maximal solution $(\overline{u}, \overline{v})$ and from below to a minimal solution $(\underline{u}, \underline{v})$ in $\overline{\Omega}_0$, respectively, and satisfy the relation*

$$\begin{aligned}(\hat{u}, \hat{v}) &\le \left(\overset{(i)}{\underline{u}}, \overset{(i)}{\underline{v}}\right) \le \left(\overset{(i+1)}{\underline{u}}, \overset{(i+1)}{\underline{v}}\right) \le \left(\overline{u}^{(i+1)}, \overset{(i+1)}{\underline{v}}\right) \\ &\le \left(\overline{u}^{(i)}, \overline{v}^{(i)}\right) \le (\tilde{u}, \tilde{v}) \\ &\le \left(\overset{(i)}{\underline{u}}, \overset{(i)}{\underline{v}}\right) \le (\tilde{u}, \tilde{v}).\end{aligned} \quad (54)$$

Moreover, if $\overline{u}(y, 0) = \underline{u}(y, 0)$ or $\overline{v}(y, 0) = \underline{v}(y, 0)$, then $(\overline{u}, \overline{v}) = (\underline{u}, \underline{v}) = (u^*, v^*)$ and (u^*, v^*) is the unique solution of (42) in $\overline{\Omega}_0$.

For problems (8) and (9), $(\tilde{u}, \tilde{v})$ and $(\hat{u}, \hat{v})$ defined in (44) are also the ordered upper and lower solutions provided the initial condition is replaced by

$$(\hat{u}, \hat{v}) \le [(\eta_1(y), \eta_2(y) \le (\tilde{u}, \tilde{v}))], \quad \text{in } \Omega_0. \quad (55)$$

Applying $(\overline{u}^{(0)}, \overline{v}^{(0)}) = (\tilde{u}, \tilde{v})$ and $(\underline{u}^{(0)}, \underline{v}^{(0)}) = (\hat{u}, \hat{v})$ as an initial iteration again, we denote the sequences generated by (49) as $\{(\underline{u}_A^{(i)}, \underline{v}_B^{(i)})\}$ and $\{(\overline{u}_A^{(i)}, \overline{v}_B^{(i)})\}$ such that

$$\left(\underline{u}_A^{(i)}, \underline{v}_B^{(i)}\right)(y, 0) = \left(\overline{u}_A^{(i)}, \overline{v}_B^{(i)}\right)(y, 0) = \{[\eta_1(y), \eta_2(y)]\}, \quad y \in \overline{\Omega}_0. \quad (56)$$

The following three lemmas follow from [43], so we omit their proofs here.

Lemma 2. *The sequences $\{(\underline{u}_A^{(i)}, \underline{v}_B^{(i)})\}$ and $\{(\overline{u}_A^{(i)}, \overline{v}_B^{(i)})\}$ converge monotonically to a unique solution $(u(y,t), v(y,t))$ of problems (7) and (8) and satisfy the relation*

$$\begin{aligned}(\hat{u}, \hat{v}) &\le \left(\underline{u}_A^{(i-1)}, \underline{v}_B^{(i-1)}\right) \le \left(\underline{u}_A^{(i)}, \underline{v}_B^{(i)}\right) \le (u, v) \\ &\le \left(\overline{u}_A^{(i)}, \overline{v}_B^{(i)}\right) \le \left(\overline{u}_A^{(i-1)}, \overline{v}_B^{(i-1)}\right) \le (\tilde{u}, \tilde{v}),\end{aligned} \quad (57)$$

on $\overline{\Omega}_0 \times [0, \infty)$.

Lemma 3. *For any i and j, if the pairs $(\overline{u}^{(i)}, \overline{v}^{(i)})$ and $(\underline{u}^{(j)}, \underline{v}^{(j)})$ are ordered upper and lower solutions to problem (42), then they are also ordered upper and lower solutions of (7) and (9) provided that $(\underline{u}^{(j)}, \underline{v}^{(j)})(y, 0) \le [\eta_1(y), \eta_2(y)] \le (\overline{u}^{(i)}, \overline{v}^{(i)})(y, 0)$ in Ω_0.*

Lemma 4. *Let $(u, v)(y, t; \eta_1, \eta_2)$ be the solution of (7) and (9) with any*

$$(\eta_1(y), \eta_2(y)) \in S_0, \quad (58)$$

where

$$\begin{aligned}S_0 = \{&(\eta_1, \eta_2) \in \mathscr{C}(\overline{\Omega}_0) : (\hat{u}, \hat{v})(y, 0) \le (\eta_1, \eta_2) \\ &\le (\tilde{u}, \tilde{v})(y, 0) \text{ on } \overline{\Omega}_0\}.\end{aligned} \quad (59)$$

Then,

$$\left(\overset{(i)}{\underline{u}}, \overset{(i)}{\underline{v}}\right)(y, t) \le (u, v)(y, t + iT; \eta_1, \eta_2) \le \left(\overline{u}^{(i)}, \overline{v}^{(i)}\right)(y, t), \quad (60)$$

on $\overline{\Omega}_0 \times [0, \infty)$.

In the next theorem, we present the attractivity of T-periodic solutions to problems (7) and (9) in relation to the maximal and minimal T-periodic solution of problems (7) and (8).

Theorem 5. *Let $(u, v)(y, t; \eta_1, \eta_2)$ be any solution of problems (7) and (9). The following assertions hold:*

(a) If $R_0(\rho) > 1$, then

$$\begin{aligned}&\lim_{i \longrightarrow \infty} (u(y, t + iT; \eta_1, \eta_2), v(y, t + iT; \eta_1, \eta_2)) \\ &= \begin{cases} (\underline{u}, \underline{v})(y, t) \text{ if } (\hat{u}, \hat{v}) \le (\eta_1, \eta_2) \le (\underline{u}, \underline{v}) \text{ in } \Omega_0, \\ (\overline{u}, \overline{v})(y, t) \text{ if } (\overline{u}, \overline{v}) \le (\eta_1, \eta_2) \le (\tilde{u}, \tilde{v}) \text{ in } \Omega_0. \end{cases}\end{aligned} \quad (61)$$

In addition, for any $(\eta_1, \eta_2) \in S_0$,

$$\begin{aligned}(\underline{u}, \underline{v})(y, t) &\le (u(y, t + iT), v(y, t + iT))(\eta_1, \eta_2) \\ &\le (\overline{u}, \overline{v})(y, t), \quad \text{on } \overline{\Omega}_0 \times [0, \infty),\end{aligned} \quad (62)$$

as $i \longrightarrow \infty$. Furthermore, if $(\underline{u}, \underline{v})(y, t) = (\overline{u}, \overline{v})(y, t) := (u^*, v^*)$, then

$$\begin{aligned}\lim_{i \longrightarrow \infty} (u, v)(y, t + iT; \eta_1, \eta_2) &= (u^*, v^*), \\ &\text{on } \overline{\Omega}_0 \times [0, \infty).\end{aligned} \quad (63)$$

(b) If $R_0(\rho) \le 1$, then for any (η_1, η_2),

$$\lim_{t \longrightarrow \infty} (u, v)(y, t; \eta_1, \eta_2) = (0, 0). \quad (64)$$

Proof. Let $(u_i, v_i)(y, t) = (u, v)(y, t + iT; \eta_1, \eta_2)$ for every $i = 1, 2, \ldots$, where $(\eta_1, \eta_2) \in S_0$ (see Lemma 4). It follows from Lemma 2 that the solution (u_i, v_i) is in $\overline{\Omega}_0 \times [0, \infty)$ and, in particular, $(\hat{u}, \hat{v})(y, t + T) \le (u_1, v_1) \le (\tilde{u}, \tilde{v})(y, t + T)$ on $\overline{\Omega}_0 \times [0, \infty)$. Next, we consider (7) with the initial condition $[\eta_1(y), \eta_2(y)]$ in Ω_0. By the iteration process in (49) for $i = 1$, we have

$$\begin{aligned}\left(\overline{u}^{(1)}, \overline{v}^{(1)}\right)(y, 0) &= \left(\overline{u}^{(0)}, \overline{v}^{(0)}\right)(y, T) = (\tilde{u}, \tilde{v})(y, T), \\ \left(\overset{(1)}{\underline{u}}, \overset{(1)}{\underline{v}}\right)(y, 0) &= \left(\overset{(1)}{\underline{u}}, \overset{(1)}{\underline{v}}\right)(y, T) = (\hat{u}, \hat{v})(y, T).\end{aligned} \quad (65)$$

Therefore, one can see that

$$\left(\underset{\_}{u}^{(1)},\underset{\_}{v}^{(1)}\right)(y,0)\le (u_1,v_1)(y,0)\le \left(\overline{u}^{(1)},\overline{v}^{(1)}\right)(y,0), \quad (66)$$

in Ω_0.

According to Lemma 2, $(\overline{u}^{(1)},\overline{v}^{(1)})(y,t)$ and $(\underline{u}^{(1)},\underline{v}^{(1)})(y,t)$ are ordered upper and lower solutions of (7), respectively, when $[\eta_1(y),\eta_2(y)]=(u_1,v_1)(y,0)$ in Ω_0. With respect to Theorem 4, we can see that

$$\left(\underset{\_}{u}^{(1)},\underset{\_}{v}^{(1)}\right)(y,t)\le (u_1,v_1)(y,t)\le \left(\overline{u}^{(1)},\overline{v}^{(1)}\right)(y,t), \quad (67)$$

on $\overline{D}$. By the principle of induction,

$$\left(\underset{\_}{u}^{(i)},\underset{\_}{v}^{(i)}\right)(y,t)\le (u_i,v_i)(y,t)\le \left(\overline{u}^{(i)},v^{(i)}\right)(y,t) \quad (68)$$

holds on $\overline{\Omega}_0\times[0,\infty)$. On the other hand, relation (63) directly follows from (62) with the assumption that $(\underline{u},\underline{v})(y,t)=(\overline{u},\overline{v})(y,t):=(u^*,v^*)$. The proof of assertion (a) is completed.

When it comes to assertion (b), in fact, it is easy to see that (N_b,A_m) and $(0,0)$ are a pair of ordered upper and lower solutions of problems (7) and (8). Using the same argument as in assertion (a), as well as the fact that $(0,0)$ is the unique solution to problems (7) and (8), we can conclude that the solution $(u,v)(y,t;\eta_1,\eta_2)$ of problem (7), associated with any nonnegative initial function pair $(\eta_1(y),\eta_2(y))$, possesses the convergence property

$$\lim_{i\longrightarrow\infty}(u,v)(y,t+iT;\eta_1,\eta_2)=(0,0), \quad (69)$$

which is equivalent to

$$\lim_{t\longrightarrow\infty}(u,v)(y,t;\eta_1,\eta_2)=(0,0). \quad (70)$$

□

6. The Impact of Evolving Domain

To better understand the impact of periodic evolving domain, in this section, we assume that $\rho(t)\equiv 1$, that is, $\Omega_t=\Omega_0$ is a fixed domain, and then problem (7) becomes

$$\begin{cases} \dfrac{\partial U}{\partial t}-D_1(y,t)\Delta U=\alpha_b(y,t)\beta_b(y,t)\dfrac{(N_b-U)}{N_b}V-\gamma_b(y,t)U, & y\in\Omega_0, t>0,\\ \dfrac{\partial V}{\partial t}=\alpha_m(y,t)\beta_b(y,t)\dfrac{(A_m-V)}{N_b}U-d_m(y,t)V, & y\in\Omega_0, t>0,\\ U(y,t)=V(y,t)=0, & y\in\partial\Omega_0, t>0, \end{cases} \quad (71)$$

with the periodic condition

$$\begin{aligned} U(y,0)&=U(y,T),\\ V(y,0)&=V(y,T), \end{aligned} \qquad y\in\Omega_0, \quad (72)$$

and under the initial condition

$$\begin{aligned} U(y,0)&=\eta_1(y)=I_{b,0}(y),\\ V(y,0)&=\eta_2(y)=I_{m,0}(y), \end{aligned} \qquad y\in\Omega_0. \quad (73)$$

By the similar arguments as in Section 2, we have the following eigenvalue problem corresponding to problems (71) and (72):

$$\begin{cases} \Phi_t-D_1\Delta\Phi=\alpha_b(y,t)\beta_b(y,t)\Psi-\gamma_b(y,t)\Phi+\lambda\Phi, & y\in\Omega_0, t>0,\\ \Psi_t=\alpha_m(y,t)\beta_b(y,t)\dfrac{A_m}{N_b}\Phi-d_m(y,t)\Psi+\lambda\Psi, & y\in\Omega_0, t>0,\\ \Phi(y,t)=\Psi(y,t)=0, & y\in\partial\Omega_0, t>0,\\ \Phi(y,0)=\Phi(y,T), \Psi(y,0)=\Psi(y,T), & y\in\Omega_0, \end{cases} \quad (74)$$

where (Φ,Ψ) is the eigenfunction corresponding to the principal eigenvalue and $R_0^*=R_0(1)$ is the principal eigenvalue of the eigenvalue problem

$$\begin{cases} \Phi_t - D_1 \Delta \Phi = \dfrac{\alpha_b(y,t)\beta_b(y,t)}{R_0^*}\Psi - \gamma_b(y,t)\Phi, & y \in \Omega_0, t > 0, \\ \Psi_t = \alpha_m(y,t)\beta_b(y,t)\dfrac{A_m}{N_b R_0^*}\Phi - d_m(y,t)\Psi, & y \in \Omega_0, t > 0, \\ \Phi(y,t) = \Psi(y,t) = 0, & y \in \partial\Omega_0, t > 0, \\ \Phi(y,0) = \Phi(y,T), \Psi(y,0) = \Psi(y,T), & y \in \Omega_0. \end{cases} \tag{75}$$

Moreover, $(\tilde{U}, \tilde{V}) = (N_b, A_m)$ and $(\hat{U}, \hat{V}) = (\delta\Phi, \delta\Psi)$ are ordered upper and lower solutions of problems (71) and (73), where δ is positive constant and small enough.

The main results of this section are given in the following two theorems which are parallel to Theorems 2–5.

Theorem 6. *The following statements are valid:*

(a) *If $R_0^* > 1$, then problems (71) and (72) possess a maximal positive T-periodic solution $(\overline{U}, \overline{V})$ and a minimal positive T-periodic solution $(\underline{U}, \underline{V})$. Besides, if $(\overline{U}, \overline{V})(y,0) = (\underline{U}, \underline{V})(y,0)$, then $(\overline{U}, \overline{V}) = (\underline{U}, \underline{V}) := (U^*, V^*)$ and (U^*, V^*) is the unique T-periodic solution of problems (71) and (72) .*

(b) *If $R_0^* \le 1$, then problems (71) and (72) have no positive T-periodic solution.*

Theorem 7. *Let $(U, V)(y, t; \eta_1, \eta_2)$ be the solution of problems (71) and (73).*

(a) *If $R_0^* > 1$, then*

$$\lim_{i \longrightarrow \infty} (U,V)(y, t+iT; \eta_1, \eta_2) = \begin{cases} (\underline{U}, \underline{V})(y,t) \text{ if } (\hat{U}, \hat{V}) \le (\eta_1, \eta_2) \le (\underline{U}, \underline{V}), & \text{in } \Omega_0, \\ (\overline{U}, \overline{V})(y,t) \text{ if } (\overline{U}, \overline{V}) \le (\eta_1, \eta_2) \le (\tilde{U}, \tilde{V}), & \text{in } \Omega_0. \end{cases} \tag{76}$$

Moreover, for any $(\eta_1, \eta_2) \in S_0^*$,

$$(\underline{U}, \underline{V})(y,t) \le (U,V)(y, t+iT; \eta_1, \eta_2) \le (\overline{U}, \overline{V})(y,t), \quad \text{on } \overline{\Omega}_0 \times [0, \infty), \tag{77}$$

as $i \longrightarrow \infty$. Additionally, if $(\underline{U}, \underline{V})(y,t) = (\overline{U}, \overline{V})(y,t) := (U^*, V^*)$, then

$$\lim_{i \longrightarrow \infty} (U,V)(y, t+iT; \eta_1, \eta_2) = (U^*, V^*)(y,t), \quad \text{on } \overline{\Omega}_0 \times [0, \infty). \tag{78}$$

(b) *If $R_0^* \le 1$, then for any (η_1, η_2),*

$$\lim_{t \longrightarrow \infty} (U,V)(y, t+iT; \eta_1, \eta_2) = (0,0), \tag{79}$$

uniformly for $y \in \Omega_0$, where

$$S_0^* = \left[(\eta_1, \eta_2) \in \mathscr{C}(\overline{\Omega}_0) : (\hat{U}, \hat{V})(y,0) \le (\eta_1, \eta_2) \le (\tilde{U}, \tilde{V})(y,0), \quad \text{in } \overline{\Omega}_0 \right]. \tag{80}$$

Thanks to the above analysis, here we adopt the integral average value $\overline{\rho^{-2}} = 1/T \int_0^T (1/\rho^2(t))dt$ generated by the evolution rate $\rho(t)$. It is easy to see that the spreading or vanishing of the virus on periodically evolving domain depends on the spatial-temporal basic reproduction number $R_0(\rho)$, while on the fixed domain, it depends on R_0^*. When $\overline{\rho^{-2}} < 1$, we have $R_0(\rho) > 1$, which means that the spreading of the virus has increased. Meanwhile, if $\overline{\rho^{-2}} > 1$, then $R_0(\rho) \le 1$, which implies that the spreading of the virus has decreased. When the domain is fixed, the parallel results hold with $R_0^* = R_0(1)$, then the virus in the case of vanishing.

7. Numerical Simulation and Discussion

In this section, we first carry out numerical simulations to illustrate the theoretical results obtained in previous sections. Our focus is the impact of periodic evolving domain on the transmission of the West Nile virus (WNv).

For simplicity, first we fix

$$\begin{aligned} \frac{A_m}{N_b} &= 20, \\ \alpha_b &= 0.88, \\ \alpha_m &= 0.16, \\ \gamma_b &= 0.01, \\ D_1 &= 0.06, \\ \lambda^* &= \pi^2, \\ \Omega_0 &= (0,1), \\ I_{b,0}(x) &= 0.3\sin(\pi x), \\ I_{m,0}(x) &= 0.2\sin(\pi x) + 0.1\sin(3\pi x), \end{aligned} \tag{81}$$

and then change the value of the evolution rate $\rho(t)$ to observe the long time behavior of problems (7) and (9).

Example 1. In systems (7) and (9), we fix $\beta_b = 0.3$ and $d_m = 0.029$ with $\rho(t) = 1$. Direct calculations show that

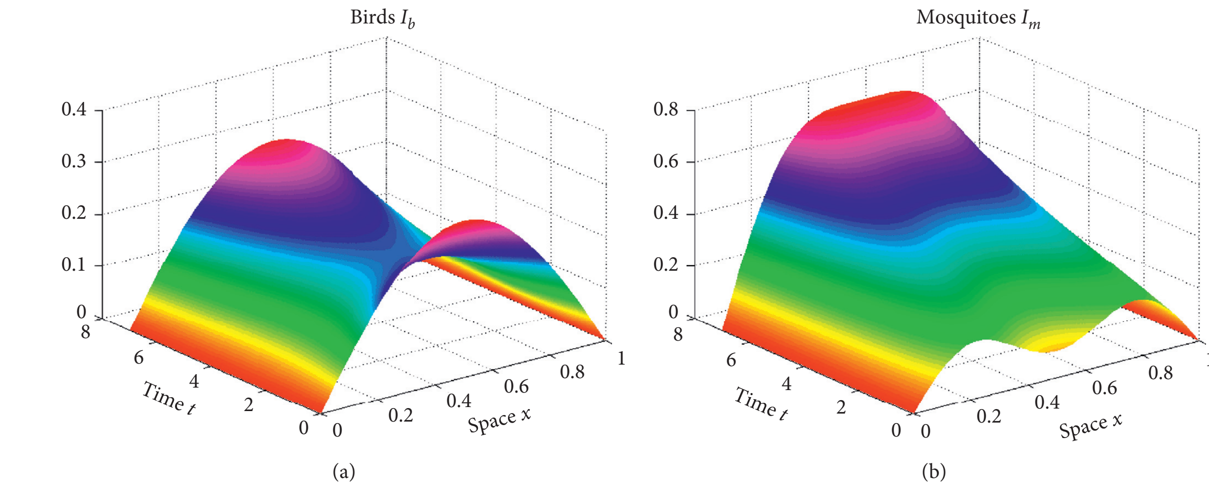


FIGURE 1: $\rho(t) = 1$. $R_0(1) > 1$, which implies that the solution tends to steady state in a fixed domain.

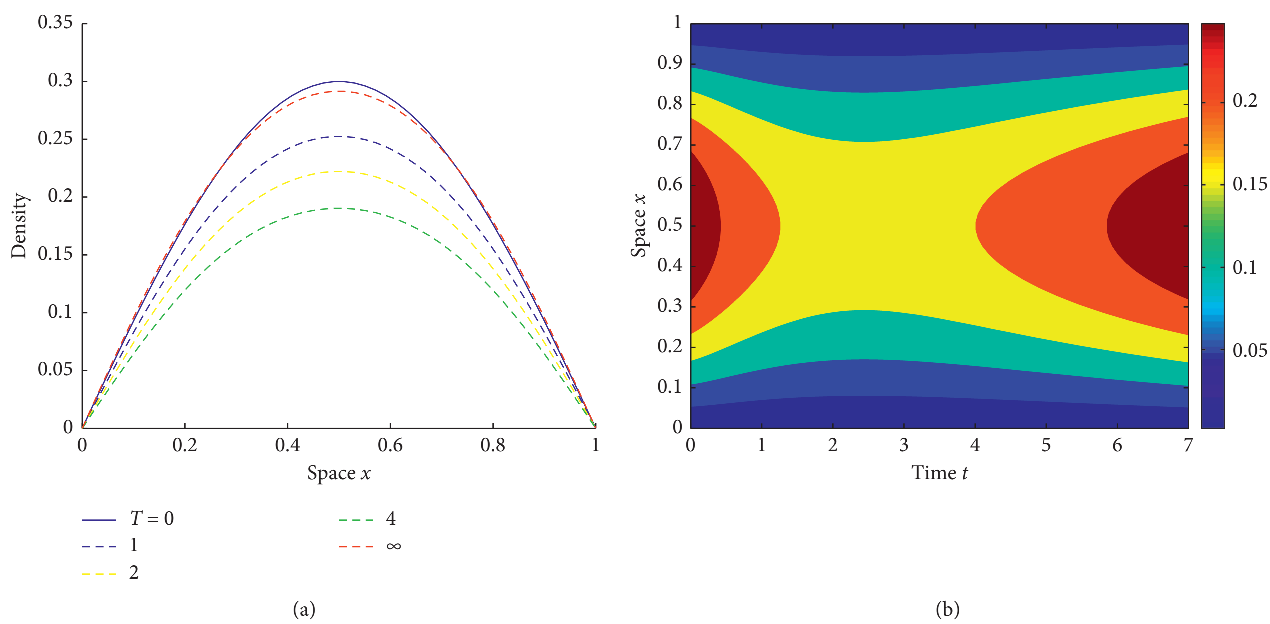


FIGURE 2: The corresponding cross-sectional view (a) and contour one (b) for the solution of problems (7) and (9), which means that the domain is fixed when the evolution rate $\rho(t) = 1$.

$$R_0(1) = \sqrt{\frac{A_m/N_b\alpha_b(\beta_b)^2\alpha_m}{d_m^*(D_1\lambda^* + \gamma_b)}} = \sqrt{\frac{20 \times 0.88 \times 0.09 \times 0.16}{0.029 \times (0.06 \times 9.8596 + 0.01)}} > 1. \tag{82}$$

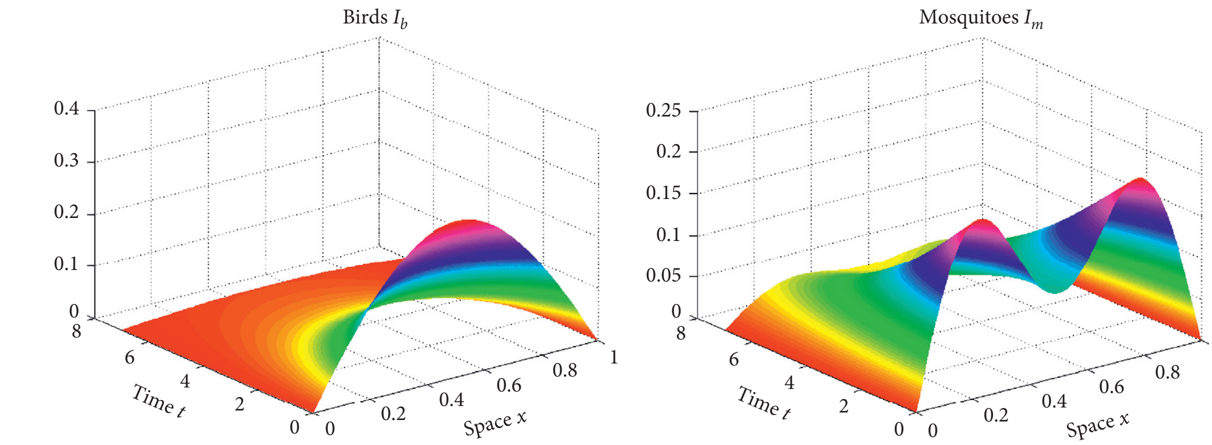
Hence, the solution of problems (7) and (9) tends to positive steady states (see Figures 1 and 2), which implies that the virus will persist in a fixed domain.

Example 2. In systems (7) and (9), we choose $\beta_b = 0.09$ and $d_m = 0.29$ with $\rho(t) = 1$. Direct calculations show that

$$R_0(1) = \sqrt{\frac{A_m/N_b\alpha_b(\beta_b)^2\alpha_m}{d_m(D_1\lambda^* + \gamma_b)}} = \sqrt{\frac{20 \times 0.88 \times 0.0081 \times 0.16}{0.29 \times (0.06 \times 9.8596 + 0.01)}} < 1. \tag{83}$$

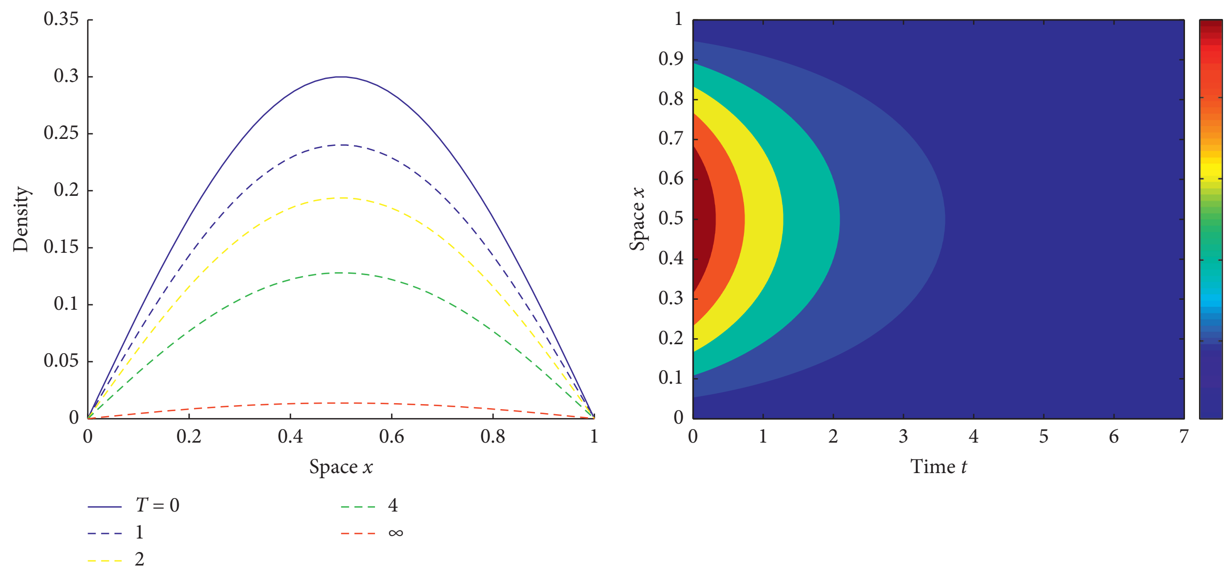
It is easy to see that the solution of problems (7) and (9) decays to zero quickly (see Figures 3 and 4), which implies that the virus will be extinct in a fixed domain.

Example 3. In systems (7) and (9), we set $\beta_b = 0.3$ and $d_m = 0.029$ with $\rho(t) = e^{0.1(1-\cos(4t))}$. Direct calculations show that



(a) (b)

FIGURE 3: $\rho(t) = 1$. $R_0(1) < 1$, which implies that the solution decays quickly to zero in a fixed domain.



(a) (b)

FIGURE 4: The corresponding cross-sectional view (a) and contour one (b) for the solution of problems (7) and (9), which implies that the domain is fixed when the evolution rate $\rho(t) = 1$.

$$\overline{\rho^{-2}} = \frac{2}{\pi}\int_0^{\pi/2} e^{0.1(1-\cos(4t))}dt \approx 0.8269 < 1,$$

$$R_0(\rho) \geq \sqrt{\frac{A_m/N_b\alpha_b(\beta_b)^2\alpha_m}{d_m\left(D_1\lambda^*\overline{\rho^{-2}} + \gamma_b\right)}} = \sqrt{\frac{20 \times 0.88 \times 0.09 \times 0.16}{0.029 \times (0.06 \times 9.8596 \times 0.8269 + 0.01)}} > 1. \tag{84}$$

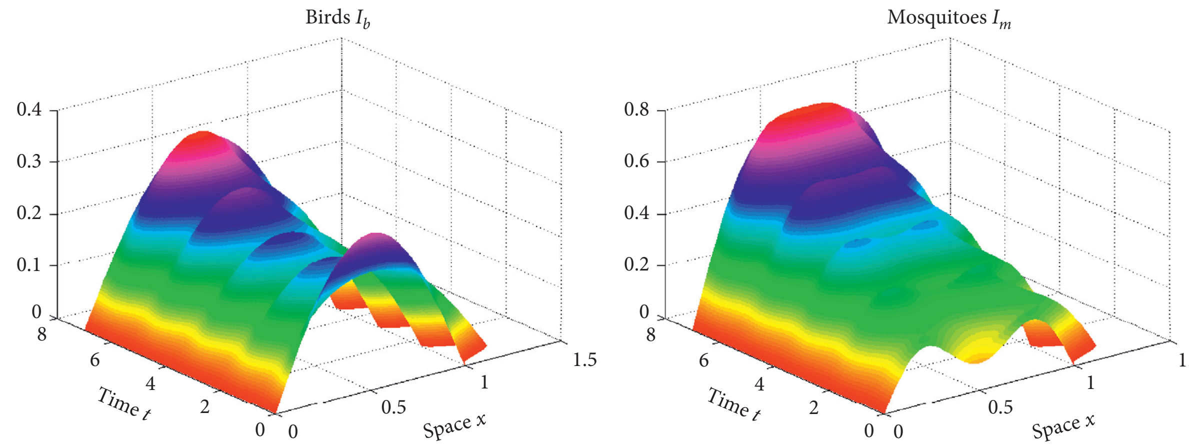
Therefore, it is easy to see that the solution of problems (7) and (9) converges to a positive periodic steady state (see Figures 5 and 6), which means that the virus with periodically evolving domain will persist. Consequently, we can see that $\overline{\rho^{-2}} < 1$ has positive effect on the persistence of WNv.

Example 4. In systems (7) and (9), we set $\beta_b = 0.09$ and $d_m = 0.29$ with $\rho(t) = e^{0.2(\cos(4t)-1)}$. Direct calculations show that

$$\overline{\rho^{-2}} = \frac{2}{\pi}\int_0^{\pi/2} e^{0.2(\cos(4t)-1)}dt \approx 1.5221 > 1,$$

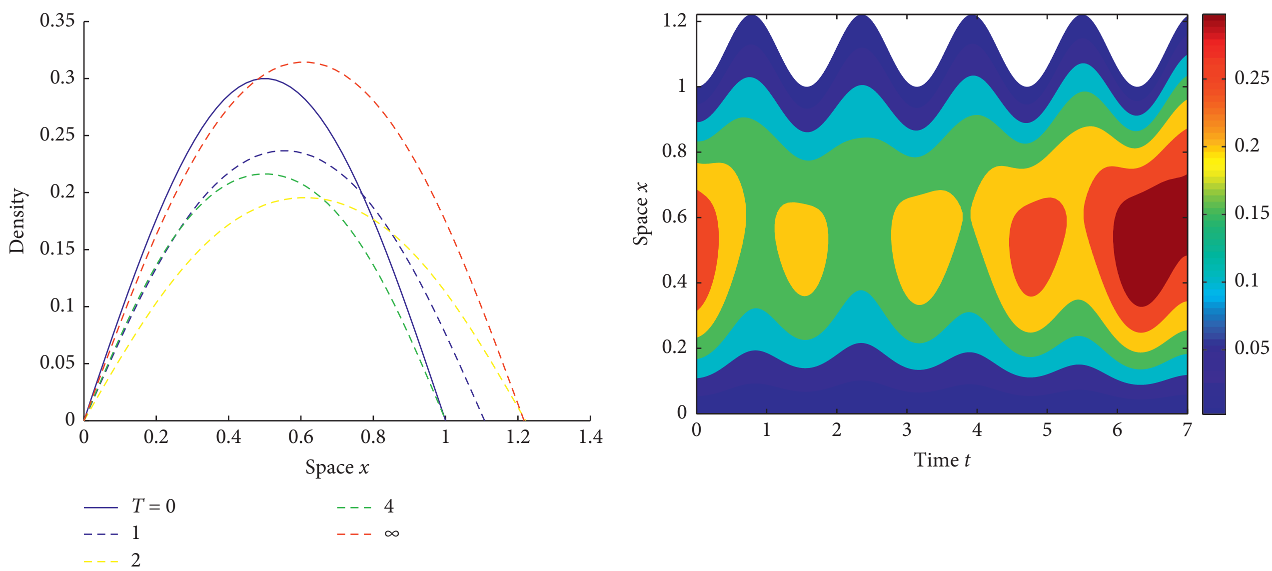
$$\sqrt{\frac{A_m/N_b\alpha_b(\beta_b)^2\alpha_m^*}{d_m\left(D_1\lambda^*\overline{\rho^{-2}} + \gamma_b\right)}} = \sqrt{\frac{20 \times 0.88 \times 0.0081 \times 0.16}{0.29 \times (0.06 \times 9.8596 \times 1.5221 + 0.01)}} < 1. \tag{85}$$

Therefore, one can easily see that the solution of problems (7) and (9) tends to zero quickly (see Figures 7 and 8), which



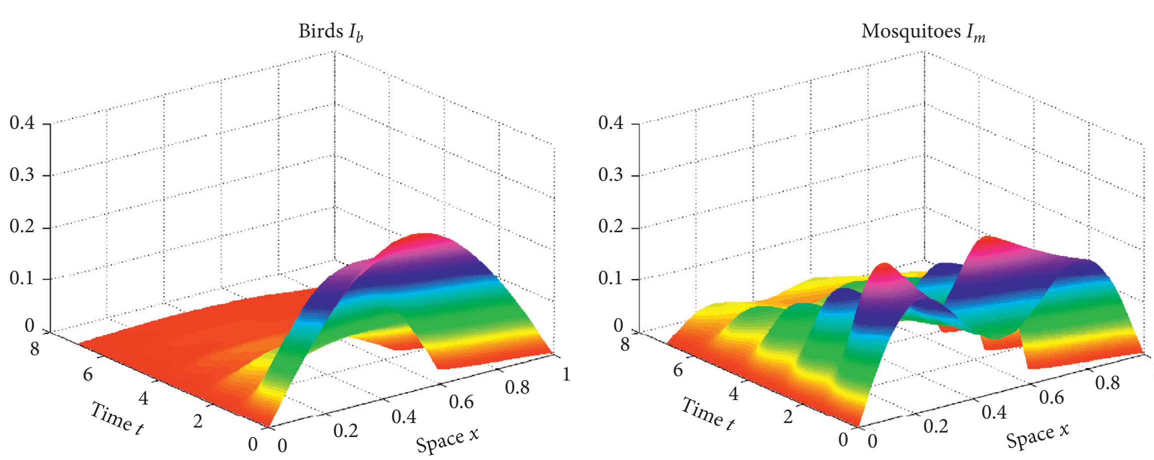
(a) (b)

Figure 5: $\rho(t) = e^{0.1(1-\cos(4t))}$. $\overline{\rho^{-2}} < 1$, $R_0(\rho) > 1$, which means that the solution of problems (7) and (9) converges to a positive periodic steady state.



(a) (b)

Figure 6: The corresponding cross-sectional view (a) and contour one (b) for the solution of problems (7) and (9), which implies that the domain is periodically evolving when the evolution rate $\rho(t) = e^{0.1(1-\cos(4t))}$.



(a) (b)

Figure 7: $\rho(t) = e^{0.2(\cos(4t)-1)}$. $\overline{\rho^{-2}} > 1$, $R_0(\rho) < 1$, which means that the solution (u, v) tends to zero.

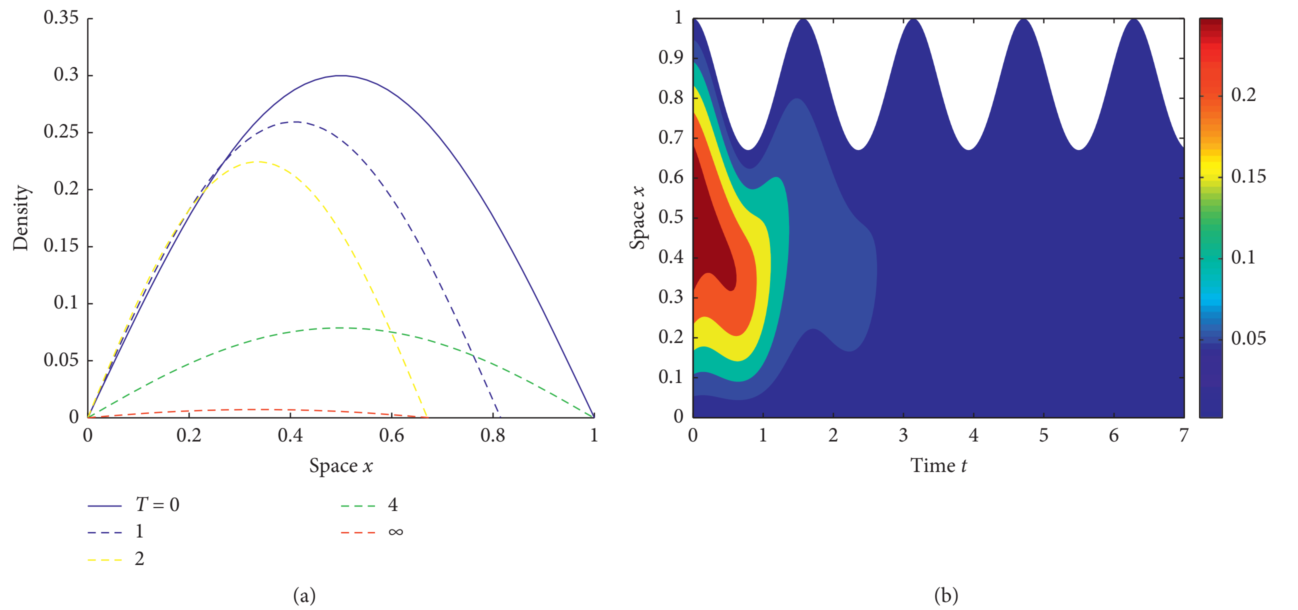


Figure 8: The corresponding cross-sectional view (a) and contour one (b) for the solution of problems (7) and (9), which implies that the domain is not periodically evolving when the evolution rate $\rho(t) = e^{0.2(\cos(4t)-1)}$.

means that the virus with periodically evolving domain will be extinct. Consequently, we can say that $\overline{\rho^{-2}} > 1$ has negative effect on the persistence of WNv.

8. Conclusions

Recently, the impact of periodic evolution domain has been attracting considerable attention. In [31], Jiang and Wang studied the impact of periodic evolution on the single-species diffusion logistic model. Asymptotic profile of a mutualistic model on a periodically evolving domain has been investigated by Adam et al. in [33]. The diffusive model for *Aedes aegypti* mosquito on a periodically evolving domain has been considered by Zhang and Lin in [32]. Zhu et al. [34] constructed a dengue fever model and studied its asymptotic profile on a periodically evolving domain. These studies indicated that the periodic domain evolution has a significant impact on the dispersal of species and transmission of infectious diseases.

In this paper, we study a diffusive West Nile virus model with periodical and isotropic domain evolution. To circumvent the difficulty induced by the advection and dilution terms, we transform the model to a reaction-diffusion model in a fixed domain. We introduce the spatial-temporal basic reproduction number $R_0(\rho)$ depending on the periodic evolution rate $\rho(t)$. In the case that all parameters are constants and $\rho(t) \equiv 1$, the explicit formula for the spatial-temporal basic reproduction number is presented (Theorem 1). Moreover, to better understand the impact of periodic evolution value on the persistence or extinction of the virus, we assume $\rho(t) \equiv 1$, that is, the periodic domain Ω_t becomes a fixed domain Ω_0. Furthermore, the notation $\overline{\rho^{-2}} = (1/T)\int_0^T 1/\rho^2(t)dt$ is utilized as an average value. Our results show that if $R_0(\rho) > 1$ depending on the evolution rate $\rho(t)$, then the virus will persist and all solutions possess the attractor $\langle(\underline{u},\underline{v}),(\overline{u},\overline{v})\rangle$, which is the sector between the maximal and minimal T-periodic solutions $(\overline{u},\overline{v})$ and $(\underline{u},\underline{v})$ of problems (7) and (8) (Theorems 2–4) (a), whereas, if $R_0(\rho) \le 1$, then any solution of problems (7) and (8) decays to (0, 0), that is, the virus is in the case of extinction (Theorem 3 and 5) (b). In the case that $\rho(t) \equiv 1$, we introduce R_0^*. For this case, if $R_0^* > 1$, the model admits a maximal and minimal T-periodic solutions, while if $R_0^* \le 1$, the model has no positive solution (Theorem 6 and 7). It is important to mention that numerical simulation in this paper is presented by using some parameters given in Lewis et al. [4], namely, $A_m/N_b = 20, \alpha_b = 0.88, \alpha_m = 0.16, \gamma_b = 0.01, \beta_b = 0.3$, and $d_m = 0.029$.

From our theoretical and numerical results, we believe that the periodic domain evolution has a significant impact on the transmission of WNv.

Data Availability

All data are provided in full in the numerical simulation and discussion section of this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally and all of them read and approved the final manuscript.

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