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Research Article

Adaptive Fuzzy Fast Finite-Time Tracking Control for Nonlinear Systems in Pure-Feedback Form with Unknown Disturbance

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In this paper, based on the fast finite-time stability theorem, an adaptive fuzzy control problem is considered for a class of nonlinear systems in pure-feedback form with unknown disturbance. In the controller design process, the mean value theorem is applied to address the nonaffine structure of the pure-feedback plant, the universal approximation capability of the fuzzy logic system (FLS) is utilized to compensate the unknown uncertainties, and the adaptive backstepping technique is used to design the controller model. Combined with the selection of the appropriate Lyapunov function at each step, a fuzzy-based adaptive tracking control scheme is proposed, which ensures that all signals in the closed-loop system are bounded and tracking error converges to a small neighborhood of the origin in fast finite-time. Finally, simulation results illustrate the validity of the proposed approach.

1. Introduction

During the recent years, the topics related to the field of nonlinear control have attracted a lot of attention [1–3]. Many approaches for controller design have been investigated, such as backstepping control, dynamic surface control, adaptive control, and so on. Among them, the adaptive control combined with the backstepping technique has provided a systematic framework model-based for control design. The adaptive backstepping control method solves the control design problem for nonlinear systems with unmatched conditions and uncertain parameters and ensures the stability of the closed-loop system successfully. Besides, it supplies an approach which can achieve the transient performance of the systems better by tuning design parameters. Until now, it has already become one of the popular control methods for nonlinear systems in [4–7]. In [8], an adaptive tracking control scheme was proposed for nonlinear strict-feedback systems with additive disturbances. In [9], the authors focused on the position control for a gear transmission servo system by using the backstepping technique. In [10], an adaptive controller was designed via backstepping for nonlinear systems with quantized states. In [11], the adaptive backstepping control approach was developed for a class of stochastic cascade nonlinear time-delay systems.

Despite the adaptive backstepping control method having a few merits, there is a need for large-enough gains to suppress uncertainties, which will degrade control performances. It is not feasible for a controlled system with unknown nonlinear functions to use this method alone. Then, the fuzzy logic system (FLS) proposed by Wang and Mendel [12] and the neural network (NN) proposed by Polycarpou [13] have solved this problem well. Because of their universal approximation capabilities, they have become a set of powerful tools to compensate the unknown nonlinear functions of closed-loop systems. During the past decades, many scholars have obtained a lot of meaningful results by using FLSs and NNs combined with the adaptive backstepping technique in [14–26]. An adaptive NN-based fault-tolerant controller for the nonlinear system was investigated in [27]. Combining the adaptive backstepping technique with FLSs and NNs, several interesting control strategies were designed for uncertain stochastic nonlinear systems in [28, 29]. In [30], the authors presented a NNs-based robust

adaptive tracking control scheme for hexacopter UAVs. In [31], an adaptive fuzzy-based event-triggered controller was considered for strict-feedback nonlinear systems.

Although the fuzzy-based or NNs-based adaptive backstepping control approaches have made great progress, the existing literatures are restricted to the strict-feedback nonlinear systems, and only few results are available about the control of nonlinear systems in pure-feedback form. Different from the strict-feedback nonlinear systems, pure-feedback nonlinear systems possess nonaffine property. It means that there are not state variables to be used as virtual control signals and the actual control input in the pure-feedback form. Therefore, it is more hard and challenging to address the problems of controller design and stability analysis in the pure-feedback systems. In [32, 33], the authors presented the adaptive neural control for nonlinear pure-feedback systems. A fuzzy-based adaptive controller was designed for pure-feedback nonlinear systems in [34]. The control schemes were proposed for pure-feedback nonlinear systems with unknown uncertainties in [35, 36], which were designed by combining dynamic surface control with adaptive backstepping algorithm.

It is worth pointing out that the abovementioned literatures are developed on the basis of the Lyapunov asymptotically stability theorem. However, in practical applications, the finite-time control method has lots of advantages such as higher tracking precision, better robustness, and the ability to achieve systems transient performance faster. Recently, plenty of meaningful research results have been produced about the finite-time control problem in [37–42]. Based on the finite-time fault-tolerant control, a new control approach was introduced for robot manipulators by utilizing time-delay estimation in [43]. The adaptive finite-time control problem was addressed for a class of nonlinear systems with the actuator faults in [44]. In [45], the authors designed an adaptive decentralized controller for time-varying output-constrained nonlinear large-scale systems in finite-time. The Lyapunov theorem of finite-time stability was proposed for the first time in [46]. Then, in order to obtain the faster convergence rate, the fast finite-time stability was introduced in [47]. However, compared with the asymptotic control design process, the procedure of adaptive finite-time or fast finite-time controller design is more complex for the nonlinear strict-feedback systems. Furthermore, it is a difficult but meaningful unsolved issue to develop an adaptive fuzzy fast finite-time tracking control scheme for nonlinear systems in pure-feedback form. It is the main motivation of this paper.

Inspired by the aforementioned observations, in this paper, the problem of adaptive fuzzy fast finite-time tracking control is considered for a class of nonlinear systems in pure-feedback form with unknown disturbance. During the controller design, the mean value theorem is applied to deal with the nonaffine problem of the pure-feedback systems, FLSs are adopted to approximate packaged unknown nonlinearities, and an improved adaptive fuzzy fast finite-time controller is designed via the backstepping technique. The stability of the closed-loop systems is guaranteed in fast finite-time. To sum up, the main contributions in this paper are listed below:

(1) The fast finite-time theorem is extended to pure-feedback systems for the first time. Also, a fuzzy-based adaptive fast finite-time tracking control scheme for nonlinear systems in pure-feedback form with unknown disturbance is proposed for the first time, too.

(2) Combined the traditional adaptive backstepping technique with the characteristics of the radial basis function of fuzzy logic systems, by applying the mean value theorem and the fast finite-time theory, the system structure is simplified so that reduces complexity of the controller design.

The remaining parts are organized as follows. Section 2 introduces problem formulation and preliminaries. Section 3 presents the controller design procedure in detail and stability analysis. Section 4 provides simulation results. Section 5 concludes this research.

2. Problem Formulation and Preliminaries

2.1. System Descriptions and Control Problem. Consider a class of nonlinear pure-feedback systems described as follows:

$$\begin{cases} \dot{x}_i(t) = f_i(\overline{x}_i, x_{i+1}), \quad 1 \le i \le n-1, \\ \dot{x}_n(t) = f_n(\overline{x}_n, u) + d(t), \\ y(t) = x_1(t), \end{cases} \tag{1}$$

in which $\overline{x}_i(t) = [x_1(t), \ldots, x_i(t)]^T \in R^i$, $i = 1, \ldots, n-1$ is the vector of the states and $\overline{x}_n(t) = [x_n(t), \ldots, x_n(t)]^T \in R^n$; $y(t) \in R$ represents the system output; $u \in R$ denotes input signal; $d(t)$ is a bounded disturbance; and $f_i(\cdot)$ and $f_n(\cdot)$ represent unknown smooth functions.

Using the mean value theorem [48], we can express $f_i(\cdot)$ and $f_n(\cdot)$ in (1) as follows:

$$f_i(\overline{x}_i, x_{i+1}) = f_i\left(\overline{x}_i, x_{i+1}^0\right) + \left.\frac{\partial f_i(\overline{x}_i, x_{i+1})}{\partial x_{i+1}}\right|_{x_{i+1}=x_{i+1}^{\delta_i}} \times\left(x_{i+1} - x_{i+1}^0\right), \quad 1 \le i \le n-1, \tag{2}$$

$$f_n(\overline{x}_n, u) = f_n\left(\overline{x}_n, u^0\right) + \left.\frac{\partial f_n(\overline{x}_n, u)}{\partial u}\right|_{u=u^{\delta_n}} \times\left(u - u^0\right), \tag{3}$$

where $x_{i+1}^{\delta_i} = \delta_i x_{i+1} + (1-\delta_i)x_{i+1}^0$, with $0 < \delta_i < 1$, and $u^{\delta_n} = \delta_n u + (1-\delta_n)u^0$, with $0 < \delta_n < 1$.

For convenience of writing, we define $h_i(\overline{x}_i, x_{i+1}^{\delta_i}) = (\partial f_i(\overline{x}_i, x_{i+1})/(\partial x_{i+1})|_{x_{i+1}=x_{i+1}^{\delta_i}}$ and $h_n(\overline{x}_n, u^{\delta_n}) = (\partial f_n(\overline{x}_n, u))/(\partial u)|_{u=u^{\delta_n}}$, which are unknown nonlinear functions. Then, substituting (2) with (3) into (4) and choosing $x_{i+1}^0 = 0$, $u^0 = 0$, we get

$$\begin{cases} \dot{x}_i(t) = f_i(\overline{x}_i, 0) + h_i\left(\overline{x}_i, x_{i+1}^{\delta_i}\right)x_{i+1}, \\ \dot{x}_n(t) = f_n(\overline{x}_n, 0) + h_n\left(\overline{x}_n, u^{\delta_n}\right)u + d(t), \\ y(t) = x_1(t). \end{cases} \tag{4}$$

The objective of this paper is to design a fuzzy-based fast finite-time tracking controller such that all signals in the closed-loop system are bounded and the output y can follow the specified desired trajectory y_d.

Throughout this paper, the following assumptions and lemmas are imposed on system (4).

Assumption 1 (see [49]). The desired trajectory signal y_d and that up to the nth derivative are smooth and bounded.

Assumption 2 (see [33]). There exists an unknown bounded constant d^* such that $|d(t)| \le d^*$.

Assumption 3 (see [50]). The function $h_i(\overline{x}_i, x_{i+1}^{\delta_i})$ satisfies $0 < \underline{h}_i \le |h_i(\cdot)| \le \overline{h}_i < \infty$, for $i = 1, \ldots, n$, where $\underline{h}_i$ and $\overline{h}_i$ are unknown constants.

Remark 1. According to Assumption 3, it is reasonable that the unknown smooth function h_i is strictly either positive or negative. Without losing generality, we assume that $h_i > 0$. For facilitating the actual controller design, $\underline{h}_n$ is known.

Lemma 1 (see [51]). *Consider the system $\dot{x} = f(x)$, if there exists continuous function $V(x)$, $\mu_1 > 0, \mu_2 > 0, 0 < \gamma < 1$, and $0 < \eta < \infty$, so that $\dot{V}(x) \le -\mu_1 V(x) - \mu_2 V^{\gamma}(x) + \eta$, then the trajectory of system $\dot{x} = f(x)$ is practical finite-time stable, and the residual set of the solution of system $\dot{x} = f(x)$ is given by*

$$\lim_{t \longrightarrow T_r} V(x) \le \min\left\{\frac{\eta}{(1-\theta_0)\mu_1}, \left(\frac{\eta}{(1-\theta_0)\mu_2}\right)^{1/\gamma}\right\}, \tag{5}$$

where θ_0 satisfies $0 < \theta_0 < 1$. The settling time is bounded as

$$T_r \le \max\left\{t_0 + \frac{1}{\theta_0\mu_1(1-\gamma)}\ln\frac{\theta_0\mu_1 V^{1-\gamma}(t_0) + \mu_2}{\mu_2}, t_0 + \frac{1}{\mu_1(1-\gamma)}\ln\frac{\mu_1 V^{1-\gamma}(t_0) + \theta_0\mu_2}{\theta_0\mu_2}\right\}. \tag{6}$$

Remark 2. For the convenience of derivation and the proof of the process in this paper, the parameter in the aforementioned inequality is chosen as $\gamma = 3/4$.

Lemma 2 (see [52]). *For any constant $\varepsilon > 0$ and variable $z \in R$, we have*

$$0 \le |z| - \frac{z^2}{\sqrt{z^2 + \varepsilon^2}} < \varepsilon. \tag{7}$$

Lemma 3 (see [53]). *For $x_i \in R, i = 1, \ldots, n$ and $\phi \in [0, 1]$, we have*

$$\left(\sum_{k=1}^{n}|x_k|\right)^{\phi} \le \sum_{k=1}^{n}|x_k|^{\phi}. \tag{8}$$

Lemma 4 (see [54]). *For $o_1 > 0, o_2 > 0, o_3 > 0, \gamma_1 \ge 0, \gamma_2 \ge 0, \gamma_3 \ge 0$, the following inequality holds:*

$$\gamma_1^{o_1}\gamma_2^{o_2}\gamma_3 \le o_3\gamma_1^{o_1+o_2} + \frac{o_2}{o_1+o_2} \times \left[\frac{o_1}{o_3(o_1+o_2)}\right]^{o_1/o_2}\gamma_2^{o_1+o_2}\gamma_3^{(o_1+o_2/o_2)}. \tag{9}$$

2.2. Fuzzy Logic Systems. An FLS is composed of the knowledge base, the fuzzifier, the fuzzy inference engine, and the defuzzifier. The knowledge base comprises a collection of fuzzy If-Then rules of the following form:

R^l: If Z_1 is $F_1^l, \ldots,$ and Z_n is F_n^l, then ϖ is G^l, $l = 1, \ldots, N$

Where $Z = [Z_1, \ldots, Z_n]^T \in R$ and ϖ represent the FLS input and output, F_i^l and G^l are fuzzy sets, their fuzzy membership functions are $\mu_{F_i^l}(Z_i)$ and $\mu_{G^l}(\varpi)$, and N is rule number of If-Then. ϖ can be expressed as

$$\varpi(Z) = \sum_{l=1}^{N}\overline{W}_j\frac{\prod_{l=1}^{n}\mu_{F_i^l}(Z_i)}{\sum_{l=1}^{N}\left[\prod_{l=1}^{n}\mu_{F_i^l}(Z_i)\right]}, \tag{10}$$

where $\overline{W}_j = \max_{\varpi \in R}\mu_{G^l}(\varpi)$.

Let

$$S_j(Z) = \frac{\prod_{l=1}^{n}\mu_{F_i^l}(Z_i)}{\sum_{l=1}^{N}\left[\prod_{l=1}^{n}\mu_{F_i^l}(Z_i)\right]}. \tag{11}$$

Denoting $W = [\overline{W}_1, \ldots, \overline{W}_N]^T$ and $S(Z) = [S_1(Z), \ldots, S_N(Z)]^T$, the FLS can be reformulated as

$$\varpi(Z) = W^T S(Z). \tag{12}$$

Lemma 5 (see [12]). *For a continuous function $f(Z)$ defined on a compact set Ω and $\forall\varepsilon > 0$, there exists an FLS (12) satisfying*

$$\sup_{Z \in \Omega}\left|f(Z) - W^T S(Z)\right| \le \varepsilon. \tag{13}$$

3. Adaptive Fuzzy Controller Design

In this section, a fuzzy-based adaptive control scheme is proposed by using the backstepping technique and FLSs for system (4). The design process of the controller contains n steps based on the following change of coordinates:

$$\begin{cases} z_1 = x_1 - y_d, \\ z_i = x_i - \alpha_{i-1}, \quad i = 2, \ldots, n, \end{cases} \tag{14}$$

where α_{i-1} is an intermediate control which will be developed for the corresponding i-subsystem combined with choosing the proper Lyapunov functions. The actual control law u will be constructed at Step n to cope with the stability problem of the closed-loop system and the unknown disturbance.

In each step, we apply an FLS $W_i^T S_i(Z_i)$ to approximate the unknown nonlinearities and define an uncertain parameter θ_i, $\widehat{\theta}_i$ is the estimate of θ_i, and $\widetilde{\theta}_i = \theta_i - \widehat{\theta}_i$. For the sake of simplicity, $h_i(\overline{x}_i, x_{i+1}^{\delta_i})$ and $h_n(\overline{x}_n, u^{\delta_n})$ will be abbreviated to h_i for $i = 1, \ldots, n-1$ and h_n.

Step 1: according to $z_1 = x_1 - y_d$, and $x_2 = z_2 + \alpha_1$, it follows from (14) that

$$\begin{aligned} \dot{z}_1 &= \dot{x}_1 - \dot{y}_d, \\ &= f_1 + h_1 x_2 - \dot{y}_d, \\ &= f_1 + h_1(z_2 + \alpha_1) - \dot{y}_d, \\ &= f_1 + h_1 z_2 + h_1\alpha_1 - \dot{y}_d. \end{aligned} \tag{15}$$

Consider Lyapunov function V_1 as

$$V_1 = \frac{1}{2}z_1^2 + \frac{h_1'\widetilde{\theta}_1^2}{2r_1}, \tag{16}$$

where $h_1' = \sqrt{\underline{h}_1}$ and the design parameter $r_1 > 0$. Then, we get

$$\begin{aligned} \dot{V}_1 &= z_1\dot{z}_1 - \frac{h_1'}{r_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1, \\ &= z_1(h_1 z_2 + h_1\alpha_1 + \Lambda_1) - \frac{z_1^2}{2} - \frac{h_1'}{r_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1, \end{aligned} \tag{17}$$

where $\Lambda_1 = f_1 - \dot{y}_d + (z_1/2)$ is an unknown smooth function, and it can be approximated by FLS such that

$$\Lambda_1(Z_1) = W_1^T S_1(Z_1) + \delta_1(Z_1), \quad |\delta_1(Z_1)| \le \varepsilon_1, \tag{18}$$

where $Z_1 = [x_1, y_d, \dot{y}_d]^T$, $\delta_1(Z_1)$ is the estimate error. By using Lemma 5 and the completion of squares, one has

$$\begin{aligned} z_1\Lambda_1(Z_1) &= z_1\left(W_1^T S_1(Z_1) + \delta_1(Z_1)\right) \\ &\le |z_1|\left(\|W_1\|\|S_1(Z_1)\| + \varepsilon_1\right) \\ &\le \frac{h_1'}{2a_1^2}z_1^2\theta_1 S_1^T S_1 + \frac{a_1^2}{2} + \frac{z_1^2}{2} + \frac{\varepsilon_1^2}{2}, \end{aligned} \tag{19}$$

where $\theta_1 = \|W_1\|^2/h_1'$ and the constant $a_1 > 0$.

Substituting (19) into (17) yields

$$\dot{V}_1 \le z_1 h_1 z_2 + z_1 h_1\alpha_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{h_1'}{2a_1^2}z_1^2\theta_1 S_1^T S_1 - \frac{h_1'}{r_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1. \tag{20}$$

The virtual control law α_1 is designed as follows:

$$\alpha_1 = -\frac{z_1\breve{\alpha}_1^2}{\sqrt{z_1^2\breve{\alpha}_1^2 + \varepsilon_1^2}}. \tag{21}$$

Combining with Lemma 2, one has

$$\begin{aligned} z_1 h_1\alpha_1 &\le -\frac{\underline{h}_1 z_1^2\breve{\alpha}_1^2}{\sqrt{z_1^2\breve{\alpha}_1^2 + \varepsilon_1^2}}. \\ &\le \varepsilon_1 - z_1\breve{\alpha}_1 h_1' \end{aligned} \tag{22}$$

Then, choose $\breve{\alpha}_1$ and adaption law $\dot{\widehat{\theta}}_1$ as

$$\breve{\alpha}_1 = \frac{1}{2}K_{11}z_1 + K_{12}\left(\frac{1}{2}\right)^{3/4}\left(z_1^2\right)^{1/4} + \frac{1}{2a_1^2}z_1\widehat{\theta}_1 S_1^T S_1, \tag{23}$$

$$\dot{\widehat{\theta}}_1 = \frac{r_1}{2a_1^2}z_1^2 S_1^T S_1 - \varepsilon_1\widehat{\theta}_1, \tag{24}$$

where the design parameters $a_1, K_{11}, K_{12} > 0$. Substituting (21)–(24) into (20) yields

$$\dot{V}_1 \le -\overline{K}_{11}\left(\frac{z_1^2}{2}\right) - \overline{K}_{12}\left(\frac{z_1^2}{2}\right)^{3/4} + z_1 h_1 z_2 + \frac{h_1'\varepsilon_1\widetilde{\theta}_1\widehat{\theta}_1}{r_1} + \sigma_1, \tag{25}$$

where $\overline{K}_{11} = K_{11}h_1'$, $\overline{K}_{12} = K_{12}h_1'$, and $\sigma_1 = (a_1^2/2) + (\varepsilon_1^2/2) + \varepsilon_1 > 0$.

Step $i\,(2 \le i \le n-1)$: utilizing the coordinate transformation in (14), the derivative of z_i is

$$\begin{aligned} \dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1}, \\ &= f_i + h_i x_{i+1} - \dot{\alpha}_{i-1}, \\ &= f_i + h_i(z_{i+1} + \alpha_i) - \dot{\alpha}_{i-1}, \\ &= f_i + h_i z_{i+1} + h_i\alpha_i - \dot{\alpha}_{i-1}, \end{aligned} \tag{26}$$

where $\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial x_j)(f_j + h_j x_{j+1}) + \sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial\widehat{\theta}_j)\dot{\widehat{\theta}}_j + \sum_{j=0}^{i-1}(\partial\alpha_{i-1}/\partial y_d^{(j)})y_d^{(j+1)}$.

Select a Lyapunov function as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{h_i'\tilde{\theta}_i^2}{2r_i}, \tag{27}$$

where $h_i' = \sqrt{\underline{h}_i}$ and the parameter $r_i > 0$.
Next, the derivative of V_i is

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + z_i\dot{z}_i - \frac{h_i'\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i},\\ &= -\sum_{j=1}^{i-1}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{i-1}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4}\\ &\quad + \sum_{j=1}^{i-1}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j} + z_i\left(h_iz_{i+1} + h_i\alpha_i + \Lambda_i\right)\\ &\quad - \frac{z_i^2}{2} - \frac{h_i'\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i} + \sigma_{i-1},\end{aligned} \tag{28}$$

where $\Lambda_i = f_i - \dot{\alpha}_{i-1} + z_{i-1}h_{i-1} + (z_i/2)$ is an unknown smooth function, and we adopt an FLS to approximate it such that

$$\Lambda_i(Z_i) = W_i^TS_i(Z_i) + \delta_i(Z_i), \left|\delta_i(Z_i)\right| \le \varepsilon_i. \tag{29}$$

Combing Lemma 5 with the completion of squares, the following result holds:

$$\begin{aligned}z_i\Lambda_i(Z_i) &= z_i\left(W_i^TS_i(Z_i) + \delta_i(Z_i)\right)\\ &\le |z_i|\left(\|W_i\|\|S_i(Z_i)\| + \varepsilon_i\right)\\ &\le \frac{h_i'}{2a_i^2}z_i^2\theta_iS_i^TS_i + \frac{a_i^2}{2} + \frac{z_i^2}{2} + \frac{\varepsilon_i^2}{2},\end{aligned} \tag{30}$$

where $\theta_i = \|W_i\|^2/h_i'$ and $a_i > 0$ is a constant.
Furthermore, substituting (30) into (28) yields

$$\begin{aligned}\dot{V}_i \le &-\sum_{j=1}^{i-1}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{i-1}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4}\\ &+ \sum_{j=1}^{i,1}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j}\\ &- \frac{h_i'\tilde{\theta}_i\dot{\hat{\theta}}_i}{r_i} + z_ih_iz_{i+1} + z_ih_i\alpha_i + \sigma_{i-1}\\ &+ \frac{h_i'}{2a_i^2}z_i^2\theta_iS_i^TS_i + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2}.\end{aligned} \tag{31}$$

The virtual control law α_i is chosen as follows:

$$\alpha_i = -\frac{z_i\breve{\alpha}_i^2}{\sqrt{z_i^2\breve{\alpha}_i^2 + \varepsilon_i^2}}. \tag{32}$$

By using Lemma 2, we have

$$\begin{aligned}z_ih_i\alpha_i &\le -\frac{\underline{h}_iz_i^2\breve{\alpha}_i^2}{\sqrt{z_i^2\breve{\alpha}_i^2 + \varepsilon_i^2}}.\\ &\le \varepsilon_i - z_i\breve{\alpha}_ih_i'\end{aligned} \tag{33}$$

Next, $\breve{\alpha}_i$ and $\dot{\hat{\theta}}_i$ are chosen as

$$\breve{\alpha}_i = \frac{1}{2}K_{i1}z_i + K_{i2}\left(\frac{1}{2}\right)^{3/4}\left(z_i^2\right)^{1/4} + \frac{1}{2a_i^2}z_i\hat{\theta}_iS_i^TS_i, \tag{34}$$

$$\dot{\hat{\theta}}_i = \frac{r_i}{2a_i^2}z_i^2S_i^TS_i - \varepsilon_i\hat{\theta}_i, \tag{35}$$

where a_i, K_{i1}, K_{i2} are positive design parameters.
By substituting (33)–(35) into (31), the following result holds:

$$\begin{aligned}\dot{V}_i \le &-\sum_{j=1}^{i}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{i}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4}\\ &+ \sum_{j=1}^{i}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j} + z_ih_iz_{i+1} + \sigma_i,\end{aligned} \tag{36}$$

where $\sum_{j=1}^{i}\overline{K}_{j1} = \sum_{j=1}^{i}K_{j1}h_j'$, $\sum_{j=1}^{i}\overline{K}_{j2} = \sum_{j=1}^{i}K_{j2}h_j'$, and $\sigma_i = \sigma_{i-1} + (a_i^2/2) + (\varepsilon_i^2/2) + \varepsilon_i > 0$.
Step n: the control input u will be designed in this step. Since $z_n = x_n - \alpha_{n-1}$ in (14), the derivative of z_n is

$$\begin{aligned}\dot{z}_n &= \dot{x}_n - \dot{\alpha}_{n-1},\\ &= f_n + h_nu + d(t) - \dot{\alpha}_{n-1}.\end{aligned} \tag{37}$$

Consider the following Lyapunov function:

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{h_n'\tilde{\theta}_n^2}{2r_n} + \frac{1}{2r_d}\tilde{d}^2, \tag{38}$$

where $h_n' = \sqrt{\underline{h}_n}$, $\tilde{d} = d^* - \hat{d}$, $\hat{d}$ is the estimate of the disturbance d^*, and r_n and r_d are positive design parameters.
Then, the time derivative of V_n is expressed as

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + z_n\dot{z}_n - \frac{h_n'\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} - \frac{1}{r_d}\tilde{d}\dot{\hat{d}}, \\ &= -\sum_{j=1}^{n-1}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{n-1}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4} + \sum_{j=1}^{n-1}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j} \\ &\quad + z_n(h_n u + \Lambda_n) - \frac{z_n^2}{2} + z_n d(t) \\ &\quad - \frac{h_n'\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} + \frac{1}{r_d}\tilde{d}\dot{\hat{d}} + \sigma_{n-1},\end{aligned} \tag{39}$$

where $\Lambda_n = f_n - \dot{\alpha}_{n-1} + z_{n-1}h_{n-1} + (z_n/2)$ is an unknown smooth function, and it can be approximated by FLS such that

$$\Lambda_n(Z_n) = W_n^T S_n(Z_n) + \delta_n(Z_n), \quad |\delta_n(Z_n)| \le \varepsilon_n. \tag{40}$$

With the help of the completion of squares and Lemma 5, one has

$$\begin{aligned}z_n\Lambda_n(Z_n) &= z_n\left(W_n^T S_n(Z_n) + \delta_n(Z_n)\right) \\ &\le |z_n|\left(\|W_n\|\|S_n(Z_n)\| + \varepsilon_n\right) \\ &\le \frac{h_n'}{2a_n^2}z_n^2\theta_n S_n^T S_n + \frac{a_n^2}{2} + \frac{z_n^2}{2} + \frac{\varepsilon_n^2}{2},\end{aligned} \tag{41}$$

where $\theta_n = \|W_n\|^2/h_n'$ and $a_n > 0$ is a constant.

Furthermore, based on Assumption 2, (39) becomes

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n-1}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{n-1}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4} + \sum_{j=1}^{n-1}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j} \\ &\quad - \frac{h_n'\tilde{\theta}_n\dot{\hat{\theta}}_n}{r_n} + z_n h_n u + \frac{h_n'}{2a_n^2}z_n^2\theta_n S_n^T S_n \\ &\quad + \frac{a_n^2}{2} + \frac{\varepsilon_n^2}{2} + \sigma_{n-1} + |z_n|d^* - \frac{1}{r_d}\tilde{d}\dot{\hat{d}}.\end{aligned} \tag{42}$$

Design the actual control signal u

$$u = -\frac{z_n\check{\alpha}_n^2}{\sqrt{z_n^2\check{\alpha}_n^2 + \varepsilon_n^2}} - \frac{\tanh(z_n/\omega)\hat{d}}{\underline{h}_n}, \tag{43}$$

where ω is a positive constant.

Combing (43) with Lemma 2, one has

$$\begin{aligned}z_n h_n u &\le -\frac{\underline{h}_n z_n^2\check{\alpha}_n^2}{\sqrt{z_n^2\check{\alpha}_n^2 + \varepsilon_n^2}} - z_n\tanh\left(\frac{z_n}{\omega}\right)\hat{d} \\ &\le \varepsilon_n - z_n\check{\alpha}_n h_n' - z_n\tanh\left(\frac{z_n}{\omega}\right)\hat{d}.\end{aligned} \tag{44}$$

Then, choose $\check{\alpha}_n$ and $\dot{\hat{\theta}}_n$ as follows:

$$\check{\alpha}_n = \frac{1}{2}K_{n1}z_n + K_{n2}\left(\frac{1}{2}\right)^{3/4}\left(z_n^2\right)^{1/4} + \frac{1}{2a_n^2}z_n\hat{\theta}_n S_n^T S_n, \tag{45}$$

$$\dot{\hat{\theta}}_n = \frac{r_n}{2a_n^2}z_n^2 S_n^T S_n - \varepsilon_n\hat{\theta}_n, \tag{46}$$

where a_n, K_{n1}, K_{n2} are positive constants.

Combing (43)–(46), V_n can be rewritten as

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n-1}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{n-1}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4} + \sum_{j=1}^{n-1}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j} \\ &\quad - \frac{1}{2}K_{n1}h_n'z_n^2 - K_{n2}h_n'\left(\frac{z_n^2}{2}\right)^{3/4} - z_n\tanh\left(\frac{z_n}{\omega}\right)\left(d^* - \tilde{d}\right) \\ &\quad + |z_n|d^* + \frac{a_n^2}{2} + \frac{\varepsilon_n^2}{2} + \sigma_{n-1} + \varepsilon_n - \frac{\tilde{d}\dot{\hat{d}}}{r_d}.\end{aligned} \tag{47}$$

Utilizing the following property of the hyperbolic tangent function

$$0 \le |z_n| - z_n\tanh\left(\frac{z_n}{\omega}\right) \le 0.2785\omega, \tag{48}$$

we have

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n-1}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right) - \sum_{j=1}^{n-1}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4} + \sum_{j=1}^{n-1}\frac{h_j'\varepsilon_j\tilde{\theta}_j\hat{\theta}_j}{r_j} \\ &\quad - \frac{1}{2}K_{n1}h_n'z_n^2 - K_{n2}h_n'\left(\frac{z_n^2}{2}\right)^{3/4} + \tilde{d}\left(z_n\tanh\left(\frac{z_n}{\omega}\right) - \frac{\dot{\hat{d}}}{r_d}\right) \\ &\quad + 0.2785\omega d^* + \frac{a_n^2}{2} + \frac{\varepsilon_n^2}{2} + \sigma_{n-1} + \varepsilon_n,\end{aligned} \tag{49}$$

and $\dot{\hat{d}}$ can be constructed as

$$\dot{\hat{d}} = r_d\left[z_n\tanh\left(\frac{z_n}{\omega}\right) - \sigma_d\hat{d}\right], \tag{50}$$

where σ_d is a positive constant.

By Young's inequality, the following inequality holds:

$$\begin{aligned}\sigma_d\tilde{d}\hat{d} &= \sigma_d\tilde{d}\left(d^* - \tilde{d}\right) \\ &\le -\frac{\sigma_d\tilde{d}^2}{2} + \frac{\sigma_d d^{*2}}{2}.\end{aligned} \tag{51}$$

Substituting (50) and (51) into (49), it yields

$$\dot{V}_n \le -\sum_{j=1}^{n}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right)-\sum_{j=1}^{n}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4} + \sum_{j=1}^{n}\frac{h_j'\varepsilon_j\widetilde{\theta}_j\widehat{\theta}_j}{r_j}+0.2785\omega d^*+\frac{\sigma_d d^{*2}}{2}-\frac{\sigma_d\widetilde{d}^2}{2}+\sigma_n, \tag{52}$$

where $\sum_{j=1}^{n}\overline{K}_{j1}=\sum_{j=1}^{n}K_{j1}h_j'$, $\sum_{j=1}^{n}\overline{K}_{j2}=\sum_{j=1}^{n}K_{j2}h_j'$, and $\sigma_n=\sigma_{n-1}+(a_n^2/2)+(\varepsilon_n^2/2)+\varepsilon_n>0$.

For $\overline{\mu}_1=\min(\overline{K}_{11},\overline{K}_{21},\ldots,\overline{K}_{n1})$, $\overline{\mu}_2=\min(\overline{K}_{12},\overline{K}_{22},\ldots,\overline{K}_{n2})$, and Lemma 3, we have

$$-\sum_{j=1}^{n}\overline{K}_{j1}\left(\frac{z_j^2}{2}\right)\le-\overline{\mu}_1\sum_{j=1}^{n}\left(\frac{z_j^2}{2}\right)\le-\overline{\mu}_1\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right), \tag{53}$$

$$-\sum_{j=1}^{n}\overline{K}_{j2}\left(\frac{z_j^2}{2}\right)^{3/4}\le-\overline{\mu}_2\sum_{j=1}^{n}\left(\frac{z_j^2}{2}\right)^{3/4}\le-\overline{\mu}_2\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{3/4}. \tag{54}$$

Furthermore, by applying Young's inequality, we obtain

$$\sum_{j=1}^{n}\frac{h_j'\varepsilon_j\widetilde{\theta}_j\widehat{\theta}_j}{r_j}\le-\sum_{j=1}^{n}\frac{h_j'\varepsilon_j\widetilde{\theta}_j^2}{2r_j}+\sum_{j=1}^{n}\frac{h_j'\varepsilon_j\theta_j^2}{2r_j}. \tag{55}$$

Substituting (53)–(55) into (52), one gets

$$\begin{aligned}\dot{V}_n\le&-\overline{\mu}_1\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)-\overline{\mu}_2\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{3/4}-\sum_{j=1}^{n}\frac{h_j'\varepsilon_j\widetilde{\theta}_j^2}{2r_j}\\&+\sum_{j=1}^{n}\frac{h_j'\varepsilon_j\theta_j^2}{2r_j}-\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}\right)^{3/4}+\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}\right)^{3/4}\\&-\left(\frac{\widetilde{d}^2}{2r_d}\right)^{3/4}+\left(\frac{\widetilde{d}^2}{2r_d}\right)^{3/4}-\frac{\sigma_d\widetilde{d}^2}{2}\\&+0.2785\omega d^*+\frac{\sigma_d d^{*2}}{2}+\sigma_n.\end{aligned} \tag{56}$$

Using Lemma 4, for $o_1=1-o_2, o_2=3/4, o_3=o_1o_2^3, \gamma_1=1, \gamma_2=\sum_{j=1}^{n}\widetilde{\theta}_j^2/2r_j, \gamma_3=1$, the following inequality holds:

$$\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}\right)^{3/4}\le o_3+\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}. \tag{57}$$

Similarly, for $o_1=1-o_2, o_2=3/4, o_3=o_1o_2^3, \gamma_1=1, \gamma_2=\widetilde{d}^2/2r_d, \gamma_3=1$, one has

$$\left(\frac{\widetilde{d}^2}{2r_d}\right)^{3/4}\le o_3+\frac{\widetilde{d}^2}{2r_d}. \tag{58}$$

Substituting (57) and (58) into (56), we get

$$\dot{V}_n\le-\overline{\mu}_1\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)-\overline{\mu}_2\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{3/4}-\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}\right)^{3/4} -\left(h_j'\varepsilon_j-1\right)\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}-\left(\frac{\widetilde{d}^2}{2r_d}\right)^{3/4}-\left(r_d\sigma_d-1\right)\frac{\widetilde{d}^2}{2r_d}+\Xi, \tag{59}$$

where $\Xi=\sum_{j=1}^{n}h_j'\varepsilon_j\theta_j^2/2r_j+0.2785\omega d^*+(\sigma_d d^{*2}/2)+2o_3+\sigma_n$.

Finally, for $\mu_1=\min\{\overline{\mu}_1,h_j'\varepsilon_j-1,r_d\sigma_d-1\}, \mu_2=\min\{\overline{\mu}_2,1\}$, according to Lemma 3, we obtain

$$\begin{aligned}\dot{V}_n\le&-\mu_1\left(\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)+\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}\right)+\left(\frac{\widetilde{d}^2}{2r_d}\right)\right)\\&-\mu_2\left(\left(\sum_{j=1}^{n}\frac{z_j^2}{2}\right)^{3/4}+\left(\sum_{j=1}^{n}\frac{\widetilde{\theta}_j^2}{2r_j}\right)^{3/4}+\left(\frac{\widetilde{d}^2}{2r_d}\right)^{3/4}\right)+\Xi\\\le&-\mu_1V_n-\mu_2V_n^{3/4}+\Xi.\end{aligned} \tag{60}$$

In the present stage, the controller design has been finished and the following theorem concludes the main result of this research:

Theorem 1. *consider the closed-loop system consisting of the plant (1) and the control input (43) with the adaptive laws (24), (35), and (46). Under Assumptions 1–3 and the bounded initial conditions, all the signals defined in the closed-loop system are fast finite-time bounded and the tracking error z_1 satisfies*

$$|z_1|\le\sqrt{\frac{2\Xi}{\theta_0\mu_1}}, \tag{61}$$

with assured settling time T_r as

$$T_r\le\max\left\{t_0+\frac{4}{\theta_0\mu_1}\ln\frac{\theta_0\mu_1V^{1/4}(t_0)+\mu_2}{\mu_2},t_0+\frac{4}{\mu_1}\ln\frac{\mu_1V^{1/4}(t_0)+\theta_0\mu_2}{\theta_0\mu_2}\right\}. \tag{62}$$

Proof. According to (60), it can be concluded that V_n is bounded, since when $V_n\ge\Xi/\mu_1, \dot{V}_n\le-\mu_2V_n^{3/4}\le0$. Also, $z_j,\widetilde{\theta}_j,\widetilde{d}$ are bounded from the boundedness of V_n. The boundedness of $\widetilde{\theta}_j$ and θ_j guarantee the boundedness of $\widehat{\theta}_j$. Similarly, $\widehat{d}$ is also bounded since $\widetilde{d}=\widehat{d}-d^*$. From (24), $\breve{\alpha}_1$ is bounded which is the result from the boundedness of z_1 and $\widehat{\theta}_1$. The boundedness of α_1 in (21) can be also inferred. As $z_2=x_2-\alpha_1$, x_2 is bounded because of the boundedness of α_1 and z_2. In an inductive manner, the boundedness of $\widehat{\theta}_2,\ldots,\widehat{\theta}_n,\alpha_2,\ldots,\alpha_n$, as well as $x_3,\ldots,x_n$, can be

guaranteed. The boundedness of actual control signal u in (43) can also be inferred. Thus, the boundedness of all closed-loop signals can be explained.

Furthermore, we transform (60) $\dot{V}_n \le -\mu_1 V_n - \mu_2 V_n^{3/4} + \Xi$ into the following form $\dot{V}_n \le -(1-\theta_0)\mu_1 V_n - \theta_0\mu_1 V_n - \mu_2 V_n^{3/4} + \Xi$, where $0 < \theta_0 < 1$. When $V_n \ge \Xi/\theta_0\mu_1$, namely, $\Xi \le \theta_0\mu_1 V_n$, one has $\dot{V}_n \le -(1-\theta_0)\mu_1 V_n - \mu_2 V_n^{3/4}$.

Based on (60) and Lemma 1, we obtain that V_n converges to the set $\Omega_V = \{V_n\colon V_n < \Xi/\theta_0\mu_1\}$ in fast finite-time with the settling-time estimation

$$T_r \le \max\left\{t_0 + \frac{4}{\theta_0\mu_1}\ln\frac{\theta_0\mu_1 V^{1/4}(t_0) + \mu_2}{\mu_2}, t_0 + \frac{4}{\mu_1}\ln\frac{\mu_1 V^{1/4}(t_0) + \theta_0\mu_2}{\theta_0\mu_2}\right\}. \tag{63}$$

Combing with the definition of V_n in (38), it can be observed that $1/2z_1^2 \le V_n < \Xi/\theta_0\mu_1$. Therefore, we have $|z_1| \le \sqrt{2\Xi/\theta_0\mu_1}$ in fast finite-time with guaranteed convergence time estimated as T_r in (63).

Through the abovementioned analysis, we have completed the proof. □

4. Simulation

This part gives an example to prove that the proposed control scheme is valid. Consider the second-order pure-feedback nonlinear system with disturbance as follows:

$$\begin{cases} \dot{x}_1 = 0.5\cos(x_1) + x_2 + 0.05\sin(x_2), \\ \dot{x}_2 = \cos(x_2^2) + 2u + 0.5\sin u + 0.1\sin(t), \\ y = x_1, \end{cases} \tag{64}$$

where x_1 and x_2 represent the state variables and y and u denote the output and input signal, respectively. $f_1(\bar{x}_1, x_2) = 0.5\cos(x_1) + x_2 + 0.05\sin(x_2)$, $f_2(\bar{x}_2, u) = \cos(x_2^2) + 2u + 0.5\sin u$, and $d(t) = 0.1\sin(t)$. The purpose is to establish a controller which can guarantee that all signals remain bounded in the closed-loop system, the system output $y = x_1$ tracks the reference signal $y_d = 0.4(\sin(t) + \sin(0.5t))$, and the tracking error $z_1 = x_1 - y_d$ is convergent.

By using Theorem 1, the virtual control signals and actual controller are expressed as follows:

$$\begin{aligned} \alpha_1 &= -\frac{z_1\breve{\alpha}_1^2}{\sqrt{z_1^2\breve{\alpha}_1^2 + \varepsilon_1^2}}, \\ \breve{\alpha}_1 &= \frac{1}{2}K_{11}z_1 + K_{12}\left(\frac{1}{2}\right)^{3/4}\left(z_1^2\right)^{1/4} + \frac{1}{2a_1^2}z_1\widehat{\theta}_1 S_1^T S_1, \\ u &= -\frac{z_2\breve{\alpha}_2^2}{\sqrt{z_2^2\breve{\alpha}_2^2 + \varepsilon_2^2}} - \frac{\tanh(z_2/\omega)\widehat{d}}{\underline{h}_2}, \\ \breve{\alpha}_2 &= \frac{1}{2}K_{21}z_2 + K_{22}\left(\frac{1}{2}\right)^{3/4}\left(z_2^2\right)^{1/4} + \frac{1}{2a_2^2}z_2\widehat{\theta}_2 S_2^T S_2, \end{aligned} \tag{65}$$

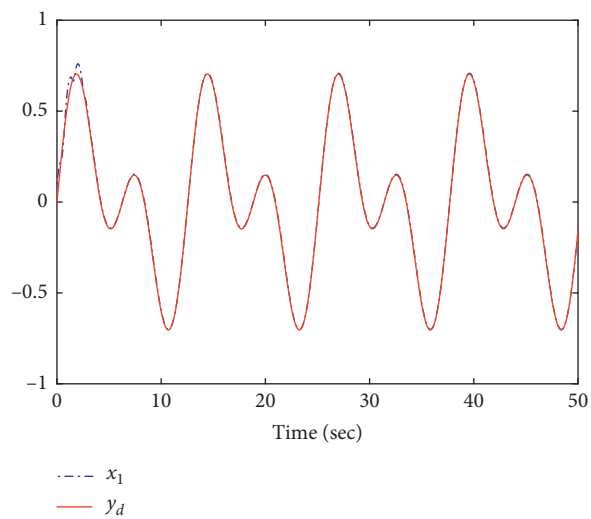

Figure 1: Reference signal y_d and system output x_1.

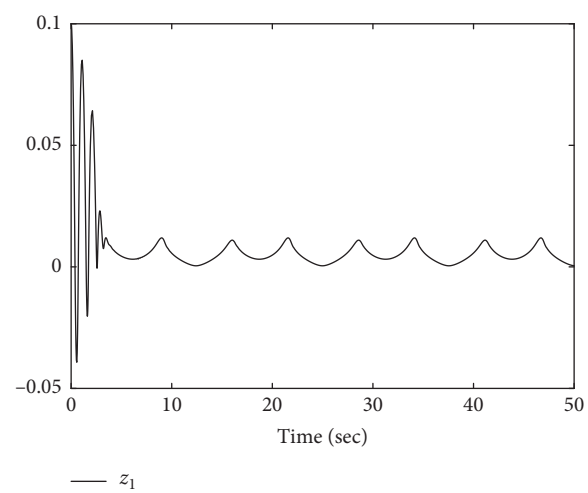

Figure 2: The tracking error z_1.

and the adaptive laws are constructed in the following forms:

$$\begin{aligned} \dot{\widehat{\theta}}_1 &= \frac{r_1}{2a_1^2}z_1^2 S_1^T S_1 - \varepsilon_1\widehat{\theta}_1, \\ \dot{\widehat{\theta}}_2 &= \frac{r_2}{2a_2^2}z_2^2 S_2^T S_2 - \varepsilon_2\widehat{\theta}_2, \\ \dot{\widehat{d}} &= r_d\left[z_2\tanh\left(\frac{z_2}{\omega}\right) - \sigma_d\widehat{d}\right]. \end{aligned} \tag{66}$$

In the simulation, the initial conditions are set as $\left[x_1(0)\, x_2(0)\, \widehat{d}(0)\, \widehat{\theta}_1(0)\, \widehat{\theta}_2(0)\right]^T = \left[0.1\, 0.1\, 0\, 0.1\, 0\right]^T$. Also, the simulation is run by taking the design parameters as $K_{11} = K_{12} = 40, K_{21} = K_{22} = 20, a_1 = a_2 = \varepsilon_1 = \varepsilon_2 = 1$, $r_1 = r_2 = \varepsilon_1 = 0.01, \omega = r_d = 0.1, \sigma_d = 10, \underline{h}_2 = 1$.

Figures 1–5 illustrate the simulation results. Figure 1 shows the desired signal y_d and the output $y = x_1$. Figure 2 depicts the trajectory of the tracking error z_1. Figure 3 shows the curve of the state x_2. From the abovementioned three

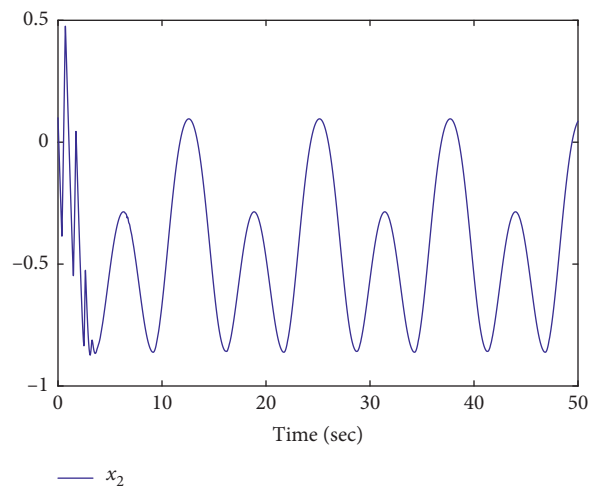

Figure 3: State variable x_2.

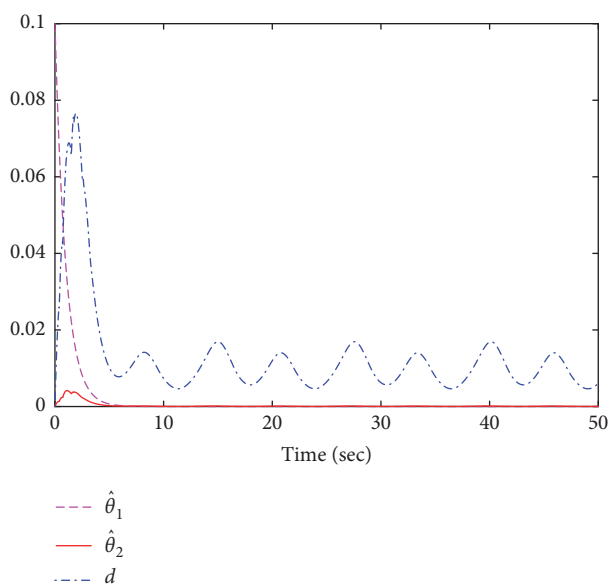

Figure 4: The adaptive parameters $\widehat{\theta}_1, \widehat{\theta}_2, \widehat{d}$.

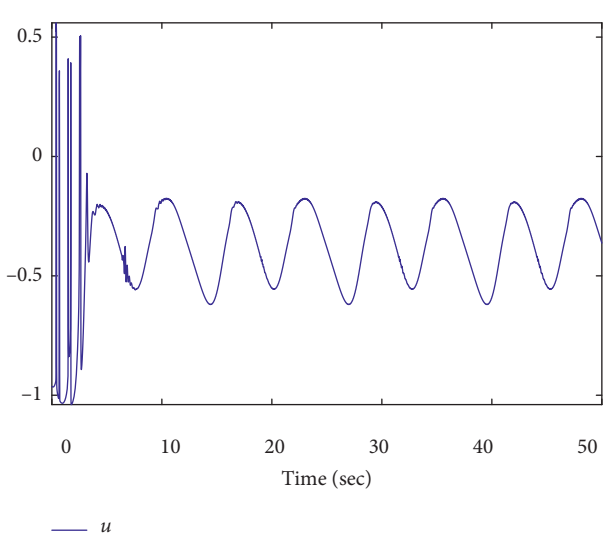

Figure 5: The input signal u.

figures, we can observe that the states $x_1(t)$ and $x_2(t)$ of (64) are bounded and x_1 can tracks y_d under our design controller in Figures 1 and 3. Moreover, the tracking error $z_1 - x_1 - y_d$ in Figure 2 is very small and converges into a small neighborhood of zero. Figure 4 displays that the adaptive laws $\widehat{\theta}_1, \widehat{\theta}_2$, and $\widehat{d}$ are bounded. The trajectory of $u(t)$ is depicted in Figure 5, from which it can be seen that $u(t)$ is bounded. Through the numerical simulation in Figures 1–5, the proposed control scheme achieves that y tracks y_d in a quick and precise way and desired convergence with the control performance.

5. Conclusions

The adaptive fuzzy-based fast finite-time tracking control via the backstepping technique has been developed for nonlinear systems in pure-feedback form with unknown disturbance. The mean value theorem was used to address the nonaffine problem of the pure-feedback systems, and FLSs were applied to approximate packaged unknown nonlinearities. All signals in the closed-loop system are bounded, and the reference signal can be tracked in fast finite-time. Simulation results have been given to prove the effectiveness of the suggested scheme. In this paper, the approximation error of the FLS is not taken into account. The future work will be concentrated on extending the results to more general nonlinear systems.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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