Hindawi
Complexity
Volume 2020, Article ID 6548142, 10 pages
https://doi.org/10.1155/2020/6548142

Research Article

Robust Parametric Control of Lorenz System via State Feedback

Da-Ke Gu [1], **Da-Wei Zhang** [1], **and Yin-Dong Liu** [1,2]

[1] *School of Automation Engineering, Northeast Electric Power University, Jilin, China*
[2] *School of Electrical Engineering, Shenyang University of Technology, Shenyang, China*

Correspondence should be addressed to Yin-Dong Liu; y.d.liu@163.com

Received 30 June 2020; Revised 29 October 2020; Accepted 16 November 2020; Published 24 November 2020

Academic Editor: Chun-Biao Li

This paper considers the parametric control to the Lorenz system by state feedback. Based on the solutions of the generalized Sylvester matrix equation (GSE), the unified explicit parametric expression of the state feedback gain matrix is proposed. The closed loop of the Lorenz system can be transformed into an arbitrary constant matrix with the desired eigenstructure (eigenvalues and eigenvectors). The freedom provided by the parametric control can be fully used to find a controller to satisfy the robustness criteria. A numerical simulation is developed to illustrate the effectiveness of the proposed approach.

1. Introduction

Lorenz system, first studied by Edward Lorenz, is a simplified mathematical model for atmospheric convection [1], which can display a phenomenon called chaos. Chaotic phenomena can be detected in many physical systems such as mechanical systems, electrical systems, and thermal systems [2]. Many scholars pay close attention to it and get a variety of positive achievements. Kim et al. put forward a robust control approach to regulate and synchronize the generalized Lorenz system based on the backstepping method, while the nonlinear and uncertain items can be estimated and canceled [3]. By using the Lyapunov function and Barbalat's lemma, Liu et al. consider the problem of synchronization and antisynchronization in the Lorenz system and apply the result to secure communication with uncertain parameters [4]. Wu et al. propose an adaptive control controller composed of a wavelet network and a proportional controller containing adaptive gain; with this controller, the coupled Lorenz system is globally stable [5]. Yu et al. present a novel approach through switched control and superheteroclinic loops to linearize two symmetrical equilibria and also give the circuit implementation [6]. Zhang et al. design a hybrid controller for the Lorenz system with a piecewise linear memristor and provide criteria to maintain that the trivial solutions are exponentially stable in the mean square [7]. Simultaneously, hidden attractors of the classical Lorenz system are also discussed [8, 9]. More results of the Lorenz system can be found in the literature [10, 11].

Robust control is an approach to design controller that explicitly deals with uncertainty. The most commonly used method in the design and synthesis of robust controllers is based on linear matrix inequalities (LMIs) (see [12–14] and the references therein). Lindemann and Dimarogonas consider the robust model predictive control that the robust optimization problem is transformed into a convex quadratic program [15]. Liu et al. investigate the robust formation of tracking control for multiple quadrotors by linear quadratic regulation and robust compensation theory [16]. Cristofaro proposes a robust sliding-mode controller to globally stabilize the tracking error system with disturbances [17]. There also are several methods to deal with robust control and optimization [18–21]. However, the goal of existed methods is to obtain a controller that satisfies some specific requirements, such as robustness. Once the design requirements change, the design needs to be repeated.

Parametric approach, creatively proposed by Duan [22, 23], solves the controller design of quasi-linear systems via state feedback and output feedback, which develops novel research areas. Furthermore, Gu et al. extend a parametric approach to second-order and high-order quasi-linear systems [24–29] and chaotic systems [30]. In this paper, based on the solutions of a class of GSE [31, 32], a whole set of parametric state feedback controllers is established and the closed-loop system is transformed into the desired eigenstructure. The parametric approach provides a group of arbitrary parameters that represent the degrees of design freedom, to implement state feedback and robust optimization.

The major contributions of this paper are summarized in two aspects. On the one hand, the complete parameterization form of state feedback is established such that closed-loop system has an expected eigenstructure, which results in that the chaos phenomenon has been eliminated effectively. On the other hand, the degrees of design freedom in arbitrary parameters are fully used to realize the robust optimization such that robustness can be improved obviously.

The remainder of this paper is organized as follows. Section 2 analyzes the dynamical behaviors of the Lorenz system. Section 3 presents the problem statement of parametric design for Lorenz system via state feedback and provides some preparations. In Section 4, the generally parameterized form of state feedback is established in two cases, and robust optimization is realized by using the degrees of freedom in arbitrary parameters. In Section 5, a numerical example is presented to prove the parametric approach is effective and feasible. Section 6 concludes the proposed results.

Notation: we propose some notations which are used in this paper. Rank$A$, det$A$, and eig$A$ represent the rank, determinant, and all eigenvalues of matrix A, respectively. deg $(A(s))$ denotes the degree n of polynomial matrix $A(s) = A_0 + sA_1 + \cdots + s^n A_n$; $\mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_n\}$ indicates the diagonal matrix with diagonal elements λ_i, $i = 1, 2, \ldots, n$. max and min represent the maximum and minimum.

2. Analysis of the Lorenz System

Consider a class of generalized Lorenz systems as follows:

$$\begin{cases} \dot{x}_1 = \sigma(x_2 - x_1), \\ \dot{x}_2 = \gamma x_1 - x_2 - x_1 x_3, \\ \dot{x}_3 = x_1 x_2 - \beta x_3, \end{cases} \tag{1}$$

where x_1, x_2, $x_3 \in \mathbb{R}$ are state variables and σ, γ, β are positive real constants. Let $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^{\mathrm{T}}$, and the system (1) can be equivalently written as

$$\dot{x} = Ax + \Phi(x), \tag{2}$$

with

$$A = \begin{bmatrix} -\sigma & \sigma & 0 \\ \gamma & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}, \tag{3}$$

and $\Phi(x)$ is a smooth nonlinear vector field satisfying

$$\begin{gathered} \Phi(x) = \begin{bmatrix} 0 \\ -x_1 x_3 \\ x_1 x_2 \end{bmatrix} = x_1 P x, \\ P = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}. \end{gathered} \tag{4}$$

The divergence of the system (1) is given as

$$\nabla V = \frac{\partial \dot{x}_1}{\partial x_1} + \frac{\partial \dot{x}_2}{\partial x_2} + \frac{\partial \dot{x}_3}{\partial x_3} = -\sigma - \beta - 1 < 0, \tag{5}$$

and $V(t) = V(0)e^{(-\sigma-\beta-1)t}$ with the rate of contraction $\mathrm{d}V/\mathrm{d}t = V(0)e^{-\sigma-\beta-1}$. Inequality (3) demonstrates the existence of chaotic attractor of the system (1).

With $\sigma = 5.46$, $\gamma = 20$, $\beta = 1$, and $x(0) = \begin{bmatrix} 0.5 & -0.3 & -0.1 \end{bmatrix}^{\mathrm{T}}$, two-dimensional phase portraits of the system (1) are shown in Figure 1.

3. Problem Statement and Preliminaries

3.1. Problem Statement. Consider the following controlled Lorenz system:

$$\dot{x} = \mathscr{A}x + \mathscr{B}u, \tag{6}$$

where $u \in \mathbb{R}^r$ is the input vector and $\mathscr{A} = A + x_1 P$ and $\mathscr{B} \in \mathbb{R}^{3\times r}$ are coefficient matrices satisfying Assumption 1.

Assumption 1. Rank $\begin{bmatrix} sI - \mathscr{A} & \mathscr{B} \end{bmatrix} = 3$, $\forall s \in \mathbb{C}$.

For the system (6), using the control input

$$u = Kx, \tag{7}$$

we can obtain the closed-loop system

$$\dot{x} = \mathscr{A}_c x, \tag{8}$$

with

$$\mathscr{A}_c = \mathscr{A} + \mathscr{B}K, \tag{9}$$

where $K \in \mathbb{R}^{r\times 3}$ is the feedback gain matrix to be determined.

The parametric control of Lorenz system (6) via state feedback (7) can be stated as follows.

Problem 1 (PCLS). Let the controlled Lorenz system (6) satisfy Assumption 1. Given an arbitrary matrix $\Lambda \in \mathbb{C}^{3\times 3}$, find for the system, a state feedback controller in the form of (7) and a nonsingular matrix $V \in \mathbb{C}^{3\times 3}$, satisfying

$$\mathscr{A}_c V = V\Lambda. \tag{10}$$

Remark 1. The above problem requirement (10) is equivalent to find a state feedback controller (7) such that the closed-loop system (8) is similar to a linear time-invariant form with the desired eigenstructure; that is, $\mathscr{A}_c$ is similar to an arbitrary matrix Λ by the proposed parametric approach.

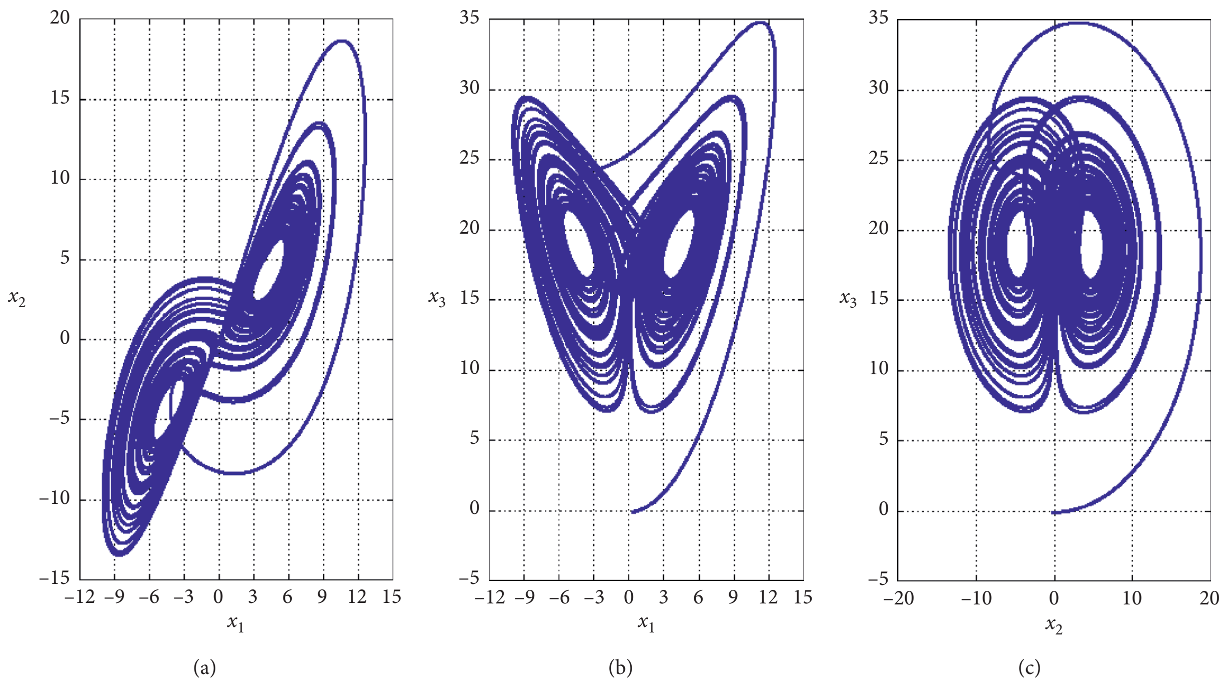

FIGURE 1: Two-dimensional portrait of Lorenz system (1).

Because the closed-loop system (8) is stable [33, 34], Λ is chosen as a Hurwitz matrix, that is,

$$\operatorname{eig}(\Lambda) \in \mathbb{C}^{-}. \tag{11}$$

3.2. Preliminaries. When Assumption 1 is met, there exists the following right coprime factorization (RCF):

$$(sI - \mathscr{A})N(s) - \mathscr{B}D(s) = 0. \tag{12}$$

Denote $D(s) = [d_{ij}(s)]_{r\times r}$, and

$$\omega = \max\left\{\deg\left(d_{ij}(s)\right),\quad i, j = 1, 2, \ldots, r\right\}. \tag{13}$$

Then,

$$\begin{cases} N(s) = \sum_{i=0}^{\omega} N_i s^i, & N_i \in \mathbb{R}^{3\times r}, \\ D(s) = \sum_{i=0}^{\omega} D_i s^i, & D_i \in \mathbb{R}^{r\times r}. \end{cases} \tag{14}$$

4. Main Results

4.1. Parametric Control of Lorenz System. With the above discussion, we propose the following Theorem 1 regarding Problem 1 (PCLS).

Theorem 1. *$N(s)$ and $D(s)$ are given by (14) and satisfy RCF (12). Problem 1 (PCLS) has a general solution if and only if there exists an arbitrary parameter matrix $Z \in \mathbb{C}^{r\times 3}$ such that*

$$\det V \neq 0, \tag{15}$$

where

$$V = \sum_{i=0}^{\omega} N_i Z \Lambda^i. \tag{16}$$

Furthermore, the feedback gain matrix K is solved by

$$K = WV^{-1}, \tag{17}$$

where

$$W = \sum_{i=0}^{\omega} D_i Z \Lambda^i. \tag{18}$$

Proof. Equation (10) is equivalent to

$$\mathscr{A}V + \mathscr{B}KV = V\Lambda. \tag{19}$$

Let

$$W = KV, \tag{20}$$

then equation (19) becomes the GSE:

$$\mathscr{A}V + \mathscr{B}W = V\Lambda. \tag{21}$$

Using (14), we have

$$\begin{aligned}(sI - \mathscr{A})N(s) &= \sum_{i=0}^{\omega} N_i s^{i+1} - \sum_{i=0}^{\omega} \mathscr{A}N_i s^i \\ &= N_\omega s^{\omega+1} + \sum_{i=1}^{\omega} N_{i-1}s^i \\ &\quad - \sum_{i=1}^{\omega} \mathscr{A}N_i s^i - \mathscr{A}N_0 \\ &= N_\omega s^{\omega+1} + \sum_{i=1}^{\omega}\left(N_{i-1} - \mathscr{A}N_i\right)s^i - \mathscr{A}N_0, \\ \mathscr{B}D(s) &= \sum_{i=0}^{\omega} \mathscr{B}D_i s^i = \sum_{i=1}^{\omega} \mathscr{B}D_i s^i + \mathscr{B}D_0.\end{aligned} \tag{22}$$

Substituting the above equations into (12) and equating the coefficients of s^i on both sides, we obtain

$$\begin{aligned}N_\omega &= 0, \\ \mathscr{A}N_0 &= -\mathscr{B}D_0, \\ N_{i-1} - \mathscr{A}N_i &= \mathscr{B}D_i, \quad i = 1, 2, \ldots, \omega.\end{aligned} \tag{23}$$

Based on equations (16), (18), and (23), we have

$$\begin{aligned}V\Lambda - \mathscr{A}V &= \sum_{i=0}^{\omega} N_i Z\Lambda^{i+1} - \sum_{i=0}^{\omega} \mathscr{A}N_i Z\Lambda^i \\ &= N_\omega Z\Lambda^{\omega+1} + \sum_{i=1}^{\omega} N_{i-1}Z\Lambda^i - \sum_{i=1}^{\omega} \mathscr{A}N_i Z\Lambda^i - \mathscr{A}N_0 \\ &= N_\omega Z\Lambda^{\omega+1} + \sum_{i=1}^{\omega}\left(N_{i-1} - \mathscr{A}N_i\right)Z\Lambda^i - \mathscr{A}N_0 \\ &= \sum_{i=1}^{\omega} \mathscr{B}D_i Z\Lambda^i + \mathscr{B}D_0 = \mathscr{B}\sum_{i=0}^{\omega} D_i Z\Lambda^i = \mathscr{B}W.\end{aligned} \tag{24}$$

This proves that the matrices V in (16) and W in (18) satisfy the GSE (21). Finally, combining equations (16) and (18) with (20), the feedback gain matrix can be obtained as equation (17).

The proof is completed. □

In application, we choose the matrix Λ in the diagonal form of

$$\Lambda = \operatorname{diag}\{\lambda_1, \lambda_2, \lambda_3\}, \tag{25}$$

where $\lambda_i \in \mathbb{C}^-, i = 1, 2, 3$. Based on Theorem 1, we can derive the following Corollary 1 to Problem 1 (PCLS).

Corollary 1. *$N(s)$ and $D(s)$ are given by (9) and satisfy RCF (8). Problem 1 (PCLS) has a general solution if and only if there exists a group of parameter vectors $z_i \in \mathbb{C}^r$, $i = 1, 2, 3$, such that condition (15) is satisfied, where matrix V can be given by columns*

$$\begin{aligned}V &= \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix}, \\ v_i &= N(\lambda_i)z_i, \quad i = 1, 2, 3, \\ Z &= \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}.\end{aligned} \tag{26}$$

Furthermore, the feedback gain matrix K is solved by

$$K = WV^{-1}, \tag{27}$$

where matrix W can be given by columns

$$\begin{aligned}W &= \begin{bmatrix} w_1 & w_2 & w_3 \end{bmatrix}, \\ w_i &= D(\lambda_i)z_i, \quad i = 1, 2, 3.\end{aligned} \tag{28}$$

4.2. Robust Optimization. Theorem 1 and Corollary 1 show that the matrix Z is an arbitrary parameter which can provide the degrees of freedom to analyse and design problems. In this subsection, robust optimization is considered such that regional pole assignment and robustness criteria are investigated.

4.2.1. Regional Pole Assignment. We aim to locate the eigenvalues λ_i, $i = 1, 2, 3$ in an admissible set to satisfy control requirements of practical applications. Actually, eigenvalues can be considered within a small interval around the expected locations to reduce the difficulty and improve the flexibility when implementing controller. In this case, the complex pole is defined as

$$\lambda_i = \lambda_i^{\mathrm{re}} + j\lambda_i^{\mathrm{im}}, \quad i = 1, 2, 3, \tag{29}$$

with

$$\begin{aligned}\lambda_i^{\mathrm{re}} &= \underline{\lambda}_i^{\mathrm{re}} + \left(\overline{\lambda}_i^{\mathrm{re}} - \underline{\lambda}_i^{\mathrm{re}}\right)\sin^2\left(\|z_i\|_2\right), \\ \lambda_i^{\mathrm{im}} &= \underline{\lambda}_i^{\mathrm{im}} + \left(\overline{\lambda}_i^{\mathrm{im}} - \underline{\lambda}_i^{\mathrm{im}}\right)\sin^2\left(\|z_i\|_2\right),\end{aligned} \tag{30}$$

where λ_i^{re} and λ_i^{im} are real and imaginary parts of complex pole and $\overline{\lambda}_i^{\mathrm{re}}$, $\underline{\lambda}_i^{\mathrm{re}}$ and $\overline{\lambda}_i^{\mathrm{im}}$, $\underline{\lambda}_i^{\mathrm{im}}$ are the upper and lower bound of λ_i^{re} and λ_i^{im}, respectively. For real pole, equation (29) can be simplified as

$$\lambda_i = \underline{\lambda}_i + \left(\overline{\lambda}_i - \underline{\lambda}_i\right)\sin^2\left(\|z_i\|_2\right), \quad i = 1, 2, 3. \tag{31}$$

Equation (29) shows the arbitrary parameter Z plays an important role in determining the location of eigenvalues, which becomes the decision factor in robust optimization.

4.2.2. Robustness Criteria. Thus, this paper fully employs the freedom to find a feedback controller to satisfy the robustness criteria. For the closed-loop system (8), a bounded disturbance Gw is considered as

$$\dot{x} = \mathscr{A}_c x + Gw, \tag{32}$$

where $w \in \mathbb{R}^p$ is the disturbance vector and $G \in \mathbb{R}^{n\times p}$ is the disturbance matrix. In this subsection, a multiobjective robustness criterion is proposed to improve the robustness of the closed-loop system (8).

First, let us introduce the robustness degree [35], a simple and useful method to measure the ability of system for overcoming external disturbances, modeling errors, and other uncertainties, defined as

$$J_1 = \min\{\psi_1, \psi_2, \psi_3\}, \tag{33}$$

where $\psi_i = |\mathrm{eig}(1/2(\mathscr{A}_c + \mathscr{A}_c^{\mathrm{T}}))|$, $i = 1, 2, 3$.

Second, we consider the disturbance attenuation which investigates the effects of external disturbances to closed-loop systems, and the objective is expressed in terms of the H_2-norm of the following function [36]:

$$J_2 = \|VG\|_2. \tag{34}$$

Third, in order to maintain the performance robustness and stability robustness when parameter uncertainty exists, a general way is to minimize the sensitivity function for closed-loop eigenvalues, and the overall eigenvalue sensitivity is defined as follows:

$$J_3 = \|V\|_2 \left\|V^{-1}\right\|_2. \tag{35}$$

Combining equations (33)–(35), the multiobjective robustness criteria are established to represent the robustness as

$$J = \varepsilon_1 J_1 + \varepsilon_2 J_2 + \varepsilon_3 J_3, \tag{36}$$

where ε_1, ε_2, and ε_3 are the weight coefficients satisfying

$$\begin{aligned} \varepsilon_1 &= \frac{J_1}{J_1 + J_2 + J_3}, \\ \varepsilon_2 &= \frac{J_2}{J_1 + J_2 + J_3}, \\ \varepsilon_3 &= \frac{J_3}{J_1 + J_2 + J_3}, \end{aligned} \tag{37}$$

which means that weight coefficients ε_1, ε_2, and ε_3 can be updated in real time. Then, we formulate the robust optimization into a multiobjective problem as

$$\begin{cases} \min J, \\ \text{s.t. } (10), (18), (22). \end{cases} \tag{38}$$

Remark 2. Based on the above results, a guideline for robust parametric control of Lorenz system (1) via state feedback is proposed as follows:

Step 1: determine a Hurwitz matrix Λ with an expected eigenstructure.

Step 2: obtain the $\{N(s), D(s)\}$ based on (12).

Step 3: seek arbitrary parameter Z by robust optimization problem (38).

Step 4: calculate the coefficient matrix of state feedback (17).

5. An Example

5.1. System Description. Consider the controlled Lorenz system (6) with $\sigma = 5.46$, $\gamma = 20$, and $\beta = 1$, and we have

$$\mathscr{A} = \begin{bmatrix} -5.46 & 5.46 & 0 \\ 20 & -1 & -x_1 \\ 0 & x_1 & -1 \end{bmatrix}. \tag{39}$$

Let

$$\mathscr{B} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{40}$$

based on RCF (8), and $N(s)$ and $D(s)$ are obtained as

$$\begin{aligned} N(s) &= \begin{bmatrix} 1 & 0 \\ \frac{50}{273}s + 1 & 0 \\ 0 & 1 \end{bmatrix}, \\ D(s) &= \begin{bmatrix} -\frac{50}{273}x_1 s - x_1 & s + 1 \\ \frac{50}{273}s^2 + \frac{323}{273}s - 19 & x_1 \end{bmatrix}. \end{aligned} \tag{41}$$

We take

$$\begin{aligned} \lambda_1 &\in [-1.2, -0.8], \\ \lambda_2 &\in [-2.1, -1.9), \\ \lambda_3 &\in [-3.3, -2.7), \end{aligned} \tag{42}$$

and a bounded disturbance

$$\begin{aligned} G &= \begin{bmatrix} 0 \\ -0.2 \\ 0.5 \end{bmatrix}, \\ w(t) &= \begin{cases} 1, & t \in [8, 8.5], \\ 0, & \text{else}. \end{cases} \end{aligned} \tag{43}$$

5.2. Nonoptimized Solution. Using the *randn* function in Matlab® to generate an arbitrary parameter matrix Z as

$$Z = \begin{bmatrix} 4.2589 & 1.5079 & 3.5294 \\ 4.6232 & 4.6535 & 1.3902 \end{bmatrix}, \tag{44}$$

and $\Lambda = \mathrm{diag}\{-1, -2, -3\}$; according to Theorem 1 or Corollary 1 with (41), we have

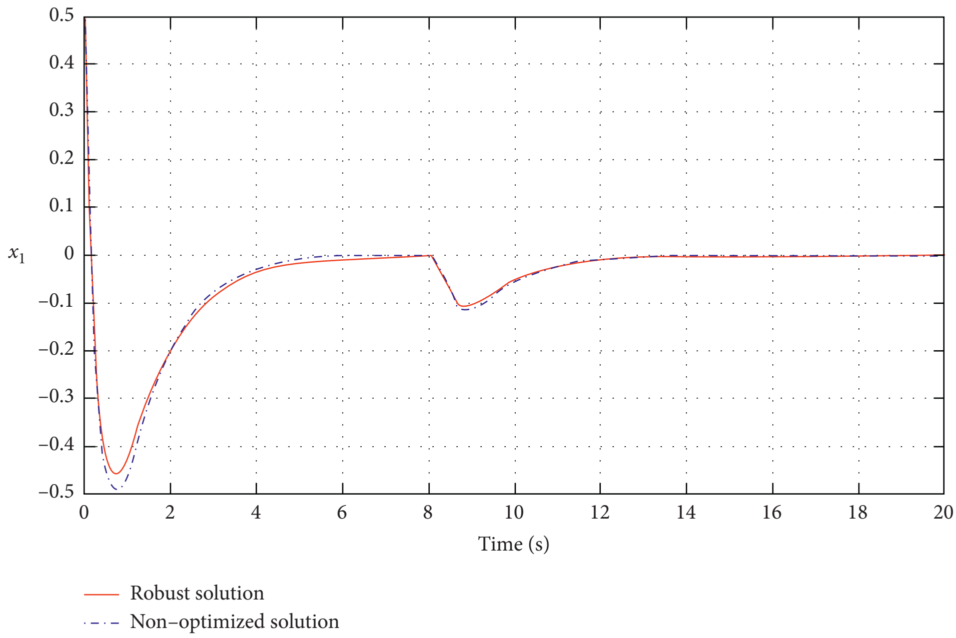

FIGURE 2: Comparison of x_1 between robust solution and nonoptimized solution.

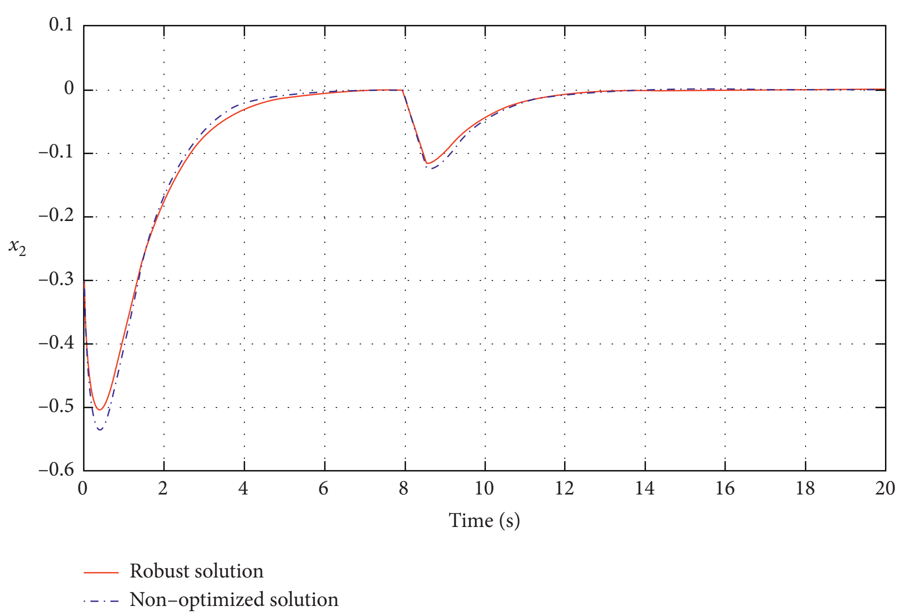

FIGURE 3: Comparison of x_2 between robust solution and nonoptimized solution.

$$V = \begin{bmatrix} 4.2589 & 1.5079 & 3.5294 \\ 3.4789 & 0.9556 & 1.5902 \\ 4.6232 & 4.6535 & 1.3902 \end{bmatrix},$$
$$W = \begin{bmatrix} -3.4789x_1 & -0.9556x_1 - 4.6535 & -1.5902x_1 - 2.7804 \\ 4.6232x_1 - 85.1780 & 4.6535x_1 - 31.1136 & 1.3902x_1 - 73.7683 \end{bmatrix}, \tag{45}$$

$$K = \begin{bmatrix} -2.2809 & -x_1 + 4.3171 & -1.1474 \\ -22.0451 & 2.6074 & x_1 - 0.0781 \end{bmatrix}. \tag{46}$$

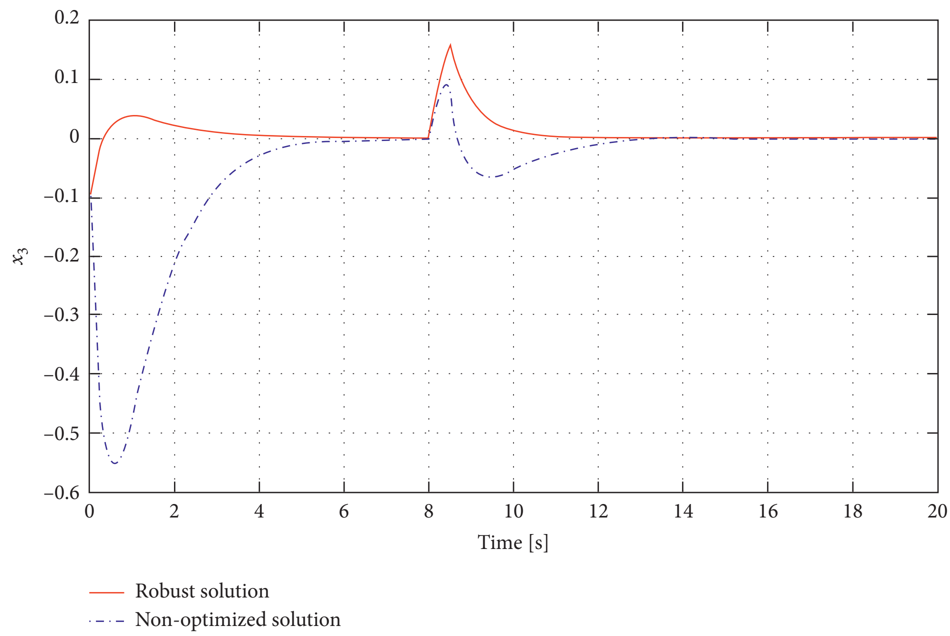

Figure 4: Comparison of x_3 between robust solution and nonoptimized solution.

By using the feedback controller (46), the closed-loop system is

$$\dot{x} = \begin{bmatrix} -5.46 & 5.46 & 0 \\ -2.0451 & 1.6074 & -0.0781 \\ -2.2809 & 4.3171 & -2.1474 \end{bmatrix} x. \tag{47}$$

5.3. Robust Solution. Consider the robust optimization problem (38). Choosing the initial value Z in (44), the optimal Z can be obtained as by the *fminsearch* function in MATLAB Optimization Toolbox:

$$Z = \begin{bmatrix} 4.3072 & 0.4714 & 3.3962 \\ -0.5399 & 6.6488 & -0.1831 \end{bmatrix}, \tag{48}$$

and $\Lambda = \text{diag}\{-0.8527, -2.0722, -3.2605\}$; according to Theorem 1 or Corollary 1 with (41), we have

$$\begin{aligned} V &= \begin{bmatrix} 4.3072 & 0.4714 & 3.3962 \\ 3.6345 & 0.2925 & 1.3681 \\ -0.5399 & 6.6488 & -0.1831 \end{bmatrix}, \\ W &= \begin{bmatrix} -0.36345x_1 - 0.0795 & -0.2925x_1 - 7.1288 & 0.4139 - 1.3681x_1 \\ -0.5399x_1 - 85.6088 & 6.6488x_1 - 9.7416 & -0.1831x_1 - 71.0166 \end{bmatrix}, \end{aligned} \tag{49}$$

$$K = \begin{bmatrix} 0.2622 & -x_1 - 0.4914 & -1.0692 \\ -21.5743 & 1.5682 & x_1 + 0.0349 \end{bmatrix}. \tag{50}$$

By using the feedback controller (50), the closed-loop system is

$$\dot{x} = \begin{bmatrix} -5.46 & 5.46 & 0 \\ -1.8558 & 1.3438 & -0.0187 \\ 0.2622 & -0.4914 & -2.0692 \end{bmatrix} x. \tag{51}$$

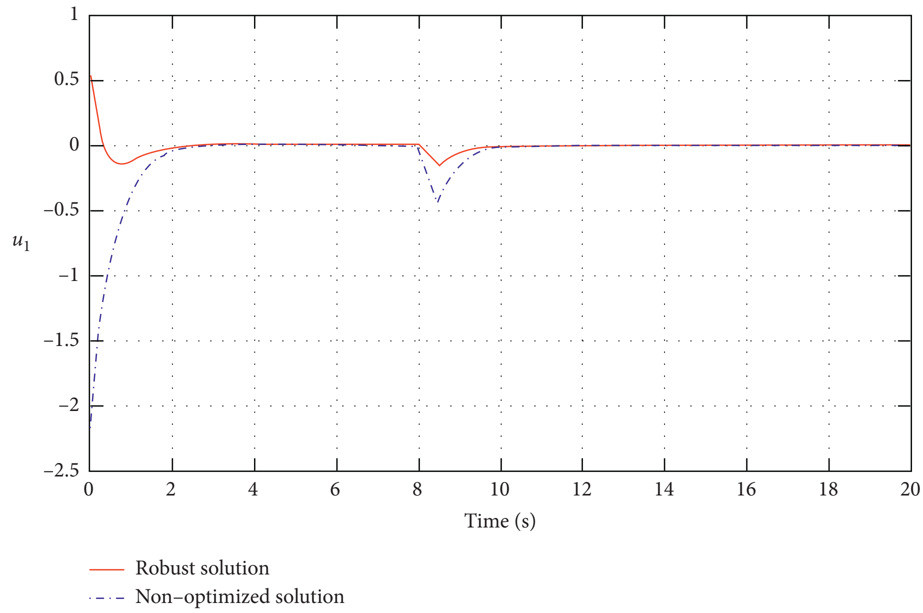

FIGURE 5: Comparison of u_1 between robust solution and nonoptimized solution.

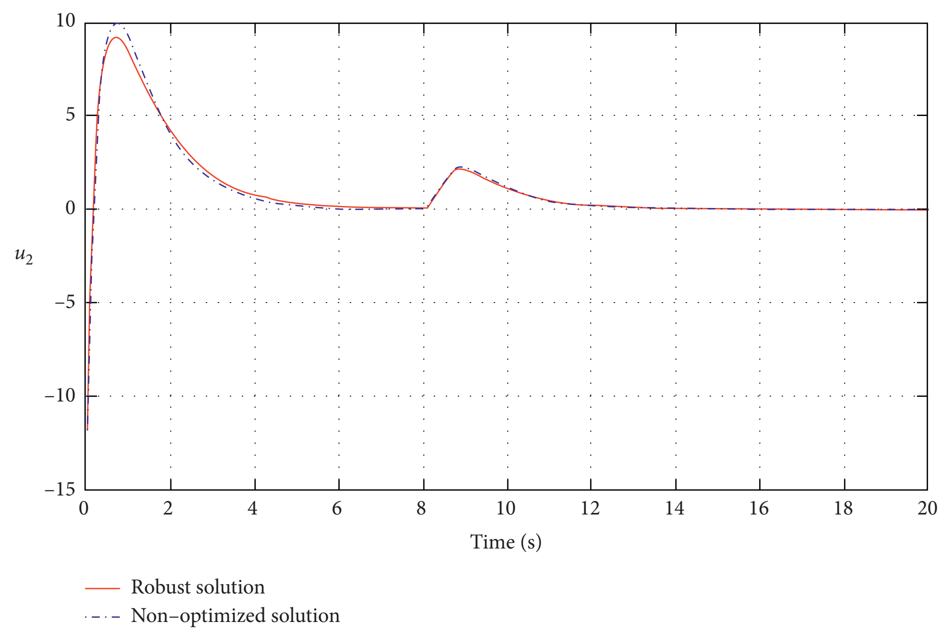

FIGURE 6: Comparison of u_2 between robust solution and nonoptimized solution.

5.4. Simulation Comparison. Choose the initial value as $x(0) = [\,0.5 \;\; -0.3 \;\; -0.1\,]^{\mathrm{T}}$, and the simulation results are shown in Figures 2–6.

Figures 2 and 3 show that robust state feedback yields a better transient performance than that of nonoptimized one. In Figure 4, although the amplitude of the robust solution is higher than nonoptimized one when the disturbance existed, it reduces the frequency of oscillation and arrives at stability rapidly. From Figures 5 and 6, we see that the robust solution consumes less energy.

Let J_o and J_n represent the optimized and nonoptimized indices, and we have

$$\begin{aligned} J_o &= 5.0952, \\ J_n &= 9.5467. \end{aligned} \tag{52}$$

$J_o < J_n$, which means the robustness of the closed-loop system is improved effectively through the robust optimization based on degrees of freedom provided by the proposed parametric approach.

6. Conclusion

A parametric approach, inspired by the solutions of GSE, is provided aiming at the Lorenz system in this study. The proposed parametric approach establishes a more unified parametric expression of state feedback concerning matrices Λ and Z and then transforms the closed-loop system into a linear constant form with the desired eigenstructure. Meanwhile, a robust optimization scheme is also considered based on degrees of design freedom given by the parametric approach. In this scheme, these arbitrary parameters to be optimized are with no physical significances; thus, the optimization interval can be greatly extended, which is a benefit to obtain an approximately globally optimal solution.

Data Availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (Grant nos. 61690210 and 61690212).

References

[1] E. N. Lorenz, "Deterministic nonperiodic flow," *Journal of The Atmospheric Sciences*, vol. 20, no. 2, pp. 130–141, 1963.

[2] R. Gao, "A novel track control for Lorenz system with single state feedback," *Chaos, Solitons & Fractals*, vol. 122, pp. 236–244, 2019.

[3] D. Kim, M. Jin, and P. H. Chang, "Control and synchronization of the generalized Lorenz system with mismatched uncertainties using backstepping technique and time-delay estimation," *International Journal of Circuit Theory and Applications*, vol. 45, no. 11, 2017.

[4] B. Liu, J.-P. Li, and W. Zheng, "Synchronization and adaptive anti-synchronization control for Lorenz systems under channel noise with applications," *Asian Journal of Control*, vol. 15, no. 3, pp. 919–929, 2013.

[5] Y. Wu, J. Braselton, Y. Jin, and A. E. Shahat, "Adaptive control of bi-directionally coupled Lorenz systems with uncertainties," *Journal of the Franklin Institute*, vol. 356, no. 3, pp. 1287–1301, 2019.

[6] S. Yu, J. Lu, X. Yu, and G. Chen, "Design and implementation of grid multiwing hyperchaotic Lorenz system family via switching control and constructing super-heteroclinic loops," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 59, no. 5, pp. 1015–1028, 2012.

[7] B. Zhang, C. Zhang, X. Zhao, and F. Deng, "Hybrid control of stochastic chaotic system based on memristive Lorenz system with discrete and distributed time-varying delays," *IET Control Theory & Applications*, vol. 10, no. 13, pp. 1513–1523, 2016.

[8] B. Munmuangsaen and B. Srisuchinwong, "A hidden chaotic attractor in the classical Lorenz system," *Chaos, Solitons & Fractals*, vol. 107, pp. 61–66, 2018.

[9] J. C. Sprott and B. Munmuangsaen, "Comment on "A hidden chaotic attractor in the classical Lorenz system"" *Chaos, Solitons & Fractals*, vol. 113, pp. 261-262, 2018.

[10] D. Ricci, M. Romero, and M. Seron, "Lyapunov-function-free backstepping design with application to the Lorenz system," *IFAC-PapersOnLine*, vol. 51, no. 33, pp. 223–228, 2018.

[11] B. Sun, M. Li, F. Zhang, H. Wang, and J. Liu, "The characteristics and self-time-delay synchronization of two-time-delay complex Lorenz system," *Journal of the Franklin Institute*, vol. 356, no. 1, pp. 334–350, 2019.

[12] L. A. Rodrigues, R. C. L. F. Oliveira, and J. F. Camino, "Parameterized LMIs for robust H2 and H∞ state feedback control of continuous-time polytopic systems," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 3, pp. 940–952, 2018.

[13] T. E. Rosa, C. F. Morais, and R. C. L. F. Oliveira, "New robust LMI synthesis conditions for mixed H2/H∞ gain-scheduled reduced-order DOF control of discrete-time LPV systems," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 18, pp. 6122–6145, 2018.

[14] Y. Wang, R. Rajamani, and A. Zemouche, "Sequential LMI approach for the design of a BMI-based robust observer state feedback controller with nonlinear uncertainties," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 4, pp. 1246–1260, 2018.

[15] L. Lindemann and D. V. Dimarogonas, "Robust control for signal temporal logic specifications using discrete average space robustness," *Automatica*, vol. 101, pp. 377–387, 2019.

[16] H. Liu, Y. Tian, F. L. Lewis, Y. Wan, and K. P. Valavanis, "Robust formation tracking control for multiple quadrotors under aggressive maneuvers," *Automatica*, vol. 105, pp. 179–185, 2019.

[17] A. Cristofaro, "Robust distributed control of quasilinear reaction-diffusion equations via infinite-dimensional sliding modes," *Automatica*, vol. 104, pp. 165–172, 2019.

[18] M. Yang, Y. Li, H. Du, C. Li, and Z. He, "Hierarchical multiobjective H-infinity robust control design for wireless power transfer system using genetic algorithm," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 4, pp. 1753–1761, 2019.

[19] C. Liu, H. Li, Y. Shi, and D. Xu, "Codesign of event trigger and feedback policy in robust model predictive control," *IEEE Transactions on Automatic Control*, vol. 65, no. 1, pp. 302–309, 2020.

[20] X. Wang, S. Li, S. Su, and T. Tang, "Robust fuzzy predictive control for automatic train regulation in high-frequency metro lines," *IEEE Transactions on Fuzzy Systems*, vol. 27, no. 6, pp. 1295–1308, 2019.

[21] C. Liu, J. Gao, H. Li, and D. Xu, "Aperiodic robust model predictive control for constrained continuous-time nonlinear systems: an event-triggered approach," *IEEE Transactions on Cybernetics*, vol. 48, no. 5, pp. 1397–1405, 2018.

[22] G.-R Duan, "Parametric control of quasi-linear systems by output feedback," in *Proceedings of 2014 14th International Conference on Control, Automation and Systems*, pp. 928–934, IEEE Computer Society, Gyeonggi-do, Korea, June 2014.

[23] G.-R Duan, "Parametric control of quasi-linear systems via state feedback," in *Proceedings of 2014 14th International Conference on Control, Automation and Systems*, pp. 23–28, IEEE Computer Society, Gyeonggi-do, Korea, June 2014.

[24] D.-K. Gu and D.-W. Zhang, "A parametric method to design dynamic compensator for high-order quasi-linear systems," *Nonlinear Dynamics*, vol. 100, no. 2, pp. 1379–1400, 2020.
[25] D.-K. Gu and D.-W. Zhang, "Parametric control to second-order linear time-varying systems based on dynamic compensator and multi-objective optimization," *Applied Mathematics and Computation*, vol. 365, Article ID 124681, 2020.
[26] D.-K. Gu and D.-W. Zhang, "Parametric control to a type of descriptor quasi-linear systems based on dynamic compensator and multi-objective optimisation," *IET Control Theory & Applications*, vol. 14, no. 10, pp. 1287–1299, 2020.
[27] D.-K. Gu, D.-W. Zhang, and G.-R. Duan, "Parametric control to a type of quasi-linear high-order systems via output feedback," *European Journal of Control*, vol. 47, pp. 44–52, 2019.
[28] D.-K Gu, D.-W. Zhang, and G.-R. Duan, "Parametric control to a type of descriptor quasi-linear systems via output feedback," *IEEE Access*, vol. 7, pp. 39911–39922, 2019.
[29] D. K. Gu, D. W. Zhang, and G. R. Duan, "Parametric control to linear time-varying systems based on dynamic compensator and multi-objective optimization," *Asian Journal of Control*, vol. 43, 2019.
[30] D.-W. Zhang and D.-K. Gu, "A parametric approach to design state feedback stabilizer for switched unified chaotic systems," in *Proceedings of 2019 Chinese Automation Congress*, pp. 471–475, IEEE Computer Society, Hangzhou, China, February 2019.
[31] G.-R. Duan, *Generalized Sylvester Equations-Unified parametric solutions*, CRC Press, Boca Raton, FL, USA, 2014.
[32] B. Zhou and G.-R. Duan, "A new solution to the generalized Sylvester matrix equation," *Systems & Control Letters*, vol. 55, no. 3, pp. 193–198, 2006.
[33] J. F. S. Dos Santos, P. C. Pellanda, and A. M Sim, "Robust pole placement under structural constraints," *Systems & Control Letters*, vol. 116, pp. 8–14, 2018.
[34] U. Konigorski, "Pole placement by parametric output feedback," *Systems & Control Letters*, vol. 61, no. 2, pp. 292–297, 2012.
[35] D. P. Kim, "Synthesis of systems with maximum robust degree of stability," *Journal of Computer and Systems Sciences International*, vol. 46, no. 5, pp. 721–725, 2007.
[36] G.-R. Duan and H.-H. Yu, *LMIs in Control Systems Analysis, Design and Applications*, CRC Press, Boca Raton, FL, USA, 2014.