

Research Article

Multilayer Network Risk Factor Pricing Model

Yu Liu,¹ Conglin Hu,² Lei Wang ,³ and Kun Yang ³

¹School of Cyber Science and Engineering, Southeast University, Nanjing 211189, China

²Antai College of Economics and Management, Shanghai Jiao Tong University, Shanghai 200030, China

³School of Economics and Management, Southeast University, Nanjing 211189, China

Correspondence should be addressed to Lei Wang; leiwang77777@163.com

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This paper proposes a multilayer network risk factor pricing model to depict the impact of interactions between stocks on excess stock returns by constructing the network risk factor based on the stock multilayer network and introducing it to the traditional three-factor pricing model. According to China's stock market data, we find that compared with the traditional three-factor model, the multilayer network risk factor pricing model can achieve higher fitting degree. Meanwhile, the multilayer network risk factor has a significant positive impact on the excess stock returns in most cases.

1. Introduction

The asset pricing puzzle always attracts the focus of the academic and practical circle. The traditional capital asset pricing model holds the view that the differences between stock returns result from their different market risk states. However, Fama and French [1] suggested that the beta coefficients cannot well explain the excess stock returns, and the size and book-to-market equity also have significant influences on the stock returns. Therefore, the three-factor model was proposed by Fama and French [1] and was widely utilized. Furthermore, it is extended by introducing more possible influencing factors, such as profitability, investment, momentum effect, and economic policy uncertainty, which contribute to explain the excess stock returns more effectively [2–4].

Stocks are affected by the common macroeconomic environment and fluctuate with the stock index as a whole. In addition, there is also a clear interaction between stocks, especially in the regional and industrial sectors. Fama and French [5] found that many stock prices have the same trend over the same period. In fact, interstock interactions can be described by network models, where stocks are network nodes, and relationships between stocks are edges between nodes. Mantegna [6] first proposed the stock network model, which leads to a series of subsequent studies. And

some studies show that structures of stock networks affect stock returns [7–9].

According to the above analysis, the residual returns of the traditional asset pricing model may contain the structural factors of stock networks. Therefore, it is necessary to construct network risk factors to test their explanations for the excess stock returns. This paper attempts to answer this question. In this paper, we construct the multilayer network risk factor based on the Fama–French three-factor model and test its explanation for the excess stock returns based on China's stock market data. The contributions of this paper are the following. Firstly, we propose the method to construct the multilayer network risk factor for stock markets. Recent study in network science has moved from single-layer networks to multiplex networks [10–12]. Therefore, this paper also extends the application of the multilayer network in stock markets. Secondly, we extend the Fama–French three-factor model from the perspective of the network theory. The empirical results in China provide strong evidence that compared with the traditional Fama–French three-factor model, the multilayer network risk factor pricing model can better explain the excess stock returns. Meanwhile, the multilayer network risk factor has a significant positive impact on the excess stock returns in most cases.

The remainder of this paper is organized as follows. Section 2 presents a review of the related literature. Section 3 outlines the multilayer network risk factor pricing model. Section 4 reports the data and empirical results. And the conclusion is drawn in Section 5.

2. Literature Review

In order to solve the asset pricing puzzle, Fama and French [1] proposed the three-factor model, where the cross-sectional variation in average portfolio returns is explained by the excess return of a portfolio, the stock size, and the book-to-market ratio. After this pioneering study, scholars proposed different types of factor models from different perspectives and tested them [3, 13–20]. For example, Lam and Tam [14] investigated the role of liquidity in pricing stock returns and found that the liquidity four-factor model is the best model to explain stock returns in the Hong Kong stock market. Fama and French [3] proposed a five-factor asset pricing model, which captures size, value, profitability, and investment patterns. Makwasha et al. [18] proposed multifactor capital asset pricing models with fixed effects and found that portfolio return forecasts generated by the six-factor panel model are superior to other multifactor capital asset pricing models.

However, it should be noted that the above studies do not consider the influences of interaction between stocks on excess stock returns. In recent years, the correlation network method is increasingly used to depict the correlation structure of stock markets by regarding stocks as network nodes and their dependence as edges [9, 21–25]. Some studies have revealed that stock network structures affect stock returns [7–9]. For example, Ahern [8] empirically documented a positive market price of centrality, i.e., more central assets earn higher expected returns. Huang et al. [9] used cross-correlations to measure the interdependence between stock prices and constructed a corresponding minimal spanning tree for 170 U.S. stocks. They found that the normalized tree length has a positive relationship with the level of stock market average return; the majority of stocks has their vertex degrees significantly positively correlated to their average returns.

Recent studies on financial networks have moved from single-layer networks to multiplex networks [10, 11, 26–29]. Therefore, it is necessary to construct multiplex network risk factors to analyze the excess stock returns. However, to our knowledge, there are no studies on this topic. Therefore, in this paper, we contribute to the literature by filling this gap.

3. Methodology

On the basis of the traditional capital asset pricing model (CAPM), Fama and French [1] proposed the three-factor model. It can be defined as

$$R_{it} - R_{ft} = a + b\text{RMRF}_t + s\text{SMB}_t + h\text{HML}_t + e_t, \quad (1)$$

where $R_{it} - R_{ft}$ states the excess stock return, in which R_{it} is the return of stock i at time t , and R_{ft} is the risk-free rate at time t ; RMRF_t is the excess market return at time t , which

can be calculated as the difference between the stock market return and the risk-free rate; SMB_t is the size factor at the time t , indicating the difference between the weighted average return series of small and big market capitalization portfolios; HML_t is the book-to-market equity factor at time t , referring to the difference between the weighted average return series of low and high book-to-market equity portfolios; b , s , and h are the marginal contributions of the excess market return, size, and book-to-market equity factors to the excess stock returns, respectively; and e_t is the residual at time t .

In order to depict the influences of stock interactions on excess stock returns, we develop a multilayer network risk factor pricing model by constructing the multilayer network risk factor and introducing it to the three-factor model. Let us construct the following formula:

$$R_{it} - R_{ft} = a + b\text{RMRF}_t + s\text{SMB}_t + h\text{HML}_t + fF_t + \varphi_t, \quad (2)$$

where φ_t denotes the individual risk return and fF_t states the correlation risk return, in which F_t and f are the multilayer network risk factor and its marginal contribution to excess returns, respectively. The construction of the multilayer network risk factor includes the following five steps:

Step 1: constructing the three-factor model to obtain the residual series of excess stock returns.

Step 2: the Pearson, Kendall, and partial correlation coefficients between residual series are calculated to construct three categories of correlation coefficient matrixes, where specific coefficient calculation methods can be seen in Kendall [30] and Baba et al. [31].

Step 3: filtering the redundant information in three correlation coefficient matrixes. In three commonly used network filtering approaches, the threshold method is subjective in the selection of thresholds, which has crucial impacts on the filtering results. The minimum spanning tree (MST) [6] method can keep the key information in financial correlation networks, while the MST network is quite different from the actual correlation states of financial markets. The planar maximally filtered graph (PMFG) [32] method not only keeps the hierarchical structure of MST networks but also reserves more useful information [33]. Therefore, we choose the PMFG method to filter the redundant information in three correlation coefficient matrixes and then obtain the corresponding adjacency matrixes.

Step 4: based on the three-layer stock correlation network, the entropy of the multiplex degree proposed by Battiston et al. [34] is calculated as the multilayer degree centrality indicator o_i , which can comprehensively reflect the structural information in three correlation networks. The mathematical expression of o_i is as follows:

$$o_i = - \sum_{\alpha=1}^3 \frac{k_i^{[\alpha]}}{k_i^{[\text{tot}]}} \log \left(\frac{k_i^{[\alpha]}}{k_i^{[\text{tot}]}} \right), \quad (3)$$

where n is the number of nodes; $k_i^{[\alpha]}$ is the degree centrality of node i at layer α , namely, $k_i^{[\alpha]} = \sum_j \alpha_{ij}^{[\alpha]}$, $1 \leq i, j \leq n, i \neq j$; and $k_i^{[\text{tot}]}$ is the sum of three degree centralities of node i , that is, $k_i^{[\text{tot}]} = \sum_{\alpha=1}^3 k_i^{[\alpha]}$.

Step 5: calculating the relative multilayer degree centralities $w_i = o_i / \sum_{i=1}^n o_i$ as stock weights, and further computing the multilayer network risk factor $F_t = ew^T$, in which e and w are the stock residual and the weight matrix, respectively.

4. Data and Empirical Results

4.1. Data Description and Sample Selection. This paper selects the data of Chinese A-share listed companies during July 2005 to June 2018 as the research sample. The sample data can be obtained from the Ruisi financial database (<http://www.resset.cn>) and the iFind database (<http://www.51ifind.com>). More specifically, the data used in this paper are stock returns, total market capitalization, and owners' equity of listed companies, risk-free rate, and stock market returns. Among them, the stock returns are 1050 stock return series weighted by market capitalization, which exclude stocks listed after 2005 and stocks with missing data for more than 42 months. Besides, the interbank offered rate and Shanghai and Shenzhen 300 index returns are used to represent the risk-free rate and the stock market return, respectively.

4.2. Portfolios and Calculation of Three Factors. In this section, we group the 1050 stocks according to their total market capitalization and the ratio of owners' equity to total market capitalization and further construct portfolios and calculate the traditional three factors. First, based on the 20%, 40%, 60%, and 80% quantiles of total market capitalization in June of t -th year, all the stocks are divided into 5 groups, which represent the different market capitalization size, respectively. Next, according to the 20%, 40%, 60%, and 80% quantiles of the ratio of owners' equity to total market capitalization in December of $(t-1)$ -th year, we can construct another 5 groups, which state the different book-to-market equity, respectively. Thus, 25 portfolios with different sizes or book-to-market equities are built. Then, regarding the monthly market capitalization of portfolio assets as weights, we can calculate the portfolio's weighted average returns during July of $(t-1)$ -th year to June of t -th year. Finally, the 156 monthly returns of 25 stock portfolios are obtained.

Similarly, we can further calculate the traditional excess market return, size, and book-to-market equity factors. Among them, the excess market return factor is defined as the difference between the stock market return and the risk-free rate, while the size and book-to-market equity factors are calculated by constructing portfolios. More specifically, first, all the stocks are divided into group B with bigger sizes and group S with smaller sizes according to the median of total market capitalization in June of t -th year. Next, based on the 30% and 70% quantiles of the ratio of owners' equity to total market capitalization in December

of $(t-1)$ -th year, all the stocks are divided into group L with lower book-to-market equity, group M with medium book-to-market equity, and group H with higher book-to-market equity. Thus, we can construct 6 portfolios and denote them as S/L , S/M , S/H , B/L , B/M , and B/H , respectively. Moreover, the portfolio's weighted average returns during July of $(t-1)$ -th year to June of t -th year can be computed, and the size and book-to-market equity factors are further calculated as equations (4) and (5), respectively. Finally, we can obtain all the monthly size and book-to-market equity factors from July 2005 to June 2018.

$$\text{SMB} = \frac{(\text{SMB}_{S/L} + \text{SMB}_{S/M} + \text{SMB}_{S/H})}{3} - \frac{(\text{SMB}_{B/L} + \text{SMB}_{B/M} + \text{SMB}_{B/H})}{3}, \quad (4)$$

$$\text{HML} = \frac{(\text{HML}_{S/H} + \text{HML}_{B/H})}{2} - \frac{(\text{HML}_{S/L} + \text{HML}_{B/L})}{2}. \quad (5)$$

4.3. Empirical Results. To begin with, we make the descriptive statistics for the excess market return factor (RMRF), the size factor (SMB), and the book-to-market equity factor (HML). The descriptive statistical results are reported in Table 1.

From Table 1, the ADF unit root tests demonstrate that all the factors are stationary at the 1% significance level, which implies that we can further utilize them to build factor pricing models. On that basis, the Fama–French three-factor model and the multilayer network risk factor pricing model of 25 portfolios are constructed, respectively. The adjusted R -squared of two types of factor pricing models is summarized in Table 2.

Table 2 manifests that although the three-factor model achieves good fitting results, the multilayer network risk factor pricing model can still improve the adjusted R -squared to some extent. It indicates that the multilayer network risk factor is helpful to explain the excess stock returns since the stocks are often affected by the changes of related stock prices. Furthermore, the regression results of the multilayer network risk factor pricing model are shown in Table 3.

From Table 3, it can be seen that the major coefficients of excess market return, size, and book-to-market equity factors are significant at the 10% level, which indicates that the traditional three factors are suitable to be applied in the Chinese stock market. Second, the coefficients of the multilayer network risk factor are significantly greater than 0 in most cases. It further confirms that the multilayer network risk factor contributes to explain the excess stock returns. Besides, the comovement of the stock market can increase the stock risk and provide corresponding risk premium. Therefore, in view of the fact that China's stock market is extremely sensitive to national policies and the international situation and there are a lot of speculative behavior and risk uncertainty, financial regulatory authorities should fully

TABLE 1: Descriptive statistical results of four factors.

Factor	Mean	Std.	Min.	Max.	ADF
RMRF	0.0109	0.0891	-0.2685	0.2950	-10.417***
SMB	0.0036	0.0563	-0.2093	0.1653	-10.393***
HML	-0.0029	0.0428	-0.2775	0.1474	-15.209***
<i>F</i>	0.0169	0.1000	-0.2738	0.3412	-10.539***

***Significance at the 1% level. ADF is the augmented Dickey–Fuller unit root test statistic.

TABLE 2: Fitting effect of different factor models.

Size quintile	Book-to-market equity quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Panel A: three-factor model					Panel B: multilayer network risk factor pricing model				
Small	0.642	0.937	0.942	0.939	0.930	0.720	0.952	0.955	0.953	0.937
2	0.906	0.931	0.940	0.950	0.954	0.922	0.960	0.962	0.972	0.967
3	0.874	0.895	0.925	0.944	0.959	0.909	0.932	0.955	0.971	0.970
4	0.860	0.889	0.900	0.938	0.942	0.900	0.947	0.934	0.958	0.946
Big	0.907	0.917	0.896	0.903	0.905	0.930	0.927	0.905	0.903	0.905

TABLE 3: Grouped regression results of the multilayer network risk factor pricing model.

Size quintile	Book-to-market equity quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	<i>b</i>					<i>t(b)</i>				
Small	4.466	0.086	0.162	0.110	0.354	8.802***	0.686	1.337	0.864	2.504**
2	0.123	-0.187	-0.012	0.005	0.207	0.789	-1.656*	-0.117	0.0489	2.045**
3	-0.284	-0.265	-0.118	-0.046	0.335	-1.748*	-1.814*	-1.044	-0.509	3.531***
4	-0.287	-0.507	-0.174	0.164	0.585	-1.808*	-4.313***	-1.221	1.490	4.704***
Big	0.126	0.462	0.472	0.991	1.098	1.097	3.561***	3.077***	5.970***	7.217***
	<i>s</i>					<i>t(s)</i>				
Small	2.885	0.543	0.597	0.546	0.595	9.223***	6.998***	7.978***	6.937***	6.821***
2	0.297	0.234	0.223	0.281	0.354	3.099***	3.367***	3.406***	4.745***	5.672***
3	0.011	-0.115	-0.022	0.018	0.266	0.106	-1.284	-0.321	0.321	4.554***
4	-0.256	-0.389	-0.168	-0.100	0.173	-2.622***	-5.365***	-1.914*	-1.471	2.256**
Big	-0.769	-0.667	-0.629	-0.343	-0.207	-10.89***	-8.342***	-6.653***	-3.347***	-2.204**
	<i>h</i>					<i>t(h)</i>				
Small	-1.537	0.018	0.043	0.130	0.161	-7.720***	0.363	0.906	2.604***	2.891***
2	-0.186	-0.138	0.029	0.185	0.345	-3.051***	-3.125***	0.693	4.915***	8.688***
3	-0.407	-0.175	0.027	0.127	0.343	-6.390***	-3.060***	0.616	3.620***	9.217***
4	-0.358	-0.263	-0.162	0.060	0.512	-5.755***	-5.694***	-2.895***	1.400	10.49***
Big	-0.736	-0.447	0.024	0.317	0.565	-16.37***	-8.774***	0.394	4.872***	9.462***
	<i>f</i>					<i>t(f)</i>				
Small	-3.284	0.865	0.785	0.881	0.610	-6.565***	6.971***	6.559***	7.005***	4.369***
2	0.868	1.180	1.003	1.018	0.802	5.657***	10.62***	9.596***	10.76***	8.044***
3	1.232	1.313	1.137	1.057	0.704	7.698***	9.131***	10.17***	11.94***	7.530***
4	1.241	1.496	1.245	0.911	0.451	7.938***	12.92***	8.879***	8.406***	3.679***
Big	0.796	0.581	0.577	0.106	-0.127	7.046***	4.546***	3.819***	0.650	-0.849

***, **, and * indicate significance at the 1%, 5%, and 10% level, respectively.

consider the relationship between excess stock returns and the multilayer network risk factor and their disturbing effects and compare the multilayer network risk factor with traditional factors, so as to determine whether it is more suitable, reasonable, and scientific regulatory measures.

5. Conclusions

In this paper, we choose the monthly data of Chinese A-share listed companies. First, the Fama–French three-

factor model is estimated to obtain stock residual series, which can be utilized to construct the stock multilayer network and further calculate multilayer degree centralities. On that basis, we propose a multilayer network risk factor pricing model by computing the multilayer network risk factor and introducing it to the three-factor model. The empirical results show that compared with the traditional Fama–French three-factor model, the multilayer network risk factor pricing model can achieve higher fitting degree. Meanwhile, the multilayer network risk factor has a

significant positive impact on the excess stock returns in most cases.

This paper complements the traditional three-factor model by constructing the multilayer network risk factor and confirms that the interactions between stocks have significant influences on the excess stock returns. It further reveals the stock pricing rules, provides important references for investors to make decisions, and offers new thought for the capital asset pricing research. In addition, from the perspective of excess stock returns and multilayer network, this paper provides a theoretical reference for financial regulatory authorities to consider the linkage and multilayer nature of stock prices and then provides index reference and theoretical basis for formulating regulatory policies to prevent excessive volatility of stock prices.

Data Availability

The sample data can be obtained from the Ruisi financial database (<http://www.resset.cn>) and the iFind database (<http://www.51ifind.com>).

Disclosure

Yu Liu, Lei Wang, and Kun Yang are the co-first authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Yu Liu, Lei Wang, and Kun Yang contributed equally to this work.

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