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Research Article

Robust H_∞ Control for the Spacecraft with Flexible Appendages

Aiping Pang,[1,2] Hui Zhu,[1] Junjie Zhou,[1] Zhen He,[3] and Jing Yang[1,2]

[1]College of Electrical Engineering, Guizhou University, Guiyang, CO 550025, China
[2]China Power Construction Group Guizhou Engineering Co., Ltd., Guiyang 550025, China
[3]School of Astronautics, Harbin Institute of Technology, Harbin 150000, China

Correspondence should be addressed to Jing Yang; jyang7@gzu.edu.cn

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Aiming at the oscillation suppression of spacecraft with large flexible appendages, we propose a control strategy using H_∞ control. The weighting functions are designed for the specific flexible modes of the spacecraft and the frequency of harmonic interference in its operating environment. Taking into account the structural uncertainty of systematic modeling and the comprehensive performance requirements of system bandwidth constraint and attitude stability, the H_∞ comprehensive performance matrix is constructed. A space telescope with a large flexible solar array is presented as an illustrative example, and a control design that meets the requirement for pointing accuracy is proposed. The simulation results show that the designed controller satisfies the requirements of attitude stability and high pointing accuracy and has effectively suppressed the disturbance of endemic frequency. The design scheme and selection method of the weight function shown in this paper can be a reference for the controller design for oscillation suppression of this type of spacecraft with flexible structures.

1. Introduction

A spacecraft with flexible appendages, such as a spacecraft carrying large flexible solar panels, has a system structure with multiple resonance modes. The resonance mode of this type of flexible system changes its amplitude characteristics greatly, and the choice of bandwidth is greatly restricted by the inherent low-frequency interference caused by the complex launch environment and the high-altitude environment during on-orbit operation and the flexible mode of the system itself [1–3]. The difficulty of the control design of this kind of flexible system is not only to suppress the inherent resonance interference but also to meet the control accuracy requirements, as well as to take into account the requirements of attitude stability and bandwidth limitation of the system [4, 5].

With the development of aerospace engineering and the increasingly diversified missions of the spacecraft, the control accuracy of the spacecraft is becoming increasingly demanding. In addition, due to the development of new composite materials and their wide application in the aerospace field, the flexible proportion of large spacecraft is continuously increasing, which makes the control of the system more complicated and difficult. The oscillation suppression of the spacecraft with large flexible appendages has always been an active research topic in the field of aerospace all time. In [6], a flexible spacecraft model with external disturbance and model uncertainty is established, and an observer is constructed to observe the attitude, angular velocity, and disturbance of the system. In addition, the stability control and disturbance suppression of the system are realized by dynamic compensation linearization. Literature [7] researched the classic control design for the Hubble Space Telescope with flexible appendages and gave a redesign scheme of the classic control. In the proposed scheme, a notch filter in series with a PID controller is used to achieve stability control and interference suppression, and an internal model controller is connected in series to suppress the unique resonance interference. The proposed controller can suppress the resonant mode of the particular frequency well, but it cannot take into account the stability and robustness of the system. The research shows that it is difficult to use classical control to balance the multiple performance requirements for the integrated control of flexible spacecraft with multiple performance requirements [8]. However, H_∞ control theory is a synthetic control theory, which can include multiple performance requirements in the design [9–18], and it solves the control problems with multiple performance requirements and takes into account the multiple performance requirements of the system under the premise of ensuring the robustness of the system. In recent years, H_∞ control theory has been widely used in the control of the spacecraft. At present, modern control theories such as robust H_∞ control, robust adaptive control, and μ synthesis control are useful for the oscillation suppression of flexible spacecraft and have been discussed in detail in [19–23].

In this paper, a space telescope with a large flexible solar array is presented as an illustrative example to study the oscillation suppression problem of the large flexible spacecraft. Aiming at the system's specific interference frequency, the corresponding H_∞ weighting function is designed to achieve disturbance attenuation and the H_∞ controller that satisfies the robustness and attitude stability of the system is given. The simulation results show that the controller designed in this paper can suppress the resonance interference in the spacecraft natural frequency and satisfies the stability and bandwidth constraints. The design scheme and selection method of the weight function shown in this paper can be a reference for the controller design for oscillation suppression of this type of spacecraft with flexible structures. The structure of this paper shows that the system model and control problems are briefly described in Section 2, and the performance requirements of the H_∞ design and the selection scheme of the weighting function are shown in Section 3; Section 4 is a performance analysis, and Section 5 is the conclusion.

2. Analysis of System Model and Control Problems

A space telescope with large flexible appendages, illustrated in Figure 1, is composed of gyros that provide speed and attitude information, precision guidance sensors and trackers that supplement the attitude information, reaction wheels that provide control torque, and two large flexible solar panels carried on the other side.

In this paper, we consider only the pitch axis of the Hubble Space Telescope control design problem, and the other axes employ the same control structure. According to the relevant data [7], the Hubble Space Telescope actuator output u to the pitch-axis pointing error output angle θ can be modeled as the sum of a rigid body module and several flexible modules:

$$\frac{\theta(s)}{u(s)} = \frac{1}{Is^2} + \sum_{i=1} \frac{K_i/I}{s^2 + 2\zeta w_i s + w_i^2}, \tag{1}$$

where s is the Laplace transforming variable, $I = 77076\,\mathrm{kg}\cdot\mathrm{m}^2$ is the spacecraft pitch inertia, K_i is the ith flexible mode gain in the pitch axis, ω_i is the ith flexible mode frequency in rad/s, and ς is the passive damping ratio assumed as 0.005. The parameters of other flexible modules are shown in Table 1.

Figure 2 shows the system block diagram of the Hubble Space Telescope. The reaction wheel has an actuator saturation limit of 2.5 Nm. The time-prolonging link is introduced to characterize the time difference between the controller output and the actuator. Therefore, the delay link parameter $T = 0.008$ sec. The rate gyro can be represented by a second-order oscillation element, where $\omega_g = 18$ Hz and $\varsigma_g = 0.7$.

The inherent low-frequency disturbances caused by the high-altitude environment of the Hubble Space Telescope on the orbit are modeled as

$$d(t) = 0.2\sin(P_1 t + \phi_1) + 0.2\sin(P_2 t + \phi_2), \tag{2}$$

where $P_1 = 2\pi(0.12)$ rad/s and $P_2 = 2\pi(0.66)$ rad/s are the frequencies of low-frequency disturbances and the phases ϕ_i are unknown parameters.

According to the Hubble Space Telescope pitch-axis model given by formula (1), the Bode magnitude plot of the loop transfer function of the system without the controller is shown in Figure 3. As can be seen in this figure, the system model of this flexible spacecraft contains multiple resonant modes, which make the frequency domain characteristics of the spacecraft vary greatly in amplitude. These flexible modes with weak damping cause great disturbance to the performance of the system, and the inherent low-frequency interference in the operating environment and the flexible modes of the system greatly limit the choice of bandwidth. For high-precision space photography missions, the system's output error is required to be no more than 0.007 arcseconds. Therefore, the designed controller should be able to effectively solve the disturbance suppression of the solar panel and the inherent flexibility suppression of the system, and it should meet certain bandwidth requirements. In summary, the control design goals can be stated as follows:

(1) Give the Hubble Space Telescope system high pointing accuracy
(2) Maintain the system bandwidth more than 1.5 Hz
(3) Provide at least 20 dB additional disturbance attenuation for the disturbance caused by the solar array
(4) Provide at least 6 dB gain suppression for the flexible structure of the system

3. Robust H_∞ Control Design

3.1. System Performance Requirements. As can be seen in Figure 3, the system has a 0.16 Hz gain crossover frequency, which is extremely low. Therefore, the bandwidth of the system is very small, and the interference suppression performance is poor. One of the performance requirements of the system is that the crossover frequency needs to be increased. The bandwidth increased, and the oscillation of the flexible solar panel is suppressed.

In addition, it can be seen from Figure 3 that there are several dominant bending modes at 13 Hz to 14 Hz in the

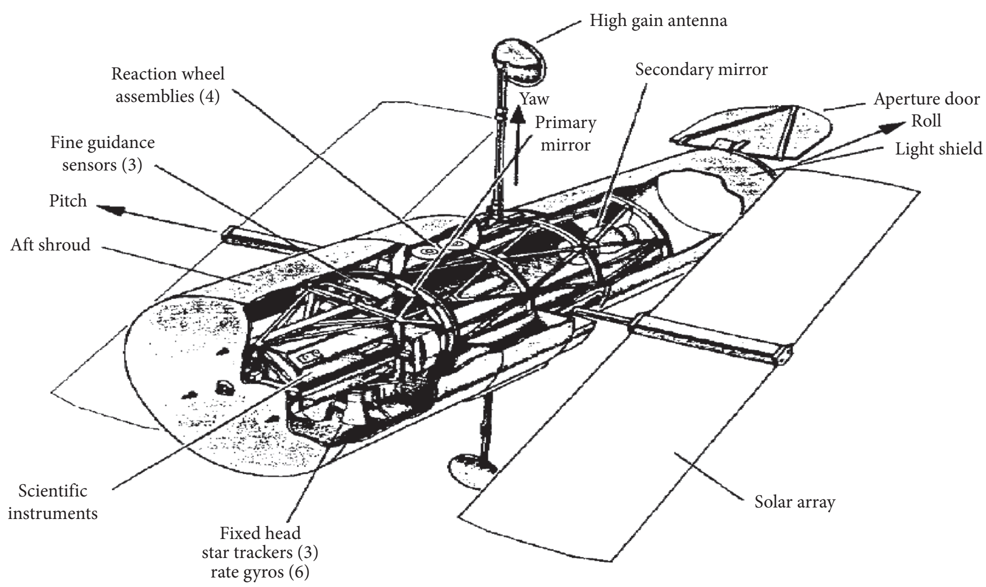

Figure 1: Diagram of Hubble Space Telescope structure.

Table 1: Hubble Space Telescope modal data [7].

K_i (kg·m^2)	ω_i (Hz)
0.018	0.110
0.012	0.432
0.057	0.912
0.024	10.834
0.155	12.133
−1.341	13.201
−1.387	14.068
−0.806	14.285
−0.134	15.264

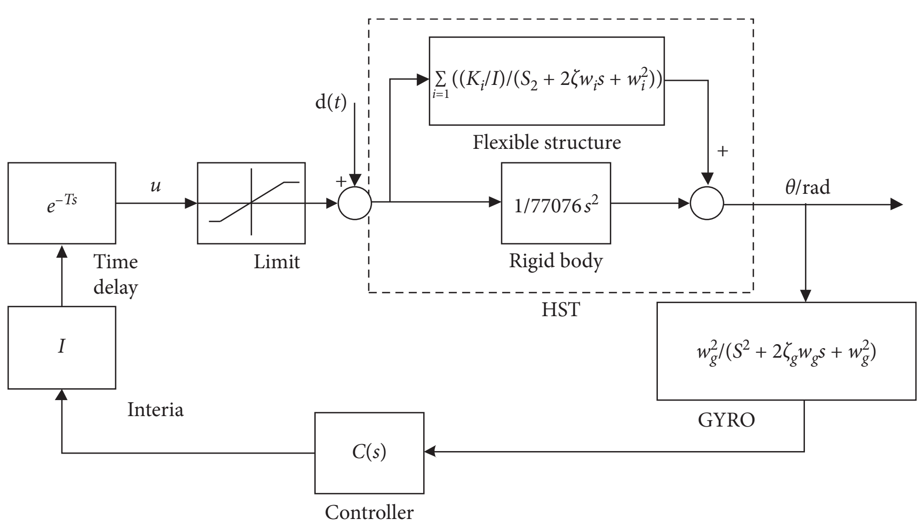

Figure 2: Block diagram of the Hubble Space Telescope control system.

system, which can be regarded as high-frequency unmodeled dynamics, namely, the uncertainty of the system. When the system has such uncertainty, the control structure block diagram of the system is shown in Figure 4. In the study of the control problem, only the rigid body module of the Hubble Space Telescope is modeled, and the other flexible

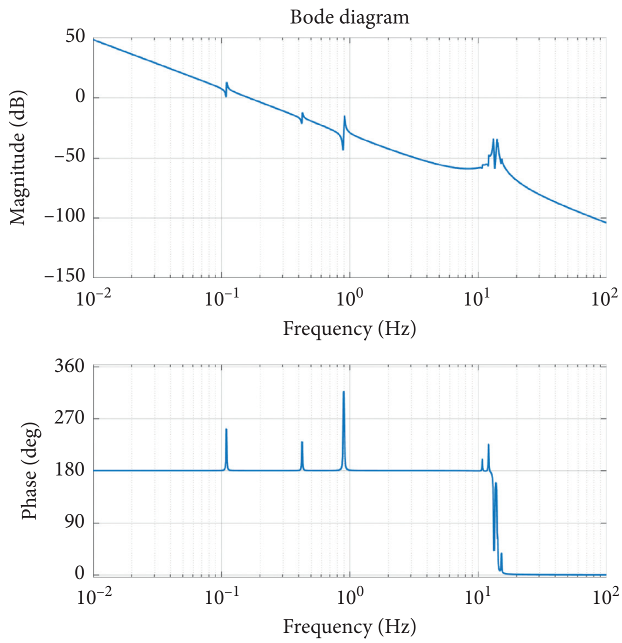

FIGURE 3: Bode magnitude plot of the loop transfer function of the Hubble Space Telescope without the controller.

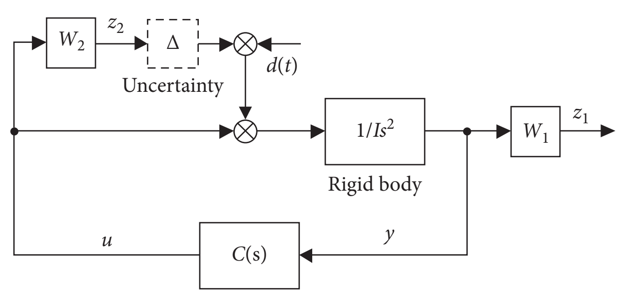

FIGURE 4: Block diagram of the system structure with uncertainty.

modules are regarded as the uncertainty of the system. The uncertainty of the system is expressed by multiplicative uncertainty:

$$G_{\text{actual}}(s) = G(s,\delta)[1 + \Delta G(s)], \quad (3)$$

where $G(s,\delta)$ is the mathematical model of the controlled object constructed based on the researched problem, $\Delta G(s)$ represents the structural uncertainty of the system, δ represents the nonstructural uncertainty of the system, and $G_{\text{actual}}(s)$ is the real mathematical model of the controlled object. Therefore, the second performance requirement of the system is robust stability.

In Figure 4, Δ represents the structured uncertainty and unstructured uncertainty of the system, W_1, W_2, and W_3 are the performance weighting function, uncertainty weighting function, and input weighting function of the system, respectively, y is the error output of the system, and Z_1 and Z_2 are the performance output of the system.

3.2. Selection of Weighting Function. According to the above analysis, the performance requirements of the control system design are system bandwidth requirements and robust stability requirements. Next, we will select appropriate weighting functions based on the two performance requirements.

According to experience, the performance weighting function $W1$ should generally include integral control laws. Aiming at the solar array oscillations at 0.12 Hz and 0.66 Hz, the system should be able to provide sufficient attenuation to the disturbance without affecting the stability of the medium-frequency domain of the system. A very small artificial damping (about 0.01) is needed for pure imaginary poles for the convenience of avoiding numerical problems in solving the H_∞ controller. After a certain amount of trial and error, we select the performance weighting function of attenuating disturbance, as shown in the following formula, and the Bode graph of this weighting function is shown in Figure 5:

$$W_1(s) = \frac{0.1225(s + 0.12 \times 2\pi)(s + 0.66 \times 2\pi)}{(s + 0.01)^2}. \quad (4)$$

It can be seen from the Bode diagram of the Hubble Space Telescope system without the controller, as shown in Figure 3, that the system performance is strongly affected by several dominant bending modes at 13 Hz to 14 Hz in the system. Therefore, the weighting function $W_2(s)$should have good notch performance. Moreover, due to the rank requirements of the generalized plant in the H_∞ control theory, the numerator and denominator of $W_2(s)$ should have the same order. Finally, we select the following robust stability weighting function:

$$W_2(s) = \frac{0.532((s/30) + 1)^2}{\left(s^2/(2\pi \times 13.8)^2\right) + (((2 \times 0.004)/(2\pi \times 13.8)s) + 1)}. \quad (5)$$

It can be seen from Figure 6 that the weighting function W_2 has an amplitude gain of 55.9 dB at 13.8 Hz and 20 dB at high frequency. The weighting function can ensure that the system has a suppression effect on flexible modules at 13~14 Hz and attenuation effect on high-frequency noise signals. Therefore, the choice of the weighting function is reasonable.

3.3. H_∞ Controller. After repeated iterative calculations, the H_∞ controller at $\gamma = 8.125$ is finally obtained, as shown in formula (8):

$$C_{h1}(s) = \frac{k(s + 0.6402)\left(s^2 + 0.002009s + 1.609 \times 10^{-6}\right)}{(s + 0.01)^2\left(s^2 + 3357s + 3.763 \times 10^{-6}\right)}, \quad (6)$$

$$C_{h2}(s) = \frac{s^2 + 0.0003605s + 0.002128}{s^2 + 0.00022s + 0.002118}, \quad (7)$$

$$C_h(s) = C_{h1}(s) \cdot C_{h2}(s), \quad (8)$$

where the parameter $k = 3.4927 \times 10^7$. The zeros and poles of the function C_{h2} are so close to each other that they can be

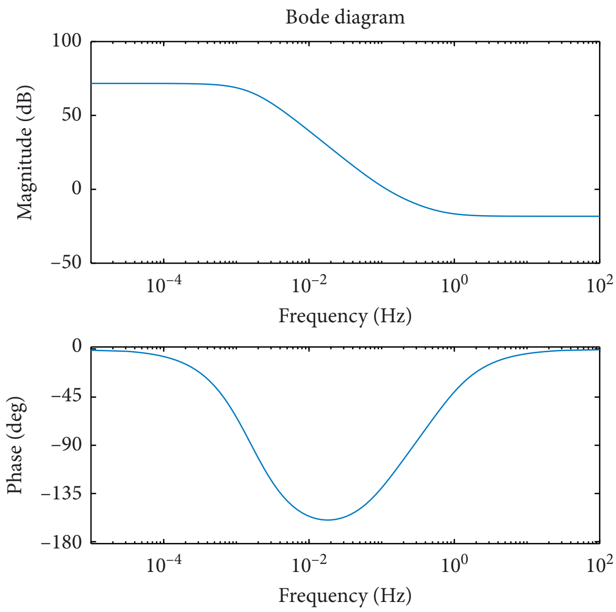

FIGURE 5: Bode diagram of the weighting function W_1.

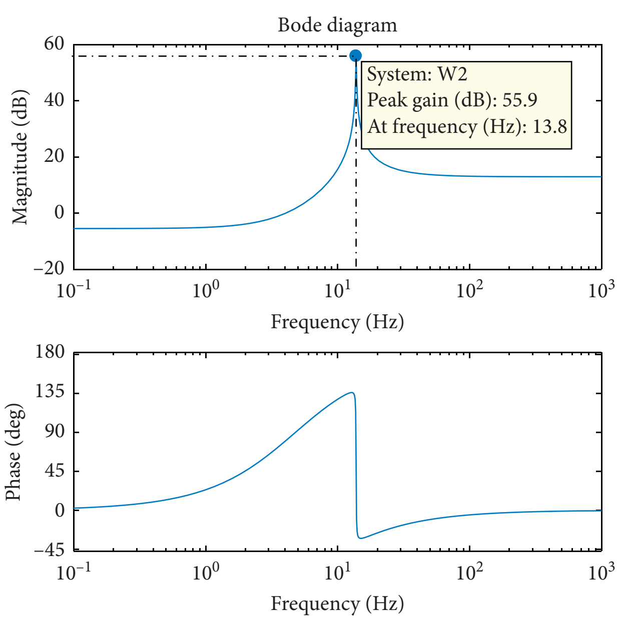

FIGURE 6: Bode diagram of the weighting function W_2.

considered as two pairs of dipoles. Thus, the simplified H_∞ controller can be obtained by pole-zero cancellations and removing the tiny perturbation introduced in the denominator:

$$\widetilde{C}_h = \frac{k\left(s + 0.6402\right)\left(s^2 + 0.002009s + 1.609 \times 10^{-6}\right)}{s^2\left(s^2 + 3357s + 3.763 \times 10^{-6}\right)}. \quad (9)$$

The designed controller needs to be verified at this point by the corresponding H_∞ norm indicator $\gamma = 8.125$. The singularity curve of the closed-loop transfer function T_{dz} from the perturbation input d to the performance output z is shown in Figure 7. It can be seen from the figure, the

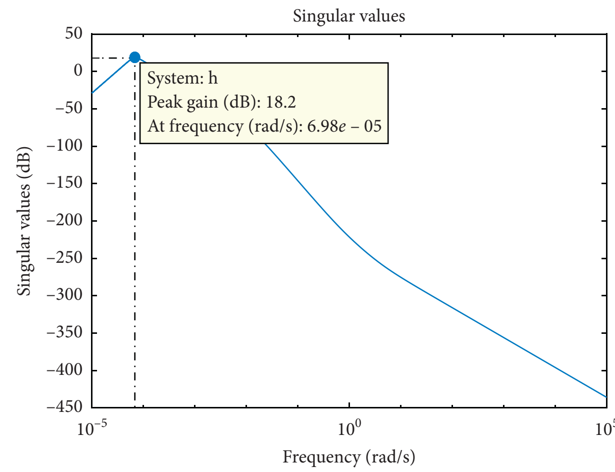

FIGURE 7: The singular performance of the system.

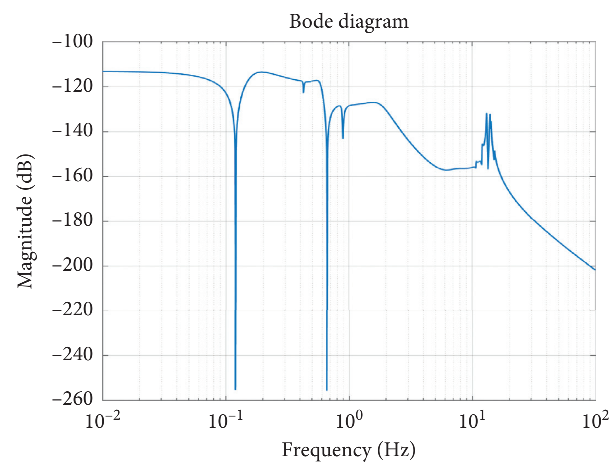

FIGURE 8: Closed-loop frequency magnitude response of the system with the H_∞ controller.

maximum singularity is about $18.2\,\text{dB} = 8.17$, which is consistent with the obtained norm indicator γ.

Furthermore, we analyze the performance of the designed controller being applied to the pitch-axis control system. From the closed-loop Bode diagram of the system shown in Figure 8, we can see that the system performs a dramatic gain attenuation for specific frequency interference.

As can be seen in the Bode magnitude plot of the open-loop transfer function (Figure 9), the system with the controller given by equation (9) has a 1.6 Hz gain crossover frequency, which meets the bandwidth requirement of not less than 1.5 Hz. Moreover, aiming at the oscillation suppression of the high-frequency bending modes, the controller provides a gain suppression of over 100 dB. The designed H_∞ controller not only suppresses the inherent resonance interference but also meets certain bandwidth requirements.

4. Performance Analysis

The time responses of the system controlled by the designed H_∞ controller are shown in Figure 10. It can be seen that an

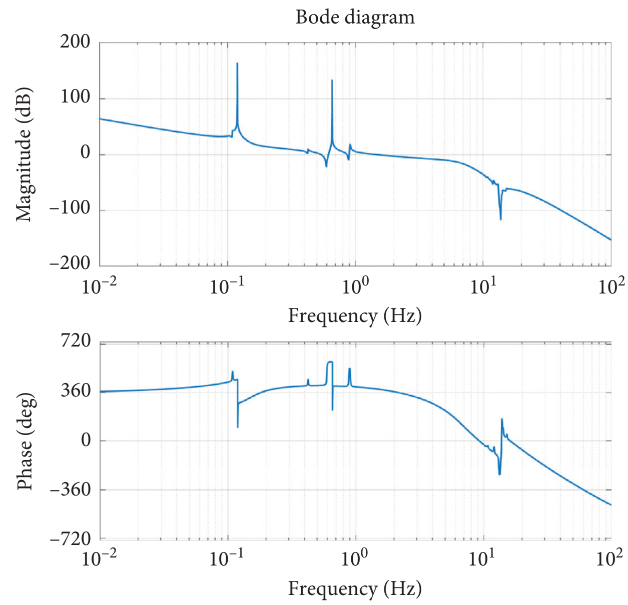

Figure 9: Open-loop frequency magnitude response of the system H_∞ controller.

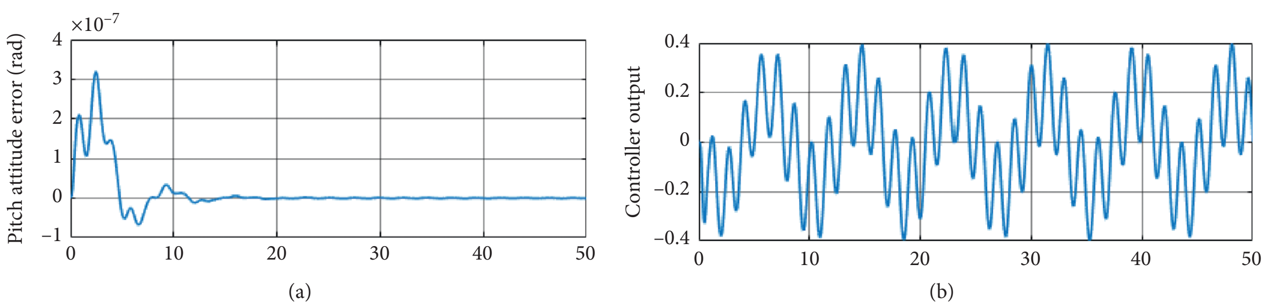

Figure 10: Time response of the system with the H_∞ controller.

effective rejection of the solar array disturbances at 0.12 Hz and 0.66 Hz has been achieved, which meets the requirements of oscillation suppression and high pointing accuracy of the system, without exceeding the actuator limit of 2.5 Nm. It can be seen from Figure 8 that the controller achieves the control goal of gain attenuation for the disturbance of the solar panel.

In this paper, the performance of the designed H_∞ controller is compared to a classical PID controller, and their system time response is shown in Figure 11. From the figure, we can see that the PID controller is able to stabilize the rigid module of the system well and suppresses perturbations at specific frequencies because of the introduction of the internal mode filter and notch filter. However, the designed H_∞ controller has better interference suppression than the PID controller, resulting in better dynamic and steady performance of the system. Furthermore, the H_∞ controller eliminates the need for an additional filter design.

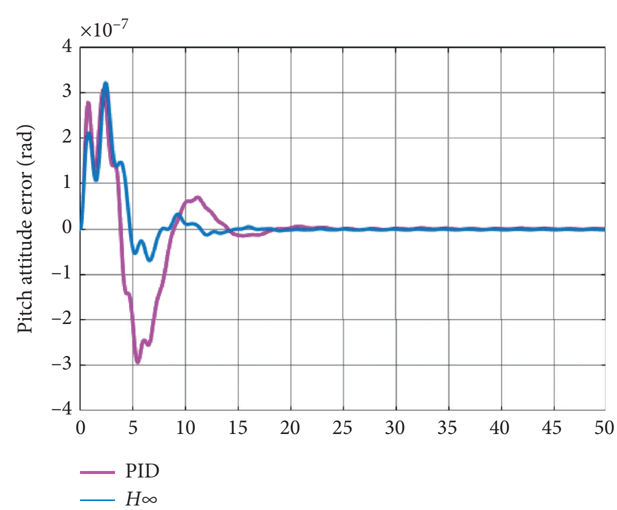

Figure 11: Time responses of the system with different controllers.

5. Conclusion

Aiming at the comprehensive problems of the spacecraft with flexible appendages, such as flexible structure vibration, deficient bandwidth, low pointing accuracy, and uncertainty caused by structural changes, the H_∞ control method is adopted in this paper. Appropriate weighting functions are selected by analyzing the performance requirements to suppress the oscillation of the flexible structure and increase the bandwidth of the system. Finally, a H_∞ controller with robust stability is given. The simulation results show that the designed controller can effectively suppress the vibration of solar panels. While maintaining the bandwidth of the system, it also dampens the vibrations of the high-frequency resonant modules, enabling the system to satisfy high pointing accuracy requirements.

Data Availability

The data used to support the findings of this study are included within the article and other data or programs used can obtain from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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