Hindawi
Complexity
Volume 2020, Article ID 6661782, 20 pages
https://doi.org/10.1155/2020/6661782

Research Article

Certain Types of Covering-Based Multigranulation $(\mathcal{I}, \mathcal{T})$-Fuzzy Rough Sets with Application to Decision-Making

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Received 18 October 2020; Revised 31 October 2020; Accepted 10 November 2020; Published 29 November 2020

Academic Editor: Ahmed Mostafa Khalil

As a generalization of Zhan's method (i.e., to increase the lower approximation and decrease the upper approximation), the present paper aims to define the family of complementary fuzzy β-neighborhoods and thus three kinds of covering-based multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets models are established. Their axiomatic properties are investigated. Also, six kinds of covering-based variable precision multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets are defined and some of their properties are studied. Furthermore, the relationships among our given types are discussed. Finally, a decision-making algorithm is presented based on the proposed operations and illustrates with a numerical example to describe its performance.

1. Introduction

Group decision-making aims at aggregating individual judgments to construct a composite group decision, which must be a true representative of individual preferences. The MAGDM methods choose among a discrete set of alternatives evaluated on multiple attributes and overall utility of the decision makers. MAGDM have some of the popular methods such as the weighted sum and the weighted product method (see, e.g., [1–7]). The theory of rough set was founded by Pawlak [8, 9] for dealing with the vagueness and granularity in information systems and data analysis. This theory has been applied to many different fields (see, e.g., [10–20]). Furthermore, we have noticed a wide range of generalized rough set models (see, e.g., [21–23]). Covering-based rough sets (CRSs) are considered to be one of the most studied generalized models. Pomykala [24, 25] obtained two pairs of dual approximation operators. Yao [26] studied these approximation operators by the concepts of neighborhood and granularity. Couso and Dubois [27] examined the two pairs within the context of incomplete information. Bonikowski et al. [28] established a CRS model based on the notion of minimal description. Zhu and Wang [29–32] presented several CRS models and discussed their relationships. Tsang et al. [33] and Xu and Zhang [34] proposed additional CRS models. Liu and Sai [35] compared Zhu′s CRS models and Xu and Zhang′s CRS models. Ma [36] constructed some types of neighborhood-related covering rough sets by using the definitions of the neighborhood and complementary neighborhood. In 2016, Ma [37] introduced the definition of fuzzy β-neighborhood. In 2017, Yang and Hu [38] constructed the definition of the fuzzy β-complementary neighborhood to establish some types of fuzzy covering-based rough sets. Zhang et al. [39], in 2019, established the fuzzy covering-based $(\mathcal{I}, \mathcal{T})$-fuzzy rough set models and applications to multiattribute decision-making. Also, in 2019, Zhan et al. [40] proposed covering-based multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough set models and applications in multiattribute group decision-making.

The concept of a family fuzzy β-neighborhoods was defined and their properties were studied by Zhan et al. [40]. Hence, to increase the lower approximation and decrease the upper approximation of Zhan's model, this article's contribution is to introduce three kinds of covering-based multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets models and explore the properties of these models with their relationships. Also, six kinds of covering-based variable precision multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets are demonstrated. An application to a practical problem illustrates their ability to help practitioners to make decisions. The outline of this paper is as follows. Section 2 gives technical preliminaries. Section 3 describes our three new types of covering-based multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets and also we introduce variable precision in order to produce the six types of covering-based variable precision multigranulation $(\mathcal{I}, \mathcal{T})$-fuzzy rough sets. Section 4 establishes relationships among our models. Section 5 puts forward a decision-making procedure that takes advantage of our theoretical framework. The conclusion is written in Section 6.

2. Preliminaries

In this section, we provide a brief survey of some notions used throughout the paper.

Definition 1 (see [1]). Suppose that the mapping $\mathcal{T}: [0,1] \times [0,1] \longrightarrow [0,1]$ is commutative, associative, and satisfies the increasing laws plus the boundary condition $\mathcal{T}(1,x) = x$ for each $x \in [0,1]$. We say that such a $\mathcal{T}$ is a t-norm (for triangular norm) of $[0,1]$.

Among the most important continuous t-norms, we can cite

(1) The minimum operator $\mathcal{T}_{\mathcal{M}}(x,y) = x \wedge y$
(2) The algebraic product $\mathcal{T}_{\mathcal{P}}(x,y) = x * y$
(3) The Lukasiewicz t-norm $\mathcal{T}_{\mathcal{L}}(x,y) = 0 \vee (x+y-1)$

Definition 2 (see [1]). Suppose that the mapping $\mathcal{S}: [0,1] \times [0,1] \longrightarrow [0,1]$ is commutative, associative, and satisfies the increasing laws plus the boundary condition $\mathcal{S}(0,x) = x$ for each $x \in [0,1]$. We say that such an $\mathcal{S}$ is an s-norm or a t-conorm of $[0,1]$.

Continuous s-norms are

(1) The maximum operator $\mathcal{S}_{\mathcal{M}}(x,y) = x \vee y$
(2) The algebraic summation $\mathcal{S}_{\mathcal{S}}(x,y) = x + y$
(3) The bounded summation $\mathcal{S}_{\mathcal{L}}(x,y) = 1 \wedge (x+y)$
(4) The probabilistic summation $\mathcal{B}_p(x,y) = x + y - x * y$

Definition 3 (see [1]). A negator operator is $\mathcal{N}: [0,1] \longrightarrow [0,1]$, an order-reversing mapping with the properties $\mathcal{N}(0) = 1$ and $\mathcal{N}(1) = 0$.

We say that $\mathcal{N}$ is involutive when $\mathcal{N}(\mathcal{N}(x)) = x$ for every $x \in [0,1]$.

The standard negator operator is defined as $\mathcal{N}(x) = 1 - x$, for any $x \in [0,1]$.

Involutive negators are continuous.

Negators produce fuzzy complements. Involutive negators assure that when $\widehat{X} \in \mathcal{F}(\Omega)$ and $x \in \Omega$, we always obtain $\mathcal{N}(\mathcal{N}(\widehat{X}(x))) = \widehat{X}(x)$.

We say that $\mathcal{T}$, a t-norm, and $\mathcal{S}$, a t-conorm, are dual with respect to negator $\mathcal{N}$, when for each $x, y \in [0,1]$, it must be the case that $\mathcal{S}(\mathcal{N}(x), \mathcal{N}(y)) = \mathcal{N}(\mathcal{T}(x,y))$ and $\mathcal{T}(\mathcal{N}(x), \mathcal{N}(y)) = \mathcal{N}(\mathcal{S}(x,y))$.

Definition 4 (see [1]). A fuzzy implicator operator is $\mathcal{I}: [0,1] \times [0,1] \longrightarrow [0,1]$, and a mapping with the properties $\mathcal{I}(0,0) = \mathcal{I}(0,1) = \mathcal{I}(1,1) = 1$ and $\mathcal{I}(1,0) = 0$.

If, in addition, $\mathcal{I}$ is such that $x \leq y \Rightarrow \mathcal{I}(x,z) \geq \mathcal{I}(y,z)$, respectively, $y \leq z \Rightarrow \mathcal{I}(x,y) \leq \mathcal{I}(x,z)$, for every $x, y, z \in [0,1]$, then $\mathcal{I}$ is left monotonic decreasing, respectively, increasing.

We say that $\mathcal{I}$ is hybrid monotonic when it is both left and right monotonic.

An implicator $\mathcal{I}$ is a border implicator when $\mathcal{I}(1,x) = x$ for each $x \in [0,1]$.

Next, we recall three relevant classes of implicator operators [1].

Definition 5

(1) The $\mathcal{S}$-implicator defined by $\mathcal{S}$ and $\mathcal{N}$ is given: for each $x, y \in [0,1]$, $\mathcal{I}_{\mathcal{S}}(x,y) = \mathcal{S}(\mathcal{N}(x), y)$
(2) The $\mathcal{R}$-implicator defined by a continuous t-norm $\mathcal{T}$ is given: for each $x, y \in [0,1]$, $\mathcal{I}_{\mathcal{R}}(x,y) = \vee\{u \in [0,1]: \mathcal{T}(x,u) \leq y\}$
(3) If $\mathcal{T}$ and $\mathcal{S}$ are dual with respect to $\mathcal{N}$, the $\mathcal{QL}$-implicator defined from $\mathcal{T}$, $\mathcal{S}$, and $\mathcal{N}$ is given: for all $x, y \in [0,1]$, $\mathcal{I}_{\mathcal{QL}}(u,v) = \mathcal{S}(\mathcal{N}(u), \mathcal{T}(u,v))$

Well-known $\mathcal{S}$-implicators are

(1) $\mathcal{I}_{\mathcal{L}}(x,y) = 1 \wedge (1-x+y)$, according to $\mathcal{S}_{\mathcal{L}}$ and $\mathcal{N}_{\mathcal{S}}$
(2) $\mathcal{I}_{\mathcal{KD}}(x,y) = (1-x) \vee y$, according to $\mathcal{S}_{\mathcal{M}}$ and $\mathcal{N}_{\mathcal{S}}$
(3) $\mathcal{I}_{\mathcal{P}}(x,y) = 1 - x + x * y$, according to $\mathcal{S}_{\mathcal{M}}$ and $\mathcal{N}_{\mathcal{S}}$

Definition 6 (see [42, 43]). Let Ω be an arbitrary universal set, and $\mathcal{F}(\Omega)$ be the fuzzy power set of Ω. We call $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_m}\}$, with $\widetilde{C}_i \in \mathcal{F}(\Omega)\,(i = 1, 2, \ldots, m)$, a fuzzy covering of Ω, if $(\cup_{i=1}^{m} \widetilde{C}_i)(x) = 1$ for each $x \in \Omega$.

As a generalization of fuzzy covering, Ma [37] defined a fuzzy β-covering by replacing 1 with a parameter $\beta\,(0 < \beta \leq 1)$, that is, we call $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_m}\}$, with $\widetilde{C}_i \in \mathcal{F}(\Omega)\,(i = 1, 2, \ldots, m)$, a fuzzy β-covering of Ω, if $(\cup_{i=1}^{m} \widetilde{C}_i)(x) \geq \beta$ for each $x \in \Omega$. Moreover, $(\Omega, \widetilde{\Gamma})$ is called a fuzzy β-covering approximation space (briefly, FβCAS).

Definition 7 (see [37]). Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS with $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_m}\}$ for some $\beta \in (0,1]$. Then, for each $x \in \Omega$, define the fuzzy β-neighborhood of x as follows:

$$\widetilde{N_x^{\beta}} = \cap \{\widetilde{C}_i \in \widetilde{\Gamma}: \widetilde{C}_i \geq \beta\}. \qquad (1)$$

Definition 8 (see [38]). Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS for some $\beta \in (0,1]$. Then, for each $x, y \in \Omega$, define the fuzzy complementary β-neighborhood of x as follows:

$$\widetilde{M_x^{\beta}}(y) = \widetilde{N_y^{\beta}}(x). \qquad (2)$$

Definition 9. (see [39]). Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS for some $\beta \in (0,1]$. For each $x \in \Omega$ and $\widehat{X} \in \mathscr{F}(\Omega)$, define the fuzzy set $\widetilde{C}_1^-(\widehat{X})$ (resp., $\widetilde{C}_1^+(\widehat{X})$, $\widetilde{C}_2^-(\widehat{X})$, $\widetilde{C}_2^+(\widehat{X})$, $\widetilde{C}_3^-(\widehat{X})$, $\widetilde{C}_3^+(\widehat{X})$, $\widetilde{C}_4^-(\widehat{X})$, and $\widetilde{C}_4^+(\widehat{X})$) which is called the first type fuzzy lower approximation (resp., the first type of the fuzzy upper approximation, the second type of the fuzzy lower approximation, the second type of the fuzzy upper approximation, the third type of the fuzzy lower approximation, the third type of the fuzzy upper approximation, the fourth type of the fuzzy lower approximation, and the fourth type of the fuzzy upper approximation), briefly, 1-FCITFLA (resp., 1-FCITFUA, 2-FCITFLA, 2-FCITFUA, 3-FCITFLA, 3-FCITFUA, 4-FCITFLA, and 4-FCITFUA), where

$$\begin{aligned}
\widetilde{C}_1^-(\widehat{X})(x) &= \wedge_{y\in\Omega} \mathscr{I}\left[\widetilde{N_x^\beta}(y)t, n\widehat{X}q(y)\right],\\
\widetilde{C}_1^+(\widehat{X})(x) &= \vee_{y\in\Omega} \mathscr{T}\left[\widetilde{N_x^\beta}(y)t, n\widehat{X}q(y)\right] \quad (\forall x \in \Omega),\\
\widetilde{C}_2^-(\widehat{X})(x) &= \wedge_{y\in\Omega} \mathscr{I}\left[\widetilde{M_x^\beta}(y)t, n\widehat{X}q(y)\right],\\
\widetilde{C}_2^+(\widehat{X})(x) &= \vee_{y\in\Omega} \mathscr{T}\left[\widetilde{M_x^\beta}(y)t, n\widehat{X}q(y)\right] \quad (\forall x \in \Omega),\\
\widetilde{C}_3^-(\widehat{X})(x) &= \wedge_{y\in\Omega} \mathscr{I}\left[\widetilde{N_x^\beta}(y)t \wedge n\widetilde{M_x^\beta}q(y)h_{,\widehat{X}x}(y)\right],\\
\widetilde{C}_3^+(\widehat{X})(x) &= \vee_{y\in\Omega} \mathscr{T}\left[\widetilde{N_x^\beta}(y)t \wedge n\widetilde{M_x^\beta}q(y)h_{,\widehat{X}x}(y)\right] \quad (\forall x \in \Omega),\\
\widetilde{C}_4^-(\widehat{X})(x) &= \wedge_{y\in\Omega} \mathscr{I}\left[\widetilde{N_x^\beta}(y)t \vee n\widetilde{M_x^\beta}q(y)h_{,\widehat{X}x}(y)\right],\\
\widetilde{C}_4^+(\widehat{X})(x) &= \vee_{y\in\Omega} \mathscr{T}\left[\widetilde{N_x^\beta}(y)t \vee n\widetilde{M_x^\beta}q(y)h_{,\widehat{X}x}(y)\right] \quad (\forall x \in \Omega).
\end{aligned} \tag{3}$$

If $\widetilde{C}_1^-(\widehat{X})$ (resp., $\widetilde{C}_2^-(\widehat{X})$, $\widetilde{C}_3^-(\widehat{X})$, $\widetilde{C}_4^-(\widehat{X})$)$\neq\widetilde{C}_1^+(\widehat{X})$ (resp., $\widetilde{C}_2^+(\widehat{X})$, $\widetilde{C}_3^+(\widehat{X})$, $\widetilde{C}_4^+(\widehat{X})$), then $\widehat{X}$ is called the first type of a fuzzy β-covering-based $(\mathscr{I},\mathscr{T})$-fuzzy rough set (resp., the second type of a fuzzy β-covering-based $(\mathscr{I},\mathscr{T})$-fuzzy rough set, the third type of a fuzzy β-covering-based $(\mathscr{I},\mathscr{T})$-fuzzy rough set, and the fourth type of a fuzzy β-covering-based $(\mathscr{I},\mathscr{T})$-fuzzy rough set), briefly, 1-FCITFRS (resp., 2-FCITFRS, 3-FCITFRS, and 4-FCITFRS).

Zhan et al. [40] defined the covering-based multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set models (briefly, CMGITFRSs). So, in the following, some basic notions related to CMGITFRSs are given.

Suppose that $\widetilde{\Gamma} = \left\{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_m}\right\}$ be m fuzzy β-coverings of Ω for some $\beta \in (0,1]$, where $\widetilde{C}_i = \left\{\widetilde{C_{i1}}, \widetilde{C_{i2}}, \ldots, \widetilde{C_{im_i}}\right\}$, for all $i = 1, 2, \ldots, n$. Then, for each $x \in \Omega$, define the family of fuzzy β-neighborhoods as follows:

$$\widetilde{N}^\beta_{\widetilde{C}_i(x)} = \left\{\mathscr{C}_{ij} \in \widetilde{C}_i\colon x \in \mathscr{C}_{ij}, j = 1, \ldots, m\right\}. \tag{4}$$

Definition 10 (see [40]). Let $(\Omega, \widetilde{\Gamma})$ be an FβCAS and $\widetilde{\Gamma} = \left\{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_m}\right\}$ be m fuzzy β-coverings of Ω for some $\beta \in (0,1]$, where $\widetilde{C}_i = \left\{\widetilde{C_{i1}}, \widetilde{C_{i2}}, \ldots, \widetilde{C_{im_i}}\right\}$, for all $i = 1, 2, \ldots, n$. For each $\widehat{X} \in \mathscr{F}(\Omega)$, the set ${}_0\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$ (resp., ${}_0\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$, ${}_0\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$, and ${}_0\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$) is called the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough lower approximation (resp., the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough upper approximation, the pessimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough lower approximation, and the pessimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough upper approximation), briefly, 0-OMGITFRLA (resp., 0-OMGITFRUA, 0-PMGITFRLA, and 0-PMGITFRUA), where

$$\begin{aligned}
{}_0\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})(x) &= \vee_{i=1}^n \wedge_{y\in\Omega} \mathscr{I}\left\{\widetilde{N}^\beta_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\},\\
{}_0\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})(x) &= \wedge_{i=1}^n \vee_{y\in\Omega} \mathscr{T}\left\{\widetilde{N}^\beta_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\}, \quad (\forall x \in \Omega),\\
{}_0\mathscr{L}_{\sum} i = 1^n\widetilde{C}_i^{\mathscr{I}(p)}(\widehat{X})(x) &= \wedge_{i=1}^n \wedge_{y\in\Omega} \mathscr{I}\left\{\widetilde{N}^\beta_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\},\\
{}_0\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})(x) &= \vee_{i=1}^n \vee_{y\in\Omega} \mathscr{T}\left\{\widetilde{N}^\beta_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\} \quad (\forall x \in \Omega).
\end{aligned} \tag{5}$$

If ${}_0\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$ (resp., ${}_0\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$)$\neq{}_0\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$ (resp., ${}_0\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$), then $\widehat{X}$ is called a covering-based optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set (resp., a covering-based pessimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set), briefly, 0-COMGITFRS (resp., 0-CPMGITFRS).

Definition 11 (see [40]). Let $(\Omega, \widetilde{\Gamma})$ be an FβCAS and $\widetilde{\Gamma} = \left\{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\right\}$ be n fuzzy β-coverings of Ω for some $\beta \in (0,1]$, where $\widetilde{C}_i = \left\{\widetilde{C_{i1}}, \widetilde{C_{i2}}, \ldots, \widetilde{C_{im_i}}\right\}$, for all $i = 1, 2, \ldots, n$. For any $\widehat{X} \in \mathscr{F}(\Omega)$ and a variable precision parameter $\gamma \in [0,1]$. Then, the set $I\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^n \widetilde{C}_i}(\widehat{X})$ (resp.,

$I\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$, $II\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$, and $II\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$) is called the first type of the variable precision multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy lower approximation (resp., the first type of the variable precision multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy upper approximation, the second type of the variable precision multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy lower approximation, and the second type of the variable precision multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy upper approximation), briefly, I-VPMGIT-FLA (resp., I-VPMGITFUA, II-VPMGITFLA, and II-VPMGITFUA), where

$$
\begin{aligned}
I\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})(x) &= \vee_{i=1}^{n}\left(\left(\underset{\widehat{X}(y)\le\gamma}{\wedge}\mathscr{I}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),\gamma\right)\right)\wedge\left(\underset{\widehat{X}(y)>\gamma}{\wedge}\mathscr{I}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),\widehat{X}(y)\right)\right)\right), \quad (\forall x,y\in\Omega),\\
I\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})(x) &= \wedge_{i=1}^{n}\left(\left(\underset{\widehat{X}(y)\ge 1-\gamma}{\vee}\mathscr{T}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),1-\gamma\right)\right)\vee\left(\underset{\widehat{X}(y)<1-\gamma}{\vee}\mathscr{T}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),\widehat{X}(y)\right)\right)\right), \quad (\forall x,y\in\Omega),\\
II\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})(x) &= \wedge_{i=1}^{n}\left(\left(\underset{\widehat{X}(y)\le\gamma}{\wedge}\mathscr{I}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),\gamma\right)\right)\wedge\left(\underset{\widehat{X}(y)>\gamma}{\wedge}\mathscr{I}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),\widehat{X}(y)\right)\right)\right), \quad (\forall x,y\in\Omega),\\
II\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})(x) &= \vee_{i=1}^{n}\left(\left(\underset{\widehat{X}(y)\ge 1-\gamma}{\vee}\mathscr{T}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),1-\gamma\right)\right)\vee\left(\underset{\widehat{X}(y)<1-\gamma}{\vee}\mathscr{T}\left(\widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y),\widehat{X}(y)\right)\right)\right), \quad (\forall x,y\in\Omega).
\end{aligned}
\tag{6}
$$

If $I\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ (resp., $II\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$)$\neq I\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ (resp., $II\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$), then $\widehat{X}$ is called a covering-based I-variable precision multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set (resp., a covering-based II-variable precision multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set), briefly, I-VPMGITFRS (resp., II-VPMGITFRS); otherwise, it is I-variable precision multigranulation fuzzy definable (resp., II-variable precision multigranulation fuzzy definable).

3. Three Types of Covering-Based Multigranulation $(\mathscr{I},\mathscr{T})$-Fuzzy Rough Sets

Here, three new kinds of covering-based multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough sets models (briefly, CMGITFRS) are introduced. Also, some of their properties are investigated.

Assume that $\widetilde{\Gamma} = \{\widetilde{C_1},\widetilde{C_2},\ldots,\widetilde{C_m}\}$ be m fuzzy β-coverings of Ω for some $\beta\in(0,1]$, where $\widetilde{C_i} = \{\widetilde{C_{i1}},\widetilde{C_{i2}},\ldots,\widetilde{C_{im_i}}\}$, for all $i = 1,2,\ldots,n$. Then, for each $x\in\Omega$, define the family of fuzzy β-neighborhoods as follows:

$$\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y) = \widetilde{N}^{\beta}_{\widetilde{C_i(y)}}(x). \tag{7}$$

Example 1. Let $(\Omega,\widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma} = \{\widetilde{C_1}, t\widetilde{C_2}\}$ be 2 fuzzy β-coverings of Ω, where $\Omega = \{x_1,x_2,x_3,x_4,x_5,x_6\}$, $\beta = 0.6$, and $\widetilde{C_1} = \{\widetilde{C_{11}},\widetilde{C_{12}},\ldots,\widetilde{C_{14}}\}$ and $\widetilde{C_2} = \{\widetilde{C_{21}},\widetilde{C_{22}},\ldots,\widetilde{C_{25}}\}$ as in Tables 1 and 2.

Then, we introduce the family of a fuzzy β-neighborhood and fuzzy complementary β-neighborhood for $\widetilde{\Gamma} = \{\widetilde{C_1}, t\widetilde{C_2}\}$ as follows in Tables 3–6.

3.1. *Three Types of the Optimistic Multigranulation $(\mathscr{I},\mathscr{T})$-Fuzzy Rough Sets.*

In the following, three kinds of COMGITFRS models are given and some of their properties are presented.

Definition 12. Let $(\Omega,\widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma} = \{\widetilde{C_1},\widetilde{C_2},\ldots,\widetilde{C_n}\}$ be n fuzzy β-coverings of Ω for some $\beta\in(0,1]$, for each $\widehat{X}\in\mathscr{F}(\Omega)$. Then, the first type of the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy lower approximation (briefly, 1-OMGITFLA) ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and the first type of the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy upper approximation (briefly, 1-OMGITFUA) ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ are, respectively, defined as follows:

$$
\begin{aligned}
{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})(x) &= \vee_{i=1}^{n}\underset{y\in\Omega}{\wedge}\mathscr{I}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y),\widehat{X}(y)\right\},\\
{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})(x) &= \wedge_{i=1}^{n}\underset{y\in\Omega}{\vee}\mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y),\widehat{X}(y)\right\}, \quad (\forall x\in\Omega).
\end{aligned}
\tag{8}
$$

If ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \neq {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$, then $\widehat{X}$ is called a covering-based optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set (briefly, 1-COMGITFRS); otherwise, it is optimistic multigranulation fuzzy definable.

Example 2. Let us consider Example 1. If $\widehat{X} = (0.6/x_1) + (0.1/x_2) + (0.7/x_3) + (0.2/x_4) + (0.5/x_5) + (0.4/x_6)$, then we have the following.

Case 1 Let us fix $\mathscr{I} = \mathscr{I}_*$ based on $\mathscr{S}_{\mathscr{P}}$ and $\mathscr{N}_{\mathscr{S}}$, and $\mathscr{T} = \mathscr{T}_{\mathscr{P}}$:

Table 1: Table for $\widetilde{C}_1$.

	$\widetilde{C}_{11}$	$\widetilde{C}_{12}$	$\widetilde{C}_{13}$	$\widetilde{C}_{14}$
x_1	0.7	0.8	0.3	0.4
x_2	0.3	0.6	0.5	0.2
x_3	0.4	0.1	0.9	0.7
x_4	0.8	0.4	0.6	0.3
x_5	0.2	0.7	0.4	0.8
x_6	0.5	0.9	0.6	0.1

Table 2: Table for $\widetilde{C}_2$.

	$\widetilde{C}_{21}$	$\widetilde{C}_{22}$	$\widetilde{C}_{23}$	$\widetilde{C}_{24}$	$\widetilde{C}_{25}$
x_1	0.6	0.3	0.5	0.4	0.7
x_2	0.9	0.7	0.1	0.3	0.4
x_3	0.3	0.6	0.4	0.8	0.3
x_4	0.4	0.2	0.9	0.7	0.6
x_5	0.2	0.8	0.4	0.6	0.5
x_6	0.7	0.4	0.3	0.9	0.2

Table 3: Table for $\widetilde{N}_x^\beta$ of $\widetilde{C_1}$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{N}_{x_1}^{0.5}$	0.7	0.3	0.1	0.4	0.2	0.5
$\widetilde{N}_{x_2}^{0.5}$	0.3	0.5	0.1	0.4	0.4	0.6
$\widetilde{N}_{x_3}^{0.5}$	0.3	0.2	0.7	0.3	0.4	0.1
$\widetilde{N}_{x_4}^{0.5}$	0.3	0.3	0.4	0.6	0.2	0.5
$\widetilde{N}_{x_5}^{0.5}$	0.4	0.2	0.1	0.3	0.7	0.1
$\widetilde{N}_{x_6}^{0.5}$	0.3	0.3	0.1	0.4	0.2	0.5

Table 4: Table for $\widetilde{N}_x^\beta$ of $\widetilde{C_2}$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{N}_{x_1}^{0.5}$	0.5	0.1	0.3	0.4	0.2	0.2
$\widetilde{N}_{x_2}^{0.5}$	0.3	0.7	0.3	0.2	0.2	0.4
$\widetilde{N}_{x_3}^{0.5}$	0.3	0.3	0.6	0.2	0.2	0.4
$\widetilde{N}_{x_4}^{0.5}$	0.4	0.1	0.3	0.6	0.4	0.2
$\widetilde{N}_{x_5}^{0.5}$	0.4	0.3	0.3	0.2	0.5	0.2
$\widetilde{N}_{x_6}^{0.5}$	0.4	0.3	0.3	0.4	0.2	0.7

Table 5: Table for $\widetilde{M}_x^\beta$ of $\widetilde{C_1}$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{M}_{x_1}^{0.5}$	0.7	0.3	0.3	0.3	0.4	0.3
$\widetilde{M}_{x_2}^{0.5}$	0.3	0.5	0.2	0.3	0.2	0.3
$\widetilde{M}_{x_3}^{0.5}$	0.1	0.1	0.7	0.4	0.1	0.1
$\widetilde{M}_{x_4}^{0.5}$	0.4	0.4	0.3	0.6	0.3	0.4
$\widetilde{M}_{x_5}^{0.5}$	0.2	0.4	0.4	0.2	0.7	0.2
$\widetilde{M}_{x_6}^{0.5}$	0.5	0.6	0.1	0.5	0.1	0.5

Table 6: Table for $\widetilde{M}_x^\beta$ of $\widetilde{C_2}$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{M}_{x_1}^{0.5}$	0.5	0.3	0.3	0.4	0.4	0.4
$\widetilde{M}_{x_2}^{0.5}$	0.1	0.7	0.3	0.1	0.3	0.3
$\widetilde{M}_{x_3}^{0.5}$	0.3	0.3	0.6	0.3	0.3	0.3
$\widetilde{M}_{x_4}^{0.5}$	0.4	0.2	0.2	0.6	0.2	0.4
$\widetilde{M}_{x_5}^{0.5}$	0.2	0.2	0.2	0.4	0.5	0.2
$\widetilde{M}_{x_6}^{0.5}$	0.2	0.4	0.4	0.2	0.2	0.7

$$
\begin{aligned}
{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) &= \frac{0.72}{x_1} + \frac{0.55}{x_2} + \frac{0.73}{x_3} + \frac{0.52}{x_4} + \frac{0.68}{x_5} + \frac{0.58}{x_6},\\
{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) &= \frac{0.3}{x_1} + \frac{0.18}{x_2} + \frac{0.42}{x_3} + \frac{0.24}{x_4} + \frac{0.25}{x_5} + \frac{0.28}{x_6}.
\end{aligned} \tag{9}
$$

Case 2 Let us fix $\mathscr{I} = \mathscr{I}_{\mathscr{KD}}$ based on $\mathscr{S}_{\mathscr{M}}$ and $\mathscr{N}_{\mathscr{S}}$, and $\mathscr{T} = \mathscr{T}_{\mathscr{M}}$. Then, we have the following results:

$$
\begin{aligned}
{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) &= \frac{0.6}{x_1} + \frac{0.5}{x_2} + \frac{0.7}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} + \frac{0.4}{x_6},\\
{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) &= \frac{0.5}{x_1} + \frac{0.3}{x_2} + \frac{0.6}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} + \frac{0.4}{x_6}.
\end{aligned} \tag{10}
$$

Theorem 1. *Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ be n fuzzy β-coverings of Ω for some $\beta \in (0,1]$, for each $\widehat{X} \in \mathscr{F}(\Omega)$. Then, we have the following properties:*

(1) If $\mathscr{I}$ is an $\mathscr{S}$-implicator based on $\mathscr{S}$, a continuous t-conorm, and $\mathscr{N}$, an involutive negator,

(i) ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) = ({}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}^{c_{\mathscr{N}}}))^{c_{\mathscr{N}}}$

(ii) ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) = ({}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}^{c_{\mathscr{N}}}))^{c_{\mathscr{N}}}$

(2) If either $\mathscr{I}$ is an $\mathscr{R}$-implicator based on $\mathscr{T}$, a continuous t-norm, and $\mathscr{N}$, a negator induced by $\mathscr{I}$, or $\mathscr{I}$ is a $\mathscr{QL}$-implicator based on $\mathscr{T}$, a continuous t-norm, and $\mathscr{N}$, an involutive negator,

(i) ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \subseteq ({}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}^{c_{\mathscr{N}}}))^{c_{\mathscr{N}}}$

(ii) ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \subseteq ({}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}^{c_{\mathscr{N}}}))^{c_{\mathscr{N}}}$

(3) If $\mathscr{I}$ satisfies left monotonicity, then

(i) ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\Omega) = \Omega$

(ii) ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\varnothing) = \varnothing$

(4) If $\mathscr{I}$ satisfies right monotonicity and $\widehat{X} \subseteq \widehat{Y}$, then

(i) ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \subseteq {}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{Y})$

(ii) ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\subseteq{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$

(5) *If* $\mathscr{I}$ *satisfies right monotonicity, then* ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y}) = {}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. *Furthermore, if* $\mathscr{I}$ *and* $\mathscr{T}$ *satisfy* $\mathscr{I}(u,\mathscr{T}(v,w))\geq\mathscr{T}(\mathscr{I}(u,v),\mathscr{I}(u,w))$*, for all* $u,v,w\in[0,1]$*, then* ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap\widehat{Y})\supseteq{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. *Also, if* $(\mathscr{T},\mathscr{T}^*)$ *satisfies* $\mathscr{T}(u,\mathscr{T}^*(v,w))\leq\mathscr{T}^*(\mathscr{T}(u,v),\mathscr{T}(u,w))$ *for all* $u,v,w\in[0,1]$*, then* ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y})\subseteq{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$.

(6) *If* $\mathscr{I}$ *and* $\mathscr{S}$ *satisfy* $\mathscr{I}(u,\mathscr{S}(v,w))\geq\mathscr{S}(\mathscr{I}(u,v),\mathscr{I}(u,w))$ *for every* $u,v,w\in[0,1]$*, then* ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cup t\widehat{Y})\supseteq{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cup{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. *Also, if* $\mathscr{T}$ *and* $\mathscr{S}$ *satisfy the weakened distributivity laws, then* ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cup t\widehat{Y})\subseteq{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cup{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. *Specially,* ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cup t\widehat{Y})={}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cup{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$.

Proof. We only need to prove (1)–(4) (i), (5), and (6), since the other proofs are similar:

(1) $({}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}^{c_{\mathscr{N}}}))^{c_{\mathscr{N}}}=\mathscr{N}(\wedge_{i=1}^{n}\vee_{y\in\Omega}\mathscr{T}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\mathscr{N}(\widehat{X}(y))\})=\mathscr{N}(\wedge_{i=1}^{n}\vee_{y\in\Omega}\mathscr{N}(\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}(y)\}))=\mathscr{N}(\mathscr{N}(\vee_{i=1}^{n}\wedge_{y\in\Omega}(\mathscr{I}\{\widetilde{M}_{\widetilde{C_i}}(x)^{\beta}(y),\widehat{X}(y)\})))=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}(y)\}={}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$.

(2) $(1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}^{c_{\mathscr{N}}}))c_{\mathscr{N}}=\mathscr{N}(\wedge_{i=1}^{n}\vee_{y\in\Omega}\mathscr{T}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\mathscr{N}(\widehat{X}(y))\})=\mathscr{N}(\wedge_{i=1}^{n}\vee_{y\in\Omega}\mathscr{N}(\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}(y)\}))=\mathscr{N}(\mathscr{N}(\vee_{i=1}^{n}\wedge_{y\in\Omega}(\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}(y)\})))\geq\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}(y)\}={}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$.

(3) Since $\mathscr{I}$ is left monotonic, we have ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\Omega)=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\Omega\}=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),1\}=1=\Omega$. Also, we have ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\varnothing)=\wedge_{i=1}^{n}\vee_{y\in\Omega}\mathscr{T}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\varnothing\}=\wedge_{i=1}^{n}\vee_{y\in\Omega}\mathscr{T}\{\widetilde{M}^{\beta}_{C_i(x)}(y),0\}=0=\varnothing$.

(4) Since $\mathscr{I}$ is right monotonic, we have ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}\}\subseteq\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{Y}\}={}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. Thus, ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\subseteq{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$ holds.

(5) Since $\mathscr{I}$ is right monotonic, we have ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y})=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}\wedge\widehat{Y}\}\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}_{\widetilde{C_i}}(x)^{\beta}(y),\widehat{X}\}\wedge\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{Y}\}={}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. Since $\mathscr{I}$ and $\mathscr{T}$ satisfy $\mathscr{I}(u,\mathscr{T}(v,w))\geq\mathscr{T}(\mathscr{I}(u,v),\mathscr{I}(u,w))$, for all $u,v,w\in[0,1]$. Then, we have ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y})=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}\wedge\widehat{Y}\}\geq\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}\}\wedge\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{Y}\}={}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. Thus, ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y})\supseteq{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. Also, since $\widehat{X}\cap\widehat{Y}\subseteq\widehat{X}$ and $\widehat{X}\cap\widehat{Y}\subseteq\widehat{Y}$, from (3) above, we have ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y})\subseteq{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\widehat{Y})\subseteq{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$. Therefore, ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cap t\,\widehat{Y})\subseteq{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cap{}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$.

(6) Since $\mathscr{I}$ and $\mathscr{S}$ satisfy $\mathscr{I}(u,\mathscr{T}(v,w))\geq\mathscr{T}(\mathscr{I}(u,v),\mathscr{I}(u,w))$, for all $u,v,w\in[0,1]$. We conclude that ${}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}\cup t\widehat{Y})=\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}\vee\widehat{Y}\}\geq\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{X}\}\vee\vee_{i=1}^{n}\wedge_{y\in\Omega}\mathscr{I}\{\widetilde{M}^{\beta}_{C_i(x)}(y),\widehat{Y}\}={}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})\cup{}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{Y})$.

Hence, ${}_1\mathscr{L}_{\sum_{i=}} \quad 1^n\widetilde{C}_i^{\mathscr{I}(o)}(\widehat{X}\cup t\widehat{Y}) \supseteq {}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \cup {}_1\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{Y})$. Also, $\forall x \in \Omega$, we have ${}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}\cup t \widehat{Y})(x) = \wedge_{i=1}^n \vee_{y\in\Omega}\mathscr{T}\left\{\widetilde{M}_{\widetilde{C_i}}\ (x)^{\beta}(y), \mathscr{S}(\widehat{X}\cup t\widehat{Y})\ (y)\right\}$ $\leq \ \wedge_{i=1}^n \vee\ {}_{y\in\Omega}\mathscr{S}\left\{\mathscr{T}(\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{X}\ \ (y)), \mathscr{T}(\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{Y}(y))\right\}$.

Since the weakened distributivity laws are satisfied,

$$\leq \mathscr{S}\left\{\wedge_{i=1}^n \bigvee_{y\in\Omega}\mathscr{T}\left(\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{X}(y)\right), \wedge_{i=1}^n \bigvee_{y\in\Omega}\mathscr{T}\left(\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{Y}(y)\right)\right\}$$
$$= \mathscr{S}\left({}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})(x), {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{Y})(x)\right) = {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})(x) \vee {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{Y})(x) = \left({}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \cup {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})\right)(x). \tag{11}$$

In particular, if $x \in \Omega$, we have

$$ {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}\cup t\widehat{Y})(x) = \wedge_{i=1}^n \bigvee_{y\in\Omega}\mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), (\widehat{X}\cup t\widehat{Y})(y)\right\} = \wedge_{i=1}^n \bigvee_{y\in\Omega}\mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{X}(y)\right\} \vee \bigwedge_{y\in\Omega}\mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{Y}(y)\right\}$$
$$= {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})(x) \vee {}_1\mathscr{U}^{\mathscr{T}}(\widehat{Y})(x) = \left({}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \cup {}_1\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{Y})\right)(x). \tag{12}$$

□

Definition 13. Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma} = \left\{\widetilde{C_1}, t\widetilde{C_2}n, q\ldots h, \widetilde{C_n}\right\}$ be n fuzzy β-coverings of Ω for some $\beta \in (0,1]$. Then, for each $\widehat{X} \in \mathscr{F}(\Omega)$, the second type of the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy lower approximation (briefly, 2-OMGITFLA) ${}_2\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})$ and the second type of the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy upper approximation (briefly, 2-OMGITFUA) ${}_2\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})$ are, respectively, defined as follows:

$$ {}_2\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})(x) = \vee_{i=1}^n \bigwedge_{y\in\Omega}\mathscr{I}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y) \wedge \widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{X}(y)\right\},$$
$$ {}_2\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})(x) = \wedge_{i=1}^n \bigvee_{y\in\Omega}\mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C_i(x)}}(y) \wedge \widetilde{N}^{\beta}_{\widetilde{C_i(x)}}(y), \widehat{X}(y)\right\},$$
$$(\forall x \in \Omega). \tag{13}$$

If ${}_2\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) \neq {}_2\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})$, then $\widehat{X}$ is called a covering-based optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set (briefly, 2-COMGITFRS); otherwise, it is optimistic multigranulation fuzzy definable.

Example 3. (continued from Example 2). We compute $(\widetilde{N}^{\beta}_{x_i} \wedge \widetilde{M}^{\beta}_{x_i})$ for all $x_i \in \Omega$, where $i = 1, 2, \ldots, 6$ for some $\beta = 0.5$, as shown in Tables 7 and 8 as follows.

Now, we calculate the 2-OMGITFLA and 2-OMGITFUA as explored in the following two cases.

Case 1 Let us fix $\mathscr{I} = \mathscr{I}_*$ based on $\mathscr{S}_{\mathscr{P}}$ and $\mathscr{N}_{\mathscr{S}}$ and $\mathscr{T} = \mathscr{T}_{\mathscr{P}}$. So,

$$ {}_2\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) = \frac{0.72}{x_1} + \frac{0.55}{x_2} + \frac{0.76}{x_3} + \frac{0.52}{x_4} + \frac{0.73}{x_5} + \frac{0.68}{x_6},$$
$$ {}_2\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) = \frac{0.3}{x_1} + \frac{0.18}{x_2} + \frac{0.42}{x_3} + \frac{0.21}{x_4} + \frac{0.25}{x_5} + \frac{0.2}{x_6}. \tag{14}$$

Case 2 Let us fix $\mathscr{I} = \mathscr{I}_{\mathscr{K}\mathscr{D}}$ based on $\mathscr{S}_{\mathscr{M}}$ and $\mathscr{N}_{\mathscr{S}}$ and $\mathscr{T} = \mathscr{T}_{\mathscr{M}}$. Thus,

$$ \begin{aligned} {}_2\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) &= \frac{0.6}{x_1} + \frac{0.5}{x_2} + \frac{0.7}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} + \frac{0.5}{x_6}, \\ {}_2\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X}) &= \frac{0.5}{x_1} + \frac{0.3}{x_2} + \frac{0.6}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} + \frac{0.4}{x_6}. \end{aligned} \tag{15}$$

Remark 1. Definition 13 satisfies Theorem 1.

Definition 14. Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ be n fuzzy β-coverings of Ω for some $\beta \in (0,1]$. For each $\widehat{X} \in \mathscr{F}(\Omega)$, the third type of the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy lower approximation (briefly, 3-OMGITFLA) ${}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})$ and the third type of the optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy upper approximation (briefly, 3-OMGITFUA) ${}_3\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^n \widetilde{C_i}}(\widehat{X})$ are, respectively, defined as follows:

Table 7: Table for $\widetilde{N}_{x_i}^{0.5} \wedge \widetilde{M}_{x_i}^{0.5}$ of $\widetilde{C}_1$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{N}_{x_1}^{0.5} \wedge \widetilde{M}_{x_1}^{0.5}$	0.7	0.3	0.1	0.3	0.2	0.3
$\widetilde{N}_{x_2}^{0.5} \wedge \widetilde{M}_{x_2}^{0.5}$	0.3	0.5	0.1	0.3	0.2	0.3
$\widetilde{N}_{x_3}^{0.5} \wedge \widetilde{M}_{x_3}^{0.5}$	0.1	0.1	0.7	0.3	0.1	0.1
$\widetilde{N}_{x_4}^{0.5} \wedge \widetilde{M}_{x_4}^{0.5}$	0.3	0.3	0.3	0.6	0.2	0.4
$\widetilde{N}_{x_5}^{0.5} \wedge \widetilde{M}_{x_5}^{0.5}$	0.2	0.2	0.1	0.2	0.7	0.1
$\widetilde{N}_{x_6}^{0.5} \wedge \widetilde{M}_{x_6}^{0.5}$	0.3	0.3	0.1	0.4	0.1	0.5

Table 8: Table for $\widetilde{N}_{x_i}^{0.5} \wedge \widetilde{M}_{x_i}^{0.5}$ of $\widetilde{C}_2$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{N}_{x_1}^{0.5} \wedge \widetilde{M}_{x_1}^{0.5}$	0.5	0.1	0.3	0.4	0.2	0.2
$\widetilde{N}_{x_2}^{0.5} \wedge \widetilde{M}_{x_2}^{0.5}$	0.1	0.7	0.3	0.1	0.2	0.3
$\widetilde{N}_{x_3}^{0.5} \wedge \widetilde{M}_{x_3}^{0.5}$	0.3	0.3	0.6	0.2	0.2	0.3
$\widetilde{N}_{x_4}^{0.5} \wedge \widetilde{M}_{x_4}^{0.5}$	0.4	0.1	0.2	0.6	0.2	0.2
$\widetilde{N}_{x_5}^{0.5} \wedge \widetilde{M}_{x_5}^{0.5}$	0.2	0.2	0.2	0.2	0.5	0.2
$\widetilde{N}_{x_6}^{0.5} \wedge \widetilde{M}_{x_6}^{0.5}$	0.2	0.3	0.3	0.2	0.2	0.7

$$
\begin{aligned}
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) &= \vee_{i=1}^{n} \wedge_{y\in\Omega} \mathscr{I}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y) \vee \widetilde{N}^{\beta}_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\},\\
{}_3\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) &= \wedge_{i=1}^{n} \vee_{y\in\Omega} \mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y) \vee \widetilde{N}^{\beta}_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\},\\
&(\forall x \in \Omega).
\end{aligned}
\tag{16}
$$

If ${}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) \neq {}_3\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$, then $\widehat{X}$ is called a covering-based optimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set (briefly, 3-COMGITFRS); otherwise, it is optimistic multigranulation fuzzy definable.

Example 4 (continued from Example 2). We compute $(\widetilde{N}^{\beta}_{x_i} \vee \widetilde{M}^{\beta}_{x_i})$ for all $x_i \in \Omega$, where $i = 1, 2, \ldots, 6$ for some $\beta = 0.5$, as shown in Tables 9 and 10.

The following two cases calculate the 3-OMGITFLA and 3-OMGITFUA, respectively.

Case 1 Let us fix $\mathscr{I} = \mathscr{I}_*$ based on $\mathscr{S}_{\mathscr{P}}$ and $\mathscr{N}_{\mathscr{S}}$ and $\mathscr{T} = \mathscr{T}_{\mathscr{P}}$. So,

$$
\begin{aligned}
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) &= \frac{0.68}{x_1} + \frac{0.64}{x_2} + \frac{0.73}{x_3} + \frac{0.52}{x_4} + \frac{0.68}{x_5} + \frac{0.58}{x_6},\\
{}_3\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) &= \frac{0.3}{x_1} + \frac{0.21}{x_2} + \frac{0.42}{x_3} + \frac{0.24}{x_4} + \frac{0.25}{x_5} + \frac{0.28}{x_6}.
\end{aligned}
\tag{17}
$$

Case 2 Let us fix $\mathscr{I} = \mathscr{I}_{\mathscr{KD}}$ based on $\mathscr{S}_{\mathscr{M}}$ and $\mathscr{N}_{\mathscr{S}}$ and $\mathscr{T} = \mathscr{T}_{\mathscr{M}}$. Thus,

$$
\begin{aligned}
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) &= \frac{0.6}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} + \frac{0.4}{x_6},\\
{}_3\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) &= \frac{0.5}{x_1} + \frac{0.4}{x_2} + \frac{0.6}{x_3} + \frac{0.4}{x_4} + \frac{0.5}{x_5} + \frac{0.4}{x_6}.
\end{aligned}
\tag{18}
$$

Remark 2. Definition 14 satisfies Theorem 1.

3.2. Three Types of the Pessimistic Multigranulation $(\mathscr{I},\mathscr{T})$-Fuzzy Rough Sets.

In the following, we introduce three kinds of CPMGITFRS models and study some of their properties.

Let $(\Omega, \widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ be n fuzzy β-coverings of Ω for some $\beta \in (0,1]$. For each $\widehat{X} \in \mathscr{F}(\Omega)$. We have three models of the pessimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy lower approximation (briefly, 1-PMGITFLA, 2-PMGITFLA, and 3-PMGITFLA) and three model of the pessimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy upper approximation (briefly, 1-PMGITFUA, 2-PMGITFUA, and 3-PMGITFUA) are, respectively, defined as follows.

Model 1:

$$
\begin{aligned}
{}_1\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) &= \wedge_{i=1}^{n} \wedge_{y\in\Omega} \mathscr{I}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\},\\
{}_1\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) &= \vee_{i=1}^{n} \vee_{y\in\Omega} \mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\}, \quad (\forall x \in \Omega).
\end{aligned}
\tag{19}
$$

Model 2:

$$
\begin{aligned}
{}_2\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) &= \wedge_{i=1}^{n} \wedge_{y\in\Omega} \mathscr{I}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y) \wedge \widetilde{N}^{\beta}_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\},\\
{}_2\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) &= \vee_{i=1}^{n} \vee_{y\in\Omega} \mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y) \wedge \widetilde{N}^{\beta}_{\widetilde{C}_i(x)}(y), \widehat{X}(y)\right\}, \quad (\forall x \in \Omega).
\end{aligned}
\tag{20}
$$

Table 9: Table for $\widetilde{N}^{\beta}_{x_i}\vee\widetilde{M}^{\beta}_{x_i}$ of $\widetilde{C}_1$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{N}^{0.5}_{x_1}\vee\widetilde{M}^{0.5}_{x_1}$	0.7	0.3	0.3	0.4	0.4	0.5
$\widetilde{N}^{0.5}_{x_2}\vee\widetilde{M}^{0.5}_{x_2}$	0.3	0.5	0.2	0.4	0.4	0.6
$\widetilde{N}^{0.5}_{x_3}\vee\widetilde{M}^{0.5}_{x_3}$	0.3	0.2	0.7	0.4	0.4	0.1
$\widetilde{N}^{0.5}_{x_4}\vee\widetilde{M}^{0.5}_{x_4}$	0.4	0.4	0.4	0.6	0.3	0.5
$\widetilde{N}^{0.5}_{x_5}\vee\widetilde{M}^{0.5}_{x_5}$	0.4	0.4	0.4	0.3	0.7	0.2
$\widetilde{N}^{0.5}_{x_6}\vee\widetilde{M}^{0.5}_{x_6}$	0.5	0.6	0.1	0.5	0.2	0.5

Table 10: Table for $\widetilde{N}^{\beta}_{x_i}\vee\widetilde{M}^{\beta}_{x_i}$ of $\widetilde{C}_2$.

	x_1	x_2	x_3	x_4	x_5	x_6
$\widetilde{N}^{0.5}_{x_1}\vee\widetilde{M}^{0.5}_{x_1}$	0.5	0.3	0.3	0.4	0.4	0.4
$\widetilde{N}^{0.5}_{x_2}\vee\widetilde{M}^{0.5}_{x_2}$	0.3	0.7	0.3	0.2	0.3	0.4
$\widetilde{N}^{0.5}_{x_3}\vee\widetilde{M}^{0.5}_{x_3}$	0.3	0.3	0.6	0.3	0.3	0.4
$\widetilde{N}^{0.5}_{x_4}\vee\widetilde{M}^{0.5}_{x_4}$	0.4	0.2	0.3	0.6	0.4	0.4
$\widetilde{N}^{0.5}_{x_5}\vee\widetilde{M}^{0.5}_{x_5}$	0.4	0.3	0.3	0.4	0.5	0.2
$\widetilde{N}^{0.5}_{x_6}\vee\widetilde{M}^{0.5}_{x_6}$	0.4	0.4	0.4	0.4	0.2	0.7

Model 3:

$$
\begin{aligned}
&{}_3\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) = \wedge_{i=1}^{n}\bigwedge_{y\in\Omega}\mathscr{I}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y)\vee\widetilde{N}^{\beta}_{\widetilde{C}_i(x)}(y),\widehat{X}(y)\right\},\\
&{}_3\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})(x) = \vee_{i=1}^{n}\bigvee_{y\in\Omega}\mathscr{T}\left\{\widetilde{M}^{\beta}_{\widetilde{C}_i(x)}(y)\vee\widetilde{N}^{\beta}_{\widetilde{C}_i(x)}(y),\widehat{X}(y)\right\},\quad (\forall x\in\Omega).
\end{aligned}
\tag{21}
$$

If ${}_1\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$ (resp., ${}_2\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$, ${}_3\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$) $\neq {}_1\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$ (resp., ${}_2\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$, ${}_3\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X})$), then $\widehat{X}$ is called a covering-based pessimistic multigranulation $(\mathscr{I},\mathscr{T})$-fuzzy rough set (briefly, 1-CPMGITFRS, 2-CPMGITFRS, and 3-CPMGITFRS); otherwise, it is pessimistic multigranulation fuzzy definable.

It is obvious that the properties of these mentioned models satisfy Theorem 1.

Example 5. (continued from Examples 2 and 3 and Remark 2). We have the following results.

Case 1 Let us fix $\mathscr{I} = \mathscr{I}_*$ based on $\mathscr{S}_{\mathscr{P}}$ and $\mathscr{N}_{\mathscr{S}}$ and $\mathscr{T} = \mathscr{T}_{\mathscr{P}}$. So,

(i) ${}_1\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.68/x_1) + (0.37/x_2) + (0.68/x_3) + (0.52/x_4) + (0.64/x_5) + (0.46/x_6)$ and ${}_1\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.42/x_1) + (0.21/x_2) + (0.49/x_3) + (0.24/x_4) + (0.28/x_5) + (0.3/x_6)$

(ii) ${}_2\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.68/x_1) + (0.37/x_2) + (0.73/x_3) + (0.52/x_4) + (0.58/x_5) + (0.58/x_6)$ and ${}_1\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.42/x_1) + (0.21/x_2) + (0.49/x_3) + (0.24/x_4) + (0.35/x_5) + (0.28/x_6)$

(iii) ${}_3\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.68/x_1) + (0.37/x_2) + (0.68/x_3) + (0.52/x_4) + (0.64/x_5) + (0.46/x_6)$ and ${}_3\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.42/x_1) + (0.24/x_2) + (0.49/x_3) + (0.28/x_4) + (0.35/x_5) + (0.3/x_6)$

Case 2 Let us fix $\mathscr{I} = \mathscr{I}_{\mathscr{KD}}$ based on $\mathscr{S}_{\mathscr{M}}$ and $\mathscr{N}_{\mathscr{S}}$ and $\mathscr{T} = \mathscr{T}_{\mathscr{M}}$. Thus,

(i) ${}_1\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.6/x_1) + (0.3/x_2) + (0.6/x_3) + (0.4/x_4) + (0.5/x_5) + (0.4/x_6)$ and ${}_1\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.6/x_1) + (0.3/x_2) + (0.7/x_3) + (0.4/x_4) + (0.5/x_5) + (0.5/x_6)$

(ii) ${}_2\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.6/x_1) + (0.3/x_2) + (0.7/x_3) + (0.4/x_4) + (0.5/x_5) + (0.4/x_6)$ and ${}_2\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C}_i}(\widehat{X}) = (0.6/x_1) + (0.3/x_2) + (0.7/x_3) + (0.4/x_4) + (0.5/x_5) + (0.4/x_6)$

(iii) ${}_3\mathcal{L}^{\mathcal{I}(p)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.5/x_1)+(0.3/x_2)+(0.6/x_3)+(0.4/x_4)+(0.5/x_5)+(0.4/x_6)$ and ${}_3\mathcal{U}^{\mathcal{T}(o)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.6/x_1)+(0.4/x_2)+(0.7/x_3)+(0.4/x_4)+(0.5/x_5)+(0.5/x_6)$

3.3. Some Types of Covering-Based Variable Precision Multigranulation $(\mathcal{I},\mathcal{T})$-Fuzzy Rough Sets. In the following, six new kinds of CVPMITFRS are defined and their properties are investigated. It is clear that the properties of these mentioned models satisfy Theorem 1. So, we only present the concepts and omit the properties.

Definition 15. Let $(\Omega,\widetilde{\Gamma})$ be a FβCAS and $\widetilde{\Gamma}=\{\widetilde{C_1},\widetilde{C_2},\ldots,\widetilde{C_n}\}$ be n fuzzy β-coverings of Ω for some $\beta\in(0,1]$. For each $\widehat{X}\in\mathcal{F}(\Omega)$ and a variable precision parameter $\gamma\in[0,1]$. Define the i-variable precision multigranulation $(\mathcal{I},\mathcal{T})$-fuzzy lower approximation (briefly, i-VPMGITFLA) and the i-variable precision multigranulation $(\mathcal{I},\mathcal{T})$-fuzzy upper approximation (briefly, i-VPMGITFUA), $\forall i\in\{\mathrm{I},\mathrm{II},\mathrm{III},\mathrm{IV},\mathrm{V},\mathrm{VI}\}$ are, respectively, defined as follows:

(1) ${}_{\mathrm{I}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\vee_{i=1}^{n}(\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),\gamma)\wedge\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$ and ${}_{\mathrm{I}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\wedge_{i=1}^{n}(\vee_{\widehat{X}(y)\geq 1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),1-\gamma)\vee\vee_{\widehat{X}(y)<1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$, $(\forall x\in\Omega)$

(2) ${}_{\mathrm{II}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\wedge_{i=1}^{n}(\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),\gamma)\wedge\wedge_{\widehat{X}(y)>\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$ and ${}_{\mathrm{II}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\vee_{i=1}^{n}(\vee_{\widehat{X}(y)\geq 1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),1-\gamma)\vee\vee_{\widehat{X}(y)<1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$, $(\forall x\in\Omega)$

(3) ${}_{\mathrm{III}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\vee_{i=1}^{n}(\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\gamma)\wedge\wedge_{\widehat{X}(y)>\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$ and ${}_{\mathrm{III}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\wedge_{i=1}^{n}(\vee_{\widehat{X}(y)\geq 1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),1-\gamma)\vee\vee_{\widehat{X}(y)<1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$, $(\forall x\in\Omega)$

(4) ${}_{\mathrm{IV}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\wedge_{i=1}^{n}(\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\gamma)\wedge\wedge_{\widehat{X}(y)>\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$ and ${}_{\mathrm{IV}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\vee_{i=1}^{n}(\vee_{\widehat{X}(y)\geq 1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),1-\gamma)\vee\vee_{\widehat{X}(y)<1-\gamma}\mathcal{T}(\widetilde{M}_{\tilde{C_i}}(x)^{\beta}(y)\vee\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$, $(\forall x\in\Omega)$

(5) ${}_{\mathrm{V}}\mathcal{L}_{\sum_{i=1}^{n}}\widetilde{C_i}^{\mathcal{I}(vp)}(\widehat{X})(x)=\vee_{i=1}^{n}(\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}_{\tilde{C_i}}(x)^{\beta}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\gamma)\wedge\wedge_{\widehat{X}(y)>\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$ and ${}_{\mathrm{V}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\wedge_{i=1}^{n}(\vee_{\widehat{X}(y)\geq 1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),1-\gamma)\vee\vee_{\widehat{X}(y)<1-\gamma}\mathcal{T}(\widetilde{M}_{\tilde{C_i}}(x)^{\beta}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$, $(\forall x\in\Omega)$

(6) ${}_{\mathrm{VI}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\wedge_{i=1}^{n}(\wedge_{\widehat{X}(y)\leq\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\gamma)\wedge\wedge_{\widehat{X}(y)>\gamma}\mathcal{I}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$ and ${}_{\mathrm{VI}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})(x)=\vee_{i=1}^{n}(\vee_{\widehat{X}(y)\geq 1-\gamma}\mathcal{T}(\widetilde{M}^{\beta}_{\tilde{C_i}(x)}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),1-\gamma)\vee\vee_{\widehat{X}(y)<1-\gamma}\mathcal{T}(\widetilde{M}_{\tilde{C_i}}(x)^{\beta}(y)\wedge\widetilde{N}^{\beta}_{\tilde{C_i}(x)}(y),\widehat{X}(y)))$, $(\forall x\in\Omega)$

If ${}_i\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})\neq{}_i\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})$, then $\widehat{X}$ is called a covering-based i-variable precision multigranulation $(\mathcal{I},\mathcal{T})$-fuzzy rough set (briefly, i-VPMGITFRS); otherwise, it is i-variable precision multigranulation fuzzy definable.

The best way to explain the above definition is to give the following example.

Example 6 (continued from Example 2). Assume that $\gamma=0.5$. Then, we have the following results:

(1) ${}_{\mathrm{I}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.8/x_1)+(0.75/x_2)+(0.82/x_3)+(0.7/x_4)+(0.75/x_5)+(0.7/x_6)$ and ${}_{\mathrm{I}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.25/x_1)+(0.15/x_2)+(0.3/x_3)+(0.2/x_4)+(0.25/x_5)+(0.25/x_6)$

(2) ${}_{\mathrm{II}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.72/x_1)+(0.65/x_2)+(0.79/x_3)+(0.7/x_4)+(0.65/x_5)+(0.65/x_6)$ and ${}_{\mathrm{II}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.35/x_1)+(0.15/x_2)+(0.35/x_3)+(0.2/x_4)+(0.35/x_5)+(0.28/x_6)$

(3) ${}_{\mathrm{III}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=0.8/x_1+0.75/x_2+0.82/x_3+0.7/x_4+0.75/x_5+0.7/x_6$ and ${}_{\mathrm{III}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=0.25/x_1+0.15/x_2+0.3/x_3+0.2/x_4+0.25/x_5+0.25/x_6$

(4) ${}_{\mathrm{IV}}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=0.72/x_1+0.65/x_2+0.79/x_3+0.7/x_4+0.65/x_5+0.65/x_6$ and ${}_{\mathrm{IV}}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=0.35/x_1+0.15/x_2+0.35/x_3+0.16/x_4+0.35/x_5+0.28/x_6$

(5) ${}_{V}\mathcal{L}^{\mathcal{I}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.8/x_1)+(0.7/x_2)+(0.8/x_3)+(0.7/x_4)+(0.75/x_5)+(0.7/x_6)$ and ${}_{V}\mathcal{U}^{\mathcal{T}(vp)}_{\sum_{i=1}^{n}\tilde{C_i}}(\widehat{X})=(0.25/$

$x_1) + (0.16/x_2) + (0.3/x_3) + (0.2/x_4) + (0.25/x_5) + (0.25/x_6)$

(6) ${}_{\mathrm{VI}}\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = (0.72/x_1) + (0.65/x_2) + (0.79/x_3) + (0.7/x_4) + (0.65/x_5) + (0.65/x_6)$ and ${}_{\mathrm{VI}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = (0.35/x_1) + (0.24/x_2) + (0.35/x_3) + (0.2/x_4) + (0.35/x_5) + (0.28/x_6)$

4. The Relationships between COMGITFRS Models and CPMGITFRS Models

In this section, we explain relationships among our models. Through the proposed study, we have the following results.

From Definitions 10 and 12, we conclude the following results.

Proposition 1. *Let* $(\Omega, \widetilde{\Gamma})$ *be a FβCAS and* $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ *be n fuzzy β-coverings of* Ω *for some* $\beta \in (0, 1]$. *Then, for each* $\widehat{X} \in \mathscr{F}(\Omega)$, *we have the following properties: (i)* ${}_{1}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cup_{i=1}^{n}\widetilde{C}_2^{-}(\widehat{X})$ *and* ${}_{1}\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cap_{i=1}^{n}\widetilde{C}_2^{-}(\widehat{X})$. *(ii)* ${}_{1}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cap_{i=1}^{n}\widetilde{C}_2^{+}(\widehat{X})$ *and* ${}_{1}\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cup_{i=1}^{n}\widetilde{C}_2^{+}(\widehat{X})$.

By Definitions 11 and 13, we have the following results.

Proposition 2. *Let* $(\Omega, \widetilde{\Gamma})$ *be a FβCAS and* $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ *be n fuzzy β-coverings of* Ω *for some* $\beta \in (0, 1]$. *Then, for each* $\widehat{X} \in \mathscr{F}(\Omega)$, *we have the following properties:*

(i) ${}_{2}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cup_{i=1}^{n}\widetilde{C}_3^{-}(\widehat{X})$ *and* ${}_{2}\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cap$ ${}_{i=1}^{n}\widetilde{C}_3^{-}(\widehat{X})$

(ii) ${}_{2}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cap_{i=1}^{n}\widetilde{C}_3^{+}(\widehat{X})$ *and* ${}_{2}\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cup_{i=1}^{n}\widetilde{C}_3^{+}(\widehat{X})$

From Definitions 11 and 14, we have the following results.

Proposition 3. *Let* $(\Omega, \widetilde{\Gamma})$ *be a FβCAS and* $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ *be n fuzzy β-coverings of* Ω *for some* $\beta \in (0, 1]$. *Then, for each* $\widehat{X} \in \mathscr{F}(\Omega)$, *we have the following properties.*

(i) ${}_{3}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cup_{i=1}^{n}\widetilde{C}_4^{-}(\widehat{X})$ *and* ${}_{3}\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cap_{i=1}^{n}\widetilde{C}_4^{-}(\widehat{X})$

(ii) ${}_{3}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cap_{i=1}^{n}\widetilde{C}_4^{+}(\widehat{X})$ *and* ${}_{3}\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = \cup_{i=1}^{n}\widetilde{C}_4^{+}(\widehat{X})$

Proposition 4. *Let* $(\Omega, \widetilde{\Gamma})$ *be a FβCAS and* $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ *be n fuzzy β-coverings of* Ω *for some* $\beta \in (0, 1]$. *Then, for each* $\widehat{X} \in \mathscr{F}(\Omega)$, *we have the following properties:*

(i) ${}_{3}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{0}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{2}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(ii) ${}_{3}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{1}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{2}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(iii) ${}_{2}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{0}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{3}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(iv) ${}_{2}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{1}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{3}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

Proposition 5. *Let* $(\Omega, \widetilde{\Gamma})$ *be a FβCAS and* $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ *be n fuzzy β-coverings of* Ω *for some* $\beta \in (0, 1]$. *Then, for each* $\widehat{X} \in \mathscr{F}(\Omega)$, *we have the following properties:*

(i) ${}_{2}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = {}_{0}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \vee {}_{1}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(ii) ${}_{2}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = {}_{0}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \wedge {}_{1}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(iii) ${}_{3}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = {}_{0}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \wedge {}_{1}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(iv) ${}_{3}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) = {}_{0}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \vee {}_{1}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

Remark 3. Let (Ω, Γ) be a FβCAS and $\widetilde{\Gamma} = \{\widetilde{C_1}, \widetilde{C_2}, \ldots, \widetilde{C_n}\}$ be n fuzzy β-coverings of Ω for some $\beta \in (0, 1]$. Then, for each $\widehat{X} \in \mathscr{F}(\Omega)$, we have the following properties:

(1) ${}_{1}\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{1}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and ${}_{1}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{1}\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(2) ${}_{2}\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{2}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and ${}_{2}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{2}\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(3) ${}_{3}\mathscr{L}^{\mathscr{I}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{3}\mathscr{L}^{\mathscr{I}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and ${}_{3}\mathscr{U}^{\mathscr{T}(o)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{3}\mathscr{U}^{\mathscr{T}(p)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(4) ${}_{\mathrm{II}}\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{\mathrm{I}}\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and ${}_{\mathrm{I}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{\mathrm{II}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

(5) ${}_{\mathrm{IV}}\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{\mathrm{III}}\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$ and ${}_{\mathrm{III}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}C_i}(\widehat{X}) \subseteq {}_{\mathrm{IV}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}C_i}(\widehat{X})$

(6) ${}_{\mathrm{VI}}\mathscr{L}^{\mathscr{I}(vp)}_{\sum_{i=1}^{n}}(\widehat{X}) \subseteq {}_{\mathrm{V}}\mathscr{L}^{\mathscr{I}(vp)}_{i=1}(\widehat{X})$ and ${}_{\mathrm{V}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X}) \subseteq {}_{\mathrm{VI}}\mathscr{U}^{\mathscr{T}(vp)}_{\sum_{i=1}^{n}\widetilde{C_i}}(\widehat{X})$

5. An Application to Decision-Making

In this section, we apply the proposed method to make a decision on a real-life problem.

5.1. Description and Process. Let $\Omega = \{u_1, u_2, \ldots, u_n\}$ be n alternatives and $\mathscr{E} = \{e_1, e_2, \ldots, e_l\}$ be l decision makers. Suppose for each v_i is a weighted vector correspondingly to e_i, where $\nu_i \geq 0$ for $i = 1, \ldots, l$ and $\sum_{i=1}^{l} \nu_i = 1$. Hence, $\widetilde{C}_i = \{\widetilde{C_{i1}}, \widetilde{C_{i2}}, \ldots, \widetilde{C_{im_i}}\}$, for all $i = 1, 2, \ldots, n$ is a set of attributes. A family of mappings $\mathscr{G} = \{g_l\}$, where $g_l\colon \Omega \times \widetilde{C}_i \longrightarrow [0,1]$. So, we construct the MAGDM with fuzzy information system $(\Omega, \widetilde{C}, \mathscr{E}, \mathscr{G})$. Based on the proposed covering methods, we present a decision-making algorithm to find the best alternative through the following steps:

Step 1: Construct the decision-making object with fuzzy information of the universe of discourse. Through the rule of fuzzy TOPSIS method, we have

$$\begin{aligned} \overline{\mathscr{P}_l} &= \left\{\widetilde{C}_{lj}, \bigvee_{1\leq i\leq n}\left(g_l\left(u_i, \widetilde{C}_{lj}\right)\right)\colon (j = 1, \ldots, m)(l = 1, \ldots, t)\right\} = \left\{\left(\widetilde{C}_{l1}, \vee\left(g_{l1}\right)\right), \ldots, \left(\widetilde{C}_{lm}, \vee\left(g_{lm}\right)\right)\right\},\\ \underline{\mathscr{P}_l} &= \left\{\widetilde{C}_{lj}, \bigwedge_{1\leq i\leq n}\left(g_l\left(u_i, \widetilde{C}_{lj}\right)\right)\colon (j = 1, \ldots, m)(l = 1, \ldots, t)\right\} = \left\{\left(\widetilde{C}_{l1}, \wedge\left(g_{l1}\right)\right), \ldots, \left(\widetilde{C}_{lm}, \wedge\left(g_{lm}\right)\right)\right\} \end{aligned} \tag{22}$$

where $\wedge$ and $\wedge$ denote "max" and "min," respectively.

Step 2: Compute the respective distances $\overline{\mathscr{D}}$ and $\underline{\mathscr{D}}$ as follows:

$$\begin{aligned} \overline{\mathscr{D}_l} &= \mathscr{S}\left(\widetilde{C}_{lj}(u_i), \widetilde{C}_{lj}(\underline{\mathscr{P}})\right) = \sqrt{\frac{1}{m}\sum_{j=1}^{m}\left(\widetilde{C}_{lj}(u_i) - \vee\left(g_{lj}\right)\right)^2},\\ \underline{\mathscr{D}_l} &= \mathscr{S}\left(\widetilde{C}_{lj}(u_i), \widetilde{C}_{lj}(\overline{\mathscr{P}})\right) = \sqrt{\frac{1}{m}\sum_{j=1}^{m}\left(\widetilde{C}_{lj}(u_i) - \wedge\left(g_{lj}\right)\right)^2}, \end{aligned} \tag{23}$$

where $\mathscr{S}(\widehat{Y}, t\widehat{Z}) = \sqrt{(1/m)\sum_{j=1}^{m}(\widehat{Y}(u_i)t - n\widehat{Z}q(u_i))^2}$ and m is the cardinality of Ω.

Step 3: Calculate the lower and upper approximations of the best and worst decision-making objects with fuzzy information by Definition 13 (2-OMGITFLA and 2-OMGITFUA).

Step 4: Calculate the closeness coefficient degree by $\mathscr{R}_l(u_i) = \underline{\mathscr{W}_l}(u_i)/\underline{\mathscr{W}_l}(u_i) + \overline{\mathscr{W}_l}(u_i)$, where

$$\begin{aligned} \underline{\mathscr{W}_l}(u_i) &= \mathscr{B}_p\left({}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\underline{\mathscr{D}_l}\right)(u_i), {}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\underline{\mathscr{D}_l}\right)(u_i)\right),\\ \overline{\mathscr{W}_l}(u_i) &= \mathscr{B}_p\left({}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\overline{\mathscr{D}_l}\right)(u_i), {}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\overline{\mathscr{D}_l}\right)(u_i)\right), \end{aligned} \tag{24}$$

be the worst and the best decision-making objects for individual ranking function of expert l for the candidates u_i, and $0 \leq \underline{\mathscr{W}_l}(u_i), \overline{\mathscr{W}_l}(u_i) \leq 1$.

Step 5: Calculate the group ranking function by the following equation $\mathscr{R}(u_i) = \sum_{l=1}^{t}\nu_l\mathscr{R}_l(u_i)$, and hence rank the alternatives.

According to these steps, we give an algorithm to solve the decision-making problems based on the 2-COMGITFRS model. The steps corresponding to it are summarized in Algorithm 1.

5.2. Applied Example. The abovementioned steps have been illustrated with a numerical example as shown next.

Example 7 (see [40]). Let $\Omega = \{u_1, u_2, \ldots, u_6\}$ be six system analysis engineers and $\widetilde{\Gamma} = \{$emotional steadiness (C_1), oral communication skill (C_2), personality (C_3), past experience (C_4), self-confidence $(C_5)\}$ be the attribute set of the basic description of the candidates. Suppose that three experts e_1, e_2, and e_3 are invited to evaluate the system analysis engineers according to their specialized knowledge. The weights of every expert are $\nu_1 = 0.4, \nu_2 = 0.1$, and $\nu_3 = 0.5$. The following steps of the stated algorithm are implemented here.

Step 1: Experts evaluate each candidate under the set of the attribute and present their judgments with the real values. These values are summarized in Tables 11–13.

Step 2: According to the importance of these five attributes, we give the following results for each expert:

$$\begin{aligned} \overline{\mathscr{P}_1} &= \left\{\left(\widetilde{C_{11}}, t0.82\right), \left(\widetilde{C_{12}}, t0.76\right), \left(\widetilde{C_{13}}, t0.74\right), \left(\widetilde{C_{14}}, t0.78\right), \left(\widetilde{C_{15}}, t0.91\right)\right\},\\ \underline{\mathscr{P}_1} &= \left\{\left(\widetilde{C_{11}}, t0.28\right), \left(\widetilde{C_{12}}, t0.32\right), \left(\widetilde{C_{13}}, t0.36\right), \left(\widetilde{C_{14}}, t0.45\right), \left(\widetilde{C_{15}}, t0.43\right)\right\},\\ \overline{\mathscr{P}_2} &= \left\{\left(\widetilde{C_{21}}, t0.85\right), \left(\widetilde{C_{22}}, t0.77\right), \left(\widetilde{C_{23}}, t0.79\right), \left(\widetilde{C_{24}}, t0.81\right), \left(\widetilde{C_{25}}, t0.71\right)\right\},\\ \underline{\mathscr{P}_2} &= \left\{\left(\widetilde{C_{21}}, t0.35\right), \left(\widetilde{C_{22}}, t0.34\right), \left(\widetilde{C_{23}}, t0.46\right), \left(\widetilde{C_{24}}, t0.26\right), \left(\widetilde{C_{25}}, t0.43\right)\right\},\\ \overline{\mathscr{P}_3} &= \left\{\left(\widetilde{C_{31}}, t0.84\right), \left(\widetilde{C_{32}}, t0.75\right), \left(\widetilde{C_{33}}, t0.74\right), \left(\widetilde{C_{34}}, t0.69\right), \left(\widetilde{C_{35}}, t0.78\right)\right\},\\ \underline{\mathscr{P}_3} &= \left\{\left(\widetilde{C_{31}}, t0.37\right), \left(\widetilde{C_{32}}, t0.36\right), \left(\widetilde{C_{33}}, t0.35\right), \left(\widetilde{C_{34}}, t0.42\right), \left(\widetilde{C_{35}}, t0.48\right)\right\}. \end{aligned} \tag{25}$$

Input: Fuzzy information systems $(\Omega, \widetilde{C}, \mathscr{E}, \mathscr{G})$.
Output: Decision-Making.
(1) Enter $\widehat{X}$, β, $\Gamma = \left\{\widetilde{C}_i\colon tin = q1h,_{\ldots}\, x, 7n\right\}$ and $\Omega = \left\{x_j\colon j = 1, \ldots, m\right\}$.
(2) From Definition 7, calculate fuzzy β-neighborhood.
(3) From **Step 2** and by Definition 8, calculate complementary fuzzy β-neighborhood
(4) From **Steps 2 and 3**, calculate $\widetilde{N}^{\beta}_{\widetilde{C_1}(u_1)} \wedge \widetilde{M}^{\beta}_{\widetilde{C_1}(u_i)}$.
(5) Enter $\overline{\mathscr{P}_l}$ and $\underline{\mathscr{P}_l}$.
(6) Calculate the distances $\overline{\mathscr{D}_l}$ and $\underline{\mathscr{D}_l}$.
(7) From Definition 13, calculate the lower approximation ${}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}(\overline{\mathscr{D}_l})$ and the upper approximation ${}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}(\underline{\mathscr{D}_l})$.
(8) Calculate the worst and the best decision-making objects $\overline{\mathscr{W}_l}$ and $\underline{\mathscr{W}_l}$ for each individual decision-maker.
(9) Calculate the individual ranking function $\mathscr{R}_l$.
(10) Calculate the group ranking function $\mathscr{R}$.
(11) Obtain the decision.

Algorithm 1: Algorithm for MAGDM with the TOPSIS method.

Step 3: If the threshold $\beta = 0.6$, it produces $\widetilde{N}^{\beta}_{\widetilde{C_i}}, \widetilde{M}^{\beta}_{\widetilde{C_i}}$ and $\widetilde{N}^{0.6}_{\widetilde{C_i}} \wedge \widetilde{M}^{0.6}_{\widetilde{C_i}}$ as displayed in Tables 14–22.

Step 4: Calculate the distances $\overline{\mathscr{D}_l}$ and $\underline{\mathscr{D}_l}$ as follows:

$$\begin{aligned}
\overline{\mathscr{D}_1} &= \frac{0.248}{u_1} + \frac{0.269}{u_2} + \frac{0.315}{u_3} + \frac{0.189}{u_4} + \frac{0.261}{u_5} + \frac{0.306}{u_6},\\
\underline{\mathscr{D}_1} &= \frac{0.307}{u_1} + \frac{0.307}{u_2} + \frac{0.241}{u_3} + \frac{0.317}{u_4} + \frac{0.247}{u_5} + \frac{0.259}{u_6},\\
\overline{\mathscr{D}_2} &= \frac{0.276}{u_1} + \frac{0.276}{u_2} + \frac{0.293}{u_3} + \frac{0.146}{u_4} + \frac{0.261}{u_5} + \frac{0.228}{u_6},\\
\underline{\mathscr{D}_2} &= \frac{0.246}{u_1} + \frac{0.278}{u_2} + \frac{0.209}{u_3} + \frac{0.352}{u_4} + \frac{0.293}{u_5} + \frac{0.278}{u_6},\\
\overline{\mathscr{D}_3} &= \frac{0.243}{u_1} + \frac{0.233}{u_2} + \frac{0.21}{u_3} + \frac{0.219}{u_4} + \frac{0.184}{u_5} + \frac{0.259}{u_6},\\
\underline{\mathscr{D}_3} &= \frac{0.225}{u_1} + \frac{0.231}{u_2} + \frac{0.25}{u_3} + \frac{0.237}{u_4} + \frac{0.232}{u_5} + \frac{0.212}{u_6}.
\end{aligned} \tag{26}$$

Step 5: Calculate the lower and upper approximations of the best and worst decision-making objects as follows.

Take $e = e_1$, and we have

$$\begin{aligned}
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\overline{\mathscr{D}_1}\right) &= \frac{0.49616}{u_1} + \frac{0.52485}{u_2} + \frac{0.55475}{u_3} + \frac{0.48907}{u_4} + \frac{0.51965}{u_5} + \frac{0.54196}{u_6},\\
{}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\overline{\mathscr{D}_1}\right) &= \frac{0.16616}{u_1} + \frac{0.17485}{u_2} + \frac{0.20475}{u_3} + \frac{0.11907}{u_4} + \frac{0.16965}{u_5} + \frac{0.20196}{u_6},\\
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\underline{\mathscr{D}_1}\right) &= \frac{0.53569}{u_1} + \frac{0.54955}{u_2} + \frac{0.50665}{u_3} + \frac{0.56971}{u_4} + \frac{0.51055}{u_5} + \frac{0.51094}{u_6},\\
{}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\widetilde{C_r}}\left(\underline{\mathscr{D}_1}\right) &= \frac{0.20569}{u_1} + \frac{0.19955}{u_2} + \frac{0.15665}{u_3} + \frac{0.19971}{u_4} + \frac{0.16055}{u_5} + \frac{0.17094}{u_6}.
\end{aligned} \tag{27}$$

Take $e = e_2$, and we have

$$
\begin{aligned}
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\overline{\mathscr{D}_2}\right) &= \frac{0.51492}{u_1} + \frac{0.5294}{u_2} + \frac{0.54045}{u_3} + \frac{0.46198}{u_4} + \frac{0.51965}{u_5} + \frac{0.49048}{u_6},\\
{}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\overline{\mathscr{D}_2}\right) &= \frac{0.18492}{u_1} + \frac{0.1794}{u_2} + \frac{0.19045}{u_3} + \frac{0.09198}{u_4} + \frac{0.16965}{u_5} + \frac{0.15048}{u_6},\\
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\underline{\mathscr{D}_2}\right) &= \frac{0.49482}{u_1} + \frac{0.5307}{u_2} + \frac{0.48585}{u_3} + \frac{0.59176}{u_4} + \frac{0.54045}{u_5} + \frac{0.52348}{u_6},\\
{}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\underline{\mathscr{D}_2}\right) &= \frac{0.16482}{u_1} + \frac{0.1807}{u_2} + \frac{0.13585}{u_3} + \frac{0.22176}{u_4} + \frac{0.19045}{u_5} + \frac{0.18348}{u_6}.
\end{aligned}
\tag{28}
$$

Take $e = e_3$, and we have

$$
\begin{aligned}
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\overline{\mathscr{D}_3}\right) &= \frac{0.49281}{u_1} + \frac{0.50145}{u_2} + \frac{0.4865}{u_3} + \frac{0.50797}{u_4} + \frac{0.4696}{u_5} + \frac{0.51094}{u_6},\\
{}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\overline{\mathscr{D}_3}\right) &= \frac{0.16281}{u_1} + \frac{0.15145}{u_2} + \frac{0.1365}{u_3} + \frac{0.13797}{u_4} + \frac{0.1196}{u_5} + \frac{0.17094}{u_6},\\
{}_3\mathscr{L}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\underline{\mathscr{D}_3}\right) &= \frac{0.48075}{u_1} + \frac{0.50015}{u_2} + \frac{0.5125}{u_3} + \frac{0.51931}{u_4} + \frac{0.5008}{u_5} + \frac{0.47992}{u_6},\\
{}_3\mathscr{U}^{\mathscr{I}(o)}_{\sum_{r=1}^{k}\tilde{C}_r}\left(\underline{\mathscr{D}_3}\right) &= \frac{0.15085}{u_1} + \frac{0.15015}{u_2} + \frac{0.1625}{u_3} + \frac{0.14931}{u_4} + \frac{0.1508}{u_5} + \frac{0.13992}{u_6}.
\end{aligned}
\tag{29}
$$

Step 6 Based on the importance of these five attributes, we give the worst and the best decision-making objects as follows:

$$
\begin{aligned}
\overline{\mathscr{W}_1} &= \frac{0.57988}{u_1} + \frac{0.60793}{u_2} + \frac{0.645915}{u_3} + \frac{0.549904}{u_4} + \frac{0.60114}{u_5} + \frac{0.63447}{u_6},\\
\underline{\mathscr{W}_1} &= \frac{0.631194}{u_1} + \frac{0.63944}{u_2} + \frac{0.58393}{u_3} + \frac{0.65564}{u_4} + \frac{0.58913}{u_5} + \frac{0.59848}{u_6},\\
\overline{\mathscr{W}_2} &= \frac{0.604621}{u_1} + \frac{0.61383}{u_2} + \frac{0.62797}{u_3} + \frac{0.52247}{u_4} + \frac{0.60114}{u_5} + \frac{0.57115}{u_6},\\
\underline{\mathscr{W}_2} &= \frac{0.578084}{u_1} + \frac{0.615503}{u_2} + \frac{0.5557}{u_3} + \frac{0.68229}{u_4} + \frac{0.62797}{u_5} + \frac{0.61091}{u_6},\\
\overline{\mathscr{W}_3} &= \frac{0.57539}{u_1} + \frac{0.57696}{u_2} + \frac{0.55659}{u_3} + \frac{0.57586}{u_4} + \frac{0.53304}{u_5} + \frac{0.59454}{u_6},\\
\underline{\mathscr{W}_3} &= \frac{0.55908}{u_1} + \frac{0.57520}{u_2} + \frac{0.58667}{u_3} + \frac{0.591082}{u_4} + \frac{0.57608}{u_5} + \frac{0.55269}{u_6}.
\end{aligned}
\tag{30}
$$

Table 11: Table for $\tilde{C}_1$.

Ω	$\tilde{C}_{11}$	$\tilde{C}_{12}$	$\tilde{C}_{13}$	$\tilde{C}_{14}$	$\tilde{C}_{15}$
u_1	0.82	0.71	0.46	0.55	0.52
u_2	0.73	0.32	0.65	0.58	0.84
u_3	0.56	0.68	0.36	0.78	0.44
u_4	0.53	0.48	0.74	0.65	0.91
u_5	0.66	0.53	0.57	0.72	0.43
u_6	0.28	0.76	0.52	0.45	0.77

Table 12: Table for $\tilde{C}_2$.

Ω	$\tilde{C}_{21}$	$\tilde{C}_{22}$	$\tilde{C}_{23}$	$\tilde{C}_{24}$	$\tilde{C}_{25}$
u_1	0.78	0.56	0.67	0.26	0.59
u_2	0.35	0.77	0.49	0.69	0.55
u_3	0.51	0.37	0.79	0.42	0.67
u_4	0.85	0.68	0.57	0.75	0.48
u_5	0.58	0.34	0.73	0.81	0.43
u_6	0.53	0.75	0.46	0.59	0.71

Thus, we evaluate a closeness coefficient as follows:

$$\begin{aligned}\mathscr{R}_1 &= \frac{0.521185}{u_1} + \frac{0.51263}{u_2} + \frac{0.45625}{u_3} + \frac{0.54385}{u_4} + \frac{0.49495}{u_5} + \frac{0.48540}{u_6},\\ \mathscr{R}_2 &= \frac{0.48878}{u_1} + \frac{0.50068}{u_2} + \frac{0.46947}{u_3} + \frac{0.57155}{u_4} + \frac{0.51091}{u_5} + \frac{0.51682}{u_6},\\ \mathscr{R}_3 &= \frac{0.49281}{u_1} + \frac{0.49924}{u_2} + \frac{0.51316}{u_3} + \frac{0.50652}{u_4} + \frac{0.51940}{u_5} + \frac{0.48176}{u_6}.\end{aligned} \tag{31}$$

Step 7 Based on these results, we calculate the group optimal index as follows. $\mathscr{R} = (0.503757/u_1) + (0.50474/u_2) + (0.486027/u_3) + (0.527955/u_4) + (0.508771/u_5) + (0.486722/u_6)$, and hence get the ranking order as $u_4 \geq u_5 \geq u_2 \geq u_1 \geq u_6 \geq u_3$.

From the calculations, we conclude that the 4th system analysis engineer is the best alternative among the others.

Furthermore, we get the solution for **Case 2** by the same analysis in **Case 1**. Therefore, we have the group optimal index as follows:

$$\mathscr{R} = \frac{0.50055}{u_1} + \frac{0.500528}{u_2} + \frac{0.500528}{u_3} + \frac{0.500636}{u_4} + \frac{0.500528}{u_5} + \frac{0.500539}{u_6}, \tag{32}$$

and hence get the ranking order as $u_4 \geq u_1 \geq u_6 \geq u_2 \geq u_3 \geq u_5$. Through the previous computation, we obtain the 4th system analysis engineer is the best alternative among the others.

5.3. Comparative Analysis. The main aim of the current work is to present a method that increases the lower approximation and decreases the upper approximation of Zhan's methods in [40]. This can be seen easily from Examples 2– 4. Moreover and by looking at Tables 23 and 24, we can see that the ranking results of the two decision-making models. It is obvious that the optimal selected alternative is the same, although there exist some differences in the ranking results because we choose different decision-making methods.

An easy way to see the effectiveness of our method and the differences between the four models (i.e., our three proposed models and Zhan's model) are shown in Figures 1 and 2.

Figure 1 explained the comparisons between the lower approximations for the four models (i.e., 0-OMGITFLA, 1-OMGITFLA, 2-OMGITFLA, and 3-OMGITFLA) for the two cases (i.e., Case 1 (resp., Case 2) is in the left (resp., right) figure). This figure justifies that the 2-OMGITFLA is better than the others.

Figure 2 clarified the differences between the upper approximations for the four models (i.e., 0-OMGITFUA, 1-OMGITFUA, 2-OMGITFUA, and 3-OMGITFUA) for the

Table 13: Table for $\widetilde{C}_3$.

Ω	$\widetilde{C}_{31}$	$\widetilde{C}_{32}$	$\widetilde{C}_{33}$	$\widetilde{C}_{34}$	$\widetilde{C}_{35}$
u_1	0.56	0.75	0.39	0.67	0.48
u_2	0.76	0.36	0.68	0.45	0.55
u_3	0.84	0.55	0.35	0.58	0.65
u_4	0.43	0.53	0.74	0.69	0.63
u_5	0.59	0.71	0.65	0.48	0.55
u_6	0.37	0.66	0.56	0.42	0.78

Table 14: Table for $\widetilde{N}^{0.6}_{\widetilde{C}_1(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{N}^{0.6}_{\widetilde{C}_1(u_1)}$	0.71	0.32	0.56	0.48	0.53	0.28
$\widetilde{N}^{0.6}_{\widetilde{C}_1(u_2)}$	0.46	0.65	0.36	0.53	0.43	0.28
$\widetilde{N}^{0.6}_{\widetilde{C}_1(u_3)}$	0.55	0.32	0.68	0.48	0.53	0.45
$\widetilde{N}^{0.6}_{\widetilde{C}_1(u_4)}$	0.46	0.58	0.36	0.65	0.43	0.45
$\widetilde{N}^{0.6}_{\widetilde{C}_1(u_5)}$	0.55	0.58	0.56	0.53	0.66	0.28
$\widetilde{N}^{0.6}_{\widetilde{C}_1(u_6)}$	0.52	0.32	0.44	0.48	0.43	0.76

Table 15: Table for $\widetilde{N}^{0.6}_{\widetilde{C}_2(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{N}^{0.6}_{\widetilde{C}_2(u_1)}$	0.67	0.35	0.51	0.57	0.58	0.46
$\widetilde{N}^{0.6}_{\widetilde{C}_2(u_2)}$	0.26	0.69	0.37	0.68	0.34	0.59
$\widetilde{N}^{0.6}_{\widetilde{C}_2(u_3)}$	0.59	0.49	0.67	0.48	0.43	0.46
$\widetilde{N}^{0.6}_{\widetilde{C}_2(u_4)}$	0.26	0.35	0.37	0.68	0.34	0.53
$\widetilde{N}^{0.6}_{\widetilde{C}_2(u_5)}$	0.26	0.49	0.42	0.57	0.73	0.46
$\widetilde{N}^{0.6}_{\widetilde{C}_2(u_6)}$	0.56	0.55	0.37	0.48	0.34	0.71

Table 16: Table for $\widetilde{N}^{0.6}_{\widetilde{C}_3(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{N}^{0.6}_{\widetilde{C}_3(u_1)}$	0.67	0.36	0.55	0.53	0.48	0.42
$\widetilde{N}^{0.6}_{\widetilde{C}_3(u_2)}$	0.39	0.68	0.35	0.43	0.59	0.37
$\widetilde{N}^{0.6}_{\widetilde{C}_3(u_3)}$	0.48	0.55	0.65	0.43	0.55	0.37
$\widetilde{N}^{0.6}_{\widetilde{C}_3(u_4)}$	0.39	0.45	0.35	0.63	0.48	0.42
$\widetilde{N}^{0.6}_{\widetilde{C}_3(u_5)}$	0.39	0.36	0.35	0.53	0.65	0.56
$\widetilde{N}^{0.6}_{\widetilde{C}_3(u_6)}$	0.48	0.36	0.55	0.53	0.55	0.66

Table 17: Table for $\widetilde{M}^{0.6}_{\widetilde{C}_1(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{M}^{0.6}_{\widetilde{C}_1(u_1)}$	0.71	0.46	0.55	0.46	0.55	0.52
$\widetilde{M}^{0.6}_{\widetilde{C}_1(u_2)}$	0.32	0.65	0.32	0.58	0.58	0.32
$\widetilde{M}^{0.6}_{\widetilde{C}_1(u_3)}$	0.56	0.36	0.68	0.36	0.56	0.44
$\widetilde{M}^{0.6}_{\widetilde{C}_1(u_4)}$	0.48	0.53	0.48	0.65	0.53	0.48
$\widetilde{M}^{0.6}_{\widetilde{C}_1(u_5)}$	0.53	0.43	0.53	0.43	0.66	0.43
$\widetilde{M}^{0.6}_{\widetilde{C}_1(u_6)}$	0.28	0.28	0.45	0.45	0.28	0.76

Table 18: Table for $\widetilde{M}^{0.6}_{C_2(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{M}^{0.6}_{C_2(u_1)}$	0.67	0.26	0.59	0.26	0.26	0.56
$\widetilde{M}^{0.6}_{C_2(u_2)}$	0.35	0.69	0.49	0.35	0.49	0.55
$\widetilde{M}^{0.6}_{C_2(u_3)}$	0.51	0.37	0.67	0.37	0.42	0.37
$\widetilde{M}^{0.6}_{C_2(u_4)}$	0.57	0.68	0.48	0.68	0.57	0.48
$\widetilde{M}^{0.6}_{C_2(u_5)}$	0.58	0.34	0.43	0.34	0.73	0.34
$\widetilde{M}^{0.6}_{C_2(u_6)}$	0.46	0.59	0.46	0.53	0.46	0.71

Table 19: Table for $\widetilde{M}^{0.6}_{C_3(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{M}^{0.6}_{C_3(u_1)}$	0.67	0.39	0.48	0.39	0.39	0.48
$\widetilde{M}^{0.6}_{C_3(u_2)}$	0.36	0.68	0.55	0.45	0.36	0.36
$\widetilde{M}^{0.6}_{C_3(u_3)}$	0.55	0.35	0.65	0.35	0.35	0.55
$\widetilde{M}^{0.6}_{C_3(u_4)}$	0.53	0.43	0.43	0.63	0.53	0.53
$\widetilde{M}^{0.6}_{C_3(u_5)}$	0.48	0.59	0.55	0.48	0.65	0.55
$\widetilde{M}^{0.6}_{C_3(u_6)}$	0.42	0.37	0.37	0.42	0.56	0.66

Table 20: Table for $\widetilde{N}^{0.6}_{C_1(u_i)} \wedge \widetilde{M}^{0.6}_{C_1(u_i)}$.

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{N}^{0.6}_{C_1(u_1)} \wedge \widetilde{M}^{0.6}_{C_1(u_1)}$	0.71	0.32	0.55	0.46	0.53	0.28
$\widetilde{N}^{0.6}_{C_1(u_2)} \wedge \widetilde{M}^{0.6}_{C_1(u_2)}$	0.32	0.65	0.32	0.53	0.43	0.28
$\widetilde{N}^{0.6}_{C_1(u_3)} \wedge \widetilde{M}^{0.6}_{C_1(u_3)}$	0.55	0.32	0.68	0.36	0.53	0.44
$\widetilde{N}^{0.6}_{C_1(u_4)} \wedge \widetilde{M}^{0.6}_{C_1(u_4)}$	0.46	0.53	0.36	0.65	0.43	0.45
$\widetilde{N}^{0.6}_{C_1(u_5)} \wedge \widetilde{M}^{0.6}_{C_1(u_5)}$	0.53	0.43	0.53	0.43	0.66	0.28
$\widetilde{N}^{0.6}_{C_1(u_6)} \wedge \widetilde{M}^{0.6}_{C_1(u_6)}$	0.28	0.28	0.44	0.45	0.28	0.76

Table 21: Table for $\widetilde{N}^{0.6}_{C_2(u_i)} \wedge \widetilde{M}^{0.6}_{C_2(u_i)}$

	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{N}^{0.6}_{C_2(u_1)} \wedge \widetilde{M}^{0.6}_{C_2(u_1)}$	0.67	0.26	0.51	0.26	0.26	0.46
$\widetilde{N}^{0.6}_{C_2(u_2)} \wedge \widetilde{M}^{0.6}_{C_2(u_2)}$	0.26	0.69	0.37	0.35	0.34	0.55
$\widetilde{N}^{0.6}_{C_2(u_3)} \wedge \widetilde{M}^{0.6}_{C_2(u_3)}$	0.51	0.37	0.67	0.37	0.42	0.37
$\widetilde{N}^{0.6}_{C_2(u_4)} \wedge \widetilde{M}^{0.6}_{C_2(u_4)}$	0.26	0.35	0.37	0.68	0.34	0.48
$\widetilde{N}^{0.6}_{C_2(u_5)} \wedge \widetilde{M}^{0.6}_{C_2(u_5)}$	0.26	0.34	0.42	0.34	0.73	0.34
$\widetilde{N}^{0.6}_{C_2(u_6)} \wedge \widetilde{M}^{0.6}_{C_2(u_6)}$	0.46	0.55	0.37	0.48	0.34	0.71

Table 22: Table for $\widetilde{N}^{0.6}_{C_3(u_i)} \wedge \widetilde{M}^{0.6}_{C_3(u_i)}$.

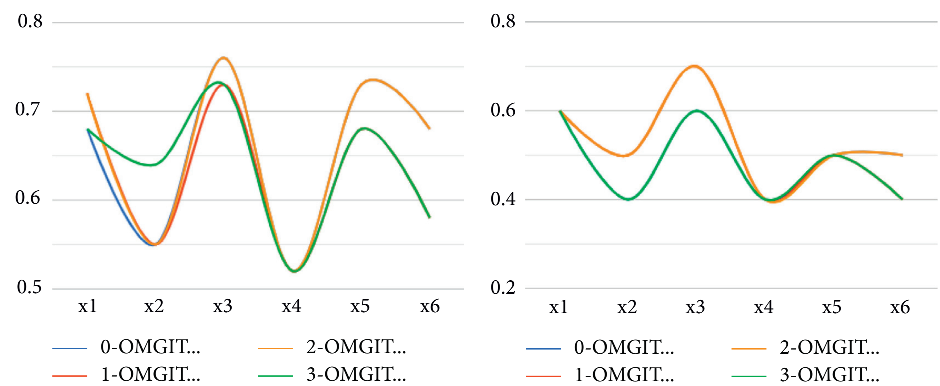
	u_1	u_2	u_3	u_4	u_5	u_6
$\widetilde{N}^{0.6}_{C_3(u_1)} \wedge \widetilde{M}^{0.6}_{C_3(u_1)}$	0.67	0.36	0.48	0.39	0.39	0.42
$\widetilde{N}^{0.6}_{C_3(u_2)} \wedge \widetilde{M}^{0.6}_{C_3(u_2)}$	0.36	0.68	0.35	0.43	0.36	0.36
$\widetilde{N}^{0.6}_{C_3(u_3)} \wedge \widetilde{M}^{0.6}_{C_3(u_3)}$	0.48	0.35	0.65	0.35	0.35	0.37
$\widetilde{N}^{0.6}_{C_3(u_4)} \wedge \widetilde{M}^{0.6}_{C_3(u_4)}$	0.39	0.43	0.35	0.63	0.48	0.42
$\widetilde{N}^{0.6}_{C_3(u_5)} \wedge \widetilde{M}^{0.6}_{C_3(u_5)}$	0.39	0.36	0.35	0.48	0.65	0.55
$\widetilde{N}^{0.6}_{C_3(u_6)} \wedge \widetilde{M}^{0.6}_{C_3(u_6)}$	0.42	0.36	0.37	0.42	0.55	0.66

Table 23: Table for the ranking results for Case 1.

Two models	Obtain a decision
Zhan model	$x_4 \geq x_1 \geq x_6 \geq x_2 = x_3 = x_5$
Our model	$x_4 \geq x_1 \geq x_6 \geq x_2 = x_3 = x_5$

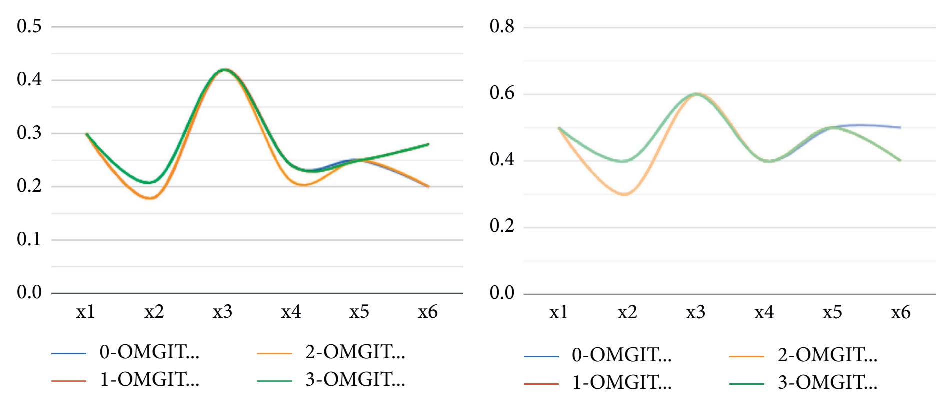
Table 24: Table for the ranking results for Case 2.

Two models	Obtain a decision
Zhan model	$x_4 \geq x_1 \geq x_5 \geq x_2 \geq x_3 \geq x_6$
Our model	$x_4 \geq x_5 \geq x_2 \geq x_1 \geq x_6 \geq x_3$



(a) (b)

Figure 1: The representations of the lower approximations by using our model and Zhan model in two cases.



(a) (b)

Figure 2: The representations of the upper approximations by using our model and Zhan model in two cases.

two cases (i.e., Case 1 (resp., Case 2) is in the left (resp., right) figure). This figure illustrates that the 2-OMGITFUA is lower than the others.

6. Conclusion and Future Work

The main aim of the present work is to increase the effectiveness of Zhan's method by increasing the lower approximation and decreasing the upper approximation. So, based on the concepts of a family of fuzzy β-neighborhood (and a family of fuzzy complementary $\mathcal{I},\mathcal{T}$-neighborhood), we introduced new three types of covering-based multigranulation $(\mathcal{I},\mathcal{T})$-fuzzy rough sets models and their properties. Furthermore, we give six kinds of covering-based variable precision multigranulation ([[1008]])-fuzzy rough sets. The relationships among these models are investigated.

Also, an illustrative example with algorithm is given. Therefore, it is clear to see that 2-COMGITFRS is better than the other models (i.e., 0-COMGITFRS, 1-COMGITFRS, and 3-COMGITFRS).

In future research, we plan to further investigate along with the following: (1) topological properties of the presented methods [44, 45], (2) combination with the soft set and the proposed methods [46, 47], and (3) combination with the neutrosophic set and the current methods [48].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Yulin University-Industry Collaboration Project (2019-75-2).

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