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Research Article

An Innovative Way to Generate Hamiltonian Energy of a New Hyperchaotic Complex Nonlinear Model and Its Control

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We are implementing a new Rabinovich hyperchaotic structure with complex variables in this research. This modern system is a real, autonomous hyperchaotic, and 8-dimensional continuous structure. Some of the characteristics of this system, as well as for invariance, dissipation, balance, and stability, are technically analyzed. Some other properties are also studied numerically, such as Lyapunov exponents, Lyapunov dimension, bifurcation diagrams, and chaotic actions. Hamiltonian energy is being studied and applying by using the innovative method. Via active control method, we inhibit our system's hyperchaotic behavior. The new system's hyperchaotic solutions are transformed into its unstable, trivial fixed point. The validity of the findings obtained is illustrated by an example of numerics and reenactment. Numerical results are plotted to display the variables of the state after and before the control to demonstrate that the control is being achieved.

1. Introduction

Chaos is an imperative fascinating wonder in the nonlinear dynamical frameworks. The most critical contrast among chaos and hyperchaos is that hyperchaotic frameworks have excess of a solitary positive Lyapunov type and show complex powerful impacts. Verifiable, Lorenz [1] first found the chaos solution in 1963, while hyperchaos was initially announced by Rössler in 1979 [2]. Since Fowler et al. [3] addressed a complex Lorenz show that is used to depict and reproduce the liquid turning and the laser being detuned. Mahmoud et al. introduced and studied the Chen and Lu frameworks' complex variables [4].

It is indeed interesting that the OGY strategy [5] for chaos control was first seen in 1990. Numerous different strategies and procedures have been proposed and produced for controlling in chaotic (or hyperchaotic) frameworks, for example, feedback control method [6], adaptive control method [7], and sliding mode and passive control method [8]. Nonetheless, for the chaotic or hyperchaotic frameworks, still some essential inquiries should have been additionally explored, for example, the complexity of the controllers and the presence of some control issues, which incompletely propels the present work [9].

The Rabinovich framework, which was initially presented in [10], shows the dynamics of the interaction of the plasma wave propagating with the acoustic ion wave and the plasma oscillation parallel to the magnetic field above the decreased plasma resonance [11]. The regulation of the hyperchaotic Rabinovich system was contemplated by [8] in 2017. Any global synchronization parameters for 4D hyperchaotic Rabinovich frameworks have been inferred from [12]:

$$\begin{aligned} \dot{x} &= -ax + hy + yz, \\ \dot{y} &= hx - by - xz + u, \\ \dot{z} &= -dz + xy, \\ \dot{u} &= -cy, \end{aligned} \tag{1}$$

where x and y are complex variables: $x = x^r + jx^i$, $y = y^r + jy^i$, $z = z^r + jz^i$, and $u = u^r + ju^i$.

The important point in using the Hamiltonian function method is to express the model concerned into a

Hamiltonian model with dissipation, which is described as the dissipative Hamiltonian realization. A dynamical system can be separated into four parts if there are energy preservation, energy production, energy dissipation, and energy exchange. To denote a dynamical system with these four parts of a dynamic system, a suitable energy function (Hamiltonian energy) can be taken into account at the beginning, and then, a generalized Hamiltonian realization concerning the dynamical system can be obtained. Recently, the energy-function-based method, which has been used to the dynamical behavior analysis and control, has been considered and some results have been achieved in the literature.

Our essential target is to sum up the hyperchaotic Rabinovich show by thinking about its qualities complex. Nonetheless, consider managing complex chaotic or hyperchaotic constructs with complex attributes. We answer it accordingly in this article. In order to form the control rules, the Lyapunov stability formula is used to shift the hyperchaotic display to its original equilibrium with complex variables.

Whatever is left with this article is written as follows in this text. In Section 2, the dynamic properties of 4D hyperchaotic Rabinovich systems are described with complex variables to understand the influence of the initial state on the frame components. The Hamiltonian energy for 4D hyperchaotic Rabinovich frameworks is available in Section 3 by using the innovative method. In Section 4, data concerning the Lyapunov steadiness, utilized to convert the hyperchaotic sort of (1) to the trivial fixed scene, is displayed. System (1) is utilized to show the viability of our outcomes. Finally, conclusions are shown in Section 5.

2. Dynamical Properties of Arrangement (1)

A few properties of system (1) are discussed in this area. For that, exponents of Lyapunov and bifurcation diagrams are used to outline the hyperchaotic system components. Isolating machine equation (1) in real components and fictional components is what we obtain:

$$\begin{aligned}
\dot{x}^r &= -ax^r + hy^r + y^r z^r - y^i z^i, \\
\dot{x}^i &= -ax^i + hy^i + y^r z^i + y^i z^r, \\
\dot{y}^r &= hx^r - by^r - x^r z^r + x^i z^i + u^r, \\
\dot{y}^i &= hx^i - by^i - x^r z^i - x^i z^r + u^i, \\
\dot{z}^r &= -dz^r + x^r y^r - x^i y^i, \\
\dot{z}^i &= -dz^i + x^r y^i + x^i y^r, \\
\dot{u}^r &= -cy^r, \\
\dot{u}^i &= -cy^i.
\end{aligned} \tag{2}$$

2.1. Symmetry and Invariant. In this structure (2), we believe that, with the transformation, this framework is invariant: $(x^r, x^i, y^r, y^i, z^r, z^i, u^r, u^i) \Longrightarrow (-x^r, -x^i, -y^r, -y^i, z^r, z^i, -u^r, -u^i)$, therefore, if $(x^r, x^i, y^r, y^i, z^r, z^i, u^r, u^i)$ is the description of system (2), next $(-x^r, -x^i, -y^r, -y^i, z^r, z^i, -u^r, -u^i)$ is the solution of the equivalent model which is known as the solution.

2.2. Dissipative. In vector notation, structure (2) can be defined as follows:

$$\dot{\mathrm{x}} = \mathbf{f}(\mathbf{x}) = \begin{pmatrix} -ax^r + hy^r + y^r z^r - y^i z^i \\ -ax^i + hy^i + y^r z^i + y^i z^r \\ hx^r - by^r - x^r z^r + x^i z^i + u^r \\ hx^i - by^i - x^r z^i - x^i z^r + u^i \\ -dz^r + x^r y^r - x^i y^i \\ -dz^i + x^r y^i + x^i y^r \\ -cy^r \\ -cy^i \end{pmatrix}. \tag{3}$$

Let $\Xi(t) \in R^8$ With a straight border and letting it also be $\Xi(t) = \Upsilon(t)$, anywhere, the $\Upsilon(t)$ becomes the flow or fluid of $\mathbf{f}$. Assign the amount or the volume of $\Xi(t)$ to be set by $V(t)$.

Through applying theorem of Liouville [13], we possess

$$\dot{V}(t) = \int_{\Xi(t)} (\nabla \cdot \mathbf{f}) \mathrm{d}x^r \mathrm{d}x^i \mathrm{d}y^r \mathrm{d}y^i \mathrm{d}z^r \mathrm{d}z^i \mathrm{d}u^r \mathrm{d}u^i. \tag{4}$$

You will find the divergence of the system vector field $\mathbf{f}$ (2) as follows:

$$\begin{aligned}
\nabla \cdot \mathbf{f} &= \frac{\partial \dot{x}^r}{\partial x^r} + \frac{\partial \dot{x}^i}{\partial x^i} + \frac{\partial \dot{y}^r}{\partial y^r} + \frac{\partial \dot{y}^i}{\partial y^i} + \frac{\partial \dot{z}^r}{\partial z^r} + \frac{\partial \dot{z}^i}{\partial z^i} + \frac{\partial \dot{u}^r}{\partial u^r} + \frac{\partial \dot{u}^i}{\partial u^i}, \\
&= -2(a + b + c + d) = -A,
\end{aligned} \tag{5}$$

where $A = 2(a + b + c + d)$. The structure is seen as being dissipative if $\nabla \cdot \mathbf{f} < 0$. Therefore, A must be greater than 0 $(A > 0)$. As a result, we have, from (4),

$$\dot{V}(t) = \int_{\Xi(t)} (-A) \mathrm{d}x^r \mathrm{d}x^i \mathrm{d}y^r \mathrm{d}y^i \mathrm{d}z^r \mathrm{d}z^i \mathrm{d}u^r \mathrm{d}u^i. \tag{6}$$

Integrating (6), we concern

$$V(t) = \exp(-At) V(0). \tag{7}$$

It concludes from (7), although $A > 0$, that it adjusts exponentially as $t \longrightarrow \infty$. This means that each volume representing this complex system's directions (2) decreases to zero. Each of the paths of the new method, in this manner, eventually hits a solution. The new hyperchaotic mechanism (2) is thus seen to be dissipative.

2.3. Fixed Points and Their Stability. Via deciding the formulas, we will presume the equilibrium point $\dot{x}^r = 0$, $\dot{x}^i = 0$, $\dot{y}^r = 0$, $\dot{y}^i = 0$, $\dot{z}^r = 0$, $\dot{z}^i = 0$, $\dot{u}^r = 0$, and $\dot{u}^i = 0$:

$$
\begin{aligned}
0 &= -ax^r + hy^r + y^r z^r - y^i z^i,\\
0 &= -ax^i + hy^i + y^r z^i + y^i z^r,\\
0 &= hx^r - by^r - x^r z^r + x^i z^i + u^r,\\
0 &= hx^i - by^i - x^r z^i - x^i z^r + u^i,\\
0 &= -dz^r + x^r y^r - x^i y^i,\\
0 &= -dz^i + x^r y^i + x^i y^r,\\
0 &= -cy^r,\\
0 &= -cy^i.
\end{aligned} \tag{8}
$$

Via resolving system (8), we find that our system only has a trivial fixed point, $E_0 = (0, 0, 0, 0, 0, 0, 0, 0)$.

2.4. Stability of E_0. The Jacobian matrix is as follows. When analyzing the stability of fixed point E_0,

$$
J_{E_0} = \begin{pmatrix}
-a & 0 & h & 0 & 0 & 0 & 0 & 0\\
0 & -a & 0 & h & 0 & 0 & 0 & 0\\
h & 0 & -b & 0 & 0 & 0 & 1 & 0\\
0 & h & 0 & -b & 0 & 0 & 0 & 1\\
0 & 0 & 0 & 0 & -d & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -d & 0 & 0\\
0 & 0 & -c & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -c & 0 & 0 & 0 & 0
\end{pmatrix}, \tag{9}
$$

and the particular polynomial is

$$
(d+\lambda)^2\left(ac + c\lambda + a\lambda^2 + b\lambda^2 - h^2\lambda + \lambda^3 + ab\lambda\right)^2 = 0. \tag{10}
$$

The eigenvalues of the computation characterization are then

$$
\begin{aligned}
\lambda_1 = \lambda_2 &= -d,\\
\left(ac + \left(c + ab - h^2\right)\lambda + (a+b)\lambda^2 + \lambda^3\right)^2 &= 0.
\end{aligned} \tag{11}
$$

So, E_0 holds stable concerning a negative root by practicing the Routh–Hurwitz theory [14] if and only if it is also stable for a root that is negative, $d > 0$, $\alpha_1 > 0$, $\alpha_2 > 0$, $\alpha_3 > 0$, and $\alpha_1\alpha_2 > \alpha_3$ where $\alpha_1 = (a + b)$, $\alpha_2 = (c + ab - h^2)$, and $\alpha_3 = ac$. Consequently, the criteria for E_0 to be stable are

$$
\begin{aligned}
d &> 0,\\
(a+b) &> 0,\\
\left(c + ab - h^2\right) &> 0,\\
ac &> 0,\\
(a+b)\left(c + ab - h^2\right) &> ac,
\end{aligned} \tag{12}
$$

otherwise, it is unstable.

2.5. Lyapunov Exponents. Simply restating model (18), as a mathematical equation,

$$
\dot{x} = \mathbf{f}(\mathbf{x}; \eta), \tag{13}
$$

the state space vector is $\mathbf{x} = [x^r, x^i, y^r, y^i, z^r, z^i, u^r, u^i]^T$ and $\mathbf{f} = [f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8]^T$. The arrangement of the group of parameters is η and the transpose matrix is $[\ldots]^T$. The formula for minor $\delta\mathbf{x}$ deviations from the $\mathbf{x}$ trajectory is

$$
\delta\dot{x} = J_{lk}(\mathbf{x}; \eta)\delta\mathbf{x}; \quad l, k = 1, 2, 3, 4, 5, 6, 7, 8, \tag{14}
$$

where $J_{lk}(\mathbf{x}; \eta) = \partial f_l / \partial \mathbf{x}_k$ is the Jacobian matrix of the form:

$$
J_{lk} = \begin{pmatrix}
-a & 0 & h + z^r & -z^i & y^r & -y^i & 0 & 0\\
0 & -a & z^i & h + z^r & y^i & y^r & 0 & 0\\
h - z^r & z^i & -b & 0 & -x^r & x^i & 1 & 0\\
-z^i & h - z^r & 0 & -b & -x^i & -x^r & 0 & 1\\
y^r & -y^i & x^r & -x^i & -d & 0 & 0 & 0\\
y^i & y^r & x^i & x^r & 0 & -d & 0 & 0\\
0 & 0 & -c & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & -c & 0 & 0 & 0 & 0
\end{pmatrix}. \tag{15}
$$

The Lyapunov exponents L_k of our system are defined by

$$
L_k = \lim_{t \longrightarrow \infty} \frac{1}{t} \log \frac{\|\partial\mathbf{x}(t)\|}{\|\partial\mathbf{x}(0)\|}, \quad k = 1, 2, 3, 4, 5, 6, 7, 8, \tag{16}
$$

where $\partial\mathbf{x}(t)$ is a derivative of $\mathbf{x}(t)$, when the difference is equal to zero, and $\partial\mathbf{x}(0)$ is the fundamental framework. Conditions (13) and (14) must be determined numerically using the Runge–Kutta order 4 formula to measure L_k [15].

Parameter values and initial conditions are set as parameter values and initial conditions for the current system (2): $a = 2.5$, $b = 12$, $c = 20$, $d = 3$, $h = 10$, and $(x^r(0), x^i(0), y^r(0), y^i(0), z^r(0), z^i(0), u^r(0), u^i(0)) = (1, 2, 3, 4, 5, 6, 7, 8)$. The corresponding exponents of Lyapunov are then shown in (Figures 1(a) and 1(b)). The numerical values of the exponents of Lyapunov are $L_1 = 0.93$, $L_2 = 0.48$, $L_3 = 0.29$, $L_4 = 0.2$, $L_5 = 0$, $L_6 = -18.3$, $L_7 = -22$, and $L_8 = -70.1$.

This ensures that, under the specified possibility of a, b, c, d, and h, our model (2) is a hyperchaotic method since 2 of the Lyapunov exponents are positive.

According to the Kaplan–Yorke dimension, the Lyapunov dimension of machine solution (2) is established as follows:

$$
D = \alpha + \frac{\sum_{k=1}^{\alpha} L_k}{\left|L_{\alpha+1}\right|}, \tag{17}
$$

such that α is the largest integer, $\sum_{k=1}^{\alpha} L_k > 0$, and $\sum_{k=1}^{\alpha+1} L_k < 0$ [16]. The Lyapunov dimension, using (17) of this hyperchaotic solution, is $D = 5.1038$.

2.5.1. Fix $b = 12, c = 20, d = 3,$ and $h = 10$ and a Is Varying where $a \in [0, 20]$. Through requirement (16) one may find out Lyapunov examples $L_1, \ldots, L_8$. Figures 1(a) and 1(b) show the direct of Lyapunov type's qualities. The components of system (2) transition among the chaotic solutions, hyperchaotic solutions, and quasiperiodic solutions. From Figures 1(a) and 1(b) when $a \in [0, 0.7]$, system (2) owns the chaotic solutions and holds one positive Lyapunov types $(+0------)$. Among 3 positive Lyapunov indexes, the hyperchaotic solutions have $(+++0----)$ and rise at the

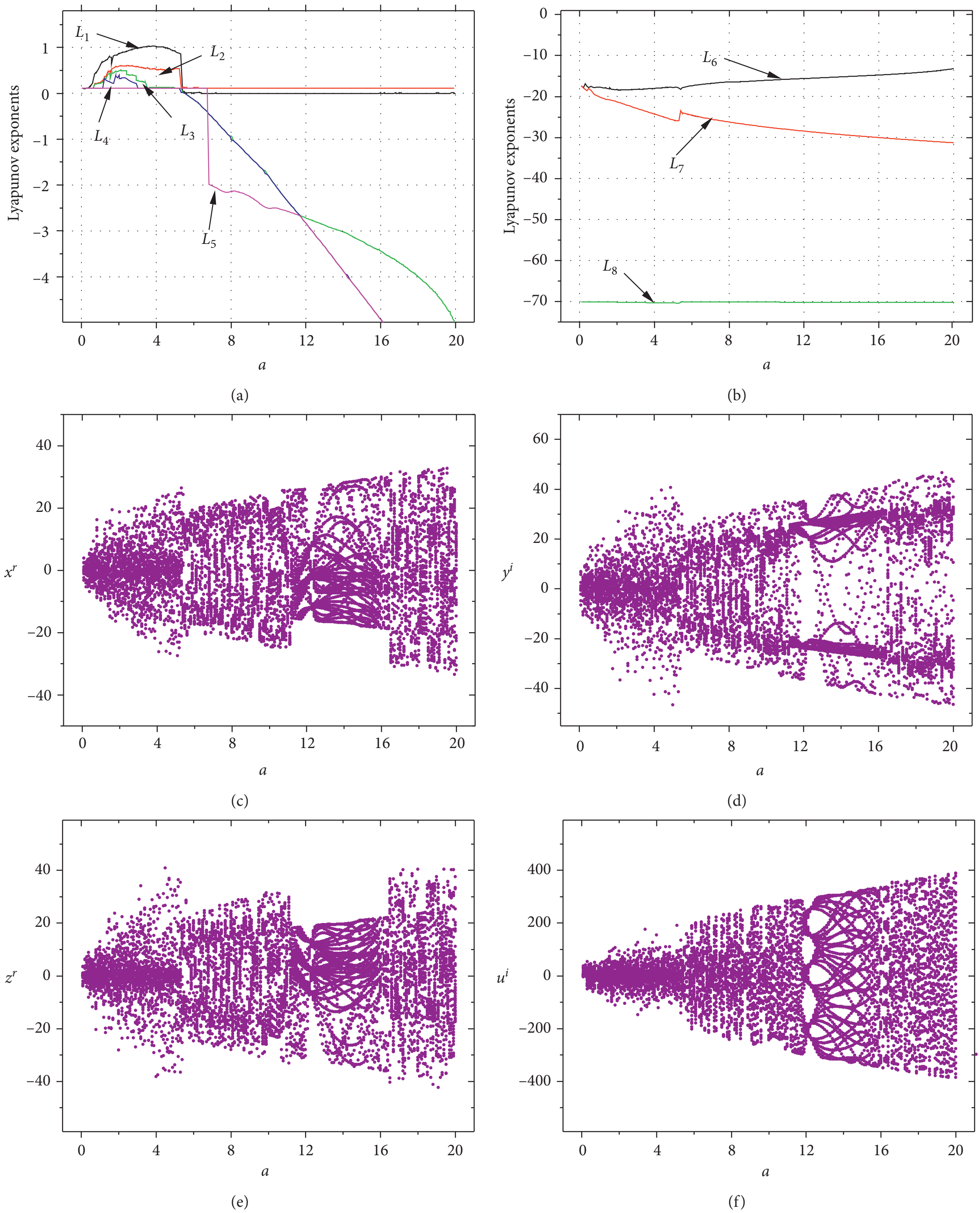

FIGURE 1: When $b = 12, c = 20, d = 3, h = 10$ and , $a \in [0, 20]$ with the primary positions $(x^r(0), x^i(0), y^r(0), y^i(0), z^r(0), z^i(0), u^r(0), u^i(0)) = (1, 2, 3, 4, 5, 6, 7, 8)$. (a) LEs of system (2): L_1, L_2, L_3, L_4, and L_5. (b) LEs of system (2): L_6, L_7, and L_8. (c) Plane bifurcation diagrams in (a, x^r). (d) Plane bifurcation diagrams (a, y^i). (e) Plane bifurcation diagrams (a, z^r). (f) Plane bifurcation diagrams (a, u^i).

period $a \in]0.7, 1.2]$. While at the interval $a \in]1.2, 3]$, the hyperchaotic solutions rise with 4 positive Lyapunov indexes $(++++0---)$. The hyperchaotic behavior with 3 positive Lyapunov types $(+++0----)$ show up at $a \in]3, 3.5]$. The hyperchaotic solutions with 2 positive Lyapunov examples $(++0-----)$ at the close $a \in]3.5, 5.3]$. The quasiperiodic solutions develop in the close $]5.3, 20]$ (Table 1).

The arrangements of system (2) are shown as follows.

We figure another apparatus to check the dynamic of system (2) by using the bifurcation charts. In Frames 1(c)–1(f), we sketch the bifurcation graph of system (2) when the parameter α is shifting or adjusting to guarantee our system is hyperchaotic and chaotic.

Frames 1(c)–1(f) display that bifurcation of system (2) for $a \in [0, 20]$; Frame 1(c) illustrates that (a, x^r) bifurcation charts for $a \in [0, 20]$. It very well may be noticed that when $a \in [0, 5.3]$, (2) has game plan chaotic and hyperchaotic solutions. Yet, when $a \in]5.3, 20]$, (2) has quasiperiodic arrangements. The results indicated in Figure 1(c) looked at to Frames 1(d)–1(f). Frames 1(d)–1(f) hold (a, y^i), (a, z^r), and (a, u^i) bifurcation charts at the point when $a \in [0, 20]$, individually. They have comparative results, which were displayed beforehand in the plane (a, x^r).

To examine this, we have explained numerically (2) (using, e.g., Mathematica 7 programming) in a couple of examples and extraordinary assertions which are obtained among the earlier events. For example, choose $b = 12$, $c = 20$, $d = 3$, $h = 10$, and the underlying conditions $(x^r(0), x^i(0), y^r(0), y^i(0), z^r(0), z^i(0), u^r(0), u^i(0)) = (1, 2, 3, 4, 5, 6, 7, 8)$. Right when $a = 0.5$ the course of action of system (2) is chaotic with one positive Lyapunov examples (see Figure 2(a)). The hyperchaotic solution with four positive Lyapunov type systems in Figure 2(b) when $a = 2.5$. In Figure 2(c), $a = 3.2$, and the plan is hyperchaotic with three positive Lyapunov types. The quasiperiodic solutions rise in Figure 2(d) for $a = 15$.

3. Hamiltonian Energy of System (2)

Hamiltonian energy of system (2) is studied by using the innovative method.

We study a continuous nonlinear system (2), implemented in the variety:

$$\dot{\mathbf{x}} = \tau(\mathbf{x})\frac{\partial \mathbf{H}}{\partial \mathbf{x}} + \sigma(\mathbf{x})\frac{\partial \mathbf{H}}{\partial \mathbf{x}}, \tag{18}$$

where $\mathbf{x} = [x^r, x^i, y^r, y^i, z^r, z^i, u^r, u^i]^T$ and $\mathbf{H}(\mathbf{x})$ continuous energy function, the column vector of $\mathbf{H}(\mathbf{x})$ is $\partial \mathbf{H}/\partial \mathbf{x}$ [15].

The quadratic energy is

$$\mathbf{H}(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T \gamma \mathbf{x}, \tag{19}$$

where γ is a constant diagonal matrix, with regard to $\partial \mathbf{H}/\partial \mathbf{x} = \gamma \mathbf{x}$, $\tau(\mathbf{x})$ is an antisymmetric matrix, and $\sigma(\mathbf{x})$ a symmetric matrix [17]:

$$\begin{aligned} \tau(\mathbf{x}) &= [-\tau(\mathbf{x})]^T, \\ \sigma(\mathbf{x}) &= [\sigma(\mathbf{x})]^T. \end{aligned} \tag{20}$$

We consider $\mathbf{H}(\mathbf{x})$ of system (2):

$$H(x) = \frac{1}{2}\left[(x^r)^2 + (x^i)^2 + (y^r)^2 + (y^i)^2 + (z^r)^2 + (z^i)^2 + \frac{(u^r)^2}{-c} + \frac{(u^i)^2}{-c}\right],$$

$$\begin{bmatrix} \dot{x}^r \\ \dot{x}^i \\ \dot{y}^r \\ \dot{y}^i \\ \dot{z}^r \\ \dot{z}^i \\ \dot{u}^r \\ \dot{u}^i \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -y^i & 0 & 0 \\ 0 & 0 & 0 & 0 & y^i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-x^r}{2} & \frac{x^i}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-x^i}{2} & \frac{-x^r}{2} & 0 & 0 \\ 0 & -y^i & \frac{x^r}{2} & \frac{x^i}{2} & 0 & 0 & 0 & 0 \\ y^i & 0 & \frac{-x^i}{2} & \frac{x^r}{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x^r \\ x^i \\ y^r \\ y^i \\ z^r \\ z^i \\ \frac{u^r}{-c} \\ \frac{u^i}{-c} \end{bmatrix} + \begin{bmatrix} -a & 0 & h & 0 & y^r & 0 & 0 & 0 \\ 0 & -a & 0 & h & 0 & y^r & 0 & 0 \\ h & 0 & -b & 0 & \frac{-x^r}{2} & \frac{x^i}{2} & -c & 0 \\ 0 & h & 0 & -b & \frac{-x^i}{2} & \frac{-x^r}{2} & 0 & -c \\ y^r & 0 & \frac{-x^r}{2} & \frac{-x^i}{2} & -d & 0 & 0 & 0 \\ 0 & y^r & \frac{x^i}{2} & \frac{-x^r}{2} & 0 & -d & 0 & 0 \\ 0 & 0 & -c & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -c & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x^r \\ x^i \\ y^r \\ y^i \\ z^r \\ z^i \\ \frac{u^r}{-c} \\ \frac{u^i}{-c} \end{bmatrix}. \tag{21}$$

Table 1: Dynamics of system (2).

L_1	L_2	L_3	L_4	L_5	L_6	L_7	L_8	**Solutions**
0	0	−	−	−	−	−	−	Quasiperiodic solutions
+	0	−	−	−	−	−	−	Chaotic Solutions
+	+	0	−	−	−	−	−	Hyperchaotic solutions with 2 PLEs
+	+	+	0	−	−	−	−	Hyperchaotic solutions with 3 PLEs
+	+	+	+	0	−	−	−	Hyperchaotic solutions with 4 PLEs

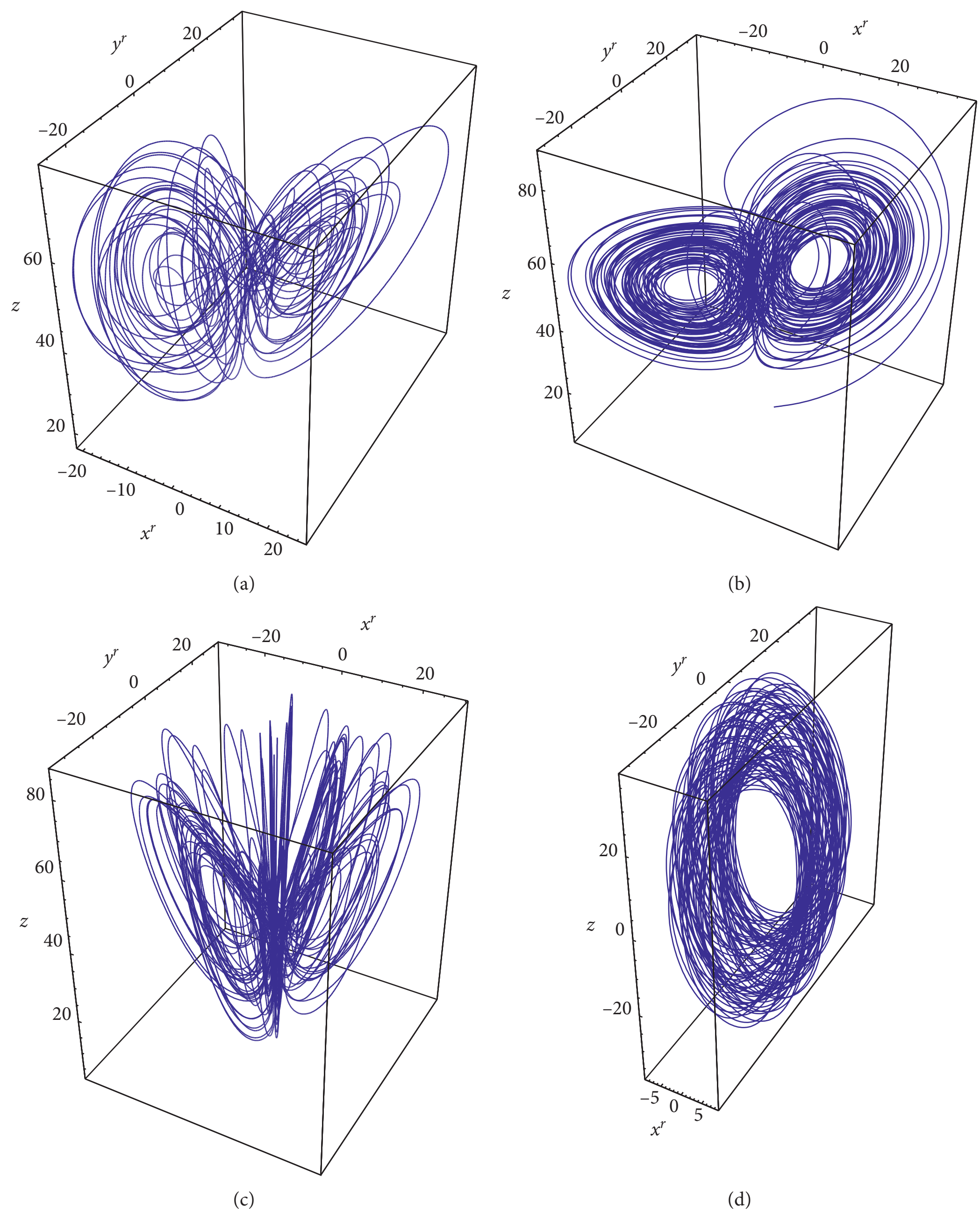

Figure 2: Clarifications of system (2) for $b = 12, c = 20, d = 3$, and $h = 10$ differ a including the primary positions in Figure 1. (a) Chaotic solution, $a = 0.5$, in (z^i, x^r, y^r). (b) Hyperchaotic solution, $a = 2.5$, in (z^i, x^r, y^r). (c) Quasiperiodic solution, $a = 3.2$, in (z^i, x^r, y^r). (d) Quasiperiodic solutions, $a = 15$, in (z^i, x^r, y^r).

Hamilton's existence in system (2) indicates the capacity to calculate physical amounts.

4. Elimination of the Hyperchaotic Features of E_0

4.1. Foundation of the Controller. Regulation or control of hyperchaos for hyperchaotic solutions requires the design of a controller that can alleviate or decrease the nonlinear systems' hyperchaotic behavior. This activity is suppressed in this section by the unstable trivial fixed point, E_0, based on the theorem of Lyapunov stability.

The formulas of the new system being regulated are given as follows:

$$\begin{aligned} \dot{x} &= -ax + hy + yz + \Psi_1, \\ \dot{y} &= hx - by - xz + u + \Psi_2, \\ \dot{z} &= -dz + xy + \Psi_3, \\ \dot{u} &- cy + \Psi_4, \end{aligned} \tag{22}$$

where $\Psi_1 = \Psi_{11} + j\Psi_{12}, \Psi_2 = \Psi_{21} + j\Psi_{22}, \ \Psi_3 = \Psi_{31} + j\Psi_{32}$, and $\Psi_4 = \Psi_{41} + j\Psi_{42}$ are complex control functions. The real machine version (22) is then obtained as a differentiation of the real and imaginary portions of these equations:

$$\begin{aligned} \dot{x^r} &= -ax^r + hy^r + y^r z^r - y^i z^i + \Psi_{11}, \\ \dot{x^i} &= -ax^i + hy^i + y^r z^i + y^i z^r + \Psi_{12}, \\ \dot{y^r} &= hx^r - by^r - x^r z^r + x^i z^i + u^r + \Psi_{21}, \\ \dot{y^i} &= hx^i - by^i - x^r z^i - x^i z^r + u^i + \Psi_{22}, \\ \dot{z^r} &= -dz^r + x^r y^r - x^i y^i + \Psi_{31}, \\ \dot{z^i} &- dz^i + x^r y^i + x^i y^r + \Psi_{32}, \\ \dot{u^r} &= -cy^r + \Psi_{41}, \\ \dot{u^i} &= -cy^i + \Psi_{42}, \end{aligned} \tag{23}$$

where Ψ_{lk} $(l = 1, 2, 3, 4, \ k = 1, 2)$ are the controllers that you need to build. These controllers should be constructed in a way that drives the system's trajectory defined by x^r, x^i, y^r, y^i, z^r, z^i, u^r, and u^i, to the unstable trivial point, E_0, of the uncontrolled system (22) (i.e., $\Psi_{lk} \longrightarrow 0$ as $t \longrightarrow \infty$).

System (23) could therefore be defined as follows:

$$\begin{aligned} \dot{x^r} &= -ax^r + hy^r + S_1^r, \\ \dot{x^i} &= -ax^i + hy^i + S_1^i, \\ \dot{y^r} &= hx^r - by^r + u^r + S_2^r, \\ \dot{y^i} &= hx^i - by^i + u^i + S_2^i, \\ \dot{z^r} &= -dz^r + S_3^r, \\ \dot{z^i} &= -dz^i + S_3^i, \\ \dot{u^r} &= -cy^r + S_4^r, \\ \dot{u^i} &= -cy^i + S_4^i, \end{aligned} \tag{24}$$

where

$$\begin{aligned} S_1^r &= \Psi_{11} + y^r z^r - y^i z^i, \\ S_1^i &= \Psi_{12} + y^r z^i + y^i z^r, \\ S_2^r &= \Psi_{21} - x^r z^r + x^i z^i, \\ S_2^i &= \Psi_{22} - x^r z^i - x^i z^r, \\ S_3^r &= \Psi_{31} + x^r y^r - x^i y^i, \\ S_3^i &= \Psi_{32} + x^r y^i + x^i y^r, \\ S_4^r &= \Psi_{41}, \\ S_4^i &= \Psi_{42}. \end{aligned} \tag{25}$$

Using the Lyapunov stability theorem, we obtain

$$\begin{bmatrix} S_1^r \\ S_1^i \\ S_2^r \\ S_2^i \\ S_3^r \\ S_3^i \\ S_4^r \\ S_4^i \end{bmatrix} = \begin{bmatrix} a-\zeta_1 & 0 & -h & 0 & 0 & 0 & 0 & 0 \\ 0 & a-\zeta_1 & 0 & -h & 0 & 0 & 0 & 0 \\ -h & 0 & b-\zeta_2 & 0 & 0 & 0 & -1 & 0 \\ 0 & -h & 0 & b-\zeta_2 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & d-\zeta_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d-\zeta_3 & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 & -\zeta_4 & 0 \\ 0 & 0 & 0 & c & 0 & 0 & 0 & -\zeta_4 \end{bmatrix} \begin{bmatrix} x^r \\ x^i \\ y^r \\ y^i \\ z^r \\ z^i \\ u^r \\ u^i \end{bmatrix}, \tag{26}$$

where $\zeta_l > 0$, $l = 1, 2, 3, 4$.

Thus, we can compute the controller, Ψ_{lk}, as follows:

$$\begin{aligned} \Psi_{11} &= (a-\zeta_1)x^r - hy^r - y^r z^r + y^i z^i, \\ \Psi_{12} &= (a-\zeta_1)x^i - hy^i - y^r z^i - y^i z^r, \\ \Psi_{21} &= -hx^r + (b-\zeta_2)y^r - u^r + x^r z^r - x^i z^i, \\ \Psi_{22} &= -hx^i + (b-\zeta_2)y^i - u^i + x^r z^i + x^i z^r, \\ \Psi_{31} &= (d-\zeta_3)z^r - x^r y^r + x^i y^i, \\ \Psi_{32} &= (d-\zeta_3)z^i - x^r y^i - x^i y^r, \\ \Psi_{41} &= cy^r - \zeta_4 u^r, \\ \Psi_{42} & cy^i - \zeta_4 u^i. \end{aligned} \tag{27}$$

By interchanging benefits from (27) into (23), we obtain

$$\begin{aligned} \dot{x^r} &= -\zeta_1 x^r, \\ \dot{x^i} &= -\zeta_1 x^i, \\ \dot{y^r} &= -\zeta_2 y^r, \\ \dot{y^i} &= -\zeta_2 y^i, \\ \dot{z^r} &= -\zeta_3 z^r, \\ \dot{z^i} &= -\zeta_3 z^i, \\ \dot{u^r} &= -\zeta_4 u^r, \\ \dot{u^i} &= -\zeta_4 u^i. \end{aligned} \tag{28}$$

Therefore, by solving the equations of the controlled system (28), the solutions, x^r, x^i, y^r, y^i, z^r, z^i, u^r, and u^i, meet the unstable fixed point E_0, $t \longrightarrow \infty$.

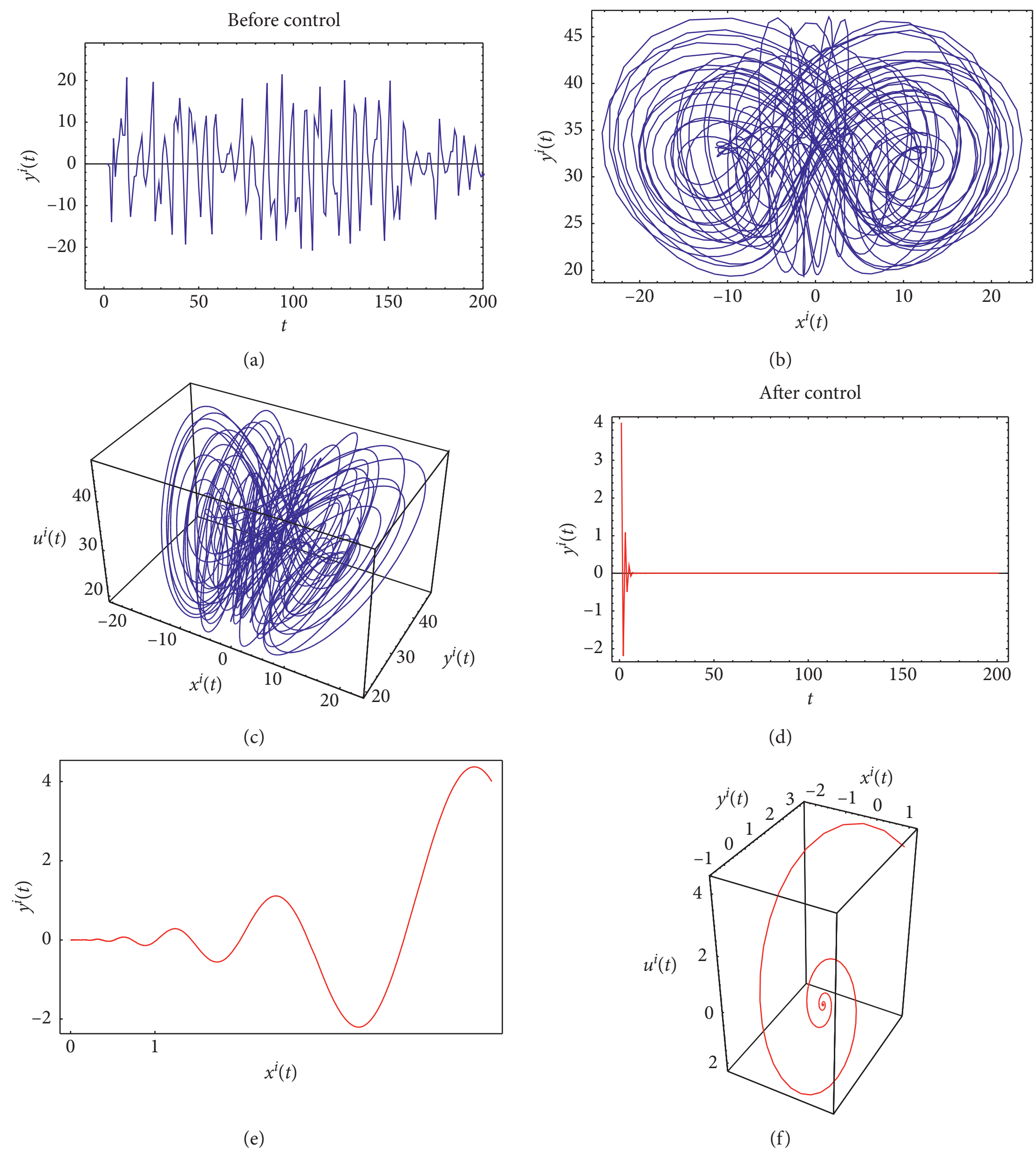

Figure 3: The system's numerical solution (23) (after and before control) for the case $b = 12, c = 20, d = 3$, and $h = 10$. (a) (t, y^i) plane (before control). (b) (x^i, y^i) plane (before control). (c) (x^i, y^i, u^i) space (before control). (d) (t, y^i) plane (after control). (e) (x^i, y^i) plane (after control). (f) (x^i, y^i, u^i) space (after control).

4.2. Numerical Simulation. We performed computer simulations to examine the adequacy of the acquired control functions. Demonstrate (23), with no controller for the situation $a = 2.5$, $b = 12$, $c = 20$, $d = 3$, and $h = 10$ (i.e., before control), for which the hyperchaotic solution of system (2) worked. The results are shown in Figures 3(a)–3(c). In Figure 3(a), we outline the state variable, y^i, for system (23) (i.e., before control). Other state variables can be plotted before as well. The hyperchaotic solutions of system (2) are outlined in Figures 3(b) and 3(c) for the y^i plane and (x^i, y^i, u^i) space, separately. Show (23) and control functions (27) are comprehended numerically (after control) with a related parameter esteem, as shown in Figures 3(a)–3(c). The acquired outline is delineated in Figures 3(d)–3(f). From these figures, plainly, the hyperchaotic arrangements are changed over to the minor settled point, obviously from the previously mentioned explanatory contemplations. Figures 3(d)–3(f) demonstrate that the control is accomplished before long interim, in this way showing the adequacy of the outcomes. Figure 4 demonstrates the numerical reproduction consequences of the controller, Ψ_{lk} $(l = 1, 2, 3, 4, k = 1, 2)$. Obviously, the control functions, Ψ_{lk}, meet at 0.

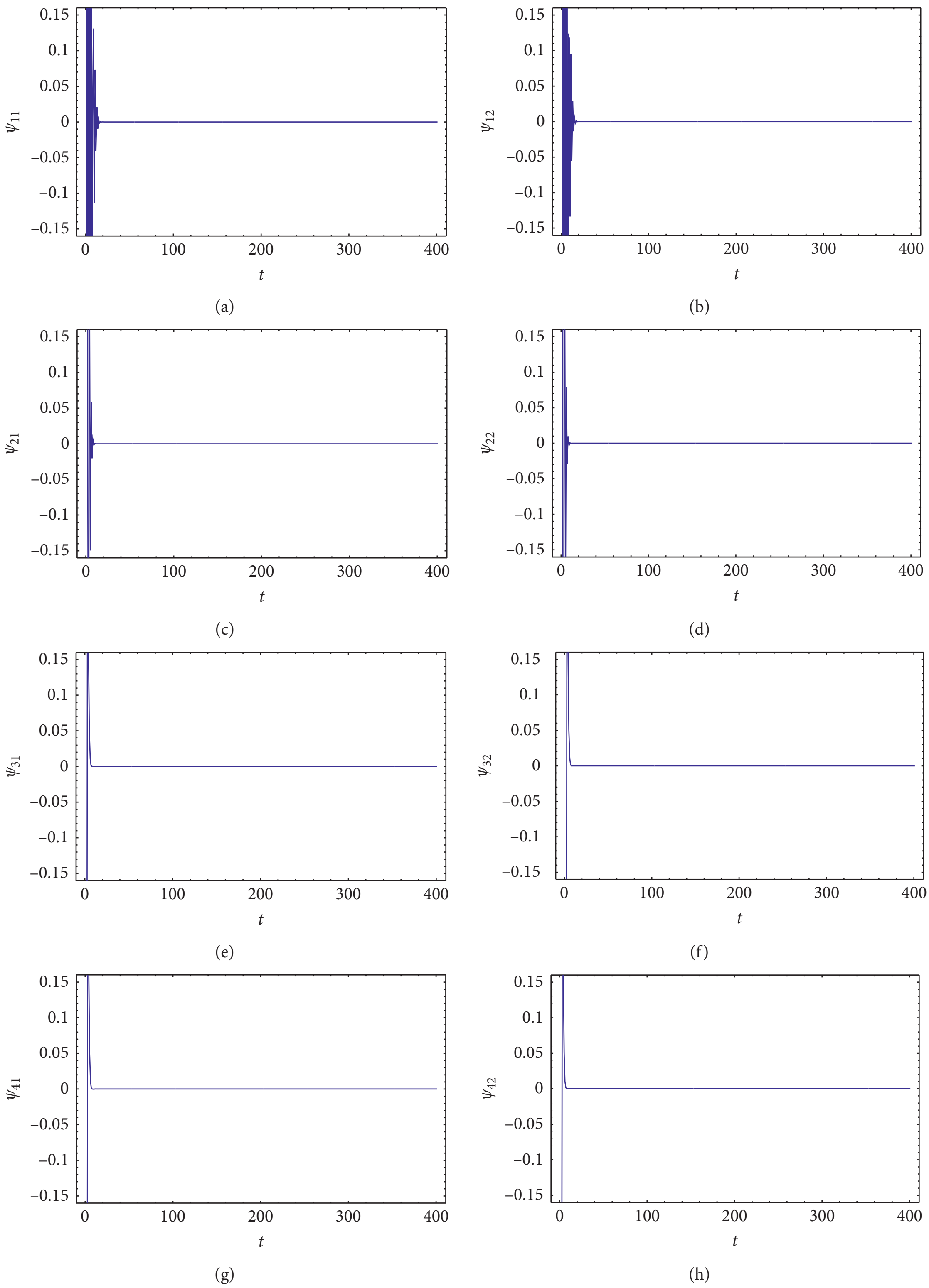

FIGURE 4: The controller's time evolution. (a) (Ψ_{11}, t) layout. (b) (Ψ_{12}, t) layout. (c) (Ψ_{21}, t) layout. (d) (Ψ_{22}, t) layout. (e) (Ψ_{31}, t) layout. (f) (Ψ_{32}, t) layout. (g) (Ψ_{41}, t) layout. (h) (Ψ_{42}, t) layout.

5. Conclusions

In this work, we displayed the significance and influence of complex variables on nonlinear systems. We exhibited this impact by concentrating the complex Rabinovich show with complex variables (2), finding that the conduct of the system was hyperchaotic with complex variables. The critical properties of system (2) were examined, and vital circumstances for it to produce tumult and hyperchaos were talked about. In light of Lyapunov types, hyperchaotic, chaotic, and quasiperiodic behaviors were indicated in our suggested framework. Bifurcation investigations were directed to confirm these practices. Also, to adjust the hyperchaotic show with complex variables to its equilibrium point, the Lyapunov stability theorem was used.

Data Availability

No data were used to support this study.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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