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Research Article

Dynamical Analysis of Dual Product Information Diffusion considering Preference in Complex Networks

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With the development of science and technology, products are being updated more and more quickly. Therefore, the diffusion of product information can make people better choose products. It is very meaningful to study the competition and diffusion of multiple product information. In this paper, the dual product information diffusion model with preference was proposed based on the mean-field equation in complex networks. The dynamic of the model was analyzed by the analysis of Markov chains. According to the Monte Carlo simulation mechanism, the critical threshold of product information was obtained. The accuracy of the theory and model results is verified by computer simulation, and the scenarios in which the two products information are mutually promoted and mutually suppressed are simulated.

1. Introduction

With the advent of the information age, product updates are getting faster and faster, and there are more and more potential alternative products [1–3]. The influences of many factors are taken into account when people purchase products, such as the quality, function, and age of the product. Faced with alternative products, such as traditional and innovative choices, individuals are often influenced by their appetite for risk. Adventurous people like to try new things and are more likely to choose innovative products, while risk-averse people prefer traditional products. Information diffusion is particularly important in the process of selecting products.

The study of information diffusion has a long history. The research on the influence of individual consciousness on communication first appeared in the spread of rumors. In 1964, Daley et al. proposed a classical rumor diffusion model (DK model) [4]. Then, the DK model was developed and a mathematical model was applied by Maki and Thomson and Murray to study the rumors [5, 6]. Many scholars have conducted research studies on information diffusion and have proposed a lot of constructive ideas in applying the diffusion theory to the practice [7–9].

Although the above research mainly focuses on theoretical analysis, complex network theory provides methods to solve several problems, such as differences in propagation rates between different individuals and differences in propagation patterns in different topologies of social networks, which promotes the research on rumor propagation [10–12]. After studying the process of rumor communication, Zanette first proposed a rumor propagation model for small world networks by applying complex network theory [13, 14]. In addition, some scholars have found that when studying the randomness of the DK model on scale-free (SF) networks, network heterogeneity has a certain impact on the rumor propagation mechanism, indicating that the heterogeneity of the network has a major impact on the rumor transmission mechanism [15–22]. However, the limitations of these studies lie in the neglect of individual behavior in composite networks [23–27].

Technological advances fuel the development of new products and services. Examples are abundant. Decades ago, black-and-white TVs were replaced by color TVs and now the market share of color TVs is replaced by high-definition TVs (HDTVs). The same phenomenon also exists in the cellular phone and software market. Two famous examples are Microsoft's Windows and Office lines of products, which usually release new versions every few years. The diffusion of product information has been well studied in the prior literature. Most of the existing multiple-generation diffusion models are inspired by the seminal Bass model [28, 29]. Kim et al. proposed a dynamic market growth model that reflected not only the proliferation of multiple-generation products within the same product category but also the complementarity and competitiveness of related product categories [30]. Glassman et al. used a random Bass model to predict product sales [31]; Bertotti and Modanese employed the mean-field method and the formulation of the Bass diffusion model for the description of the innovation diffusion process [32].

However, there is very little research on product information diffusion considering preference in complex networks. By combining the information diffusion model with the complex network theory, the mean-field theory is extended to the dual product information diffusion system. Through the analysis of Markov chain, the dynamic of the model is established. This paper analyzes the diffusion of product information in complex networks from the perspective of people's risk appetite for products. The rest of this paper is organized as follows. In Section 2, we introduce the model of product information diffusion with preference. In Section 3, we establish the average field equation and analyze the dynamic model. In Section 4, we analyze the critical threshold value of production diffusion. In Section 5, we perform a numerical simulation in a phase diagram to verify the theoretical predictions in Section 4. Finally, we give the conclusion in Section 6.

2. Dual Product Information Diffusion Model

2.1. Model Assumptions. It is considered that the spread of information and the change of individual behavior are complex socio-psychological processes. We use a network model to formalize and simplify these propagation mechanisms. Each product information diffusion through the same mechanism and the same network of contacts can be seen in Figure 1.

People exchange product information through social media. Individual behavior status is divided into risk preference state and risk aversion state. We assume the product information diffusion process according to the following rules:

(i) Product information type: these are innovative product 1 and traditional product 2. For example, software product 1 is an updated version and software product 2 is a traditional version.

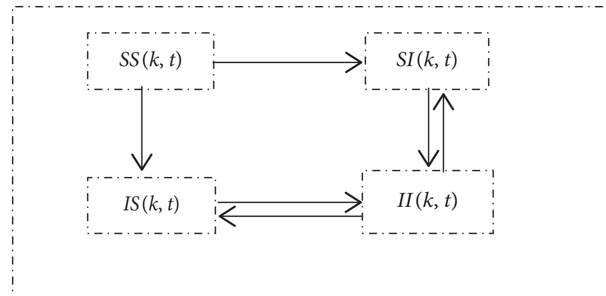

Figure 1: Dynamic process of dual product information diffusion.

(ii) Product information diffusion: the spreading of product information satisfies the susceptible-infected-susceptible (SIS) process. Here, we use "infected" (abbreviated as "I") to indicate that an individual has been informed of a product, and we use "susceptible" (abbreviated as "S") to indicate that an individual is not aware of product information. According to the individual's acceptance of information, the population is divided into four categories. The individual does not know the information of two products, which is represented by $\mathrm{SS}(k,t)$, the individual knows the information of the two products, which is represented by $\mathrm{II}(k,t)$, and the individual does not know about product 1 (2) information but knows that product 2 (1) information is represented by $\mathrm{SI}(k,t)$ ($\mathrm{IS}(k,t)$).

(iii) Individual behavior: the node at risk preference state is affected by all its neighbors, and the node in the risk aversion state is only affected by the risk preference state node in its neighbors. But beyond that, we give a risk rate α of each node. That is to say, the probability that a node is in risk preference is α and in risk aversion is $1-\alpha$.

2.2. Dynamic Model. With these assumptions, there are four primary states: $\mathrm{SS}(k,t)$, $\mathrm{II}(k,t)$, $\mathrm{SI}(k,t)$, and $\mathrm{II}(k,t)$. Assuming risk appetites are likely to accept new products, risk averse is more likely to accept traditional product information. Since the product is replaceable, because the individual's preferences are different, the information receiving process is also in a sequential order, so the individual cannot be directly converted into II state by the state SS. The transition between node states, at time step t, is given by the outward arrow from the given state of the node, which points to its possible successor state at time step $t+1$.

And eight substates are as follows: $\mathrm{SS}(k,t)^a$, $\mathrm{SS}(k,t)^d$, $\mathrm{SI}(k,t)^a$, $\mathrm{SI}(k,t)^d$, $\mathrm{IS}(k,t)^a$, $\mathrm{IS}(k,t)^d$, $\mathrm{II}\ (k,t)^a$, and $\mathrm{II}(k,t)^d$. We introduce an individual's choice of product risk to consider the influence of node activity on diffusion dynamics. Different investors have different attitudes towards

Table 1: Definition of model parameters.

Parameter	Definition
λ_1	A node individual from susceptible product information to infected product information
λ_2	A node individual from susceptible product information to infected product information
β_1^m	The information transmission rate of product 1 due to spreader also infected the information of product 2
β_1^n	The information diffusion probability of product 1 on account of the node has already obtained the information of product 2
β_2^m	The information transmission rate of product 2 due to spreader also infected the information of product 1
β_2^n	The information diffusion probability of product 2 on account of the node has already obtained the information of product 1

Set of transitions allowed in the model. The variables $[\mathrm{SS}(k,t), \mathrm{SI}(k,t), \mathrm{IS}(k,t), \mathrm{II}(k,t)]$ represent the densities of individuals of each type in a system with k neighbors in the network.

risks; we divide them into risk preference and risk aversion, which are represented by a and d, respectively.

The model thus contains two basic infection probabilities, λ_1 and λ_2, as well as four special rates, β_1^m, β_1^n, β_2^m, and β_2^n, one for each product information, as we explain in detail in Table 1.

Therefore, the information diffusion model is given by the following.

(i) Behavior state change is

$$\begin{aligned}&\mathrm{SS}^a \xrightarrow{1-\alpha} SS^d,\ \mathrm{SS}^d \xrightarrow{\alpha} \mathrm{SS}^a,\\&\mathrm{IS}^a \xrightarrow{1-\alpha} IS^d,\ \mathrm{IS}^d \xrightarrow{\alpha} \mathrm{IS}^a,\\&\mathrm{SI}^a \xrightarrow{1-\alpha} SI^d,\ \mathrm{SI}^d \xrightarrow{\alpha} \mathrm{SI}^a,\\&\mathrm{II}^a \xrightarrow{1-\alpha} II^d,\ \mathrm{II}^d \xrightarrow{\alpha} \mathrm{II}^a.\end{aligned} \tag{1}$$

(ii) The dynamic change process of individual nodes from SS state to IS state is given by the following:

The risk preference state of the node is

$$\begin{aligned}&\mathrm{SS}^a + \mathrm{IS} \xrightarrow{\lambda_1} \mathrm{IS}^a + \mathrm{IS},\\&\mathrm{SS}^a + \mathrm{II} \xrightarrow{\beta_1^m\lambda_1} \mathrm{IS}^a + \mathrm{II}.\end{aligned} \tag{2}$$

The risk aversion state of the node is

$$\begin{aligned}&\mathrm{SS}^d + \mathrm{IS}^a \xrightarrow{\lambda_1} \mathrm{IS}^d + \mathrm{IS}^a,\\&\mathrm{SS}^d + \mathrm{II}^a \xrightarrow{\beta_1^m\lambda_1} \mathrm{IS}^d + \mathrm{II}^a.\end{aligned} \tag{3}$$

(iii) The dynamic change process of individual nodes from SS state to SI state is given by the following.

The risk preference state of the node is

The risk aversion state of the node is

$$\begin{aligned}&\mathrm{SS}^d + \mathrm{SI}^a \xrightarrow{\lambda_2} \mathrm{SI}^d + \mathrm{SI}^a,\\&\mathrm{SS}^d + \mathrm{II}^a \xrightarrow{\beta_2^m\lambda_2} \mathrm{SI}^d + \mathrm{II}^a.\end{aligned} \tag{4}$$

(iv) The dynamic change process of individual nodes from IS state to II state is given by the following.

The risk preference state of the node is

$$\begin{aligned}&\mathrm{IS}^a + \mathrm{SI} \xrightarrow{\beta_2^n\lambda_2} \mathrm{II}^a + \mathrm{SI},\\&\mathrm{IS}^a + \mathrm{II} \xrightarrow{\beta_2^m\beta_2^n\lambda_2} \mathrm{II}^a + \mathrm{II}.\end{aligned} \tag{5}$$

The risk aversion state of the node is

$$\begin{aligned}&\mathrm{IS}^d + \mathrm{SI}^a \xrightarrow{\beta_2^n\lambda_2} \mathrm{II}^d + \mathrm{SI}^a,\\&\mathrm{IS}^d + \mathrm{II}^a \xrightarrow{\beta_2^m\beta_2^n\lambda_2} \mathrm{II}^a + \mathrm{II}^d.\end{aligned} \tag{6}$$

(v) The dynamic change process of individual nodes from SI state to II state is given by the following.

The risk preference state of the node is

$$\begin{aligned}&\mathrm{SI}^a + \mathrm{IS} \xrightarrow{\beta_1^n\lambda_1} \mathrm{II}^a + \mathrm{IS},\\&\mathrm{SI}^a + \mathrm{II} \xrightarrow{\beta_1^m\beta_1^n\lambda_1} \mathrm{II}^a + \mathrm{II}.\end{aligned} \tag{7}$$

The risk aversion state of the node is

$$\begin{aligned}&\mathrm{SI}^d + \mathrm{IS}^a \xrightarrow{\beta_1^n\lambda_1} \mathrm{II}^d + \mathrm{IS}^a,\\&\mathrm{SI}^d + \mathrm{II}^a \xrightarrow{\beta_1^m\beta_1^n\lambda_1} \mathrm{II}^d + \mathrm{II}^a.\end{aligned} \tag{8}$$

(vi) The dynamic process of product risk forgetting is as follows:

$$\begin{aligned}&\mathrm{IS}(k,t) \xrightarrow{\mu_1} \mathrm{SS}(k,t),\\&\mathrm{SI}(k,t) \xrightarrow{\mu_2} \mathrm{SS}(k,t),\\&\mathrm{II}(k,t) \xrightarrow{\eta_1\mu_1} \mathrm{SI}(k,t),\\&\mathrm{II}(k,t) \xrightarrow{\eta_2\mu_2} \mathrm{IS}(k,t).\end{aligned} \tag{9}$$

We consider the baseline scenario in which the isolated dynamics of each product information, when the second is absent, is described by a simple SIS scheme. In addition, $\mathrm{SS}(k,t), \mathrm{II}(k,t), \mathrm{SI}(k,t)$, and $\mathrm{IS}(k,t)$, respectively, represent the proportion of node individuals in states SS,II, SI, and IS within the composed degree class (k,t) at time t. Thus, $\mathrm{SS}(k,t) + \mathrm{SI}(k,t) + \mathrm{II}(k,t) + \mathrm{IS}(k,t) = 1, \forall (k,t)$. Then, we given that the node has degree $<k> = \sum_{k,t} P(k,t)k$. The $P(k,t)$ represents the composed degree distribution which gives the proportion of nodes having k links in network.

3. Mean-Field Equation Theory Analysis

In this section, we give the mean-field equation analysis of product information diffusion. We give the total transition probability of the node individual with state $SS(k,t)$ as

$$p_{SS\longrightarrow SS} = 1 - p_{SS\longrightarrow IS} - p_{SS\longrightarrow SI}. \quad (10)$$

$$\begin{aligned} S_{ss}(k,t+\Delta t) &= S_{SS}(k,t) - (1 - p_{SS\longrightarrow SS})S_{SS}(k,t) \\ &= S_{SS}(k,t) - (p_{SS\longrightarrow IS} + p_{SS\longrightarrow SI})S_{SS}(k,t). \end{aligned} \quad (11)$$

Here, $p_{SS\longrightarrow SS}$, $p_{SS\longrightarrow IS}$, and $p_{SS\longrightarrow SI}$, respectively, represent the transition probability of individual j from state $SS(k,t)$ to $SS(k,t)$,$IS(k,t)$, and $SI(k,t)$.

Next, according to the model given in the second part, we analyze the communication process of two kinds of product information between node individuals.

Transition probability from SS to IS can be given by the following equations.

Node in the risk preference state is

$$p^{a}_{SS\longrightarrow IS} = k\lambda_1\Delta t\theta_{IS} + k\beta_1^m\lambda_1\theta_{II}\Delta t. \quad (12)$$

Node in risk aversion state is

$$p^{d}_{SS\longrightarrow IS} = k\lambda_1\Delta t\alpha\theta_{IS} + k\beta_1^m\lambda_1\alpha\theta_{II}\Delta t. \quad (13)$$

Here, the θ parameters represent the nodes' link probability. That is, the probability that a given node links to an IS node is

$$\theta_{IS} = \frac{\sum_{k,t}P(k,t)kIS}{\sum_{k,t}P(k,t)k} = \frac{\sum_{k,t}P(k,t)kIS}{\langle k\rangle}. \quad (14)$$

The probability that a given node links to an II node is

$$\theta_{II} = \frac{\sum_{k,t}P(k,t)kII}{\sum_{k,t}P(k,t)k} = \frac{\sum_{k,t}P(k,t)kII}{\langle k\rangle}. \quad (15)$$

Similarly, we can get the following:

$$\theta_{SI} = \frac{\sum_{k,t}P(k,t)kSI}{\sum_{k,t}P(k,t)k} = \frac{\sum_{k,t}P(k,t)kSI}{\langle k\rangle}. \quad (16)$$

Thus, we can get the total transition probability from SS to IS as follows:

$$\begin{aligned} p_{SS\longrightarrow IS} &= \alpha(p^{a}_{SS\longrightarrow IS}) + (1-\alpha)(p^{d}_{SS\longrightarrow IS}), \\ &= 2k\lambda_1\Delta t\alpha\theta_{IS} + 2k\beta_1^m\lambda_1\alpha\theta_{II}\Delta t - \alpha^2k\lambda_1\Delta t\theta_{IS} \\ &\quad - \alpha^2k\beta_1^m\lambda_1\Delta t\theta_{II}. \end{aligned} \quad (17)$$

Transition probability from SS to SI can be given by the following equation.

Node in the risk preference state is

$$p^{a}_{SS\longrightarrow SI} = k\lambda_2\Delta t\theta_{SI} + k\beta_2^m\lambda_2\theta_{II}\Delta t. \quad (18)$$

Node in the risk aversion state is

$$p^{d}_{SS\longrightarrow SI} = k\lambda_2\Delta t\alpha\theta_{SI} + k\beta_2^m\lambda_2\alpha\theta_{II}\Delta t. \quad (19)$$

Thus, we can get the total transition probability from SS to SI as follows:

$$\begin{aligned} p_{SS\longrightarrow SI} &= \alpha(p^{a}_{SS\longrightarrow SI}) + (1-\alpha)(p^{d}_{SS\longrightarrow SI}), \\ &= 2k\lambda_2\Delta t\alpha\theta_{SI} + 2k\beta_2^m\lambda_2\alpha\theta_{II}\Delta t - \alpha^2k\lambda_2\Delta t\theta_{SI} \\ &\quad - \alpha^2k\beta_2^m\lambda_2\Delta t\theta_{II}. \end{aligned} \quad (20)$$

At the same time, we added the risk forgetting probability μ_1 and μ_2; then, we substitute the value of equation (17) and (20) into equation (11), when $\Delta t \longrightarrow 0$, and then, we get the following:

$$\frac{\partial S_{SS}(k,t)}{\partial t} = \mu_1 IS(k,t) + \mu_2 SI(k,t) - k(\sigma_1 + \sigma_2)S_{SS}(k,t). \quad (21)$$

Then, $\sigma_1 = \alpha\lambda_1(2-\alpha)(\theta_{IS} + \beta_1^m\theta_{II})$ and $\sigma_2 = \alpha\lambda_2(2-\alpha)(\theta_{SI} + \beta_2^m\theta_{II})$ represent the average probability of each node linked to obtain product information 1 and product information 2, respectively.

Similarly, we analyze that transition probability from IS to II can be given by the following equation.

Node in the risk preference state is

$$p^{a}_{IS\longrightarrow II} = k\beta_2^n\lambda_2\Delta t\theta_{SI} + k\beta_2^m\beta_2^n\lambda_2\theta_{II}\Delta t. \quad (22)$$

Node in risk aversion state is

$$p^{d}_{IS\longrightarrow II} = k\beta_2^n\lambda_2\Delta t\alpha\theta_{SI} + k\beta_2^m\beta_2^n\lambda_2\alpha\theta_{II}\Delta t. \quad (23)$$

So, we can get the total transition probability from IS to II:

$$\begin{aligned} p_{IS\longrightarrow II} &= \alpha(p^{a}_{IS\longrightarrow II}) + (1-\alpha)(p^{d}_{IS\longrightarrow II}), \\ &= 2k\alpha\beta_2^n\lambda_2\Delta t\theta_{SI} + 2k\alpha\beta_2^m\beta_2^n\lambda_2\theta_{II}\Delta t - k\beta_2^n\lambda_2\Delta t\alpha^2\theta_{SI} \\ &\quad - k\beta_2^m\beta_2^n\lambda_2\alpha^2\theta_{II}\Delta t. \end{aligned} \quad (24)$$

Adding the risk forgetting probability $\eta_1\mu_1$ and η_2, form equations (17) and (24); meanwhile, $\Delta t \longrightarrow 0$, and we can get the following:

$$\begin{aligned} \frac{\partial S_{IS}(k,t)}{\partial t} &= \eta_2\mu_2 II(k,t) - \mu_1 IS(k,t) + k\sigma_1 S_{SS}(k,t) \\ &\quad - k\beta_2^n\sigma_2 S_{IS}(k,t). \end{aligned} \quad (25)$$

In the same way, we can get the remaining two equations:

$$\frac{\partial S_{SI}(k,t)}{\partial t} = \eta_1\mu_1 II(k,t) - \mu_2 SI(k,t) + k\sigma_2 S_{SS}(k,t) - k\beta_1^n\sigma_1 S_{SI}(k,t), \quad (26)$$

$$\begin{aligned} \frac{\partial S_{II}(k,t)}{\partial t} &= k\beta_2^n\sigma_2 S_{IS}(k,t) + k\beta_1^n\sigma_1 S_{SI}(k,t) - \eta_1\mu_1 II(k,t) \\ &\quad - \eta_2\mu_2 II(k,t). \end{aligned} \quad (27)$$

From equations (25)–(27), we get the following:

$$
\begin{aligned}
\mathrm{SI} = {} & -(k\sigma_1(-\eta_1\mu_1-\eta_2\mu_2)(k\beta_2^n\sigma_2(-k\sigma_2+\eta_1\mu_1)-k\sigma_2(-\eta_1\mu_1-\eta_2\mu_2)-k\sigma_2 \\
& \cdot(-\eta_1\mu_1-\eta_2\mu_2)((-k\sigma_1-k\beta_2^n\sigma_2-\mu_1)(-\eta_1\mu_1-\eta_2\mu_2)-k\beta_2^n\sigma_2(-k\sigma_1+\eta_2\mu_2)))/ \\
& ((-k\beta_2^n\sigma_2(-k\sigma_2+\eta_1\mu_1)-k\sigma_2(-\eta_1\mu_1-\eta_2\mu_2))(t-nkq\sigma_1 h(-\eta_1\mu_1-\eta_2\mu_2)- \\
& k\beta_1^n\sigma_1(-k\sigma_2-\mu_2)(-\eta_1\mu_1-\eta_2\mu_2))((-k\sigma_1-k\beta_2^n\sigma_2-\mu_1)(-\eta_1\mu_1-\eta_2\mu_2)-k\beta_2^n\sigma_2(-k\sigma_1+\eta_2\mu_2)))), \\
\mathrm{IS} = {} & -\left(-k^2\beta_1^n\sigma_1^2\eta_2\mu_2-k^2\beta_1^n\sigma_1\sigma_2\eta_2\mu_2-k\sigma_1\eta_1\mu_1\mu_2-k\sigma_1\eta_2\mu_2^2\right)/ \\
& \left(k^3\beta_1^n\beta_2^n\sigma_1^2\sigma_2+k^3\beta_1^n\beta_2^n\sigma_1\sigma_2^2+k^2\beta_1^n\sigma_1\sigma_2\mu_1+k^2\beta_2^n\sigma_1\sigma_2\eta_1\mu_1+k^2\beta_2^n\sigma_2^2\eta_1\mu_1\right. \\
& +k\sigma_2\eta_1\mu_1^2+k^2\beta_2^n\sigma_1\sigma_2\mu_2+k^2\beta_1^n\sigma_1^2\eta_2\mu_2+k^2\beta_1^n\sigma_1\sigma_2\eta_2\mu_2+k\sigma_1\eta_1\mu_1\mu_2+k\beta_2^n\sigma_2\eta_1\mu_1 \\
& \left.+k\beta_1^n\sigma_1\eta_2\mu_1\mu_2+k\sigma_2\eta_2\mu_1\mu_2+\eta_1\mu_1^2\mu_2+k\sigma_1\eta_2\mu_2^2+k\beta_2^n\sigma_2\eta_2\mu_2^2+\eta_2\mu_1\mu_2^2\right), \\
\mathrm{II} = {} & -\left(-k^3\beta_1^n\beta_2^n\sigma_1^2\sigma_2-k^3\beta_1^n\beta_2^n\sigma_1\sigma_2^2-k^2\beta_1^n\sigma_1\sigma_2\mu_1-k^2\beta_2^n\sigma_1\sigma_2\mu_2\right)/ \\
& \left(k^3\beta_1^n\beta_2^n\sigma_1^2\sigma_2+k^3\beta_1^n\beta_2^n\sigma_1\sigma_2^2+k^2\beta_1^n\sigma_1\sigma_2\mu_1+k^2\beta_2^n\sigma_1\sigma_2\eta_1\mu_1+k^2\beta_2^n\sigma_2^2\eta_1\mu_1\right. \\
& +k\sigma_2\eta_1\mu_1^2+k^2\beta_2^n\sigma_1\sigma_2\mu_2+k^2\beta_1^n\sigma_1^2\eta_2\mu_2+k^2\beta_1^n\sigma_1\sigma_2\eta_2\mu_2+k\sigma_1\eta_1\mu_1\mu_2 \\
& \left.+k\beta_2^n\sigma_2\eta_1\mu_1\mu_2+k\beta_1^n\sigma_1\eta_2\mu_1\mu_2+k\sigma_2\eta_2\mu_2\mu_1+\eta_1\mu_1^2\mu_2+k\sigma_1\eta_2\mu_2^2+\eta_2\mu_1\mu_2^2\right).
\end{aligned}
\tag{28}
$$

Only three of these four equations are linearly independent for each composed connectivity class (k,t). Considering this, we next analyze the time evolution of the vector $[\mathrm{SI}(k,t), \mathrm{IS}(k,t), \mathrm{SS}(k,t)]$ to have $\mathrm{II}(k,t) = 1 - \mathrm{SI}(k,t) - \mathrm{IS}(k,t) - \mathrm{SS}(k,t)$.

4. Information Transmission Threshold Analysis

In order to analyze the critical threshold of the system, we need to find a value to satisfy the steady state $[\mathrm{SI}(k,t), \mathrm{IS}(k,t), \mathrm{II}(k,t)] = (0,0,0) \forall \in (k,t)$. In other words, all individuals in the system have access to both types of information. Therefore, we focus to consider the additional parameters 1 and 2, which are the other variables of linear combination. Therefore, it is possible to obtain self-consistent equations for σ_1 and σ_2 as

$$
\begin{aligned}
\sigma_1 &= f_1(\sigma_1,\sigma_2), \\
&= k\alpha\lambda_1(2-\alpha)\left(\theta_{\mathrm{IS}}+\beta_1^m\theta_{\mathrm{II}}\right), \\
&= \frac{k\alpha\lambda_1(2-\alpha)}{\langle k\rangle}\sum_{k,t}P(k,t)k\left[\mathrm{IS}(k,t,\sigma_1,\sigma_2)+\beta_1^m\mathrm{II}(k,t,\sigma_1,\sigma_2)\right].
\end{aligned}
\tag{29}
$$

$$
\begin{aligned}
\sigma_2 &= f_2(\sigma_1,\sigma_2), \\
&= k\alpha\lambda_2(2-\alpha)\left(\theta_{\mathrm{SI}}+\beta_2^m\theta_{\mathrm{II}}\right), \\
&= \frac{k\alpha\lambda_2(2-\alpha)}{\langle k\rangle}\sum_{k,t}P(k,t)k\left[\mathrm{SI}(k,t,\sigma_1,\sigma_2)+\beta_2^m\mathrm{II}(k,t,\sigma_1,\sigma_2)\right].
\end{aligned}
\tag{30}
$$

Then, $\sigma_1 = f_1(\sigma_1,\sigma_2)$ or $\sigma_2 = f_1(\sigma_1,\sigma_2)$ indicates that product 1 or product 2 information diffusion has reached a stable state. Given the symmetry of equations (29) and (30), we only need to study equation $\sigma_1 = f_1(\sigma_1,\sigma_2)$. In fact, $(\partial^2 f_1(\sigma_1,\sigma_2)/\partial\sigma_1^2) < 0$ always exists. So, we must verify that $[\partial^2 f_1(\sigma_1,\sigma_2)/\partial\sigma_1^2]_{\sigma_1=0} > 1$. After some algebraic calculations, this condition produces the following expression:

$$
\frac{\lambda_1\sum_{k,t}P(k,t)k^2k^2\sigma_2^2\beta_2^n\beta_1^n\beta_1^m+k\sigma_2\left[\eta_2\mu_2\beta_1^n+\beta_1^m\left(\beta_1^n\mu_1+\beta_2^n\mu_2\right)\right]+\mu_2\left(\eta_1\mu_1+\eta_2\mu_2\right)/k^2\sigma_2^2\beta_2^n\eta_1+k\sigma_2\left(\eta_1\mu_1+\eta_2\mu_2+\beta_2^n\mu_1\mu_2\right)+\mu_2\left(\eta_1\mu_1+\eta_2\mu_2\right)}{\mu_1\langle k\rangle} > 1.
\tag{31}
$$

Then, we get the critical threshold of product risk diffusion:

$$
\lambda_1^c(\sigma_2) = \frac{\mu_1\langle k\rangle}{\sum_{k,t}P(k,t)k^2k^2\sigma_2^2\beta_2^n\beta_1^n\beta_1^m+k\sigma_2\left[\eta_2\mu_2\beta_1^n+\beta_1^m\left(\beta_1^n\mu_1+\beta_2^n\mu_2\right)\right]+\mu_2\left(\eta_1\mu_1+\eta_2\mu_2\right)/k^2\sigma_2^2\beta_2^n\eta_1+k\sigma_2\left(\eta_1\mu_1+\eta_2\mu_2+\beta_2^n\mu_1\mu_2\right)+\mu_2\left(\eta_1\mu_1+\eta_2\mu_2\right)}.
\tag{32}
$$

From formula 32, we can see that the threshold of product 1 information transmission is affected by product 2 information. If we set $\sigma_2 = 0$, we get a result $\lambda_1^c(\sigma_2 = 0) = (\mu_1\langle k\rangle/\langle k^2\rangle)$, namely, the critical threshold

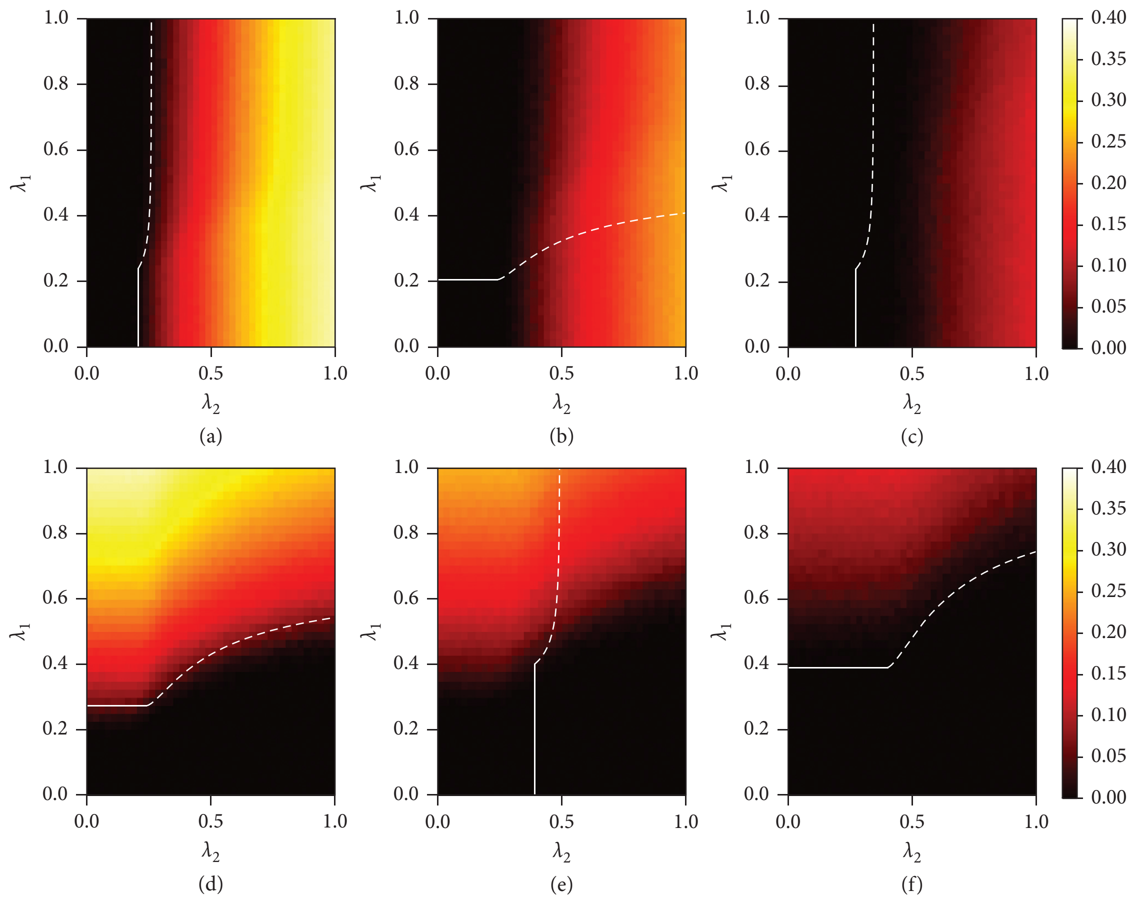

FIGURE 2: Product information diffusion mutual suppressed process.

under the diffusion of single product information. Thus, $\lambda_1^c(\sigma_2 = 0)$ is denoted as the main threshold, and more generally, what we call a second-order threshold, i.e., $\lambda_1^c(\sigma_2)$(with $\sigma_2 > 0$).

5. Numerical Simulations

In this part, we simulate the model with a computer, show the simulation results with a phase diagram, and verify the theoretical analysis. The Monte Carlo simulation mechanism was designed in the simulation process, that is, only one state can be changed in each diffusion process, and no two state changes can occur in a time interval. Product 1 represents the new product, and product 2 represents the old product. For example, Microsoft product 1 represents the new version and Microsoft product 2 represents the old version.

On the one hand, in order to avoid direct double infection from the SS state to the II state, it is assumed that if the number of product 1 information carried by the neighbor node of an individual who has no knowledge of either product information is larger than the number of product 2 information carried by the neighbor node, it will be easier for the individual to obtain product 1 information rather than product 2 information. Just as individuals do not have any product information, if there are more neighbor nodes carrying new product information than neighbor nodes carrying old product information, then it will be easier to obtain information about innovative products, and vice versa. On the other hand, in order to avoid the individual directly returning to the SS state from the II state, it is assumed that only one product information can be forgotten at each time interval. The forgetting of the product information depends on the forgetting probability $(\eta_1\mu_1/(\eta_1\mu_1 + \eta_2\mu_2))$ value of the product 1 information and the forgetting probability $(1 - \eta_1\mu_1/(\eta_1\mu_1 + \eta_2\mu_2))$ worth of the product 2 information.

After the simulation parameter adjustment, we find that the change of the simulation result mainly depends on the size relationship between the coefficients β,η, and "1," which is independent of the specific value between $\beta_1^m, \beta_2^m, \beta_1^n, \beta_2^n, \eta_1$, and η_2. Therefore, there are mainly two opposite simulation scenarios: first, when $\beta < 1$ and $\eta > 1$, they are called mutual damage scenarios; second, when $\beta > 1$ and $\eta < 1$, they are called mutual promotion scenarios. In the first mutual damage scenario, $\beta < 1$ indicates that information diffusion is mutually suppressed. For example, it is more difficult for an individual who has acquired product 1 information to acquire information of product 2 than an individual who does not acquire product 1 information. In addition, $\eta > 1$ indicates that product information forgetting promotes each other. For example, the forgetting time of the individual who has obtained the product 2 information is

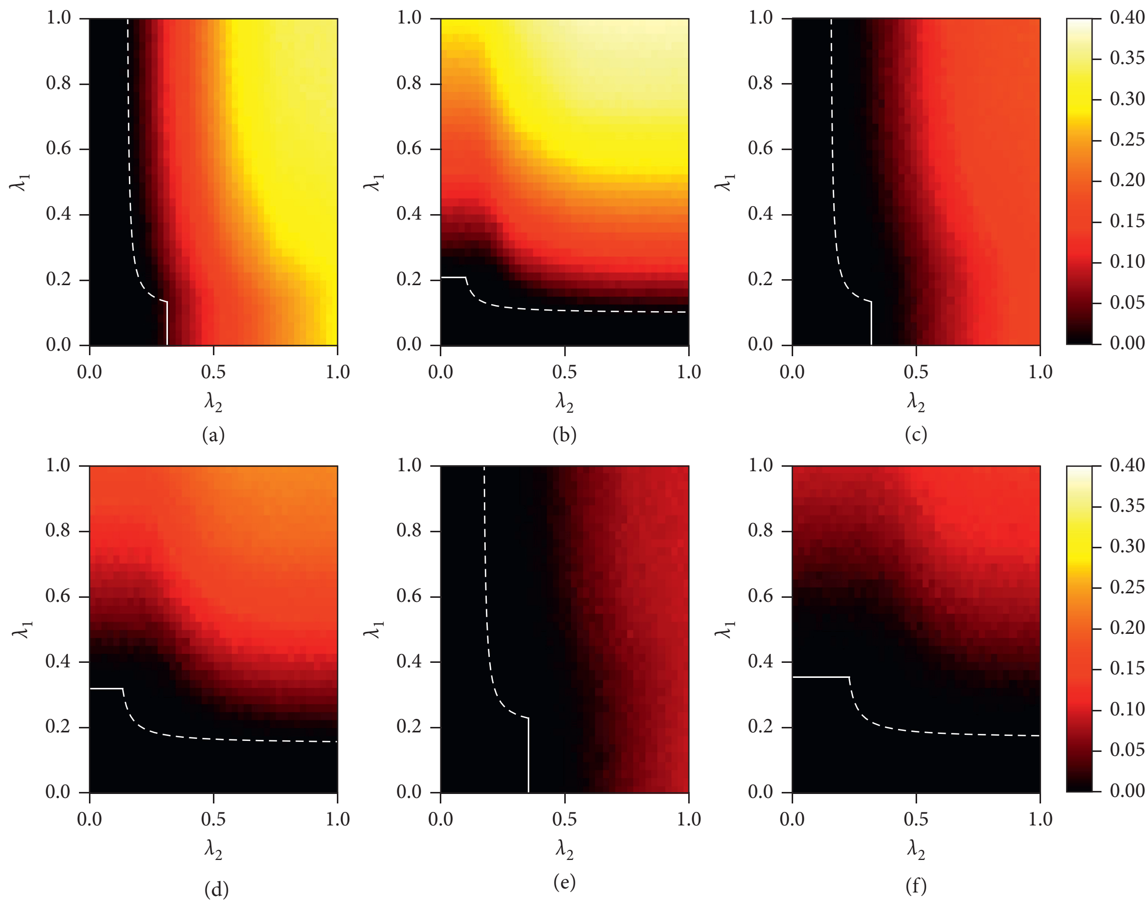

Figure 3: Product information diffusion mutual reinforcing process.

shortened due to the information of the product 1. We have combined these two factors and found that the information diffusion of product 2 in the system will promote the diffusion of product 1 information. At the same time, due to interactions, the role of product information diffusion is the same. Finally, for the second mutual promotion scenario, the diffusion and forgetting of product information are completely opposite to the first case; that is, the diffusion of one product information promotes the diffusion of another product information.

Figure 2 describes the scenario where product information diffusion is mutually suppressed. The first line in the figure shows the final diffusion of the product 2 information system. The second line in the figure shows the final diffusion of the product 1 information system. We set the simulation parameters to $\beta_1^m = \beta_2^m = \beta_1^n = \beta_2^n = 0.8\eta_1 = \eta_2 = 1.2$, and $\mu_1 = \mu_2 = 0.775$. The number of network nodes is 5000, and a new node is added with 4 edges each time to generate a scale-free network. The probability of introducing product 1 and 2 information individuals initially is IS = SI = 0.3 and for (a) and (d), the activity rate is $\alpha = 1$;for (b) and (e), the activity rate is $\alpha = 0.5$ and for (c) and (f), the activity rate is $\alpha = 0.3$ The solid and dashed lines in the figure are critical thresholds analyzed according to model theory, where the solid line represents the primary threshold and the dashed line represents the second threshold. In this scenario, the primary threshold is always below the second threshold, namely, $\lambda_1^c(\sigma_2) > \lambda_1 > \lambda_1^c(0)$ and $\lambda_2^c(\sigma_1) > \lambda_2 > \lambda_2^c(0)$. As the system activity is reduced, the ability of product information diffusion in the system is also decreasing, and information diffusion is becoming less and less obvious.

Figure 3 describes the scenarios in which product information diffusion is mutually reinforcing. The simulation network and graphical representation are the same as in Figure 1. The simulation parameters are set to $\beta_1^m = \beta_2^m = \beta_1^n = \beta_2^n = 1.1\eta_1 = \eta_2 = 1.1$, and $\mu_1 = \mu_2 = 0.775$. The probability of introducing product 1 and 2 information individuals initially is IS = SI = 0.3, and for (a) and (d), the activity rate is $\alpha = 1$; for (b) and (e), the activity rate is $\alpha = 0.5$; (c) and (f), the activity rate is $\alpha = 0.3$. The solid and dashed lines in the figure are critical thresholds analyzed according to model theory, where the solid line represents the primary threshold that $\lambda_1^c(0)$ and $\lambda_2^c(0)$ and the dashed line represents the second threshold that $\lambda_1^c(\sigma_2)$ and $\lambda_1^c(\sigma_1)$. It can be seen from Figure 3 that the critical threshold analyzed by the theory agrees with the boundary line in the simulated image. Moreover, in the case where product information diffusion is mutually promoted, the second threshold is below the main threshold, namely, $\lambda_1^c(\sigma_2) < \lambda_1 < \lambda_1^c(0)$ and $\lambda_2^c(\sigma_1) < \lambda_2 < \lambda_2^c(0)$. At the same time, with the decrease of activity, the critical thresholds λ_1^c and λ_2^c of product information diffusion are also decreasing, which indicates that reducing the activity of individuals has a certain control effect on information diffusion.

Compared with previous studies, this paper considers the mutual influence of information between products and also considers the individual's forgetting of product information over time. The numerical simulation of this part is consistent with the theoretical analysis, indicating the accuracy of the model. In the field of research product information, a new perspective has been opened to help operators better understand how product information is transmitted among consumers.

6. Conclusion

In this paper, the theory of complex network is used to study the competitive information of products. The mean-field equation describing the dynamic diffusion process is established by SIS diffusion mechanism and Monte Carlo simulation. The risk preference is introduced into the information diffusion and competition of products, and the diffusion mechanism of two kinds of product information is analyzed. In contrast to previous studies, the authors take into account the mutual influence of information between products and the individual's forgetting of information about the product over time. The research results shown that when an individual has a state of risk preference at a certain time, it will interact with all neighboring nodes. However, if someone is in the state of risk aversion at some time, it can only interact with the neighbor node in risk preference. In the first mutual damage scenario, the information diffusion is suppressed. In the second mutually reinforcing scenario, the diffusion of one product information promotes the diffusion of the other. After considering these two factors, the SIS diffusion mechanism is used to analyze and calculate the diffusion dynamics process and critical threshold. Then, through computer numerical simulation, the correctness of the theoretical analysis is verified, which has strong practicability. The interesting results can be used to predict the state of the market for goods or technologies.

Data Availability

The data used to support the findings of this study are currently under embargo while the research findings are commercialized. Requests for data, 12 months after publication of this article, will be considered by the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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