

Research Article

Dynamic Analysis of a Heterogeneous Diffusive Prey-Predator System in Time-Periodic Environment

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In this paper, a heterogeneous diffusive prey-predator system is first proposed and then studied analytically and numerically. Some sufficient conditions are derived, including permanence and extinction of system and the boundedness of the solution. The existence of periodic solution and its stability are discussed as well. Furthermore, numerical results indicate that both the spatial heterogeneity and the time-periodic environment can influence the permanence and extinction of the system directly. Our numerical results are consistent with the analytical analysis.

1. Introduction

Due to the complexity of ecosystems, prey-predator dynamics have always drawn interest among mathematical ecologists, as well as experimental ecologists [1–3]. The significance of studying prey-predator dynamics is to gain insights into the complex ecological processes. Prey-predator models, as the base of researching prey-predator dynamics, have attracted increasing attention [4–7]. Since Holling [8] introduced the concept of the functional response, a lot of studies have been devoted to the understanding of the effect of functional response on prey-predator dynamics [9]. Usually, the functional response is assumed to be either prey dependent or ratio dependent in prey-predator models [10, 11].

A classical general prey-predator system can be written as follows [12]:

$$\frac{dN}{dt} = f(N)N - g(N, P)P, \quad (1a)$$

$$\frac{dP}{dt} = h(g(N, P), P)P, \quad (1b)$$

where N and P denote the prey and predator densities, respectively, $f(N)$ is the prey growth rate, $g(N, P)$ is the functional response, and $h(g(N, P), P)$ is the per capita growth rate of predators. Let $h(g(N, P), P) = eg(N, P) - m(P)$, then equation (1b) can be rewritten as follows:

$$\frac{dP}{dt} = (eg(N, P) - m(P))P, \quad (2)$$

where e is the conversion efficiency and $m(P)$ is the specific mortality of predators in absence of prey. For the function $m(P)$, the most widely accepted assumption [13] is

$m(P) = \mu$, where μ is a constant describing the death rate of the predator. However, Cavani and Farkas [14] introduced another function for $m(P)$:

$$m(P) = \frac{\gamma + \delta P}{1 + P}, \quad (3)$$

where γ is the mortality at low density and δ is the limiting, maximal mortality (obviously, $\gamma < \delta$). The specific mortality (3) depends on the quantity of predators, which suggests that the predator mortality is neither a constant nor an unbounded function, and increasing with quantity. Obviously, when $\gamma = \delta$, equation (3) can be simplified to a constant death rate type. Prey-predator systems with this nonconstant death rate have been studied by some researchers [15–17].

Additionally, in order to understand patterns and the mechanisms of spatial distribution of interacting species, the dispersal process is taken into consideration [18–20]. Thus, the spatiotemporal dynamics of a prey-predator system can be presented by a couple of reaction-diffusion equations based on equations (1a) and (2) [10, 21, 22]:

$$\frac{\partial N}{\partial t} = f(N)N - g(N, P)P + D_N \Delta N, \quad (4a)$$

$$\frac{\partial P}{\partial t} = (eg(N, P) - m(P))P + D_P \Delta P, \quad (4b)$$

where D_N and D_P are the prey and predator diffusion coefficients, respectively, and the Laplace operator Δ describes the spatial dispersal.

Because of the emergence of Lotka–Volterra models [23, 24], a logistic type growth $f(N)$ is usually assumed for the prey species in the models. Some functional response $g(N, P)$ are taken into account in many works, such as Holling type [25], Michaelis–Menten type [26, 27], and Beddington–DeAngelis type [28, 29]. Especially, many biologists argued that the ratio-dependent theory is more suitable for describing prey-predator systems in many situations [13, 30–32]. Since Ardini and Ginzburg proposed the ratio-dependent prey-predator system, the prey-predator systems with ratio-dependent functional response are widely studied [13, 33–36], and many interesting results are obtained.

Based on model (4a) and (4b), in this paper, we employ the ratio-dependent functional response and the nonconstant death rate (i.e., equation (3)) and assume that the growth rate of prey population follows the logistic growth type. Moreover, let u and v be the prey density and the predator density, respectively. Then, the resulting system is

$$\frac{\partial u}{\partial t} = ru \left(1 - \frac{u}{K}\right) - \frac{auv}{bv + u} + \mu_1 \Delta u, \quad x \in \Omega, t > 0, \quad (5a)$$

$$\frac{\partial v}{\partial t} = \frac{euv}{bv + u} - \frac{c + \delta v}{1 + v} v + \mu_2 \Delta v, \quad x \in \Omega, t > 0, \quad (5b)$$

$$\frac{\partial u}{\partial n} = \frac{\partial v}{\partial n} = 0, \quad x \in \partial \Omega, t > 0, \quad (5c)$$

where $\Omega \in \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial \Omega$.

In system (5a)–(5c), when $c = \delta = \mu$, the system without diffusion is so-called the Michaelis–Menten ratio-dependent predator-prey system, which has been studied by many researchers. Kuang and Beretta [37] systematically studied the global behaviors of solutions and obtained some new and significant results, but many important open questions remain to be unsolved. For these open questions, Hsu et al. [38] resolved the global stability of all equilibria in various cases and the uniqueness of limit cycles by transforming the Michaelis–Menten-type ratio-dependent model. Xiao and Ruan [39] investigated the qualitative behavior of the Michaelis–Menten-type ratio-dependent model at the origin in the interior of the first quadrant and confirmed that the origin is indeed a critical point inducing rich and complicated dynamics. Additionally, when the diffusion process is considered, the Michaelis–Menten ratio-dependent predator-prey system with diffusion can produce rich spatial patterns, which makes it a widely studied system for pattern formation [10, 40–43].

While $c < \delta$, Kovács et al. [44] incorporated delays into system (5a)–(5c) and studied the qualitative behaviour of the system without diffusion. Yun et al. [45] presented an efficient and accurate numerical method for solving system (5a)–(5c) with a Turing instability and studied the existence of nonconstant stationary solutions. Aly et al. [46] studied Turing instability for system (5a)–(5c) and showed that diffusion-driven instability occurs at a certain critical value analytically. In these works, parameters in system (5a)–(5c) are always considered as constants.

However, it seems that there is no research for considering spatial heterogeneity and time-periodic environment in system (5a)–(5c). It is well known that spatial heterogeneity occurs at all scales of the environment [47]. Additionally, interactive populations often live in a fluctuating environment [48], where some environmental conditions such as temperature, light, availability of food, and other resources usually vary in time. Specially, some data depending on season in systems may be periodic functions of time. Thus, more realistic models to describe ecosystem should be nonautonomous systems with spatial heterogeneity. With this mind, we propose the following system to study effects of spatial heterogeneity and time-periodic environment on prey-predator dynamics:

$$\frac{\partial u(t, x)}{\partial t} = r(t, x)u(t, x) \left(1 - \frac{u(t, x)}{K(t, x)}\right) - \frac{a(t, x)u(t, x)v(t, x)}{b(t, x)v(t, x) + u(t, x)} + \mu_1 \Delta u(t, x), \quad (6a)$$

$$\frac{\partial v(t, x)}{\partial t} = \frac{e(t, x)u(t, x)v(t, x)}{b(t, x)v(t, x) + u(t, x)} - \frac{c(t, x) + \delta(t, x)v(t, x)}{1 + v(t, x)} v(t, x) + \mu_2 \Delta v(t, x), \quad (6b)$$

$$\frac{\partial u(t, x)}{\partial n} = \frac{\partial v(t, x)}{\partial n} = 0, \quad x \in \partial \Omega, t > 0, \quad (6c)$$

where $u(t, x)$ and $v(t, x)$ represent the densities of the prey and predator, respectively, at a space point x and time t ; for simplification, $u(t, x)$ and $v(t, x)$ are rewritten as u and v in the rest of this paper, respectively; $r(t, x)$ is the intrinsic growth rate of prey population; $K(t, x)$ denotes the environmental carrying capacity of prey population; $a(t, x)$ is the capturing rate of the predator; $b(t, x)$ is the half saturation; and $e(t, x)$ denotes the conversion rate. The term $c(t, x) + \delta(t, x)v(t, x)/1 + v(t, x)$ describes the specific mortality of predators in absence of prey population, where $c(t, x)$ is the mortality at low density and $\delta(t, x)$ is the limiting, maximal mortality. The terms $\mu_1\Delta u(t, x)$ and $\mu_2\Delta v(t, x)$ with positive diffusion coefficients μ_1 and μ_2 represent the nonhomogeneous dispersion of the prey and the predator, respectively. Neumann boundary conditions (see equation (6c)) are employed, which characterize the absence of migration. Here, we assume that prey and predator populations are confined to a fixed bounded space domain $\Omega \in \mathbb{R}^n$ with smooth boundary $\partial\Omega$ and $\bar{\Omega} = \Omega \cup \partial\Omega$.

The rest of the paper is organized as follows. In Section 2, some conditions and definitions are given. In Section 3, dynamics of system (6a)–(6c) are studied, including boundedness, permanence, extinction, and periodic solution. Moreover, a series of numerical simulations are carried out for further study of the dynamics of system (6a)–(6c) in Section 4. Finally, the paper ends with conclusion in Section 5.

2. Preliminaries

Let \mathbb{R} , \mathbb{Z} , and \mathbb{N} be the sets of all real numbers, integers, and positive integers, respectively, and $\mathbb{R}_+ = [0, +\infty)$. We assume that the following condition holds throughout the paper:

(H) The functions $r(t, x)$, $K(t, x)$, $a(t, x)$, $b(t, x)$, $c(t, x)$, $e(t, x)$, $\delta(t, x)$ are bounded positive-valued functions on $\mathbb{R} \times \bar{\Omega}$, continuously differentiable in t and x , and are periodic in t with period $\tau > 0$.

Moreover, for a continuous function $\phi(t, x)$, we denote $\phi^L = \inf_{(t,x)} \phi(t, x)$ and $\phi^M = \sup_{(t,x)} \phi(t, x)$.

Definition 1. Solutions of system (6a)–(6c) are ultimately bounded if there exist positive constants N_1 and N_2 such that for every solution $(u(t, x, u_0, v_0), v(t, x, u_0, v_0))$, there exists a moment of time $T = T(u_0, v_0) > 0$ such that

$$\begin{aligned} u(t, x, u_0, v_0) &\leq N_1, \\ v(t, x, u_0, v_0) &\leq N_2, \end{aligned} \quad (7)$$

for all $x \in \bar{\Omega}$ and $t \geq T$.

Definition 2. System (6a)–(6c) is permanent if there exist positive constants ζ and η such that for every solution with nonnegative initial functions $u_0(x) \neq 0$ and $v_0(x) \neq 0$, there exists a moment of time $\hat{t} = \hat{t}(u_0, v_0)$ such that

$$\begin{aligned} \zeta &\leq u(t, x, u_0, v_0) \leq \eta, \\ \zeta &\leq v(t, x, u_0, v_0) \leq \eta, \end{aligned} \quad (8)$$

for all $x \in \bar{\Omega}$ and $t \geq \hat{t}$.

Consider the following equations:

$$\frac{\partial u}{\partial t} - d\Delta u + f(t, x, u) = 0, \quad (t, x) \in (0, T] \times \Omega, \quad (9a)$$

$$\frac{\partial u}{\partial n} = 0, \quad (t, x) \in (0, T] \times \partial\Omega. \quad (9b)$$

Then, we have the following definition.

Definition 3. A function $\hat{u}: (0, T] \times \Omega \rightarrow \mathbb{R}$ is called a lower solution of equations (9a) and 9b if it satisfies

$$\frac{\partial \hat{u}}{\partial t} - d\Delta \hat{u} + f(t, x, \hat{u}) \leq 0, \quad (t, x) \in (0, T] \times \Omega, \quad (10a)$$

$$\frac{\partial \hat{u}}{\partial n} \leq 0, \quad (t, x) \in (0, T] \times \partial\Omega. \quad (10b)$$

To analyze dynamics of system (6a)–(6c), the following results will be needed.

Theorem 1 (Walter [49]). *Suppose that vector-functions $v(t, x) = (v_1(t, x), \dots, v_m(t, x))$ and $w(t, x) = (w_1(t, x), \dots, w_m(t, x))$, $m \geq 1$, satisfy the following conditions:*

(i) *They are of class C^2 in x , $x \in \Omega$ and of class C^1 in $(t, x) \in [a, b] \times \bar{\Omega}$, where $\Omega \in \mathbb{R}^n$ is a bounded domain with a smooth boundary;*

(ii) *$v_i - \mu\Delta v - g(t, x, v) \leq w_i - \mu\Delta w - g(t, x, w)$, where $(t, x) \in [a, b] \times \Omega$, $\mu = (\mu_1, \dots, \mu_m) > 0$ (inequalities between vectors are satisfied coordinate-wise), and vector function $g(t, x, u) = (g_1(t, x, u), \dots, g_m(t, x, u))$ is continuously differentiable and quasimonotonically increasing with respect to $u = (u_1, \dots, u_m)$:*

$$\frac{\partial g_i(t, x, u_1, \dots, u_m)}{\partial u_j} \geq 0, \quad i, j = 1, \dots, m, i \neq j; \quad (11)$$

(iii) $\partial v/\partial n = \partial w/\partial n = 0$, $(t, x) \in [a, b] \times \partial\Omega$.

Then, $v(t, x) \leq w(t, x)$ for $(t, x) \in [a, b] \times \bar{\Omega}$.

Theorem 2 (Smith [50]). *Assume that T and d are positive real numbers, a function $u(t, x)$ is continuous on $[0, T] \times \bar{\Omega}$, continuously differentiable in $x \in \bar{\Omega}$, with continuous derivatives $\partial^2 u/\partial x_i \partial x_j$ and $\partial u/\partial t$ on $(0, T] \times \Omega$, and $u(t, x)$ satisfies the following inequalities:*

$$\frac{\partial u}{\partial t} - d\Delta u + c(t, x)u \geq 0, \quad (t, x) \in (0, T] \times \Omega,$$

$$\frac{\partial u}{\partial n} \geq 0, \quad (t, x) \in (0, T] \times \partial\Omega, \quad (12)$$

$$u(0, x) \geq 0, \quad x \in \Omega,$$

where $c(t, x)$ is bounded on $(0, T] \times \Omega$. Then, $u(t, x) \geq 0$ on $(0, T] \times \bar{\Omega}$.

Moreover, $u(t, x)$ is strictly positive on $(0, T] \times \bar{\Omega}$ if $u(t, x)$ is not identically zero.

3. Main Results

3.1. Boundedness. From the biological and ecological viewpoint, we are always interested in the nonnegative solutions. Thus, the following theorem is given first in system (6a)–(6c).

Theorem 3. *Suppose that the condition (H) holds, then nonnegative and positive quadrants of \mathbb{R}^2 are positively invariant for system (6a)–(6c).*

Proof. Let $(u(t, x, u_0, v_0), v(t, x, u_0, v_0))$ be a solution of system (6a)–(6c) with initial condition $u_0(x) \geq 0 (\neq 0)$, $v_0(x) \geq 0 (\neq 0)$. Additionally, \hat{u} is a solution of the following system:

$$\frac{\partial \hat{u}}{\partial t} - \mu_1 \Delta \hat{u} - \hat{u} \left(r^L - \frac{a^M}{b^L} - \frac{r^M}{K^L} \hat{u} \right) = 0, \quad \hat{u}(0, x) = u_0(x). \quad (13)$$

From system (6a), we can obtain

$$\begin{aligned} \frac{\partial u}{\partial t} - \mu_1 \Delta u - r(t, x)u \left(1 - \frac{u}{K(t, x)} \right) + \frac{a(t, x)uv}{b(t, x)v + u} \\ \leq \frac{\partial u}{\partial t} - \mu_1 \Delta u - u \left(r^L - \frac{a^M}{b^L} - \frac{r^M}{K^L} u \right), \end{aligned} \quad (14)$$

which implies $\hat{u}(t, x)$ is a lower solution of system (6a). According to Theorem 2, it is obvious that $\hat{u}(t, x) \geq 0$ for all $x \in \bar{\Omega}$ and $t > 0$. Furthermore, due to $u_0(x) \geq 0 (\neq 0)$, $\hat{u}(t, x) > 0$ holds for all $x \in \bar{\Omega}$ and $t > 0$. Thus, $u(t, x) > 0$ holds because $u(t, x)$ is bounded from below by positive function $\hat{u}(t, x)$.

For system (6b), it can be simply verified that $\hat{v}(t, x)$ is a lower solution of system (6b), where $\hat{v}(t, x)$ satisfies

$$\begin{aligned} \frac{\partial \hat{v}}{\partial t} - \mu_2 \Delta \hat{v} + \delta^M \hat{v} = 0, \\ \hat{v}(0, x) = v_0(x). \end{aligned} \quad (15)$$

By the similar argument to $u(t, x)$, we can prove the positiveness of $v(t, x)$.

This completes the proof.

Based on Theorem 3, we will discuss ultimate boundedness of solutions in system (6a)–(6c), and then the following theorem can be obtained. \square

Theorem 4. *If the condition (H) holds, then all solutions of system (6a)–(6c) with nonnegative initial conditions are ultimately bounded.*

Proof. From system (6a), it can be found that the following inequality holds:

$$\begin{aligned} 0 = \frac{\partial u}{\partial t} - \mu_1 \Delta u - r(t, x)u \left(1 - \frac{u}{K(t, x)} \right) + \frac{a(t, x)uv}{b(t, x)v + u} \\ \geq \frac{\partial u}{\partial t} - \mu_1 \Delta u - u \left(r^M - \frac{r^L}{K^M} u \right). \end{aligned} \quad (16)$$

Let $\bar{u}(t, x, u_0)$ be a solution of

$$\frac{\partial \bar{u}}{\partial t} - \mu_1 \Delta \bar{u} - \bar{u} \left(r^M - \frac{r^L}{K^M} \bar{u} \right) = 0, \quad (17)$$

then

$$\begin{aligned} \frac{\partial \bar{u}}{\partial t} - \mu_1 \Delta \bar{u} - \bar{u} \left(r^M - \frac{r^L}{K^M} \bar{u} \right) = 0 \\ \geq \frac{\partial u}{\partial t} - \mu_1 \Delta u - u \left(r^M - \frac{r^L}{K^M} u \right). \end{aligned} \quad (18)$$

According to Theorem 1, we can get $u(t, x, u_0, v_0) \leq \bar{u}(t, M_u)$, where M_u satisfies $\|u_0(x)\|_C = \max_{x \in \bar{\Omega}} |u_0(x)| \leq M_u$. By the uniqueness theorem, it is obvious that the solution $\bar{u}(t, M_u)$ with initial conditions independent of x does not depend on x for $t > 0$. Therefore, $\bar{u}(t, M_u)$ is the solution of the following ordinary differential equation:

$$\frac{d\bar{u}}{dt} = \bar{u} \left(r^M - \frac{r^L}{K^M} \bar{u} \right), \quad \bar{u}(0, M_u) = M_u. \quad (19)$$

Hence, we have

$$u(t, x, u_0, v_0) \leq \bar{u}(t, M_u) \longrightarrow \frac{r^M K^M}{r^L}, \quad \text{as } t \longrightarrow \infty. \quad (20)$$

Thus, there exists a positive constant M_1 in system (6a)–(6c) such that $u(t, x) \leq M_1$, starting with some moment of time.

For predator population v , by system (6b), we have

$$\begin{aligned} 0 = \frac{\partial v}{\partial t} - \mu_2 \Delta v - v \left(\frac{e(t, x)u}{b(t, x)v + u} - \frac{c(t, x) + \delta(t, x)v}{1 + v} \right) \\ = \frac{\partial v}{\partial t} - \mu_2 \Delta v - v \left(\frac{e(t, x)u}{b(t, x)v + u} - \delta(t, x) - \frac{c(t, x) - \delta(t, x)}{1 + v} \right) \\ \geq \frac{\partial v}{\partial t} - \mu_2 \Delta v + c^L v - \frac{e^M M_1}{b^L}, \end{aligned} \quad (21)$$

which implies that $v(t, x, u_0, v_0) \leq \bar{v}(t, M_v)$, where $\bar{v}(t, M_v)$ is a solution of the following initial value problem:

$$\frac{d\bar{v}}{dt} = -c^L \bar{v} + \frac{e^M M_1}{b^L}, \quad \bar{v}(0, M_v) = M_v, \quad (22)$$

and M_v satisfies $\|v_0(x)\|_C = \max_{x \in \bar{\Omega}} |v_0(x)| \leq M_v$. Obviously, we can obtain that

$$\bar{v}(t, M_v) = M_v e^{-c^L t} + \frac{e^M M_1}{b^L c^L} \longrightarrow \frac{e^M M_1}{b^L c^L}, \quad \text{as } t \longrightarrow \infty. \quad (23)$$

Therefore, $v(t, x, u_0, v_0)$ is also ultimately bounded.

This completes the proof. \square

3.2. Permanence

Theorem 5. Under the condition (H), if the following inequalities

$$r^L - \frac{a^M}{b^L} > 0, \quad (24a)$$

$$e^L - \delta^M > 0, \quad (24b)$$

hold, then system (6a)–(6c) is permanent, i.e., there exist positive constants m_i and M_i ($i = 1, 2$) such that any solution of system (6a)–(6c) with nonnegative initial functions $u_0(x) (\neq 0)$ and $v_0(x) (\neq 0)$ satisfies $(u(t, x), v(t, x)) \in S = \{(u, v): m_1 \leq u(t, x) \leq M_1, m_2 \leq v(t, x) \leq M_2\}$, starting with a certain time.

Proof. Under the condition (H), we can know from Theorem 4 that there exists M_i ($i = 1, 2$) such that $u(t, x) \leq M_1, v(t, x) \leq M_2$, starting with some moment of time. By comparison principle, if $u_0(x) \geq 0 (\neq 0)$ and $v_0(x) \geq 0 (\neq 0)$, then $u(t, x, u_0, v_0) > 0$ and $v(t, x, u_0, v_0) > 0$ for all $x \in \bar{\Omega}$ and $t > 0$.

Thus, for some small $\varepsilon > 0$, we can get initial conditions $(u(\varepsilon, x, u_0, v_0), v(\varepsilon, x, u_0, v_0))$ separated from zero by the solution on the interval $t \geq \varepsilon$. Without loss of generality, we assume that $\min_{x \in \bar{\Omega}} u_0(x) = m_u, \min_{x \in \bar{\Omega}} v_0(x) = m_v$. Then, the following inequality holds:

$$\begin{aligned} 0 &= \frac{\partial u}{\partial t} - \mu_1 \Delta u - u \left(r(t, x) - \frac{r(t, x)}{K(t, x)} u \right) + \frac{a(t, x)uv}{b(t, x)v + u} \\ &\leq \frac{\partial u}{\partial t} - \mu_1 \Delta u - u \left(r^L - \frac{a^M}{b^L} - \frac{r^M}{K^L} u \right). \end{aligned} \quad (25)$$

Obviously, we can get

$$\begin{aligned} 0 &= \frac{\partial \hat{u}}{\partial t} - \mu_1 \Delta \hat{u} - \hat{u} \left(r^L - \frac{a^M}{b^L} - \frac{r^M}{K^L} \hat{u} \right) \\ &\leq \frac{\partial \hat{u}}{\partial t} - \mu_1 \Delta \hat{u} - \hat{u} \left(r^L - \frac{a^M}{b^L} - \frac{r^M}{K^L} \hat{u} \right). \end{aligned} \quad (26)$$

Consequently, for $t \geq 0$, we have

$$u(t, x, u_0, v_0) \geq \hat{u}(t, m_u). \quad (27)$$

Thus, the solution $u(t, x, u_0, v_0)$ is bounded from below by a solution of the following logistic equation:

$$\frac{d\hat{u}}{dt} = \hat{u} \left(r^L - \frac{a^M}{b^L} - \frac{r^M}{K^L} \hat{u} \right), \quad \hat{u}(0) = m_u. \quad (28)$$

Thus, by Theorem 1 and condition (24a) and (24b), we have

$$u(t, x, u_0, v_0) \geq \hat{u}(t, x) \longrightarrow \frac{K^L (r^L - a^M/b^L)}{r^M}, \quad \text{as } t \longrightarrow \infty \quad (29)$$

Therefore, there exists a positive constant m_1 such that $u(t, x, u_0, v_0) \geq m_1$ for t large enough.

By system (6b), the following inequality holds:

$$\begin{aligned} \frac{\partial v}{\partial t} - \mu_2 \Delta v - v \left(\frac{e(t, x)u}{b(t, x)v + u} - \frac{c(t, x) + \delta(t, x)v}{1 + v} \right) \\ \leq \frac{\partial v}{\partial t} - \mu_2 \Delta v + (\delta^M - e^L)v + \frac{b^M e^L}{m_1} v^2. \end{aligned} \quad (30)$$

By a similar analysis to u , we have $v(t, x, u_0, v_0) \geq \hat{v}(t, m_v)$, where $\hat{v}(t, m_v)$ is a solution of the following system:

$$\frac{\partial \hat{v}}{\partial t} - \mu_2 \Delta \hat{v} + (\delta^M - e^L)\hat{v} + \frac{b^M e^L}{m_1} \hat{v}^2, \quad \hat{v}(0) = m_v. \quad (31)$$

According to condition (24b), we can obtain that there exists a positive m_2 such that $v(t, x, u_0, v_0) \geq m_2$ for t large enough. Thus, system (6a)–(6c) is permanent, starting with a certain time.

This completes the proof. \square

3.3. Extinction. In this section, we will discuss the extinction of predator species, and then the following theorem arrives in system (6a)–(6c).

Theorem 6. If the condition (H) holds, and

$$e^M - c^L < 0, \quad (32)$$

then, $v(t, x) \longrightarrow 0$ as $t \longrightarrow \infty$.

Proof. Suppose M_v is a fixed positive constant guaranteeing $M_v \leq v_0(x)$, and $\bar{v}(t, M_v)$ is the solution of the following initial value problem:

$$\frac{\partial \bar{v}}{\partial t} = \bar{v}(e^M - c^L), \quad (33)$$

$$\bar{v}(0, M_v) = M_v.$$

By system (6b), we have

$$\begin{aligned} 0 &= \frac{\partial v}{\partial t} - \mu_2 \Delta v + v \left(-\frac{e(t, x)u}{b(t, x)v + u} + \frac{c(t, x) + \delta(t, x)v}{1 + v} \right) \\ &\geq \frac{\partial v}{\partial t} - \mu_2 \Delta v + (c^L - e^M)v. \end{aligned} \quad (34)$$

Thus, according to Theorem 1, we can deduce that $v(t, x, u_0, v_0) \leq \bar{v}(t, M_v) \longrightarrow 0$ as $t \longrightarrow \infty$ if inequality (32) holds.

This completes the proof. \square

3.4. Periodic Solution. In this section, we will study the periodic solutions in system (6a)–(6c) by constructing a proper Lyapunov function.

Theorem 7. Under the condition (H), assume that system (6a)–(6c) is permanent, that is, there exist positive constants N and M such that an arbitrary solution of system (6a)–(6c)

with nonnegative initial functions not identically equal to zero satisfies the condition:

$$(u(t, x), v(t, x)) \in E = \{(u, v): N \leq u(t, x) \leq M, \\ N \leq v(t, x) \leq M\}, \quad (35)$$

starting with a certain moment of time. If

$$\lambda_M(W) < 0, \quad (36)$$

where λ_M is the maximal eigenvalue of the following matrix:

$$\begin{pmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{pmatrix}, \quad (37)$$

where

$$E_{11} = 2 \left(r^M - \frac{r^L}{K^M} N - \frac{a^L b^L N^2}{(b^M M + M)^2} \right),$$

$$E_{22} = 2 \left(-\delta^L + \frac{\delta^M - c^L}{(1 + N)^2} + \frac{e^M M^2}{(b^L N + N)^2} \right), \quad (38)$$

$$E_{12} = E_{21} = a^M + \frac{e^M}{b^L}.$$

Then system (6a)–(6c) has a unique and strictly positive τ -periodic solution, which is globally asymptotically stable.

Proof. Let $(u(t, x), v(t, x))$ and $(\bar{u}(t, x), \bar{v}(t, x))$ be two solutions of system (6a)–(6c) bounded by constants N and

M from below and above, respectively. Consider the following function:

$$L(t) = \int_{\Omega} [(u(t, x) - \bar{u}(t, x))^2 + (v(t, x) - \bar{v}(t, x))^2] dx. \quad (39)$$

By system (6a)–(6c), we can get its derivative:

$$\begin{aligned} \frac{dL(t)}{dt} &= 2 \int_{\Omega} (u - \bar{u}) \left(\frac{\partial u}{\partial t} - \frac{\partial \bar{u}}{\partial t} \right) dx + 2 \int_{\Omega} (v - \bar{v}) \left(\frac{\partial v}{\partial t} - \frac{\partial \bar{v}}{\partial t} \right) dx \\ &= 2\mu_1 \int_{\Omega} (u - \bar{u}) \Delta(u - \bar{u}) dx + 2\mu_2 \int_{\Omega} (v - \bar{v}) \Delta(v - \bar{v}) dx \\ &\quad + 2 \int_{\Omega} (u - \bar{u}) \left[\left(u \left(r - \frac{r}{K} u \right) - \frac{auv}{bv+u} \right) - \left(\bar{u} \left(r - \frac{r}{K} \bar{u} \right) - \frac{a\bar{u}\bar{v}}{b\bar{v}+\bar{u}} \right) \right] dx \\ &\quad + 2 \int_{\Omega} (v - \bar{v}) \left[\left(\frac{euv}{bv+u} - \frac{c+\delta v}{1+v} \right) - \left(\frac{e\bar{u}\bar{v}}{b\bar{v}+\bar{u}} - \frac{c+\delta\bar{v}}{1+\bar{v}} \right) \right] dx \\ &= I_1 + I_2 + I_3 + I_4. \end{aligned} \quad (40)$$

Then, from the boundary condition (6c),

$$\begin{aligned} I_1 + I_2 &= -2\mu_1 \int_{\Omega} \nabla^2(u - \bar{u}) dx - 2\mu_2 \int_{\Omega} \nabla^2(v - \bar{v}) dx \\ &\leq -2\mu_1 \int_{\Omega} |\nabla(u - \bar{u})|^2 dx - 2\mu_2 \int_{\Omega} |\nabla(v - \bar{v})|^2 dx \leq 0. \end{aligned} \quad (41)$$

For other terms I_3 and I_4 ,

$$\begin{aligned} I_3 + I_4 &= 2 \int_{\Omega} (u - \bar{u}) \left(r(u - \bar{u}) - \frac{r}{K} (u - \bar{u})(u + \bar{u}) + \frac{a\bar{u}\bar{v}}{b\bar{v}+\bar{u}} - \frac{auv}{bv+u} \right) dx \\ &\quad + 2 \int_{\Omega} (v - \bar{v}) \left(-\delta(v - \bar{v}) + \frac{c - \delta}{1 + \bar{v}} \bar{v} - \frac{c - \delta}{1 + v} v + \frac{euv}{bv+u} - \frac{e\bar{u}\bar{v}}{b\bar{v}+\bar{u}} \right) dx \\ &= 2 \int_{\Omega} (u - \bar{u})^2 \left(r - \frac{r}{K} (u - \bar{u}) - \frac{abv\bar{v}}{(bv+u)(b\bar{v}+\bar{u})} \right) dx \\ &\quad + 2 \int_{\Omega} (v - \bar{v})^2 \left(\frac{e\bar{u}\bar{v}}{(bv+u)(b\bar{v}+\bar{u})} - \frac{c - \delta}{(1+v)(1+\bar{v})} - \delta \right) dx \\ &\quad + 2 \int_{\Omega} (u - \bar{u})(v - \bar{v}) \left(\frac{ebv\bar{v}}{(bv+u)(b\bar{v}+\bar{u})} - \frac{a\bar{u}\bar{v}}{(bv+u)(b\bar{v}+\bar{u})} \right) dx \\ &\leq 2 \int_{\Omega} (u - \bar{u})^2 \left(r^M - \frac{r^L}{K^M} N - \frac{a^L b^L N^2}{(b^M M + M)^2} \right) dx \\ &\quad + 2 \int_{\Omega} (v - \bar{v})^2 \left(\frac{e^M M^2}{(b^L N + N)^2} + \frac{\delta^M - c^L}{(1 + N)^2} - \delta^L \right) dx \\ &\quad + 2 \int_{\Omega} |u - \bar{u}| |v - \bar{v}| \left(a^M + \frac{e^M}{b^L} \right) dx \\ &\leq \lambda_M \int_{\Omega} [(u - \bar{u})^2 + (v - \bar{v})^2] dx. \end{aligned} \quad (42)$$

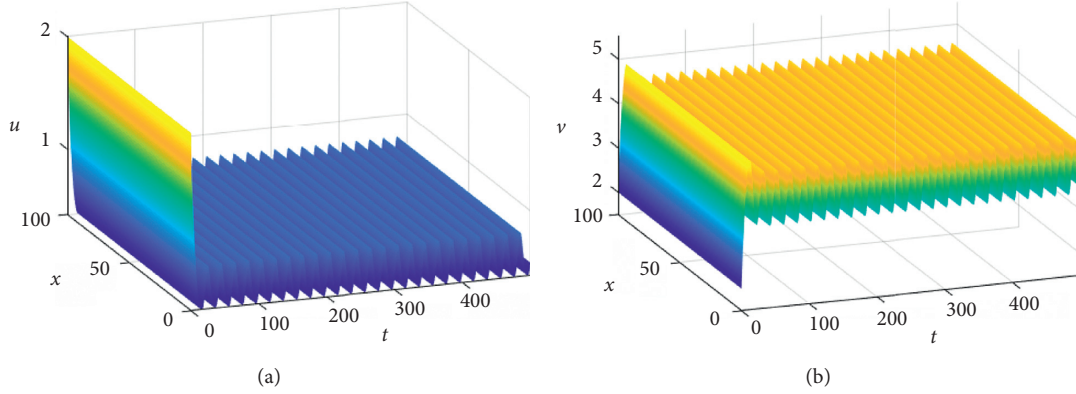


FIGURE 1: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v , where $r(t, x) = 1.1 + 0.1 \sin(\pi * t/10)$, $K(t, x) = 2 + 0.5 * \cos(\pi * t/10)$, $a(t, x) = 0.8 + 0.005 * \cos(\pi * t/10)$, $b(t, x) = 0.9 + 0.005 * \cos(\pi * t/10)$, $e(t, x) = 0.7 - 0.002 * \cos(\pi * t/10)$, $c(t, x) = 0.02 + 0.005 * \cos(\pi * t/10)$, $\delta(t, x) = 0.12 + 0.05 * \cos(\pi * t/10)$, $\mu_1 = 1$, and $\mu_2 = 1$.

By condition (36), we have

$$L(t) \leq L(0)e^{\lambda_M t} \rightarrow 0, \quad \text{as } t \rightarrow \infty, \quad (43)$$

which implies that $\|u(t, x) - \bar{u}(t, x)\| \rightarrow 0$ and $\|v(t, x) - \bar{v}(t, x)\| \rightarrow 0$ as $t \rightarrow \infty$, where $\|\cdot\|$ is the norm of the space $L_2(\Omega)$. Additionally, by condition (35), solutions of system (6a)–(6c) are bounded in the space $C^{1+\nu}(\bar{\Omega}, \mathbb{R}^2)$, where $0 < \nu < 2l - (n/p)$ and $(1/2) + (n/2p) < l < 1$.

Therefore,

$$\begin{aligned} \lim_{t \rightarrow \infty} \sup_{x \in \bar{\Omega}} |u(t, x) - \bar{u}(t, x)| &= 0, \\ \lim_{t \rightarrow \infty} \sup_{x \in \bar{\Omega}} |v(t, x) - \bar{v}(t, x)| &= 0. \end{aligned} \quad (44)$$

Consider the sequence $(u(k\tau, x, u_0, v_0), v(k\tau, x, u_0, v_0)) = W(k\tau, W_0)$, $k \in \mathbb{N}$. Then, $\{W(k\tau, W_0), k \in \mathbb{N}\}$ is compact in the space $C(\bar{\Omega}) \times C(\bar{\Omega})$. Let \bar{W} be a limit of this sequence, then $W(\tau, \bar{W}) = \bar{W}$.

Actually, because $W(\tau, W(k_n\tau, W_0)) = W(k_n\tau, W(\tau, W_0))$ and $W(k_n\tau, W(\tau, W_0)) - W(k_n\tau, W_0) \rightarrow 0$ as $k_n \rightarrow \infty$, we have

$$\begin{aligned} \|W(\tau, \bar{W}) - \bar{W}\|_C &\leq \|W(\tau, \bar{W}) - W(\tau, W(k_n\tau, W_0))\|_C \\ &+ \|W(\tau, W(k_n\tau, W_0)) - W(k_n\tau, W_0)\|_C \\ &+ \|W(k_n\tau, W_0) - \bar{W}\|_C \rightarrow 0 \text{ as } n \rightarrow \infty. \end{aligned} \quad (45)$$

Thus, the sequence $\{W(k\tau, W_0), k \in \mathbb{N}\}$ has a unique limit point. Otherwise, suppose that the sequence has two limit points $\bar{W} = \lim_{n \rightarrow \infty} W(k_n\tau, W_0)$ and $\tilde{W} = \lim_{n \rightarrow \infty} W(k_n\tau, W_0)$, then we can get the following result from (45) and $\bar{W} = W(k_n\tau, \bar{W})$:

$$\begin{aligned} \|\bar{W} - \tilde{W}\|_C &\leq \|\bar{W} - W(k_n\tau, W_0)\|_C \\ &+ \|W(k_n\tau, W_0) - \tilde{W}\|_C \rightarrow 0, \quad n \rightarrow \infty. \end{aligned} \quad (46)$$

Hence, $\bar{W} = \tilde{W}$. The solution $(u(t, x, \bar{u}, \bar{v}), v(t, x, \bar{u}, \bar{v}))$ is the unique periodic solution of system (6a)–(6c), and it is asymptotically stable using equation (44).

This completes the proof. \square

4. Numerical Results

In the previous section, we have obtained some interesting results of system (6a)–(6c). However, due to the complexity of system (6a)–(6c), it becomes much more difficult to provide in-depth analysis. Thus, here, we perform some numerical simulations to investigate prey-predator dynamics further.

According to Theorem 5, when $r^L - a^M/b^L > 0$ and $e^L - \delta^M > 0$ holds, system (6a)–(6c) is permanent under condition (H). Figure 1 shows that system (6a)–(6c) is permanent, where $r^L - a^M/b^L \approx 0.1 > 0$ and $e^L - \delta^M = 0.528 > 0$. When $e = 0.005 - 0.002 \sin(\pi * t/10)$, other parameters are the same as the ones in Figure 1, and we can get a numerical solution of system (6a)–(6c) (see Figure 2). It is obvious that predator population v is extinct ultimately, which is consistent with Theorem 6 because $e^M - c^L = -0.008 < 0$.

In section 3.4, the existence of periodic solution was discussed, and its stability and uniqueness were analyzed as well. In fact, Figure 1 has shown the existence of a periodic solution. Yet, we here take another set of function corresponding to the parameters of system (6a)–(6c), which is only the periodic function of time t with period 200. The corresponding numerical solutions are shown in Figure 3. Clearly, the numerical solution is periodic in t with the period of 200 (see Figures 3(c) and 3(d)), but it is homogeneous in space (see Figures 3(a) and 3(b)). Compared to Figure 3, we consider another situation that the parameters of system (6a)–(6c) are functions with respect to both time t and space x . We find the solution is still periodic, but it is heterogeneous in space (see Figure 4). It is evident that the spatial heterogeneity is the reason giving rise to the oscillation of the solution in space.

Additionally, we find that the spatial heterogeneity can promote the permanence of the system. Figure 5 indicates that the extinction occurs in system (6a)–(6c). However,

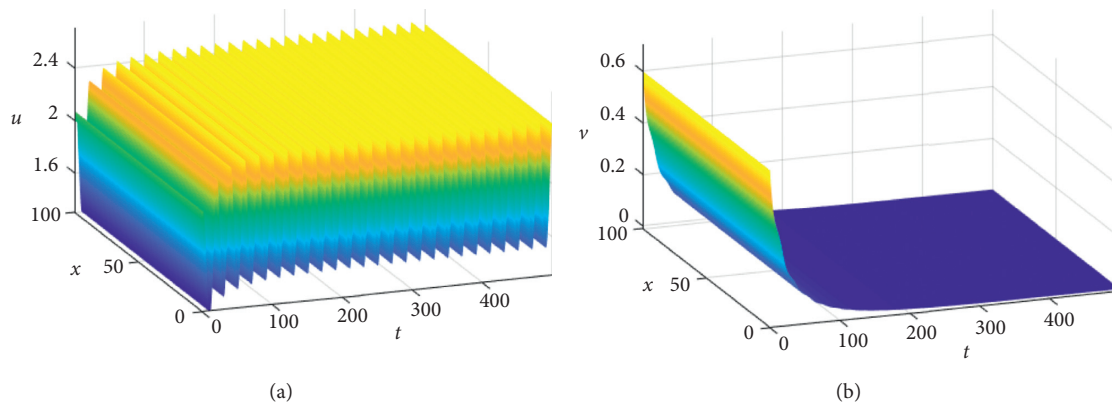


FIGURE 2: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v .

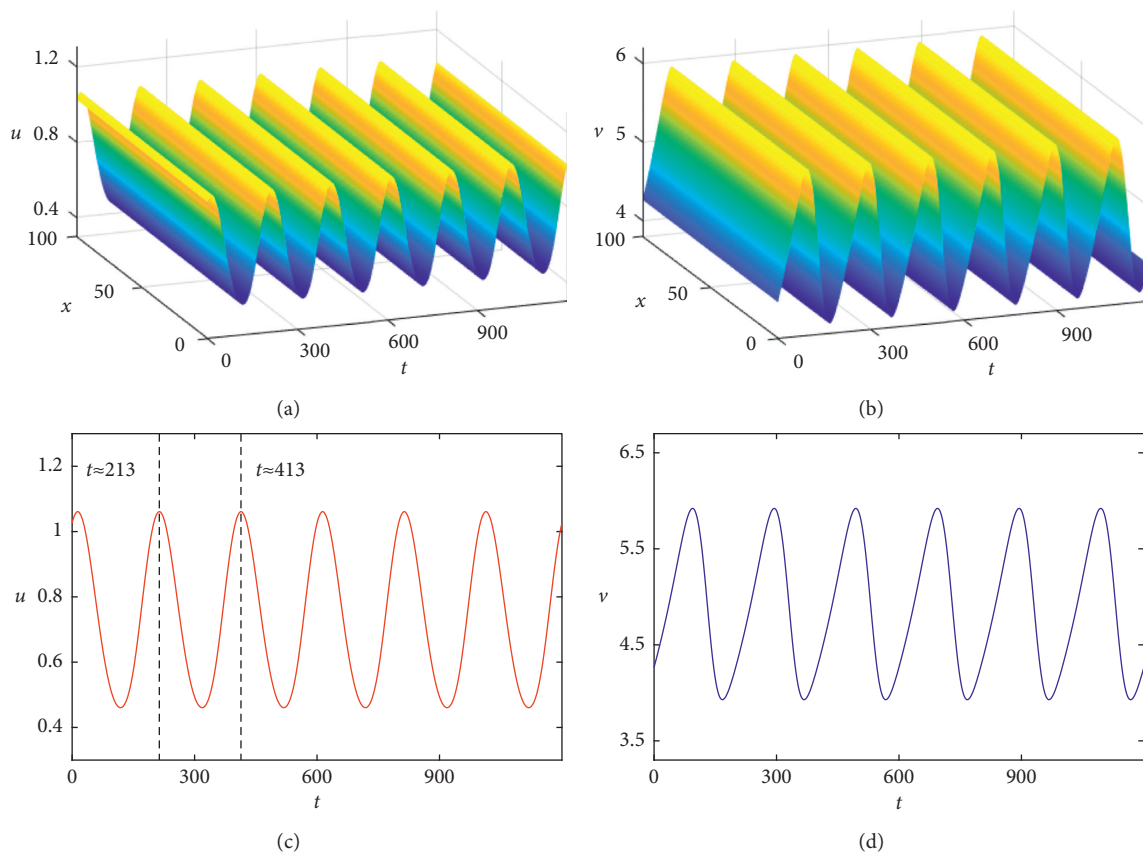


FIGURE 3: Numerical solutions of system (6a)–(6c): (a) prey population u , (b) predator population v , (c) profile of $u(t)$ at $x = 50$, and (d) profile of $v(t)$ at $x = 50$. Here, $r(t, x) = 1.2 + 0.1 \sin(\pi * t/100)$, $K(t, x) = 2 + 0.5 \cos(\pi * t/100)$, $a(t, x) = 0.8 + 0.005 \cos(\pi * t/100)$, $b(t, x) = 0.9 + 0.005 \cos(\pi * t/100)$, $e(t, x) = 0.7 - 0.005 \sin(\pi * t/100)$, $c(t, x) = 0.02 + 0.01 \cos(\pi * t/100)$, $\delta(t, x) = 0.12 + 0.05 \cos(\pi * t/100)$, $\mu_1 = 1$, and $\mu_2 = 1$.

when we set $r(t, x) = 0.9 + 0.001 \sin(5 * \pi * t/100) + 0.1 \cos(5 * \pi * x/100)$ (other parameters are the same as the ones in Figure 5), we get a very interesting result, that is, system (6a)–(6c) becomes permanent (see Figure 6). Likewise, other parameters are explored by repeating the same procedure, and similar results are obtained, which are

omitted here. Obviously, the spatial heterogeneity plays an important role in dynamics of system (6a)–(6c).

Let all the parameters be constant, then there exists a nonconstant stationary solution in system (6a)–(6c), as shown in Figure 7. Furthermore, we consider the parameters depending on time t based on Figure 7, but the result shows

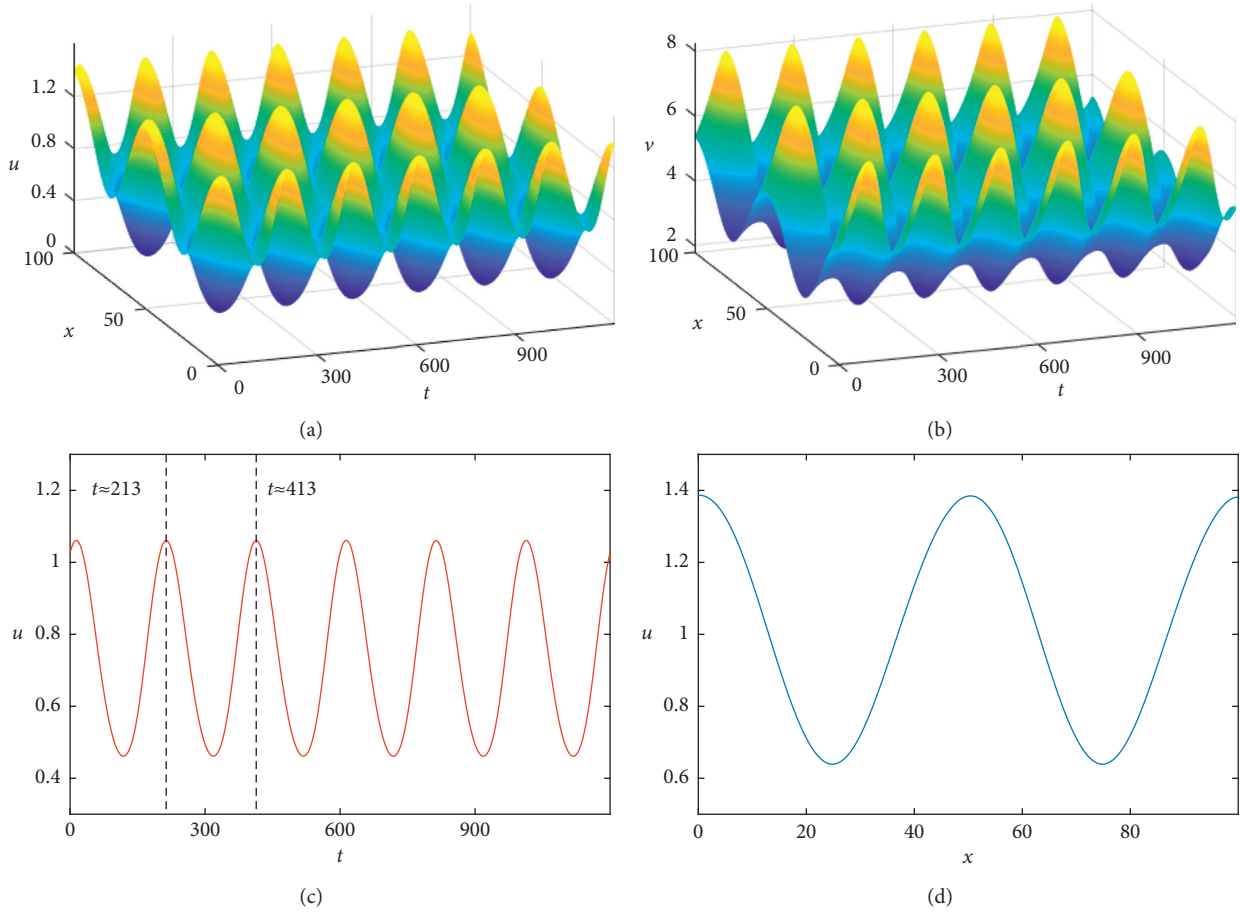


FIGURE 4: Numerical solutions of system (6a)–(6c): (a) prey population u , (b) predator population v , (c) profile of $u(t)$ at $x = 50$, and (d) profile of $u(x)$ at $t = 1200$. Here, $r(t, x) = 1.2 + 0.1 \sin(\pi * t/100) + 0.01 \cos(4 * \pi * x/100) * \sin(4 * \pi * x/100)$, $K(t, x) = 2 + 0.5 \cos(\pi * t/100) + \cos(4 * \pi * x/100)$, $a(t, x) = 0.8 + 0.005 \cos(\pi * t/100) + 0.1 \cos(4 * \pi * x/100)$, $b(t, x) = 0.9 + 0.005 \cos(\pi * t/100) + 0.1 \cos(4 * \pi * x/100)$, $e(t, x) = 0.7 - 0.005 \sin(\pi * t/100) + 0.1 \cos(4 * \pi * x/100)$, $c(t, x) = 0.02 + 0.01 \cos(\pi * t/100) + 0.001 \cos(4 * \pi * x/100)$, $\delta(t, x) = 0.12 + 0.05 \cos(\pi * t/100) + 0.01 \cos(4 * \pi * x/100)$, $\mu_1 = 1$, and $\mu_2 = 1$.

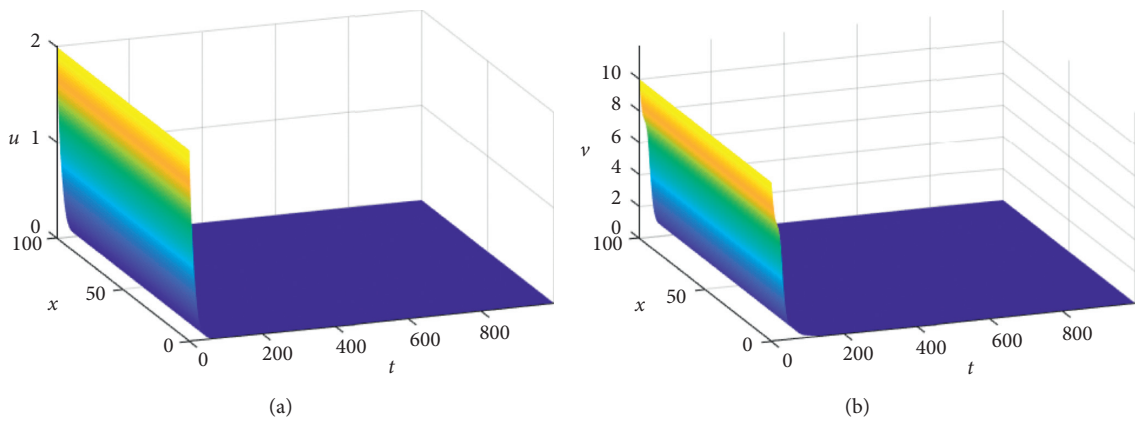


FIGURE 5: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v , where $r(t, x) = 0.9 + 0.001 \sin(5 * \pi * t/100)$, $K(t, x) = 10 + 0.5 \cos(5 * \pi * t/100)$, $a(t, x) = 0.8 + 0.005 \cos(5 * \pi * t/100)$, $b(t, x) = 0.78 + 0.005 \cos(5 * \pi * t/100)$, $e(t, x) = 0.7 - 0.005 \sin(5 * \pi * t/100)$, $c(t, x) = 0.02 + 0.001 \cos(5 * \pi * t/100)$, $\delta(t, x) = 0.12 + 0.05 \cos(5 * \pi * t/100)$, $\mu_1 = 1$, and $\mu_2 = 1$.

that the extinction of both prey and predator occurs (see Figure 8). However, when the parameter r depends on space x except for time t , it can be found from Figure 9 that system

(6a)–(6c) is still permanent. Although both Figures 7 and 9 show the spatial heterogeneity of population distribution, their natures are different. The spatial heterogeneity in

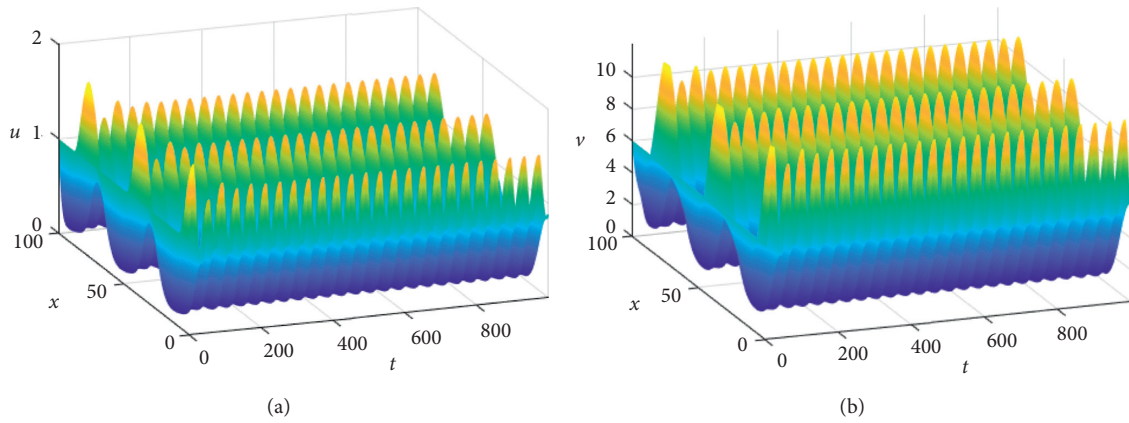


FIGURE 6: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v .

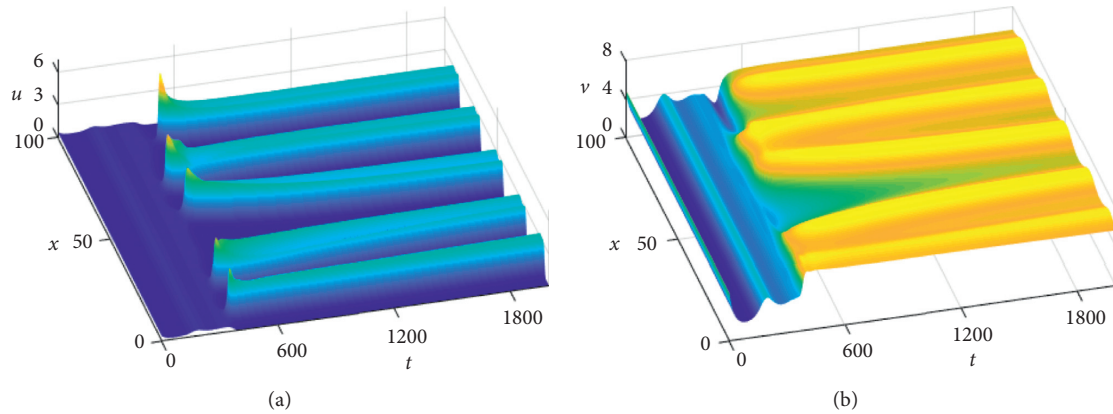


FIGURE 7: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v , where $r(t, x) = 0.9$, $K(t, x) = 10$, $a(t, x) = 0.8$, $b(t, x) = 0.8$, $e(t, x) = 0.7$, $c(t, x) = 0.02$, $\delta(t, x) = 0.12$, $\mu_1 = 0.01$, and $\mu_2 = 20$.

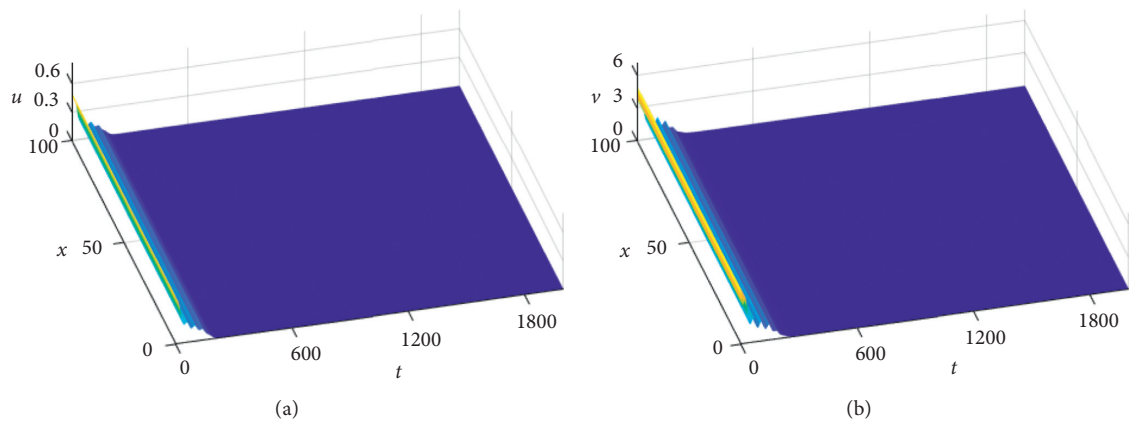


FIGURE 8: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v , where $r(t, x) = 0.9 + 0.001 \sin(5 * \pi * t/100)$, $K(t, x) = 10 + 0.5 \sin(5 * \pi * t/100)$, $a(t, x) = 0.8 + 0.005 \sin(5 * \pi * t/100)$, $b(t, x) = 0.8 + 0.005 \sin(5 * \pi * t/100)$, $e(t, x) = 0.7 - 0.005 \sin(5 * \pi * t/100)$, $c(t, x) = 0.02 + 0.001 \sin(5 * \pi * t/100)$, $\delta(t, x) = 0.12 + 0.05 \sin(5 * \pi * t/100)$, $\mu_1 = 0.01$, and $\mu_2 = 20$.

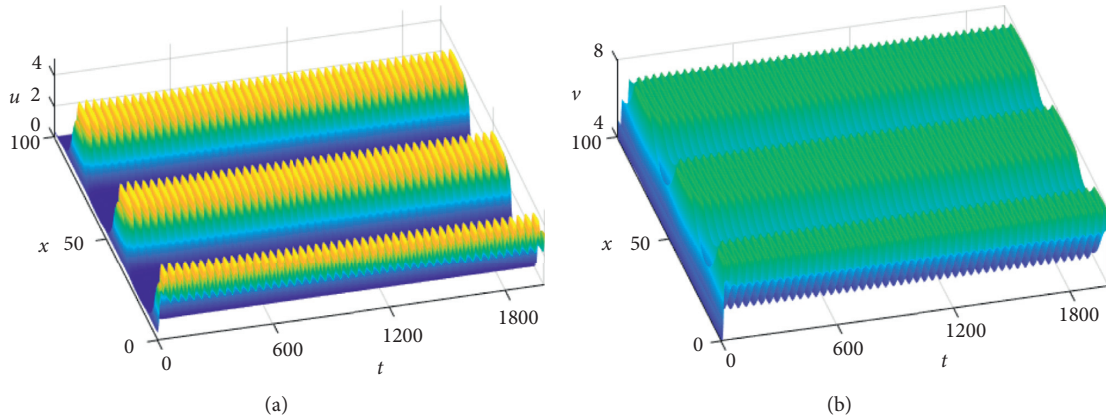


FIGURE 9: Numerical solutions of system (6a)–(6c): (a) prey population u and (b) predator population v , where $r(t, x) = 0.9 + 0.001 \sin(5 * \pi * t/100) + 0.1 \cos(t * \pi * x/100)$ and other parameters are the same as the ones in Figure 8.

Figure 7 is induced by the diffusion, while in Figure 9 it depends on the space variation of parameters. For other parameters of system (6a)–(6c), we can obtain similar results, which are omitted here.

5. Conclusion

In this paper, we first propose a reaction-diffusion system (6a)–(6c) to describe the interaction between the prey and the predator, where the spatial heterogeneity and the time-periodic environment are considered. In order to study the boundedness of solution, the positive invariance of system (6a)–(6c) is discussed, and the results demonstrate that nonnegative and positive quadrants of \mathbb{R}^2 are always positively invariant for system (6a)–(6c) when the condition (H) holds. Based on this, we find that all solutions of system (6a)–(6c) are ultimately bounded as long as the initial conditions are nonnegative. Also, we discuss the permanence of system (6a)–(6c) and obtain the sufficient conditions. Moreover, we derive the sufficient conditions for the extinction of predator population. Obviously, these conditions are very significant for studies of permanence and extinction of the system. When system (6a)–(6c) is permanent, we discuss the existence of a periodic solution, which suggests that a unique and strictly positive periodic solution with fixed period exists under certain conditions.

According to theoretical analysis, some numerical results are given, which show further dynamics in system (6a)–(6c). Results from literature [45, 46] indicate that Turing patterns can exist in system (5a)–(5c) (i.e., system (6a)–(6c) without spatial heterogeneity and time-periodic environment). After taking time-periodic environment into account, we find that both prey population and predator population are extinct. However, when the combination of spatial heterogeneity and time-periodic environment is considered, it is demonstrated that both prey population and predator population are permanent, which means that spatial heterogeneity tends to enhance the persistence of prey and predator population. Additionally, when prey population and predator population are permanent, our results show that solutions of system (6a)–(6c) seem to be periodic

because of the time-periodic environment and the spatial heterogeneity. Thus, we want to emphasize that spatial heterogeneity and time-periodic environment indeed play a significant role in prey-predator dynamics.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

Acknowledgments

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