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Research Article

An Interval Efficiency Measurement in DEA When considering Undesirable Outputs

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Data envelopment analysis (DEA) is a popular mathematical tool for analyzing the relative efficiency of homogenous decision-making units (DMUs). However, the existing DEA models cannot tackle the newly confronted applications with imprecise and negative data as well as undesirable outputs simultaneously. Thus, we introduce undesirable outputs into modified slack-based measure (MSBM) model and propose an interval-modified slack-based measure (IMSBM) model, which extends the application of interval DEA (IDEA) in fields that concern with less undesirable outputs. The novelties of the model are that it considers the undesirable outputs while dealing with imprecise and negative data, and it is slack-based. Furthermore, the model with undesirable outputs is proven translation-invariant and unit-invariant. Moreover, a numerical example is provided to illustrate the changes of the lower and upper bounds of the efficiency score after considering the undesirable outputs. The empirical results show that, without considering undesirable outputs, most of the lower bounds of the efficiency scores will be overestimated when the DMUs are weakly efficient and inefficient. The upper bound will also change after considering undesirable outputs when the DMU is inefficient. Finally, an improved degree of preference approach is introduced to rank the DMUs.

1. Introduction

Data envelopment analysis (DEA) is a popular mathematical tool for analyzing the relative efficiency of homogenous decision-making units (DMUs). With multiple inputs and outputs, DEA can measure the relative efficiency of DMUs by using a ratio of the weighted sum of outputs to the weighted sum of inputs. An efficient DMU always consumes less input to produce a specific amount of outputs or produces more outputs by consuming an equal amount of inputs. However, the conventional DEA models of CCR [1] and BCC [2] are based on two priori assumptions that limit their application: the input and output data should be first precise and, second, nonnegative.

Obtaining precise data in real-life situations is not always possible, so bounded (interval), ordinal, and ratio-bounded data are often used in applications [3, 4]. This precise data assumption can, in some cases, limit the applications of conventional DEA models. Cooper et al. [5] first introduced the imprecise (interval) DEA (IDEA) to cope with imprecise data, and many scholars have since contributed to the theoretical development of this method. Despotis and Smirlis [6] transformed a CCR model to handle interval data, and it gave a natural outcome in the form of lower and upper bounds of efficiency scores. However, the transformation was only applied to variables. Entani et al. [7] formulated dual models of the IDEA with an interval efficiency obtained from both optimistic and pessimistic viewpoints. Based on this, Wang et al. [8] developed a pair of interval models to convert ordinal preference information and fuzzy data into interval data through scale transformation and an α-level set, respectively. Wang et al. [9] further introduced a virtual anti-ideal DMU into a bounded DEA model to unify the best and the worst relative efficiencies under optimistic and pessimistic situations. However, Azizi and Jahed [10] pointed out that this assumed virtual anti-ideal DMU will make no sense when the input is zero and proposed a pair of improved IDEA models that make it possible to conduct a DEA analysis using the concepts of the best and worst relative efficiencies. Toloo et al. [11] constructed a pair of IDEA models based on pessimistic and optimistic standpoints to identify the unique status of each imprecise dual-role factor. Amir et al. [12] addressed the managerial and technical issues in allocating weights and in handling imprecise data through a total cost of ownership- (TCO-) based DEA approach. However, these models are not slack-based and can only deal with nonnegative data, indicating that these models can only measure radial efficiency with nonnegative data.

In addition to the assumption of precise data, conventional DEA models assume that all DMU inputs and outputs are nonnegative. However, this is not always possible in real-life problems when loss occurs, such as with profit or noninterest income. Traditionally, negative data are eliminated or transformed to positive through data transformation [13, 14]. However, eliminating the negative data will lose some DMU information, and the solution of the object function will be affected through the data transformation. Pastor [15] was the first to use the translation invariance property of DEA models when addressing negative data, which does not require the data to be eliminated or transformed. Halme et al. [16] introduced the property to radial models for dealing with interval data, including negative data. Hatamimarbini et al. [17] developed the interval semioriented radial measure (SORM) model to evaluate efficiency in the presence of interval data without sign restrictions. Cheng et al. [18] developed a variant of radial measure (VRM) to address variables, which could be negative or nonnegative, for different DMUs, but the efficiencies produced by the input-oriented VRM model may be negative [19] and those from the output-oriented VRM model can be in the range of [0.5, 1] [20]. To avoid such drawbacks, Tung [20] further defined two efficiency measures for input-oriented and output-oriented VRM models. Although the models mentioned above can deal with negative data, and some are translation-invariant and/or unit-invariant, they are still not slack-based models and ignore the inefficiency caused by nonradial slacks.

Thus, most developed models have addressed only imprecise data or only negative data rather than both simultaneously, and none are slack-based. Tone [21] proposed a slack-based measure SBM (a) of efficiency that puts aside assumptions about proportionate changes in inputs and outputs and deals directly with the input excesses and the output shortfalls of DMUs. Lotfi et al. [22] integrated the SBM (a) model into IDEA to address interval data from the optimistic perspective and defined the upper and lower bounds of the SBM-efficiency scores, to classify DMUs into three subsets. Azizi et al. [23] formulated SBM (a) models in IDEA from both optimistic and pessimistic perspectives to measure the overall performance of DMUs. These SBM (a)-based IDEA models measure the nonradial efficiency with interval data, but do not consider negative data. Sharp et al. [24] introduced the idea of the range-possible improvement into the SBM (a) model and developed a modified slack-based measure (MSBM) model to evaluate DMUs with negative data. The MSBM considers input and output slacks and possesses the property of being translation invariant. Tone et al. [25] proposed base point SBM (BP-SBM) models, which are consistent with ordinary SBM (a) models, to deal with negative data. Both MSBM model and BP-SBM models are slack-based and can handle negative data. However, they ignore imprecise data. Yang and Mo [26] considered these three characters simultaneously and extended the MSBM model to the interval MSBM (IMSBM) model, to evaluate the efficiency of particular DMUs with imprecise and negative data, and is also slack-based. However, the IMSBM model does not consider undesirable outputs. Tone [27] developed a new SBM (b) model from the SBM (a) model to measure efficiency in the presence of undesirable outputs. However, SBM (b) cannot yet deal with imprecise data.

This study develops the IMSBM model to address undesirable outputs, which extends the application of IDEA in fields that concern with less undesirable outputs, such as air pollutants, hazardous wastes, and nonperforming loans. Our new IMSBM model is based on SBM (b), unlike the current IDEA models, and thus it considers undesirable outputs and both radial and nonradial efficiencies from the perspectives of slacks. We also confirm that the new model is unit-invariant and translation-invariant. In Table 1, we compare the new IMSBM model with the other DEA models mentioned above.

The remainder of this paper is organized as follows. In Section 2, the IMSBM model with undesirable outputs is presented. Section 3 classifies DMUs into three subsets, and an improved degree of preference approach is introduced to rank the interval efficiencies. The IMSBM model with undesirable outputs is applied to evaluate the interval efficiency of Chinese city commercial banks in Section 4. The final section presents our conclusions.

2. The MSBM and IMSBM Models with Undesirable Outputs

Färe et al. [28] pointed out that the assumption of the constant returns to scale (CRS) suggested that any DMU could be radially expanded or contracted to form other feasible DMUs, which causes inconsistency with negative data. However, this is not the case under a variable returns to scale (VRS), so the models mentioned below are therefore assigned under the VRS.

2.1. The MSBM Model with Undesirable Outputs. First, we extend the MSBM proposed by Sharp et al. [24] to deal with undesirable outputs. Consider a set of n homogenous units under analysis, and each consumes varying amounts of m different inputs to produce s different outputs ($s = s_r + s_l$), where s_r is the number of good outputs and s_l is the number of bad (undesirable) outputs. Specifically,

Table 1: Comparison of the new IMSBM model with other DEA models.

	IDEA	SBM (a)-based IDEA	Interval SORM	SBM (b)	BP-SBM	MSBM	IMSBM	New IMSBM
Interval data	√	√	√	×	×	×	√	√
Negative data	×	×	√	×	√	√	√	√
Slacks-based	×	√	×	√	√	√	√	√
Undesirable outputs	×	×	×	√	×	×	×	√

DMU_j $(j = 1, 2, \ldots, n)$ consumes x_{ij} $(i = 1, 2, \ldots, m)$ of each input to produce y^g_{rj} $(r = 1, 2, \ldots, s_r)$ of each good output and y^b_{lj} $(l = 1, 2, \ldots, s_l)$ of each bad output. The inputs, good outputs, and bad outputs can be represented by three vectors $X = (x_{ij}) \in R^{m\times n}$ $(j = 1, 2, \ldots, n,\ i = 1, 2, \ldots, m)$, $Y^g = (y^g_{rj}) \in R^{s_r\times n}$ $(r = 1, 2, \ldots, s_r)$, and $Y^b = (y^b_{lj}) \in R^{s_l\times n}$ $(l = 1, 2, \ldots, s_l)$, respectively. Then, the production possibility set (P) under VRS assumption is defined as

$$P = \left\{ \left(x, y^g, y^b\right) | x \ge \sum_{j=1}^{n} x_j\lambda_j,\ y^g \le \sum_{j=1}^{n} y^g_j\lambda_j,\ y^b \ge \sum_{j=1}^{n} y^b_j\lambda_j,\quad \lambda \ge 0 \ \sum_{j=1}^{n}\lambda_j = 1 \right\}, \tag{1}$$

where $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T$ is the intensity vector and $\sum_{j=1}^{n}\lambda_j = 1$ keeps P under VRS assumption. A DMU_k (x_k, y^g_k, y^b_k) is efficient in the presence of undesirable outputs if there is no vector $(x_k, y^g_k, y^b_k) \in P$ such that $x_k \ge x$, $y^g_k \le y^g$ and $y^b_k \ge y^b$ with at least one strict inequality [27]. When considering input and output slacks, i.e., input exceeds (s^-), good output shortfalls (s^{g+}), and undesirable outputs exceeds (s^{b-}), the production possibility set (P') under VRS assumption can be defined as

$$P' = \left\{ \left(x, y^g, y^b\right) \middle| x = \sum_{j=1}^{n} x_j\lambda_j + s^-,\ y^g = \sum_{j=1}^{n} y^g_j\lambda_j - s^{g+},\ y^b = \sum_{j=1}^{n} y^b_j\lambda_j + s^{b-},\quad \lambda \ge 0 \ \sum_{j=1}^{n}\lambda_j = 1 \right\}. \tag{2}$$

We now introduce the ideal point into the MSBM model with undesirable outputs. For a given dataset, the ideal point is considered as $I = (\min_j x_{ij} (i = 1, 2, \ldots, m), \max_j y^g_{rj} (r = 1, 2, \ldots, s_r), \min_j y^b_{lj} (l = 1, 2, \ldots, s_l))$. Therefore, for DMU_k, the range of possible improvement is defined as

$$\begin{aligned} R^-_{ik} &= x_{ik} - \min_j\left\{x_{ij}\right\}, \quad i = 1, 2, \ldots, m, \\ R^{g+}_{rk} &= \max_j\left\{y^g_{rj}\right\} - y^g_{rk}, \quad r = 1, 2, \ldots, s_r, \\ R^{b-}_{lk} &= y^b_{lk} - \min_j\left\{y^b_{lj}\right\}, \quad l = 1, 2, \ldots, s_l. \end{aligned} \tag{3}$$

Obviously, $R^-_{ik}, R^{g+}_{rk}, R^{b-}_{lk} \ge 0$. Replacing the corresponding terms in the SBM (b) model with R^-_{ik}, R^{g+}_{rk} and R^{b-}_{lk}, the MSBM model with undesirable outputs is thus

$$\min \rho_k = \frac{1 - \sum_{i=1}^{m} \left(w_i s^-_{ik}\right)/R^-_{ik}}{1 + \sum_{r=1}^{s_r}\left(v_r s^{g+}_{rk}\right)/R^{g+}_{rk} + \sum_{l=1}^{s_l}\left(v_l s^{b-}_{lk}\right)/R^{b-}_{lk}}, \tag{4}$$

$$\text{subject to} \begin{cases} x_{ik} = \sum_{j=1}^{n} x_{ij}\lambda_j + s^-_{ik}, & i = 1, 2, \ldots, m, \\ y^g_{rk} = \sum_{j=1}^{n} y^g_{rj}\lambda_j - s^{g+}_{rk}, & r = 1, 2, \ldots, s_r, \\ y^b_{lk} = \sum_{j=1}^{n} y^b_{lj}\lambda_j + s^{b-}_{lk}, & l = 1, 2, \ldots, s_l, \\ \sum_{j=1}^{n}\lambda_j = 1, & j = 1, 2, \ldots, n, \\ s^-_{ik}, s^{b+}_{rk}, s^{b-}_{lk}, \lambda_j \ge 0. \end{cases} \tag{5}$$

According to Tone [29] and Cooper et al. [30], in formula (4), the minimization of the numerator can be interpreted as the MSBM-input-efficiency, that is, $\rho^I_k = \min[1 - \sum_{i=1}^{m}(w_i s^-_{ik})/R^-_{ik}]$. In addition, the reciprocal of the maximization of the denominator can be interpreted as the MSBM-output-efficiency, that is, $\rho^O_k = 1/\max[1 + \sum_{r=1}^{s_r}(v_r s^{g+}_{rk})/R^{g+}_{rk} + \sum_{l=1}^{s_l}(v_l s^{b-}_{lk})/R^{b-}_{lk}]$. Therefore, the MSBM nonoriented efficiency can be defined as $\min \rho_k$ through multiplying ρ^I_k by ρ^O_k, and $\min \rho_k$ subjects to P'. In formulas (4) and (5), s^-_{ik}, s^{g+}_{rk} and s^{b-}_{lk} are slacks in the i^{th} input, r^{th} good output and l^{th} bad output of DMU_k, respectively. The weights of each input w_i, good output v_r, and bad output v_l are determined subjectively by decision-makers and subject to $\sum_{i=1}^{m} w_i = 1$, $\sum_{r=1}^{s_r} v_r + \sum_{l=1}^{s_l} v_l = 1$, w_i,v_r, and $v_l > 0$.

Note that when $R^-_{ik} = 0$, $R^{g+}_{rk} = 0$, or $R^{b-}_{lk} = 0$, it is assumed that the corresponding $w_i s^-_{ik}/R^-_{ik}$, $v_r s^{g+}_{rk}/R^{g+}_{rk}$, or $v_l s^{b-}_{lk}/R^{b-}_{lk}$ is dropped from the numerator or denominator [24].

2.2. The IMSBM Model with Undesirable Outputs. The IMSBM model with undesirable outputs can be defined based on the MSBM model with undesirable outputs.

For the IMSBM model with undesirable outputs, the inputs, good outputs, and bad outputs are assumed to be interval variables denoted as $x_{ij} \in [\underline{x}_{ij}, \overline{x}_{ij}]$, $y^g_{rj} \in [\underline{y}^g_{rj}, \overline{y}^g_{rj}]$, and $y^b_{lj} \in [\underline{y}^b_{lj}, \overline{y}^b_{lj}]$, where $\underline{x}_{ij}$ is the lower bound of x_{ij}, $\overline{x}_{ij}$ is the upper bound of x_{ij}, $\underline{y}^g_{rj}$ is the lower bound of y^g_{rj}, $\overline{y}^g_{rj}$ is the upper bound of y^g_{rj}, $\underline{y}^b_{lj}$ is the lower bound of y^b_{lj}, and $\overline{y}^b_{lj}$

is the upper bound of y^b_{lj}. In this case, the ideal point in the IMSBM model with undesirable outputs is considered as $I = (\min_j \underline{x}_{ij} (i = 1, 2, \ldots, m), \max_j \overline{y}^g_{rj} (r = 1, 2, \ldots, s_r), \min_j \underline{y}^b_{lj} (l = 1, 2, \ldots, s_l))$. Consequently, for DMU_k, the range of possible improvement is defined as

$$\begin{aligned}
\left[\underline{R}^-_{ik}, \overline{R}^-_{ik}\right] &= \left[\underline{x}_{ik} - \min_j\left\{\underline{x}_{ij}\right\}, \overline{x}_{ik} - \min_j\left\{\underline{x}_{ij}\right\}\right], \quad i = 1, 2, \ldots, m, \\
\left[\underline{R}^{g+}_{rk}, \overline{R}^{g+}_{rk}\right] &= \left[\max_j\left\{\overline{y}^g_{rj}\right\} - \overline{y}^g_{rk}, \max_j\left\{\overline{y}^g_{rj}\right\} - \underline{y}^g_{rk}\right], \quad r = 1, 2, \ldots, s_r, \\
\left[\underline{R}^{b-}_{lk}, \overline{R}^{b-}_{lk}\right] &= \left[\underline{y}^b_{lk} - \min_j\left\{\underline{y}^b_{lj}\right\}, \overline{y}_{lk} - \min_j\left\{\underline{y}^b_{lj}\right\}\right], \quad l = 1, 2, \ldots, s_l.
\end{aligned} \tag{6}$$

Obviously, $\underline{R}^-_{ik}, \overline{R}^-_{ik}, \underline{R}^{g+}_{rk}, \overline{R}^{g+}_{rk}, \underline{R}^{b-}_{lk}, \overline{R}^{b-}_{lk} \geq 0$. Therefore, the IMSBM model with undesirable outputs for DMU_k is defined as

$$\min \rho_k = \frac{1 - \sum_{i=1}^m w_i \left[\underline{s}^-_{ik}, \overline{s}^-_{ik}\right] / \left[\underline{R}^-_{ik}, \overline{R}^-_{ik}\right]}{1 + \left(\sum_{r=1}^{s_r} v_r \left[\underline{s}^{g+}_{rk}, \overline{s}^{g+}_{rk}\right] / \left[\underline{R}^{g+}_{rk}, \overline{R}^{g+}_{rk}\right] + \sum_{l=1}^{s_l} v_l \left[\underline{s}^{b-}_{lk}, \overline{s}^{b-}_{lk}\right] / \left[\underline{R}^{b-}_{lk}, \overline{R}^{b-}_{lk}\right]\right)}, \tag{7}$$

$$\text{subject to} \begin{cases}
\left[\underline{x}_{ik}, \overline{x}_{ik}\right] = \displaystyle\sum_{j=1}^n \left[\underline{x}_{ij}, \overline{x}_{ij}\right]\lambda_j + \left[\underline{s}^-_{ik}, \overline{s}^-_{ik}\right], \quad i = 1, 2, \ldots, m, \\
\left[\underline{y}^g_{rk}, \overline{y}^g_{rk}\right] = \displaystyle\sum_{j=1}^n \left[\underline{y}^g_{rj}, \overline{y}^g_{rj}\right]\lambda_j - \left[\underline{s}^{g+}_{rk}, \overline{s}^{g+}_{rk}\right], \quad r = 1, 2, \ldots, s_r, \\
\left[\underline{y}^b_{lk}, \overline{y}^b_{lk}\right] = \displaystyle\sum_{j=1}^n \left[\underline{y}^b_{lj}, \overline{y}^b_{lj}\right]\lambda_j + \left[\underline{s}^{b-}_{lk}, \overline{s}^{b-}_{lk}\right], \quad l = 1, 2, \ldots, s_l, \\
\displaystyle\sum_{j=1}^n \lambda_j = 1, \quad j = 1, 2, \ldots, n, \\
\underline{s}^-_{ik}, \overline{s}^-_{ik}, \underline{s}^{g+}_{rk}, \overline{s}^{g+}_{rk}, \underline{s}^{b-}_{lk}, \overline{s}^{b-}_{lk}, \lambda_j \geq 0,
\end{cases} \tag{8}$$

where ρ_k is interval data denoted as $[\underline{\rho}_k, \overline{\rho}_k]$. Similarly, when $\underline{R}^-_{ik} (\overline{R}^-_{ik})$, $\underline{R}^{g+}_{rk} (\overline{R}^{g+}_{rk})$ or $\underline{R}^{b-}_{lk} (\overline{R}^{b-}_{lk})$ is zero, the corresponding term is assumed to be dropped from the numerator or denominator.

The lower bound of the interval efficiency $\underline{\rho}_k$ is under the most unfavourable situation for DMU_k. Thus, DMU_k consumes $\overline{x}_{ik}$ to produce $\underline{y}^g_{rk}$ and $\overline{y}^b_{lk}$, while DMU_j consumes $\underline{x}_{ij}$ to produce $\overline{y}^g_{rj}$ and $\underline{y}^b_{lj} (j \neq k)$. Symmetrically, the upper bound of the efficiency $\overline{\rho}_k$ is the most favourable situation for DMU_k. Thus, DMU_k consumes $\underline{x}_{ik}$ to produce $\overline{y}^g_{rk}$ and $\underline{y}^b_{lk}$, while DMU_j consumes $\overline{x}_{ij}$ to produce $\underline{y}^g_{rj}$ and $\overline{y}^b_{lj} (j \neq k)$. Therefore, models (7) and (8) interpret the IMSBM model with undesirable outputs as a whole, including the relative efficiencies under the most unfavourable and favourable situations. Subsequently, they can be divided into a pair of precise models, the lower efficiency models, and the upper efficiency models. Models (9) and (10) interpret the lower efficiency under the most unfavourable situation for DMU_k; inversely, models (11) and (12) interpret the upper efficiency under the most favourable situation [26, 31]:

$$\min \underline{\rho}_k = \frac{1 - \sum_{i=1}^m w_i \overline{s}^-_{ik} / \overline{R}^-_{ik}}{1 + \left(\sum_{r=1}^{s_r} \left(v_r \underline{s}^{g+}_{rk}\right) / \underline{R}^{g+}_{rk} + \sum_{l=1}^{s_l} \left(v_l \overline{s}^{b-}_{lk}\right) / \overline{R}^{b-}_{lk}\right)}, \tag{9}$$

$$\text{subject to} \begin{cases}
\overline{x}_{ik} = \displaystyle\sum_{j=1, j\neq k}^n \underline{x}_{ij}\lambda_j + \overline{x}_{ik}\lambda_k + \overline{s}^-_{ik}, \quad i = 1, 2, \ldots, m, \\
\underline{y}^g_{rk} = \displaystyle\sum_{j=1 j\neq k}^n \overline{y}^g_{rj}\lambda_j + \underline{y}^g_{rk}\lambda_k - \underline{s}^{g+}_{rk}, \quad r = 1, 2, \ldots, s_r, \\
\overline{y}^b_{lk} = \displaystyle\sum_{j=1, j\neq k}^n \underline{y}^b_{lj}\lambda_j + \overline{y}^b_{lk}\lambda_k + \overline{s}^{b-}_{lk}, \quad l = 1, 2, \ldots, s_l, \\
\displaystyle\sum_{j=1}^n \lambda_j = 1, \quad j = 1, 2, \ldots, n, \\
\overline{s}^-_{ik}, \underline{s}^{g+}_{rk}, \overline{s}^{b-}_{lk}, \lambda_j \geq 0,
\end{cases} \tag{10}$$

$$\min \overline{\rho}_k = \frac{1 - \sum_{i=1}^m w_i \underline{s}^-_{ik} / \underline{R}^-_{ik}}{1 + \left(\sum_{r=1}^{s_r} \left(v_r \overline{s}^{g+}_{rk}\right) / \overline{R}^{g+}_{rk} + \sum_{l=1}^{s_l} \left(v_l \underline{s}^{b-}_{lk}\right) / \underline{R}^{b-}_{lk}\right)}, \tag{11}$$

$$\text{subject to}\begin{cases}\underline{x}_{ik}=\sum_{j=1,j\neq k}^{n}\overline{x}_{ij}\lambda_j+\underline{x}_{ik}\lambda_k+\underline{s}_{ik}^{-}, & i=1,2,\ldots,m,\\ \overline{y}_{rk}^{g}=\sum_{j=1\,j\neq k}^{n}\underline{y}_{rj}^{g}\lambda_j+\overline{y}_{rk}^{g}\lambda_k-\overline{s}_{rk}^{g+}, & r=1,2,\ldots,s_r,\\ \underline{y}_{lk}^{b}=\sum_{j=1,j\neq k}^{n}\overline{y}_{lj}^{b}\lambda_j+\underline{y}_{lk}^{b}\lambda_k+\underline{s}_{lk}^{b-}, & l=1,2,\ldots,s_l,\\ \sum_{j=1}^{n}\lambda_j=1, & j=1,2,\cdots,n,\\ \underline{s}_{ik}^{-},\overline{s}_{rk}^{g+},\underline{s}_{lk}^{b-},\lambda_j\geq 0.\end{cases}\tag{12}$$

According to the Charnesa and Cooper transformation [32] and referring to [33–35], the IMSBM model with undesirable outputs can be transformed into a linear programming form. We multiply a scalar variable $t\,(t>0)$ for both the numerator and the denominator of the objective function of (9) which does not impact $\underline{\rho}_k$. By adjusting t and if the denominator equals 1, then the denominator can be regarded as a constraint, and the objective function minimises the corresponding numerator. The lower bound of the IMSBM model with undesirable outputs is

$$\min\underline{\tau}_k=t-\sum_{i=1}^{m}\frac{w_i t\overline{s}_{ik}^{-}}{\overline{R}_{ik}^{-}},\tag{13}$$

$$\text{subject to}\begin{cases}1=t+\sum_{r=1}^{s_r}\frac{\left(v_r t\underline{s}_{rk}^{g+}\right)}{\underline{R}_{rk}^{g+}}+\sum_{l=1}^{s_l}\frac{\left(v_l t\overline{s}_{lk}^{b-}\right)}{\overline{R}_{lk}^{b-}},\\ t\overline{x}_{ik}=\sum_{j=1,j\neq k}^{n}\underline{x}_{ij}t\lambda_j+\overline{x}_{ik}t\lambda_k+t\overline{s}_{ik}^{-}, & i=1,2,\ldots,m,\\ t\underline{y}_{rk}^{g}=\sum_{j=1\,j\neq k}^{n}\overline{y}_{rj}^{g}t\lambda_j+\underline{y}_{rk}^{g}t\lambda_k-t\underline{s}_{rk}^{g+}, & r=1,2,\ldots,s_r,\\ t\overline{y}_{lk}^{b}=\sum_{j=1,j\neq k}^{n}\underline{y}_{lj}^{b}t\lambda_j+\overline{y}_{lk}^{b}t\lambda_k+t\overline{s}_{lk}^{b-}, & l=1,2,\ldots,s_l,\\ \sum_{j=1}^{n}t\lambda_j=t, & j=1,2,\ldots,n,\\ \overline{s}_{ik}^{-},\underline{s}_{rk}^{g+},\overline{s}_{lk}^{b-},\lambda_j\geq 0.\end{cases}\tag{14}$$

Formula (14) is a nonlinear programming problem due to its nonlinear terms, and some definitions are needed to transform it into a linear programming problem. Assume

$$\overline{S}_{ik}^{-}=t\overline{s}_{ik}^{-},\underline{S}_{rk}^{g+}=t\underline{s}_{rk}^{g+},\overline{S}_{lk}^{b-}=t\overline{s}_{lk}^{b-},\Lambda_j=t\lambda_j.\tag{15}$$

Obviously, $\overline{S}_{ik}^{-},\underline{S}_{rk}^{g+},\overline{S}_{lk}^{b-},\Lambda_j\geq 0$, and the transformed problem is

$$\min\underline{\tau}_k=t-\sum_{i=1}^{m}\frac{w_i\overline{S}_{ik}^{-}}{\overline{R}_{ik}^{-}},\tag{16}$$

$$\text{subject to}\begin{cases}1=t+\sum_{r=1}^{s_r}\frac{v_r\underline{S}_{rk}^{g+}}{\underline{R}_{rk}^{g+}}+\sum_{l=1}^{s_l}\frac{v_l\overline{S}_{lk}^{b-}}{\overline{R}_{lk}^{b-}},\\ t\overline{x}_{ik}=\sum_{j=1,j\neq k}^{n}\underline{x}_{ij}\Lambda_j+\overline{x}_{ik}\Lambda_k+\overline{S}_{ik}^{-}, & i=1,2,\ldots,m,\\ t\underline{y}_{rk}^{g}=\sum_{j=1\,j\neq k}^{n}\overline{y}_{rj}^{g}\Lambda_j+\underline{y}_{rk}^{g}\Lambda_k-\underline{S}_{rk}^{g+}, & r=1,2,\ldots,s_r,\\ t\overline{y}_{lk}^{b}=\sum_{j=1,j\neq k}^{n}\underline{y}_{lj}^{b}\Lambda_j+\overline{y}_{lk}^{b}\Lambda_k+\overline{S}_{lk}^{b-}, & l=1,2,\ldots,s_l,\\ \sum_{j=1}^{n}\Lambda_j=t, & j=1,2,\ldots,n,\\ \overline{S}_{ik}^{-},\underline{S}_{rk}^{g+},\overline{S}_{lk}^{b-},\Lambda_j\geq 0.\end{cases}\tag{17}$$

Assuming the optimal solution of (16) and (17) to be $(\underline{\tau}_k^{*},t^{*},\Lambda^{*},\overline{S}_{ik}^{-*},\underline{S}_{rk}^{g+*},\overline{S}_{lk}^{b-*})$, then, according to (15), the optimal solution of (9) and (10) can be obtained numerically as

$$\underline{\rho}_k^{*}=\underline{\tau}_k^{*},\lambda^{*}=\frac{\Lambda^{*}}{t^{*}},\overline{s}_{ik}^{-*}=\frac{\overline{S}_{ik}^{-*}}{t^{*}},\underline{s}_{rk}^{g+*}=\frac{\underline{S}_{rk}^{g+*}}{t^{*}},\overline{s}_{lk}^{b-*}=\frac{\overline{S}_{lk}^{b-*}}{t^{*}}.\tag{18}$$

Symmetrically, the transformed problem of (11) and (12) is

$$\min\overline{\tau}_k=t-\sum_{i=1}^{m}\frac{w_i\underline{S}_{ik}^{-}}{\underline{R}_{ik}^{-}},\tag{19}$$

$$\text{subject to}\begin{cases}1=t+\sum_{r=1}^{s_r}\frac{\left(v_r\overline{S}_{rk}^{g+}\right)}{\overline{R}_{rk}^{g+}}+\sum_{l=1}^{s_l}\frac{\left(v_l\underline{S}_{lk}^{b-}\right)}{\underline{R}_{lk}^{b-}},\\ t\underline{x}_{ik}=\sum_{j=1,j\neq k}^{n}\overline{x}_{ij}\Lambda_j+\underline{x}_{ik}\Lambda_k+\underline{S}_{ik}^{-}, & i=1,2,\ldots,m,\\ t\overline{y}_{rk}^{g}=\sum_{j=1\,j\neq k}^{n}\underline{y}_{rj}^{g}\Lambda_j+\overline{y}_{rk}^{g}\Lambda_k-\overline{S}_{rk}^{g+}, & r=1,2,\ldots,s_r,\\ t\underline{y}_{lk}^{b}=\sum_{j=1,j\neq k}^{n}\overline{y}_{lj}^{b}\Lambda_j+\underline{y}_{lj}^{b}\Lambda_k+\underline{S}_{lk}^{b-}, & l=1,2,\ldots,s_l,\\ \sum_{j=1}^{n}\Lambda_j=t, & j=1,2,\ldots,n,\\ \underline{s}_{ik}^{-},\overline{s}_{rk}^{g+},\underline{s}_{lk}^{b-},\lambda_j\geq 0.\end{cases}\tag{20}$$

Assuming the optimal solution of (19) and (20) to be $(\overline{\tau}_k^*, t^*, \Lambda^*, \underline{S}_{ik}^{-*}, \overline{S}_{rk}^{g+*}, \underline{S}_{lk}^{b-*})$, then, according to (15), the optimal solution of (11) and (12) can be obtained numerically as

$$\begin{aligned} \overline{\rho}_k^* &= \overline{\tau}_k^*, \lambda^* = \frac{\Lambda^*}{t^*}, \\ \underline{s}_{ik}^{-*} &= \frac{\underline{S}_{ik}^{-*}}{t^*}, \\ \overline{s}_{rk}^{+*} &= \frac{\overline{S}_{rk}^{g+*}}{t^*}, \\ \underline{s}_{lk}^{-*} &= \frac{\underline{S}_{lk}^{b-*}}{t^*}. \end{aligned} \tag{21}$$

If the lower bound of DMU_k is inefficient, it can be improved to become efficient by

$$\begin{aligned} \overline{x}_{ik} &\longleftarrow \overline{x}_{ik} - \overline{s}_{ik}^{-*}, \quad i = 1, 2, \ldots, m, \\ \underline{y}_{rk} &\longleftarrow \underline{y}_{rk} + \underline{s}_{rk}^{+*}, \quad r = 1, 2, \ldots, s_r, \\ \overline{y}_{lk} &\longleftarrow \overline{y}_{lk} - \overline{s}_{lk}^{-*}, \quad l = 1, 2, \ldots, s_l. \end{aligned} \tag{22}$$

Symmetrically, if the upper bound of DMU_k is inefficient, it can be improved to become efficient by

$$\begin{aligned} \underline{x}_{ik} &\longleftarrow \underline{x}_{ik} - \underline{s}_{ik}^{-*}, \quad i = 1, 2, \ldots, m, \\ \overline{y}_{rk} &\longleftarrow \overline{y}_{rk} + \overline{s}_{rk}^{+*}, \quad r = 1, 2, \ldots, s_r, \\ \underline{y}_{lk} &\longleftarrow \underline{y}_{lk} - \underline{s}_{lk}^{-*}, \quad l = 1, 2, \ldots, s_l. \end{aligned} \tag{23}$$

2.3. Properties of the IMSBM Model with Undesirable Outputs. The following properties are considered the bases of designing an efficiency measure [1].

Property 1 (translation-invariant). This is critical, particularly when input-output data contain zero or negative values.

Property 2 (units-invariant). This is considered an important property in DEA, and in general mathematical terms, this property is referred to as dimensionless.

Theorem 1. *The IMSBM model with undesirable outputs is translation-invariant.*

Proof. A measure is translation-invariant if and only if the model is equivalent before and after the translation [36].

Transform the input data $\underline{x}_{ij}$ and $\overline{x}_{ij}$ by the real number $z_i\,(i = 1, 2, \ldots, m)$, transform the good output data $\underline{y}_{rj}$ and $\overline{y}_{rj}\,(r = 1, 2, \ldots, s_r)$ by the real number $t_r\,(r = 1, 2, \ldots, s_r)$, and transform the bad output data $\underline{y}_{lj}$ and $\overline{y}_{lj}\,(l = 1, 2, \ldots, s_l)$ by the real number $t_l\,(l = 1, 2, \ldots, s_l)$, where z_i subjects to $\underline{x}_{ij} + z_i \geq 0\,(\forall j = 1, 2, \ldots, n)$, t_r subjects to $\overline{y}_{rj} + t_r \geq 0\,(\forall j = 1, 2, \ldots, n)$, and t_l subjects to $\underline{y}_{lj} + t_l \geq 0\,(\forall j = 1, 2, \ldots, n)$ (without loss of generality, z_i, t_r, and t_l are assumed to be nonnegative). Models (9) and (10) for the translated data are

$$\min \underline{\rho}_k' = \frac{1 - \sum_{i=1}^{m} \left(w_i \overline{s}_{ik}^{-'} \right) / \overline{R}_{ik}^{-'}}{1 + \left(\sum_{r=1}^{s_r} \left(v_r \underline{s}_{rk}^{g+'} \right) / \underline{R}_{rk}^{g+'} + \sum_{l=1}^{s_l} \left(v_l \overline{s}_{lk}^{b-'} \right) / \overline{R}_{lk}^{b-'} \right)}, \tag{24}$$

$$\text{subject to} \begin{cases} \overline{x}_{ik}' = \sum_{j=1, j\neq k}^{n} \underline{x}_{ij}' \lambda_j + \overline{x}'_{ik} \lambda_k + \overline{s}_{ik}^{-'}, & i = 1, 2, \ldots, m, \\ \underline{y}_{rk}^{g'} = \sum_{j=1 j\neq k}^{n} \overline{y}_{rj}^{g'} \lambda_j + \underline{y}_{rk}^{g'} \lambda_k - \underline{s}_{rk}^{g+'}, & r = 1, 2, \ldots, s_r, \\ \overline{y}_{lk}^{b'} = \sum_{j=1, j\neq k}^{n} \underline{y}_{lj}^{b'} \lambda_j + \overline{y}_{lk}^{b'} \lambda_k + \overline{s}_{lk}^{b-'}, & l = 1, 2, \ldots, s_l, \\ \sum_{j=1}^{n} \lambda_j = 1, \quad j = 1, 2, \ldots, n, \\ \sum_{i=1}^{m} w_i = 1, \sum_{r=1}^{s_r} v_r + \sum_{l=1}^{s_l} v_l = 1, \\ w_i, v_r, v_l, \overline{s}_{ik}^{-}, \underline{s}_{rk}^{g+}, \overline{s}_{lk}^{b-}, \lambda_j \geq 0. \end{cases} \tag{25}$$

where $\underline{x}_{ij}' = \underline{x}_{ij} + z_i$, $\overline{x}_{ik}' = \overline{x}_{ik} + z_i$, $\underline{y}_{rj}^{g'} = \underline{y}_{rj}^{g} + t_r$, $\overline{y}_{rj}^{g'} = \overline{y}_{rj}^{g} + t_r$, $\underline{y}_{lj}^{b'} = \underline{y}_{lj}^{b} + t_l$, and $\overline{y}_{lj}^{b'} = \overline{y}_{lj}^{b} + t_l$. It can be verified that $\overline{R}_{ik}^{-'} = \overline{R}_{ik}^{-}$, $\underline{R}_{rk}^{g+'} = \underline{R}_{rk}^{g+}$ and $\overline{R}_{rk}^{b-'} = \overline{R}_{rk}^{b-}$. As $\sum_{j=1}^{n} \lambda_j = 1$, the constraints in (25) imply $\overline{s}_{ik}^{-'} = \overline{s}_{ik}^{-}$, $\underline{s}_{rk}^{g+'} = \underline{s}_{rk}^{g+}$, and $\overline{s}_{lk}^{b-'} = \overline{s}_{lk}^{b-}$. Therefore models (9) and (10) and (24) and (25) are equivalent problems, and thus models (9) and (10) are translation-invariant. Models (11) and (12) can similarly be proven to be translation-invariant. Therefore, the IMSBM model with undesirable outputs is translation-invariant. This proof is thus complete. □

Theorem 2. *IMSBM model with undesirable outputs is unit-invariant.*

Proof. Consider the g^{th} input, the h^{th} good output, and the q^{th} bad output in the models (9) and (10), rescale both bounds of the g^{th} input by multiplying it by a scalar $\alpha > 0$, rescale both bounds of the h^{th} good output by multiplying it by a scalar $\beta > 0$, and rescale both bounds of the q^{th} bad output by multiplying it by a scalar $\gamma > 0$. The ideal point is $I_{\alpha,\beta,\gamma} = (\min_j \underline{x}_{ij}\,(i = 1, 2, \ldots, m, i \neq g), \min_j (\alpha \underline{x}_{gj}), \max_j \overline{y}_{rj}\,(r = 1, 2, \ldots, s_1, r \neq h), \max_j (\beta \overline{y}_{hj}), \min_j \underline{y}_{lj}\,(l = 1, 2, \ldots, s_2, l \neq q), \min_j (\gamma \underline{y}_{qj}))$. It can be proven that $\overline{R}_{ik,\alpha}^{-} = \overline{R}_{ik}^{-}\,(i \neq g)$, $\overline{R}_{gk,\alpha}^{-} = \alpha \overline{R}_{gk}^{-}$, $\underline{R}_{rk,\beta}^{+} = \underline{R}_{rk}^{+}\,(r \neq h)$, $\underline{R}_{hk,\beta}^{+} = \beta \underline{R}_{hk}^{+}$, $\overline{R}_{lk,\gamma}^{-} = \overline{R}_{lk}^{-}\,(l \neq q)$, and $\overline{R}_{qk,\gamma}^{-} = \gamma \overline{R}_{qk}^{-}$. From the constraints, we have $\overline{s}_{ik,\alpha}^{-} = \overline{s}_{ik}^{-}\,(i \neq g)$, $\overline{s}_{gk,\alpha}^{-} = \alpha \overline{s}_{gk}^{-}$, $\underline{s}_{rk,\beta}^{+} = \underline{s}_{rk}^{+}\,(r \neq h)$, $\underline{s}_{hk,\beta}^{+} = \beta \underline{s}_{hk}^{+}$, $\overline{s}_{lk,\gamma}^{-} = \overline{s}_{lk}^{-}\,(l \neq q)$, and $\overline{s}_{qk,\gamma}^{-} = \gamma \overline{s}_{qk}^{-}$. The rescaling does not impact $\underline{\rho}_k$. Thus, models (9) and (10) are unit-invariant.

Models (11) and (12) can be similarly proven. Therefore, the IMSBM with undesirable outputs is unit-invariant. This proof is thus complete. □

3. Classification and Ranking of the DMUs

The efficiency scores measured by the IMSBM model with undesirable outputs are calculated in an interval form, and thus a simple and practical approach is required to compare and rank the performance of the DMUs.

Haghighat and Khorram [37] noted that DMUs can be classified into three subsets according to the interval efficiency. The first is the strictly efficient subset, with $E^{++} = \left\{\mathrm{DMU}_j | \underline{\rho}_j = 1, \overline{\rho}_j = 1, j = 1, 2, \ldots, n\right\}$. The second is the weakly efficient subset, with $E^{+} = \left\{\mathrm{DMU}_j | \underline{\rho}_j < 1, \overline{\rho}_j = 1, j = 1, 2, \ldots, n\right\}$. The third is the inefficient subset, with $E^{-} = \left\{\mathrm{DMU}_j | \underline{\rho}_j < 1, \overline{\rho}_j < 1, j = 1, 2, \ldots, n\right\}$. Ranking the DMUs in the same subset is obviously difficult when the DMU number is greater than one. Wang et al. [38] proposed the degree of preference approach for ranking interval data. However, although this approach is suitable for a pairwise comparison, it is less convenient in a complex system. Thus, we introduce an improved degree of preference approach to rank interval efficiency scores.

Suppose there are two interval efficiencies, denoted as $\rho_i = [\underline{\rho}_i, \overline{\rho}_i]$ and $\rho_j = [\underline{\rho}_j, \overline{\rho}_j]$. Then, the degree of preference of ρ_i over ρ_j $(\rho_i \succ \rho_j)$ can be defined as $P_{ij} = P(\rho_i \succ \rho_j)$, which reflects the interrelationship among ρ_i and ρ_j:

$$P_{ij} = P\left(\rho_i \succ \rho_j\right) = \begin{cases} 1, & \underline{\rho}_i \geq \overline{\rho}_j, \\ \dfrac{\overline{\rho}_i - \overline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i} + \dfrac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i} \cdot \dfrac{\underline{\rho}_i - \underline{\rho}_j}{\overline{\rho}_j - \underline{\rho}_j} & \\ + \dfrac{1}{2} \cdot \dfrac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i} \cdot \dfrac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_j - \underline{\rho}_j}, & \underline{\rho}_j < \underline{\rho}_i < \overline{\rho}_j \leq \overline{\rho}_i, \\ \dfrac{\overline{\rho}_i - \overline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i} + \dfrac{1}{2} \cdot \dfrac{\overline{\rho}_j - \underline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i}, & \underline{\rho}_i \leq \underline{\rho}_j < \overline{\rho}_j \leq \overline{\rho}_i. \end{cases} \tag{26}$$

Accordingly, the degree of preference of ρ_j over ρ_i $(\rho_j \succ \rho_i)$ can be defined as

$$P_{ji} = P\left(\rho_j \succ \rho_i\right) = \begin{cases} 0, & \underline{\rho}_i \geq \overline{\rho}_j, \\ \dfrac{1}{2} \cdot \dfrac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i} \cdot \dfrac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_j - \underline{\rho}_j}, & \underline{\rho}_j < \underline{\rho}_i < \overline{\rho}_j \leq \overline{\rho}_i, \\ \dfrac{\underline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i} + \dfrac{1}{2} \cdot \dfrac{\overline{\rho}_j - \underline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i}, & \underline{\rho}_i \leq \underline{\rho}_j < \overline{\rho}_j \leq \overline{\rho}_i. \end{cases} \tag{27}$$

Besides the above two options in (26) and (27) ($\rho_i \succ \rho_j$ and $\rho_i \prec \rho_j$), the interrelationship among ρ_i and ρ_j exists in the third option, that is, $\rho_i = \rho_j$ ($\underline{\rho}_i = \underline{\rho}_j$ and $\overline{\rho}_i = \overline{\rho}_j$). It is easy to verify that if $\rho_i = \rho_j$, then $P_{ij} = P_{ji} = 0$. According to (26) and (27), if $\forall i \neq j$, such that $P_{ij} + P_{ji} = 1$, then the following $n \times n$ matrix that consists of P_{ij} is an antisymmetric matrix.

$$P_{n \times n} = \begin{bmatrix} - & P_{12} & \cdots & P_{1n} \\ P_{21} & - & \cdots & P_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ P_{n1} & P_{n2} & \cdots & - \end{bmatrix}, \tag{28}$$

where $P_{ik} = P(\rho_i \succ \rho_k)$ and $P_{ik} = P(\rho_i = \rho_k) = 0$, $i, k = 1, 2, \ldots, n, i \neq k$.

If $P_{\rho_i \succ \rho_j} \geq 0.5$ and $P_{\rho_j \succ \rho_k} \geq 0.5$ $(k = 1, 2, \ldots, n, k \neq i, j)$, then $P_{\rho_i \succ \rho_k} \geq 0.5$, indicating that the degree of preference satisfies transitivity [38].

According to transitivity, it can be verified that the degree of preference approach possesses the following property.

Property 3. If $\rho_i \succ \rho_j$, $i, j = 1, 2, \ldots, n, i \neq j$, then $r_i > r_j$; inversely, if $\rho_i \prec \rho_j$, $i, j = 1, 2, \ldots, n, i \neq j$, then $r_i < r_j$; and if $\rho_i = \rho_j$, then $r_i = r_j$, $i, j = 1, 2, \ldots, n, i \neq j$.

Here, r_i and r_j denote the sum value of the degree of preference of the deferent rows in matrix, that is,

$$\begin{aligned} r_i &= \sum_{k=1}^{n} P_{ik}, \quad i, k = 1, 2, \ldots, n, i \neq k, \\ r_j &= \sum_{k=1}^{n} P_{jk}, \quad j, k = 1, 2, \ldots, n, j \neq k, \end{aligned} \tag{29}$$

where r_i and r_j can be denoted by the vector $R = (r_1, r_2, \ldots, r_n)^T$. We can verify from the property that if $\rho_i \succ \rho_j$, then $r_i > r_j$, and if $\rho_i = \rho_j$, then $r_i = r_j$. Therefore, the different interval efficiency scores in subsets E^{+} and E^{-} can be ranked through vector $R = (r_1, r_2, \ldots, r_n)^T$ due to the transitivity.

Proof. For a 2×2 matrix, as $P_{12} + P_{21} = 1$, if $\rho_1 \geq \rho_2$, then $P_{12} \geq P_{21}$ and $r_1 \geq r_2$, and thus it possesses the property.

For an $(n-1) \times (n-1)$ $(n \geq 3)$ matrix, if $\rho_i = \rho_j$, then $P_{ij} = 0$, and $r_i = r_j$. If $\rho_i \succ \rho_j$, then $0 < P_{ij} \leq 1$ (when $\rho_i > \rho_j$, $P_{ij} = 1$), specifically.

(1) If $\underline{\rho}_i \leq \underline{\rho}_j < \overline{\rho}_j \leq \overline{\rho}_i$, according to (26), then

$$P_{ij} = \frac{\overline{\rho}_i - \overline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i} + \frac{1}{2} \cdot \frac{\overline{\rho}_j - \underline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i}. \tag{30}$$

When $\overline{\rho}_i - \overline{\rho}_j = \underline{\rho}_j - \underline{\rho}_i$, thus $P_{ij} = \overline{\rho}_j + \underline{\rho}_j - 2\underline{\rho}_i / 2(\overline{\rho}_i - \underline{\rho}_i) = \overline{\rho}_i - \underline{\rho}_i / 2(\overline{\rho}_i - \underline{\rho}_i) = 0.5$.

When $\overline{\rho}_i - \overline{\rho}_j > \underline{\rho}_j - \underline{\rho}_i$, thus $P_{ij} = 2\overline{\rho}_i - (\overline{\rho}_j + \underline{\rho}_j)/2(\overline{\rho}_i - \underline{\rho}_i) = \overline{\rho}_i / \overline{\rho}_i - \underline{\rho}_i - \overline{\rho}_j + \underline{\rho}_j / 2(\overline{\rho}_i - \underline{\rho}_i)$, as $\overline{\rho}_i / \overline{\rho}_i - \underline{\rho}_i > 1$ and $\overline{\rho}_j + \underline{\rho}_j / 2(\overline{\rho}_i - \underline{\rho}_i) < 1/2$, then $P_{ij} > 0.5$.

Likewise, when $\overline{\rho}_i - \overline{\rho}_j < \underline{\rho}_j - \underline{\rho}_i$, it can be verified that $P_{ij} < 0.5$.

(2) If $\underline{\rho}_j < \underline{\rho}_i < \overline{\rho}_j \leq \overline{\rho}_i$, according to (26), then

$$P_{ij} = \frac{\overline{\rho}_i - \overline{\rho}_j}{\overline{\rho}_i - \underline{\rho}_i} + \frac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i} \cdot \frac{\underline{\rho}_i - \underline{\rho}_j}{\overline{\rho}_j - \underline{\rho}_j} + \frac{1}{2} \cdot \frac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_i - \underline{\rho}_i} \cdot \frac{\overline{\rho}_j - \underline{\rho}_i}{\overline{\rho}_j - \underline{\rho}_j}. \quad (31)$$

When $\overline{\rho}_i = \overline{\rho}_j$, thus $P_{ij} = 1/2 \cdot (\overline{\rho}_i - \underline{\rho}_j) + (\underline{\rho}_i - \underline{\rho}_j)/\overline{\rho}_i - \underline{\rho}_j$, as $(\overline{\rho}_i - \underline{\rho}_j) + (\underline{\rho}_i - \underline{\rho}_j)/\overline{\rho}_i - \underline{\rho}_j > 1$, then $P_{ij} > 0.5$.

When $\underline{\rho}_j < \underline{\rho}_i < \overline{\rho}_j < \overline{\rho}_i$, thus $P_{ij} = 1/2\,(\overline{\rho}_i - \overline{\rho}_j) + (\overline{\rho}_i - \underline{\rho}_i)/\overline{\rho}_i - \underline{\rho}_i$, as $(\overline{\rho}_i - \overline{\rho}_j) + (\overline{\rho}_i - \underline{\rho}_i)/\overline{\rho}_i - \underline{\rho}_i > 1$, then $P_{ij} > 0.5$.

Therefore, if $\rho_i \succ \rho_j$, it can be verified that $0 < P_{ij} \leq 1$, according to the transitivity property, then $r_i = \sum_{k=1}^{n} P_{ik} = P_{i1} + P_{i2} + \cdots + P_{in} > r_j = \sum_{k=1}^{n} P_{jk} = P_{j1} + P_{j2} + \cdots + P_{jn}$, $i, k = 1, 2, \ldots, n$, $i \neq j$, $i \neq k$, $j \neq k$. This proof is thus complete. □

4. Application to Chinese City Commercial Banks

In this section, we implement the proposed IMSBM model with undesirable outputs to evaluate the efficiency scores and the classification of Chinese city commercial banks in 2017. Our study represents the first attempt to measure the interval efficiency of these banks with both negative data and undesirable outputs.

Based on the availability of data, we evaluate the interval efficiency scores of 99 city commercial banks, each of which is associated with two inputs (staff costs (COST) and total assets (ASST)) and three outputs (noninterest income (NINT), interest income (INTE), and nonperforming loan (NPL)). To simplify the problem, the inputs and outputs are weighted equally, as presented in Table 2 in the parentheses. Due to space limitations, we only give the DMUs with negative data in Table 2. Four DMUs have negative outputs in all of the samples. DMU_{87} (Cang Zhou bank) has negative noninterest income at both the lower and the upper bound. Three other banks (DMU_{66} (Xia Men bank), DMU_{77} (Ying Kou bank), and DMU_{97} (Gui Zhou bank)) have negative noninterest income at the lower bounds. The remaining 95 banks have positive inputs and outputs at both bounds and are not included in Table 2.

The resulting interval efficiency scores and corresponding classification evaluated by the IMSBM model with undesirable outputs are shown in Table 3, and those for the model without undesirable outputs are given in the two adjacent columns for comparison.

Table 3 shows that, for the IMSBM model with undesirable outputs, only DMU_{18} (Liang Shan Zhou bank) is strictly efficient, 82 banks are weakly efficient, and the remaining 16 are inefficient.

A comparison of the efficiency scores evaluated by the IMSBM models with and without undesirable outputs shows that the strictly efficient DMUs in the two models are the same (i.e., DMU_{18}). However, it is important to note that the interval efficiency scores of the weakly efficient and the inefficient DMUs changed after considering the undesirable outputs. When the DMUs are weakly efficient, the lower bound of the efficiency score decreased after considering undesirable outputs except DMU_3, DMU_{22}, DMU_{71}, and DMU_{85}, while the upper bounds of the efficiency score remained unchanged. When the DMUs are inefficient, the lower bound of the efficiency score also decreased after considering undesirable outputs, while the change of the upper bound of the efficiency score is complicated. The upper bounds of the efficiency score of 8 banks increased, and for the other 13 banks, the opposite is observed. Therefore, without considering the undesirable outputs, the lower bound of the efficiency score will be overestimated as a whole, when the DMUs are weakly efficient and inefficient. In addition, the upper bound of the efficiency score will change when considering the undesirable outputs when the DMUs are inefficient.

The details of the performance ranking are required, in addition to the classification. According to (26) and (27), the interrelationship among DMUs can be established through the degree of preference P_{ij}, which constitutes a 99×99 matrix. Due to space limitations, the matrix is not shown in this paper. The sum value r of the degree of preference can then be calculated according to (29), and all of the DMUs are ranked based on the value. The r value and the corresponding rank of each DMU are shown in Table 4, where r_i^* and R^* denote the sum values of the degree of preference and the rank of each DMU, respectively, with the IMSBM model with undesirable outputs, and the contrasting r_i and R with the model without undesirable outputs are given in the adjacent two columns.

As shown in Table 4, DMU_{18} is strictly efficient under both models; therefore, the sum values r_i^* and r_i are both equal to 98, excluding the value on the leading diagonal. From r_i^* and r_i, DMU_{18} is found to be ranked in the top position under both models. We can then examine the ranks of the other 10 DMUs below DMU_{18}. With the IMSBM model with undesirable outputs, the relationship among the 10 DMUs is established as $DMU_{71} \succ DMU_{85} \succ DMU_2 \succ DMU_1 \succ DMU_4 \succ DMU_{42} \succ DMU_{47} \succ DMU_{72} \succ DMU_{96} \succ DMU_{91}$. For the IMSBM model without undesirable outputs, the relationship is established as $DMU_{71} \succ DMU_{85} \succ DMU_2 \succ DMU_{72} \succ DMU_1 \succ DMU_{42} \succ DMU_4 \succ DMU_{91} \succ DMU_{70} \succ DMU_{96}$. This indicates that the IMSBM model with undesirable outputs leads the ranks of the weakly efficient and inefficient DMUs to change.

Table 2: The DMUs with negative outputs.

DMU	Inputs		Outputs		
	COST ($w_1 = 0.5$)	ASST ($w_2 = 0.5$)	NINT ($v_{r_1} = 0.33$)	INTE ($v_{r_2} = 0.33$)	NPL ($v_{l_1} = 0.33$)
DMU66	[628.00, 649.11]	[188589.00, 212413.91]	[−539.00, 158.95]	[8726.00, 9881.56]	[731.07, 878.96]
DMU77	[615.00, 678.96]	[126756.00, 156344.22]	[−55.90, 104.27]	[5428.00, 7279.38]	[665.09, 822.17]
DMU87	[676.00, 875.34]	[111723.00, 123832.19]	[−23.08, −22.94]	[4524.00, 5671.94]	[850.01, 1084.49]
DMU97	[1601.00, 1771.00]	[229489.00, 286760.00]	[−91.46, 123.97]	[11193.74, 12870.00]	[1303.00, 1375.00]

Table 3: Results of the interval efficiency and classification.

DMU	IMSBM with undesirable Output	Class	IMSBM	Class	DMU	IMSBM with undesirable Output	Class	IMSBM	Class
DMU1	[0.564, 1]	E+	[0.596, 1]	E+	DMU51	[0.382, 1]	E+	[0.488, 1]	E+
DMU2	[0.565, 1]	E+	[0.611, 1]	E+	DMU52	[0.287, 0.6]	E−	[0.366, 0.744]	E−
DMU3	[0.34, 1]	E+	[0.31, 1]	E+	DMU53	[0.293, 1]	E+	[0.363, 1]	E+
DMU4	[0.554, 1]	E+	[0.547, 1]	E+	DMU54	[0.253, 1]	E+	[0.318, 0.986]	E−
DMU5	[0.396, 1]	E+	[0.481, 1]	E+	DMU55	[0.267, 1]	E+	[0.331, 1]	E+
DMU6	[0.39, 1]	E+	[0.458, 1]	E+	DMU56	[0.297, 0.826]	E−	[0.385, 0.949]	E−
DMU7	[0.336, 1]	E+	[0.393, 0.914]	E−	DMU57	[0.313, 1]	E+	[0.387, 1]	E+
DMU8	[0.383, 1]	E+	[0.469, 1]	E+	DMU58	[0.293, 1]	E+	[0.335, 1]	E+
DMU9	[0.198, 0.763]	E−	[0.244, 0.838]	E−	DMU59	[0.241, 0.771]	E−	[0.306, 0.853]	E−
DMU10	[0.248, 0.935]	E−	[0.312, 0.936]	E−	DMU60	[0.268, 0.595]	E−	[0.345, 0.705]	E−
DMU11	[0.37, 1]	E+	[0.464, 1]	E+	DMU61	[0.325, 1]	E+	[0.421, 1]	E+
DMU12	[0.316, 1]	E+	[0.34, 1]	E+	DMU62	[0.248, 1]	E+	[0.313, 1]	E+
DMU13	[0.214, 0.98]	E−	[0.268, 0.977]	E−	DMU63	[0.284, 1]	E+	[0.352, 1]	E+
DMU14	[0.246, 1]	E+	[0.255, 1]	E+	DMU64	[0.333, 1]	E+	[0.395, 1]	E+
DMU15	[0.279, 1]	E+	[0.347, 1]	E+	DMU65	[0.325, 1]	E+	[0.412, 1]	E+
DMU16	[0.372, 1]	E+	[0.461, 1]	E+	DMU66	[0.404, 1]	E+	[0.481, 1]	E+
DMU17	[0.286, 1]	E+	[0.311, 1]	E+	DMU67	[0.323, 1]	E+	[0.374, 1]	E+
DMU18	[1, 1]	E++	[1, 1]	E++	DMU68	[0.318, 0.802]	E−	[0.406, 0.954]	E−
DMU19	[0.226, 1]	E+	[0.257, 1]	E+	DMU69	[0.325, 1]	E+	[0.405, 1]	E+
DMU20	[0.233, 0.845]	E−	[0.299, 0.862]	E−	DMU70	[0.415, 1]	E+	[0.508, 1]	E+
DMU21	[0.403, 1]	E+	[0.48, 1]	E+	DMU71	[0.753, 1]	E+	[0.713, 1]	E+
DMU22	[0.386, 1]	E+	[0.386, 1]	E+	DMU72	[0.5, 1]	E+	[0.603, 1]	E+
DMU23	[0.384, 1]	E+	[0.469, 1]	E+	DMU73	[0.209, 1]	E+	[0.27, 1]	E+
DMU24	[0.235, 0.765]	E−	[0.299, 0.833]	E−	DMU74	[0.245, 0.775]	E−	[0.299, 0.876]	E−
DMU25	[0.303, 1]	E+	[0.375, 1]	E+	DMU75	[0.229, 1]	E+	[0.244, 0.997]	E−
DMU26	[0.364, 1]	E+	[0.454, 1]	E+	DMU76	[0.337, 1]	E+	[0.402, 1]	E+
DMU27	[0.282, 1]	E+	[0.326, 1]	E+	DMU77	[0.278, 1]	E+	[0.344, 1]	E+
DMU28	[0.295, 1]	E+	[0.357, 1]	E+	DMU78	[0.344, 1]	E+	[0.43, 1]	E+
DMU29	[0.199, 0.667]	E−	[0.255, 0.844]	E−	DMU79	[0.363, 1]	E+	[0.432, 1]	E+
DMU30	[0.316, 1]	E+	[0.399, 1]	E+	DMU80	[0.251, 1]	E+	[0.305, 1]	E+
DMU31	[0.405, 1]	E+	[0.499, 1]	E+	DMU81	[0.346, 1]	E+	[0.426, 1]	E+
DMU32	[0.327, 0.737]	E−	[0.41, 0.857]	E−	DMU82	[0.347, 1]	E+	[0.416, 1]	E+
DMU33	[0.293, 1]	E+	[0.374, 1]	E+	DMU83	[0.371, 1]	E+	[0.457, 1]	E+
DMU34	[0.267, 1]	E+	[0.333, 1]	E+	DMU84	[0.372, 1]	E+	[0.454, 1]	E+
DMU35	[0.36, 1]	E+	[0.419, 1]	E+	DMU85	[0.635, 1]	E+	[0.635, 1]	E+
DMU36	[0.382, 1]	E+	[0.466, 1]	E+	DMU86	[0.35, 1]	E+	[0.385, 1]	E+
DMU37	[0.315, 1]	E+	[0.324, 1]	E+	DMU87	[0.244, 0.902]	E−	[0.309, 0.883]	E−

Table 3: Continued.

DMU	IMSBM with undesirable Output	Class	IMSBM	Class	DMU	IMSBM with undesirable Output	Class	IMSBM	Class
DMU38	[0.25, 1]	E+	[0.303, 1]	E+	DMU88	[0.322, 1]	E+	[0.37, 1]	E+
DMU39	[0.296, 1]	E+	[0.333, 1]	E+	DMU89	[0.273, 1]	E+	[0.32, 1]	E+
DMU40	[0.406, 1]	E+	[0.425, 1]	E+	DMU90	[0.282, 0.915]	E−	[0.357, 0.862]	E−
DMU41	[0.376, 1]	E+	[0.442, 1]	E+	DMU91	[0.419, 1]	E+	[0.519, 1]	E+
DMU42	[0.548, 1]	E+	[0.581, 1]	E+	DMU92	[0.278, 1]	E+	[0.351, 1]	E+
DMU43	[0.326, 1]	E+	[0.38, 1]	E+	DMU93	[0.22, 1]	E+	[0.262, 1]	E+
DMU44	[0.379, 1]	E+	[0.435, 1]	E+	DMU94	[0.317, 1]	E+	[0.354, 1]	E+
DMU45	[0.261, 1]	E+	[0.333, 0.954]	E−	DMU95	[0.268, 1]	E+	[0.323, 1]	E+
DMU46	[0.247, 1]	E+	[0.311, 1]	E+	DMU96	[0.428, 1]	E+	[0.506, 1]	E+
DMU47	[0.522, 1]	E+	[0.482, 1]	E+	DMU97	[0.284, 1]	E+	[0.345, 1]	E+
DMU48	[0.334, 1]	E+	[0.406, 1]	E+	DMU98	[0.247, 1]	E+	[0.293, 1]	E+
DMU49	[0.294, 0.596]	E−	[0.373, 0.772]	E−	DMU99	[0.264, 1]	E+	[0.322, 1]	E+
DMU50	[0.25, 1]	E+	[0.315, 0.978]	E−					

Table 4: The sum values of the degree of preference and ranks of the DMUs.

DMU	r_i^*	R^*	r_i	R	DMU	r_i^*	R^*	r_i	R	DMU	r_i^*	R^*	r_i	R
DUM1	67.614	5	66.477	6	DUM34	46.561	60	46.003	57	DUM67	50.244	43	48.869	44
DUM2	67.687	4	67.716	4	DUM35	52.820	30	52.222	31	DUM68	35.789	77	47.615	48
DUM3	51.417	35	44.491	67	DUM36	54.388	22	55.896	18	DUM69	50.381	42	51.159	35
DUM4	66.886	6	62.451	8	DUM37	49.699	47	45.403	61	DUM70	56.769	12	59.275	10
DUM5	55.395	17	57.096	15	DUM38	45.511	65	44.045	69	DUM71	81.049	2	76.080	2
DUM6	54.962	18	55.261	21	DUM39	48.429	50	46.003	57	DUM72	62.947	9	67.055	5
DUM7	51.139	37	43.453	71	DUM40	56.117	13	52.683	29	DUM73	43.138	71	42.047	75
DUM8	54.459	21	56.135	17	DUM41	53.958	24	54.002	24	DUM74	29.352	80	34.134	85
DUM9	26.096	83	28.222	91	DUM42	66.448	7	65.240	7	DUM75	44.267	68	40.360	79
DUM10	40.860	74	39.648	82	DUM43	50.449	40	49.305	42	DUM76	51.209	36	50.933	36
DUM11	53.530	27	55.737	19	DUM44	54.173	23	53.456	25	DUM77	47.258	57	46.752	54
DUM12	49.767	46	46.478	55	DUM45	46.186	62	42.422	74	DUM78	51.696	34	53.069	27
DUM13	42.036	72	40.179	80	DUM46	45.330	66	44.555	66	DUM79	53.032	29	53.224	26
DUM14	45.270	67	41.192	78	DUM47	64.551	8	57.176	14	DUM80	45.572	64	44.172	68
DUM15	47.322	56	46.959	52	DUM48	51.001	38	51.234	34	DUM81	51.835	33	52.760	28
DUM16	53.672	25	55.498	20	DUM49	19.360	85	30.506	89	DUM82	51.905	32	51.993	32
DUM17	47.775	53	44.555	66	DUM50	45.511	65	43.114	73	DUM83	53.601	26	55.181	22
DUM18	98.000	1	98.000	1	DUM51	54.388	22	57.659	13	DUM84	53.672	25	54.944	23
DUM19	44.094	69	41.304	77	DUM52	19.243	86	27.784	92	DUM85	72.728	3	69.697	3
DUM20	33.659	78	33.029	86	DUM53	48.232	52	48.081	46	DUM86	52.116	31	49.671	41
DUM21	55.900	16	57.015	16	DUM54	45.694	63	43.928	70	DUM87	38.306	75	35.288	84
DUM22	54.675	19	49.744	40	DUM55	46.561	60	45.869	58	DUM88	50.176	44	48.581	45
DUM23	54.531	20	56.135	17	DUM56	36.151	76	45.662	59	DUM89	46.939	58	45.139	64
DUM24	28.095	82	30.738	88	DUM57	49.564	48	49.818	39	DUM90	41.548	73	36.710	83
DUM25	48.893	49	48.941	43	DUM58	48.232	52	46.138	56	DUM91	57.058	11	60.168	9
DUM26	53.103	28	54.944	23	DUM59	28.846	81	32.730	87	DUM92	47.258	57	47.236	51
DUM27	47.516	55	45.535	60	DUM60	17.848	87	23.376	93	DUM93	43.752	70	41.587	76
DUM28	48.363	51	47.656	47	DUM61	50.381	41	52.376	30	DUM94	49.835	45	47.446	49
DUM29	19.527	84	29.226	90	DUM62	45.390	66	44.684	65	DUM95	46.624	59	45.337	62
DUM30	49.767	46	50.709	37	DUM63	47.645	54	47.306	50	DUM96	57.711	10	59.113	11
DUM31	56.045	14	58.546	12	DUM64	50.932	39	50.410	38	DUM97	47.645	54	46.821	53
DUM32	31.646	79	40.082	81	DUM65	50.381	41	51.689	33	DUM98	45.330	66	43.422	72
DUM33	48.232	52	48.869	44	DUM66	55.972	15	57.096	15	DUM99	46.373	61	45.271	63

5. Conclusion and Discussion

This study develops the IMSBM model to address undesirable outputs, which extends the application of IDEA in fields that concern with less undesirable outputs, such as air pollutants, hazardous waste, and nonperforming loan. Several models in the literature have been developed to handle problems of imprecise and (or) negative data, but few models consider handling imprecise and negative data simultaneously. These models also ignore undesirable outputs. Thus, we first propose the IMSBM model with undesirable outputs. The model is novel as it considers undesirable outputs while dealing with imprecise and negative data, and it is slack-based, which ensures efficiency is obtained when considering both radial and nonradial slacks.

This study establishes that the IMSBM model with undesirable outputs is translation-invariant and unit-invariant. The model is applied to evaluate the interval efficiency scores of Chinese city commercial banks, which are compared with those evaluated by the IMSBM model without considering undesirable outputs. The empirical results show that the IMSBM model with undesirable outputs reduces the lower bounds of the efficiency scores of the weakly and inefficient DMUs as a whole. Therefore, without considering undesirable outputs, most of the lower bounds of the efficiency scores will be overestimated when the DMUs are weakly efficient and inefficient. In addition, the model leads to changes in the upper bounds of the efficiency scores of inefficient DMUs. Finally, the interval efficiency scores are ranked with an improved degree of preference approach.

The proposed IMSBM model with undesirable outputs is assigned under the VRS, but not the CRS. Therefore, the interval efficiency scores evaluated by the right model are pure technical efficiencies (PTE). In addition, the resulting interval efficiency scores are in the range of [0, 1], and the upper bound of each cannot be greater than one, so the strictly efficient DMUs cannot be ranked. Thus, in future studies, we will focus our attention on the IMSBM model with undesirable outputs under the CRS to evaluate the technical efficiency (TE). In addition, we will develop a superefficiency model from our model to rank the strictly efficient DMUs.

Data Availability

All of the data used to support the application of the model were collected by the authors from the annual reports and audit reports of Chinese city commercial banks.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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