

Research Article

Higher-Order Multifractal Detrended Partial Cross-Correlation Analysis for the Correlation Estimator

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In this paper, we develop a new method to measure the nonlinear interactions between nonstationary time series based on the detrended cross-correlation coefficient analysis. We describe how a nonlinear interaction may be obtained by eliminating the influence of other variables on two simultaneous time series. By applying two artificially generated signals, we show that the new method is working reliably for determining the cross-correlation behavior of two signals. We also illustrate the application of this method in finance and aeroengine systems. These analyses suggest that the proposed measure, derived from the detrended cross-correlation coefficient analysis, may be used to remove the influence of other variables on the cross-correlation between two simultaneous time series.

1. Introduction

There are numerous real-world systems where the output signals are nonstationary and exhibit complex self-correlation or cross-correlation over a broad range of time scales. The output signals can be characterized by power-law correlations. One method, which has proved to be quite useful to detect the degree of interrelation between two stationary variables, is Pearson's correlation coefficient [1]:

$$r = \frac{\langle (X - \langle X \rangle) \cdot (Y - \langle Y \rangle) \rangle}{\sigma_X \cdot \sigma_Y}, \quad (1)$$

where $\langle X \rangle$ is the arithmetic average of X and σ_X is its standard deviation and likewise for Y . Proposition of Pearson's correlation coefficient (PCC) has achieved great success in multivariate analysis, such as the principal component analysis [2], random matrix theory [3], and singular value decomposition [4].

Nevertheless, in real-world systems, nonlinear and nonstationary characteristics are present. Therefore, PCC may not be suitable to describe the interrelation between two variables that are nonlinear and nonstationary. For dealing with the drawbacks of PCC, the detrended cross-correlation

analysis (DCCA) method and the DCCA coefficient are proposed by Stanley and Podobnik [5, 6]. The advantage of the DCCA method is that it allows the detection of cross-correlations between noisy signals with embedded polynomial trends, which can mask the true cross-correlations in the fluctuations of signals. The DCCA method is widely applied to measure the cross-correlations in different fields, such as social sciences [7], biology [8], climatology [9], geophysics [10, 11], transportation [12, 13], seismic signals [11, 14], economics [15–20], and aeroengine dynamics [21–24].

Recently, multifractal analysis is one of the major interests for researchers from interdisciplinary domains to uncover the scaling properties and understand the hidden information. Among these researchers, many of them applied the multifractal analysis to meteorology [25–27], electroencephalography [28], and economics [29–31]. Later, as some researchers thought of extending the research of multifractal analysis to the detrended cross-correlations between time series, the multifractal detrended cross-correlation analysis (MFDXA) was proposed [32–34].

The cross-correlation between two variables may be influenced by other variables. Hence, we have to be alert to

the possibilities of spurious correlation while investigating the cross-correlation. Then, the methods of partial correlation and partial correlation coefficient are therefore proposed to measure the degree of association between two random variables [35, 36]. The linear effect may be removed using the partial correlation coefficient (partial CC):

$$r_{XY,\xi} = \frac{\langle (X' - \langle X' \rangle) \cdot (Y' - \langle Y' \rangle) \rangle}{\sigma_{X'} \cdot \sigma_{Y'}} = \frac{r_{XY} - r_{X\xi} \cdot r_{Y\xi}}{\sqrt{(1 - r_{X\xi}^2) \cdot (1 - r_{Y\xi}^2)}} \quad (2)$$

where $X' = X - L_X(\xi)$ and $L_X(\xi) = c_0 + c_1\xi$ to minimize the mean $E(X - L_X(\xi))^2$ and likewise for Y' . If n additional variables are to be accounted for, say $\xi_1, \xi_2, \dots, \xi_n$, the n th-order partial CC can be computed by [36]

$$r_{XY.12\dots n} = \frac{r_{XY.12\dots n-1} - r_{Xn.12\dots n-1} r_{Yn.12\dots n-1}}{\sqrt{(1 - r_{Xn.12\dots n-1}^2)(1 - r_{Yn.12\dots n-1}^2)}} \quad (3)$$

Lately, the detrended partial cross-correlation analysis and multifractal detrended partial cross-correlation analysis (MFDPCA) which can measure cross-correlations between nonlinear time series influenced by common external forces is proposed [37, 38].

In order to remove the spurious correlation and improve the estimation performance for quantifying the intrinsic interactions between two nonstationary time series, this paper proposes the method of n th-order multifractal detrended partial cross-correlation analysis by incorporating the partial correlation coefficient with the multifractal detrended cross-correlation analysis.

The rest of the paper is organized as follows. In the next section, we introduce the multifractal DCCA coefficient method and propose the method of n th-order multifractal detrended partial cross-correlation analysis. In Section 3, we show the data results for the randomly generated dataset and stock and engine dataset by the proposed methods. Finally, we draw some conclusions in Section 4.

2. Methodologies

2.1. Multifractal Detrended Partial Cross-Correlation Analysis. For the sake of clarity, we begin with a summary of the multifractal DCCA coefficient algorithm. For two series $\{r_i(t)\}$ and $\{r_j(t)\}$ with equal length N , where $t = 1, 2, \dots, N$, the computational procedure of the multifractal DCCA coefficient is as follows:

Step 1: construct the profile of each series by eliminating the mean value:

$$\begin{aligned} R_i(t) &= \sum_{k=1}^t (r_i(k) - \langle r_i \rangle), \\ R_j(t) &= \sum_{k=1}^t (r_j(k) - \langle r_j \rangle), \end{aligned} \quad (4)$$

$t = 1, 2, \dots, N,$

where $\langle r_i \rangle$ and $\langle r_j \rangle$ are the average values of $\{r_i(t)\}$ and $\{r_j(t)\}$, respectively.

Step 2: divide the profiles $\{R_i(k)\}$ and $\{R_j(k)\}$ into $N_s = \text{int}(N/s)$ nonoverlapping units of equal length s . Considering that N is usually not a multiple of the time scale s , we repeat the same procedure by starting from the opposite end of the sequence in order to take the whole series into account. Thus, we obtain $2N_s$ segments of equal length s . In this paper, we follow the previous literature practice and set $10 \leq s \leq N/4$.

Step 3: for each segment $\nu (\nu = 1, 2, \dots, N_s, N_s + 1, \dots, 2N_s)$, the local trends $\{\tilde{R}_i^\nu(k)\}$ and $\{\tilde{R}_j^\nu(k)\}$ are estimated on the basis of a least-squares fit of the sequences $\{R_i(k)\}$ and $\{R_j(k)\}$, respectively. The corresponding detrended covariance for $\nu = 1, 2, \dots, N_s$ is

$$f_{\text{DCCA}}^2(s, \nu) = \frac{1}{s} \sum_{t=1}^s (R_i^{(\nu-1)s+t}(t) - \tilde{R}_i^\nu(t)) (R_j^{(\nu-1)s+t}(t) - \tilde{R}_j^\nu(t)), \quad (5)$$

and for $\nu = N_s + 1, N_s + 2, \dots, 2N_s$ is

$$\begin{aligned} f_{\text{DCCA}}^2(s, \nu) &= \frac{1}{s} \sum_{t=1}^s (R_i^{N-(\nu-N_s)s+t}(t) - \tilde{R}_i^\nu(t)) \\ &\quad \cdot (R_j^{N-(\nu-N_s)s+t}(t) - \tilde{R}_j^\nu(t)), \end{aligned} \quad (6)$$

where $\{\tilde{R}_i^\nu(k)\}$ and $\{\tilde{R}_j^\nu(k)\}$ are the fitting polynomials in the segment ν .

Step 4: calculate the average of multifractal detrended covariance fluctuation function $F_{\text{DCCA}}^q(s, \nu)$ over all segments:

$$F_{\text{DCCA}}^q(s, \nu) = \left\{ \frac{1}{2N_s} \sum_{\nu=1}^{2N_s} [f_{\text{DCCA}}^2(s, \nu)]^{q/2} \right\}^{1/q}. \quad (7)$$

Generally, q can take any real value, except zero. For $q = 0$, the equation becomes

$$F_0(s) = \exp\left(\frac{1}{2N_s} \sum_{m=1}^{N_s} \ln F(s, m)\right). \quad (8)$$

For $q = 2$, $F_{\text{DCCA}}^q(s, \nu)$ is equal to the detrended cross-correlation fluctuation function $F_{\text{DCCA}}^2(s)$.

Step 5: estimate the multifractal DCCA coefficient:

$$\rho_{ij}^q(s) = \frac{F_{\text{DCCA}}^q(s)}{F_{\text{DFA}}^q\{r_i(t)\}(s) F_{\text{DFA}}^q\{r_j(t)\}(s)}. \quad (9)$$

For $q = 2$, the standard DCCA coefficient $\rho_{ij}(s)$ is retrieved.

Step 6: compute the multifractal detrended partial cross-correlation coefficient between X and Y by eliminating the influence of the controlling variable ξ_1 on X and Y analogous to the generalization of the correlation coefficient to partial correlation coefficient:

$$\rho_{XY,1}^q = \frac{\rho_{XY}^q - \rho_{X\xi_1}^q \rho_{Y\xi_1}^q}{\sqrt{(1 - (\rho_{X\xi_1}^q)^2)(1 - (\rho_{Y\xi_1}^q)^2)}} \quad (10)$$

named the first-order multifractal detrended partial cross-correlation coefficient (first-order MFDPPCC coefficient), where X, Y are random variables, ξ_1 is the controlling variable, and $\rho_{XY}^q, \rho_{X\xi_1}^q, \rho_{Y\xi_1}^q$ represent the mean of MFDCCA coefficients for X and Y , X and ξ_1 , and Y and ξ_1 , respectively.

For $q = 2$, the first-order detrended partial cross-correlation coefficient (first-order DPCC coefficient) is retrieved.

2.2. The n th-Order Multifractal Detrended Partial Cross-Correlation Analysis and n -Controlling-Variables Detrended Partial Cross-Correlation Coefficient. Considering the cross-correlation between X and Y affected by more than one variable in complex systems, we define the second-order multifractal detrended partial cross-correlation coefficient (second-order MFDPPCC coefficient) by using the partial correlation method [36]:

$$\rho_{XY,12}^q = \frac{\rho_{XY,1}^q - \rho_{X\xi_1}^q \rho_{Y\xi_2}^q}{\sqrt{(1 - (\rho_{X\xi_1}^q)^2)(1 - (\rho_{Y\xi_2}^q)^2)}} \quad (11)$$

where X, Y are random variables, controlling variables ξ_1, ξ_2 are not related to each other, and $\rho_{XY,1}, \rho_{X\xi_1},$ and $\rho_{Y\xi_2}$ are first-order MFDPPCC coefficients.

Generally, the n th-order multifractal detrended partial cross-correlation coefficient (n th-order MFDPPCC coefficient) is as follows:

$$\rho_{XY,12\dots n}^q = \frac{\rho_{XY,12\dots n-1}^q - \rho_{X\xi_1,12\dots n-1}^q \rho_{Y\xi_n,12\dots n-1}^q}{\sqrt{(1 - (\rho_{X\xi_1,12\dots n-1}^q)^2)(1 - (\rho_{Y\xi_n,12\dots n-1}^q)^2)}} \quad (12)$$

where $\rho_{XY,12\dots n-1}^q, \rho_{X\xi_1,12\dots n-1}^q,$ and $\rho_{Y\xi_n,12\dots n-1}^q$ are $(n-1)$ th-order MFDPPCC coefficients and controlling variables $\xi_1, \xi_2, \dots, \xi_n$ are not related to each other. For $q = 2$, the n th-order detrended partial cross-correlation coefficient (n th-order DPCC coefficient) is retrieved.

In general, the n th-order partial cross-correlation is necessary when these controlling variables $\xi_1, \xi_2, \dots, \xi_n$ are not related to each other. Nevertheless, in real-world systems, the variables $\xi_1, \xi_2, \dots, \xi_n$ generated by large number of interacting units are cross-correlated. Therefore, we define the n -controlling-variables multifractal detrended partial

cross-correlation coefficient (n -variables MFDPPCC) by equation (12) for related controlling variables $\xi_1, \xi_2, \dots, \xi_n$. Note that when the controlling variables $\xi_1, \xi_2, \dots, \xi_n$ are not related to each other, the n -variables MFDPPCC is equivalent to the n th-order MFDPPCC.

3. Data and Analysis

3.1. Two-Component ARFIMA Process. In order to test the robustness of the proposed n -controlling-variables MFDPPCC coefficient method, power-law cross-correlated time series $\{u_i\}$ and $\{v_i\}$ are generated by using the two-component ARFIMA stochastic process in this section [18, 39, 40]. In this model, the series is defined by

$$\begin{cases} u_i = [WU_i + (1 - W)V_i] + \varepsilon_i, \\ v_i = [WV_i + (1 - W)U_i] + \bar{\varepsilon}_i, \\ U_i = \sum_{j=1}^{\infty} a_j(\rho_1)u_{i-j}, \\ V_i = \sum_{j=1}^{\infty} a_j(\rho_2)v_{i-j}, \end{cases} \quad (13)$$

where $a_j(\rho) = \Gamma(j - \rho)/\Gamma(-\rho)\Gamma(i + j)$ ($0 < \rho < 0.5$) is weight, W is a free parameter to control the coupling strength between $\{u_i\}$ and $\{v_i\}$ ($0.5 \leq W \leq 1$), and ε_i and $\bar{\varepsilon}_i$ are independent and identically distributed (*i.i.d.*) Gaussian variables with $\langle \varepsilon_i \rangle = \langle \bar{\varepsilon}_i \rangle = 0$ and $\langle \varepsilon_i^2 \rangle = \langle \bar{\varepsilon}_i^2 \rangle = 1$ [18, 39]. For different values of W , the different coupling strength between the variables $\{u_i\}$ and $\{v_i\}$ is $1 - W$. In this section, the two-component ARFIMA series $\{u_i\}$ and $\{v_i\}$ with parameter $\rho_1 = \rho_2 = 0.3$ and $W = 0.5$, denoted by X and Y , are employed to detect the interactions between two time series. Then, the effect of white noise sequence ξ_1 on the cross-correlation of the two series X and Y is tested to investigate the validity of the n -controlling-variables MFDPPCC coefficient analysis mentioned in this paper. For this purpose, we study the difference between the mean of the MFDCCA coefficient and the n -controlling-variables MFDPPCC coefficient for any parameter q by using the influence degree function $I(n, q)$. The influence degree function is defined as

$$I(n, q) = |\rho_{XY,12\dots n}^q - \rho_{XY}^q|. \quad (14)$$

We calculate the influence degree function $I(n, q)$ of the synthetical signals using the proposed first-order MFDPPCC coefficient and present the influence degree function $I(1, q)$ vs. parameter q in Figure 1. The results of the influence degree values of different q are just about nil, which indicates that there is hardly any effect of white noise sequence on cross-correlation of the two series X and Y .

3.2. Stock Market. To further exemplify the potential utility of the n -controlling-variables MFDPPCC coefficient method for analyzing real-world data, we study daily closing prices of fifteen stock markets including the São Paulo Index (IBOV), the Dow Jones Index (DJI), the NASDAQ Index (IXIC), the Standard & Poor 500 Composite Stock Price Index (SPX),

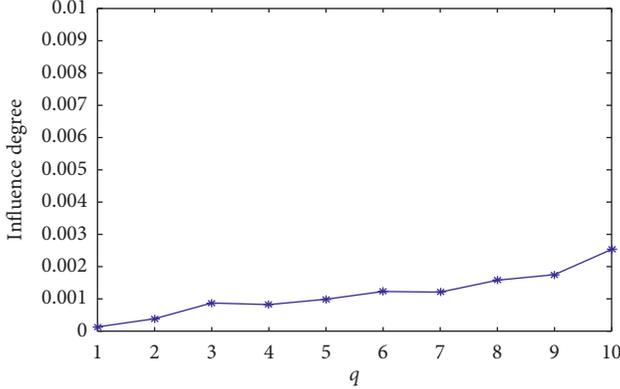


FIGURE 1: The effect of white noise sequence ξ_1 on cross-correlation of the two series X and Y .

the FTSE Global Equity Index Series (FISE), the French CAC 40 (FCHI), German DAX Index (GDAXI), Nikkei 255 Index (N225), Korea Composite Index (KS11), Hang Seng Index (HSI), Australian Standard & Poor's 200 (AS51), Mumbai Index (SENSEX), Russian Index (RTS), Shanghai Composite Index (SSEC), and Shenzhen Composite Index (SZI). Datasets are from January 04, 1993, to January 03, 2019.

Figure 2 shows the mean of DCCA coefficients for the stock series. The mean of DCCA coefficient between DJI and SPX is 0.97, which performs relevantly different from other DCCA coefficients. It indicates the close cross-correlation between the American stock markets. The next largest DCCA coefficient $\rho = 0.92$ is obtained by SSEC and SZI, which indicates the close cross-correlation in Chinese mainland stock markets.

The mean of DCCA coefficients between SZI and stock markets in developed countries (GDAXI, N225, KS11, and AS51) is less than 0.3. It shows that SZI has a weak relationship with stock markets in developed countries. The mean of DCCA coefficients between SZI and HSI is in an intermediate state, which indicates the existence of cross-correlation in Chinese stock markets.

Next, we analyze the effect of the other thirteen stock markets on cross-correlation characteristics between SSEC and SZI, by applying the influence degree of first-order DPCC coefficient. For the effect on cross-correlation characteristics between the SSEC and SZI, the largest influence degree $I = 0.05$ is obtained by HSI, which shows the information exchange between the Chinese stock market, as seen in Figure 3. The next largest $I = 0.04$ is acquired by SENSEX, which indicates the association between the stock markets in developing countries (Indian and Chinese stock markets). The I values of other stock time series are less than 0.1, which indicates little information exchange between the Chinese mainland stock market and other stock markets. The influence degree values of 13 stock markets for first-order MFDPCC coefficient with $q = 1, 2, \dots, 10$ are also demonstrated in the upper left of Figure 3.

During the analysis, we observe the effect of HSI on cross-correlation characteristics between SSEC and SZI from influence degree function $I(1, q)$ that decreases as the scale q increases. And this infers the change of multifractal cross-correlation.

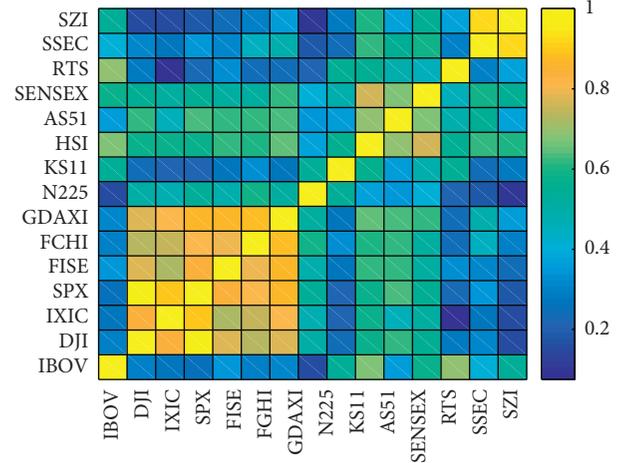


FIGURE 2: The mean of DCCA coefficients between the stock series.

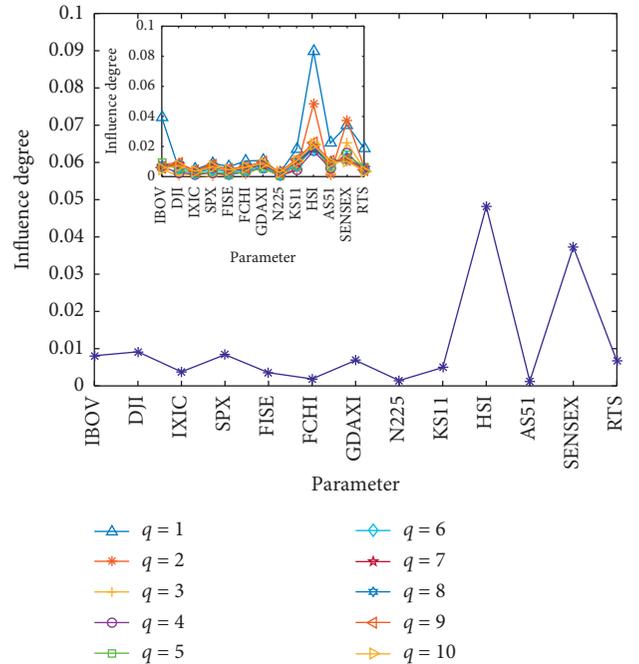


FIGURE 3: The influence degree of the first-order detrended partial cross-correlation in stock market and the influence degree of the first-order multifractal detrended partial cross-correlation coefficient in stock market (inset).

In order to capture the change of multifractal cross-correlation between two nonstationary time series influenced by common external forces, multifractal detrended partial cross-correlation analysis (MFDPXA) is employed [38]. We also investigate the multifractal behavior between the bivariate time series through MFDPXA method for comparison. The result shows that both the corresponding spectra $f_{xy}(\alpha)$ and $f_{xy,z}(\alpha)$ are wide, but the latter is narrower than the former, which is presented in Figure 4.

Here, we perform cross-correlation analysis using MFDPXA method and give the multifractal spectrum for SZI and SSEC time series in which HSI shows significant

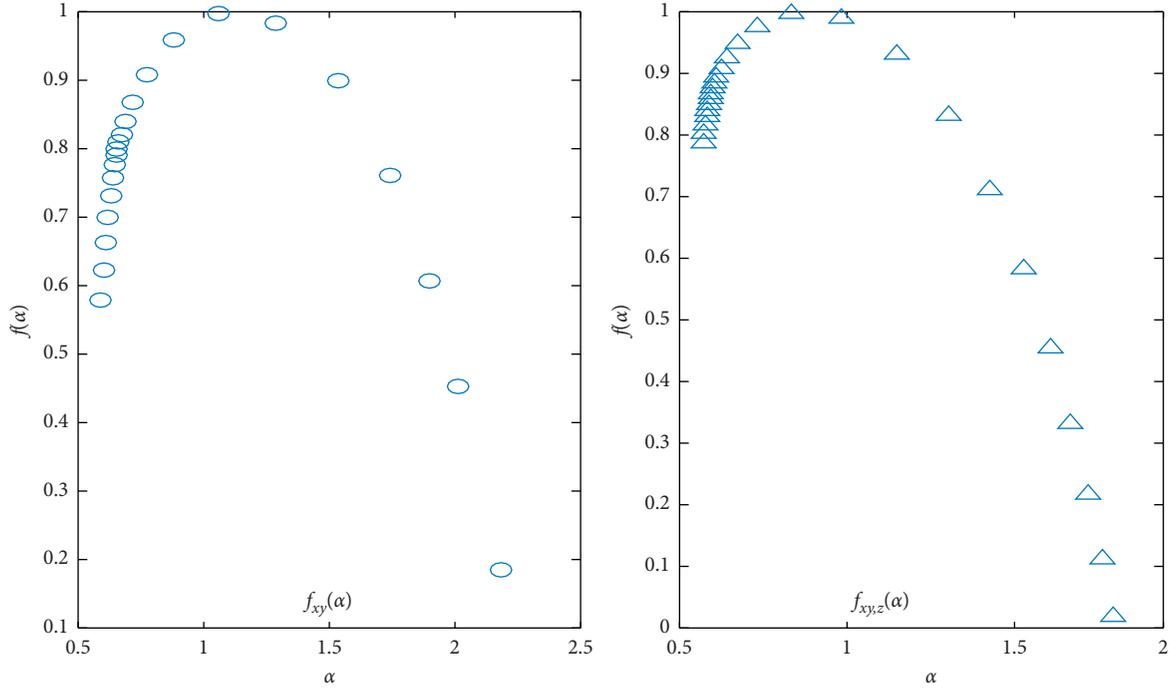


FIGURE 4: The multifractal spectra $f_{xy}(\alpha)$ of bivariate time series obtained through MFDXA method and $f_{xy,z}(\alpha)$ of bivariate time series obtained through MFDPCA, where x , y , and z denote the SSEC, SZI, and HSI.

influence on multifractal spectrum, as seen in Figure 4. We compare the obtained influence degrees with the aforementioned method and infer that the HSI has significant influence on SZI and SSEC time series. These similar results imply that the partial cross-correlation method is quite efficient in eliminating external common influence factor.

Applied to scalar variables, the first-order MFDPCA will detect the intrinsic interactions by removing the correlations of controlling variables. When variables are time series, this application is equivalent to removal of zero delay correlations, whereas delayed correlations are not considered [36, 37, 40, 41]. Therefore, we investigate the delayed effect of variable ξ_1 on the correlation between variables X and Y . Because the two variables X and Y in question may themselves be correlated at nonzero delays, we write the multifractal detrended partial cross-correlation between X and Y , given ξ_1 , as a function of two time delays:

$$\rho_{XY(\tau_1),1(\tau_2)}^q = \frac{\rho_{XY(\tau_1)}^q - \rho_{X1(\tau_2)}^q \rho_{Y1(\tau_1)}^q}{\sqrt{\left(1 - \left(\rho_{X1(\tau_2)}^q\right)^2\right) \left(1 - \left(\rho_{Y1(\tau_1)}^q\right)^2\right)}} \quad (15)$$

where τ_1 is the delay between variables X and Y and τ_2 is the delay between variables X and ξ_1 .

In this section, we estimate the delayed effect of HSI on the correlation between SSEC and SZI by using the time delay influence degree $I(q, \tau_1, \tau_2) = |\rho_{XY(\tau_1),1(\tau_2)}^q - \rho_{XY}^q|$. Figure 5 shows the time delay influence degree for $q = 2$. The effect of τ_1 on influence degree is weaker than that of τ_2 on influence degree.

We now analyze the 2-controlling-variables effect of the other thirteen stock markets on cross-correlation characteristics between SSEC and SZI, by giving a set of two controlling variables. In Figure 6, we illustrate the comparative relation of the influence degree of 2-controlling-variables DPCC coefficients for 13×13 elements by the matrix diagram.

We note that the structure of the matrix is symmetrical and that element at the intersection of row i and column j represents the influence of controlling variables ξ_i, ξ_j on the cross-correlation of SSEC and SZI, where the 2-controlling-variables ξ_i, ξ_j ($i, j = 1, 2, \dots, 13$) are the stock time series from IBOV, DJI, IXIC, SPX, FISE, FCHI, GDAXI, N255, KS11, HSI, AS51, SENSEX, and RTS. Therefore, we analyze the top left corner of the matrix. It can be seen that the largest element is the intersection of row 2 and column 10, i.e., SENSEX and HSI, which indicates the association between the Indian and Chinese stock markets. This is consistent with our result of first-order MFDPCA coefficient method.

Concerning the influence degree $I(2, q)$ of the 2-controlling-variables MFDPCA, we demonstrate 5 cases (HSI and SENSEX, HSI and RTS, SENSEX and KS11, FCHI and N255, and IXIC and FISE) for $q = 1, 2, \dots, 10$ in Figure 7. The largest influence degree is the case SENSEX and HSI, which is consistent with the 2-controlling-variables DPCC method.

3.3. Aeroengine Time Series. Previous research studies show that the aeroengine gas path parameters such as low-pressure rotor speed ($N1$), high-pressure rotor speed ($N2$),

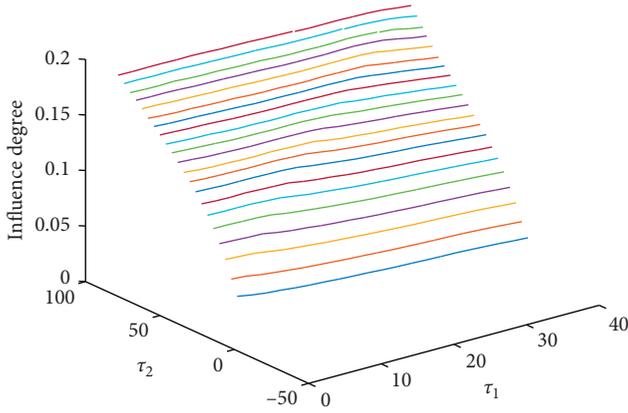


FIGURE 5: The time delay influence degree for delay τ_1 between SSEC and SZI and delay τ_2 between SSEC and HSI.

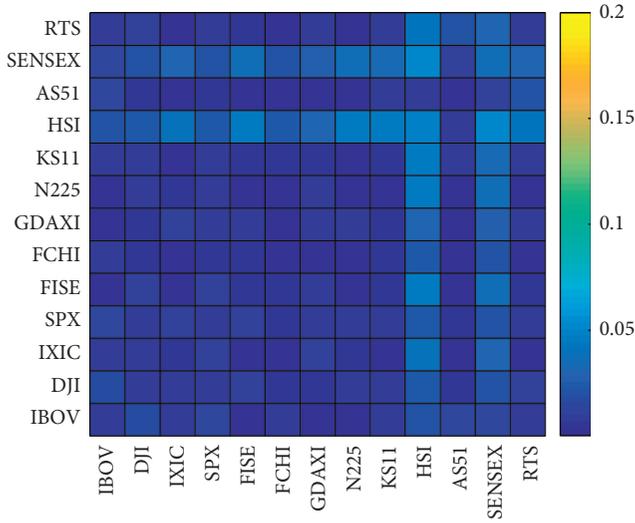


FIGURE 6: The influence degree of the 2-controlling-variables detrended partial cross-correlation.

and fuel flow (WF) play an important role in understanding the aeroengine system [21, 42]. The mean of DCCA coefficients for the aeroengine time series is shown in Figure 8, where the average DCCA coefficient between $N1$ and $N2$ is 0.85, which shows the close cross-correlation between $N1$ and $N2$.

We here investigate the partial correlation between $N1$ and $N2$ given a set of eight controlling variables, including WF, exhaust gas temperature (EGT), $N2$ tracked vibration channel B ($N2TB$), inlet air pressure ($P2$), outlet temperature of high-pressure compressor ($T3$), outlet temperature of low-pressure compressor ($T2.5$), and other temperatures ($T2$ and $T2.95$).

In Figure 9, we plot the influence degree of first-order DPCC coefficient, investigating the effect of the other eight controlling variables on cross-correlation characteristics between $N1$ and $N2$. The largest influence degree $I=0.51$, obtained by $T3$, shows the information exchange between the outlet temperature of high-pressure compressor and the rotor speed system. The next largest $I=0.22$ is acquired by

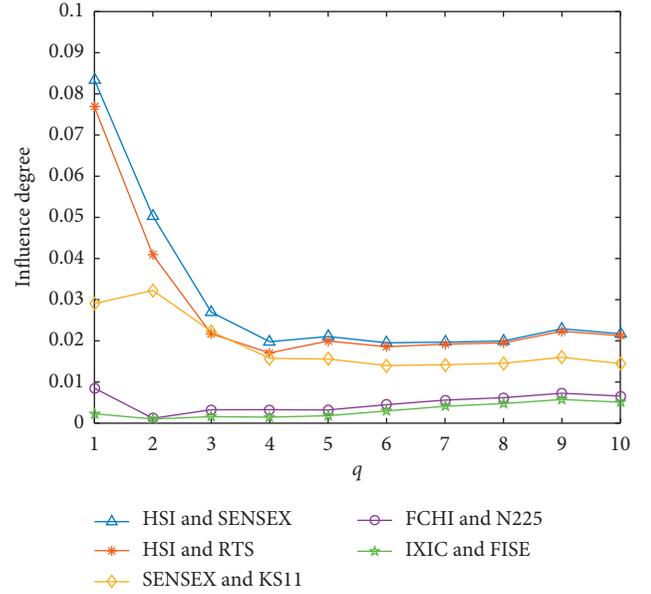


FIGURE 7: The influence degree of the 2-controlling-variables multifractal detrended partial cross-correlation.

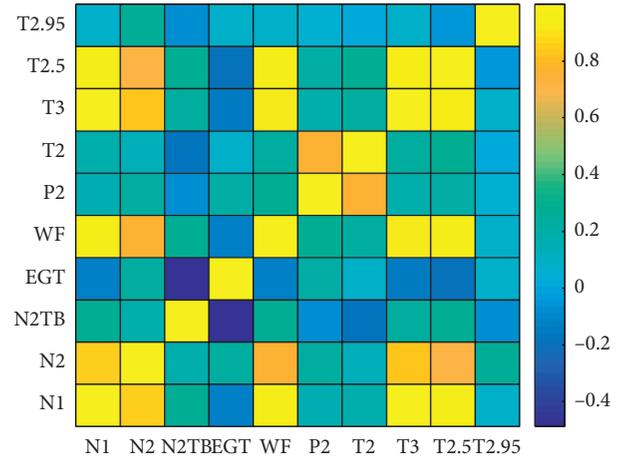


FIGURE 8: The mean of standard DCCA coefficients for the time series of aeroengine system gas path parameters.

WF, which indicates the association between the fuel flow system and rotor speed system.

The result of the influence degree $I(1, q)$ for eight aeroengine parameters applying by first-order MFDPC coefficient with $q = 1, 2, \dots, 10$ is also demonstrated in the upper left of Figure 9. The effect of $T3$ on cross-correlation characteristics, observed from influence degree function $I(1, q)$, decreases as the scale q increases. It indicates that the multifractal cross-correlation differs across values of q .

Further, we apply the MFDPCA method on aforementioned $N1$ and $N2$ time series considering the $T3$ as common influencing factor. It is observed from Figure 10 that the corresponding spectra $f_{xy}(\alpha)$ and $f_{xy,z}(\alpha)$ are wide which shows the strength of multifractal behavior in analyzed time series. We observe that the width of singularity spectrum $f_{xy,z}(\alpha)$ is narrower, and this implies the strength

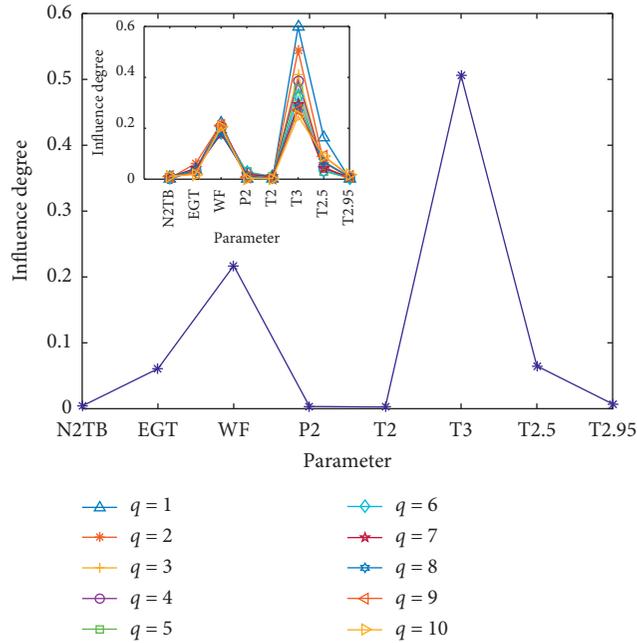


FIGURE 9: The influence degree of the first-order detrended partial cross-correlation in aeroengine system and the influence degree of the first-order multifractal detrended partial cross-correlation coefficient in aeroengine system (the upper left figure).

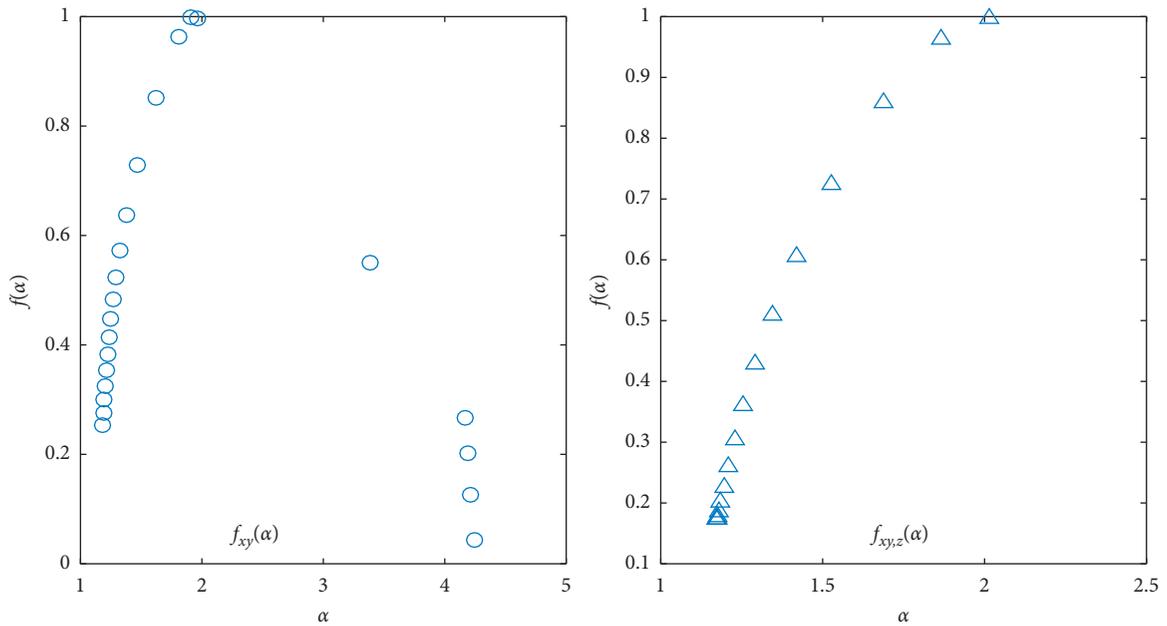


FIGURE 10: The multifractal spectrum $f_{xy}(\alpha)$ of bivariate time series obtained through MFDXA method and $f_{xy,z}(\alpha)$ of bivariate time series obtained through MFDPXA, where x , y , and z denote N1, N2, and T3.

of multifractal nature is weak in analyzed bivariate time series.

Here, we estimate the delayed effect of T3 on the correlation between N1 and N2 by using the time delay influence degree $I(q, \tau_1, \tau_2)$. Figure 11 shows the time delay influence degree for $q = 2$. It is obvious that the time delay influence degree gradually increases and then declines as a single-peak curve when τ_2 remains constant. As τ_2 increases,

the peak value of time delay influence degree shifts rightward.

The next observation concerns the influence degree of 2-controlling-variables DPCC coefficient in the aeroengine system. We now analyze the influence of two controlling parameters on the cross-correlation between N1 and N2. In Figure 12, we illustrate the comparative relation of the influence degree of 2-controlling-variables DPCC

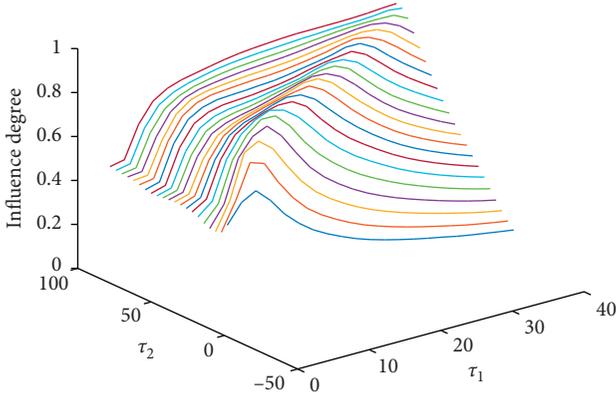


FIGURE 11: The time delay influence degree for delay τ_1 between $N1$ and $N2$ and delay τ_2 between $N1$ and $T3$.

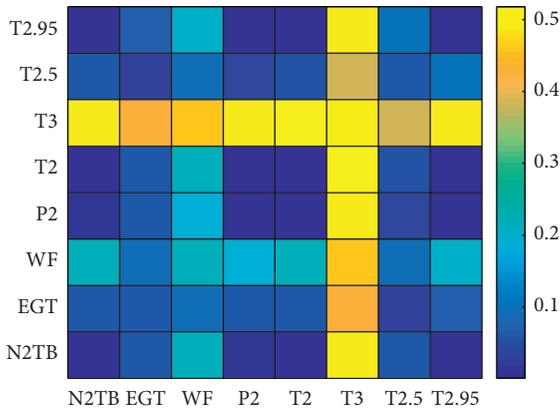


FIGURE 12: The influence degree of the 2-controlling-variables detrended partial cross-correlation for aeroengine system.

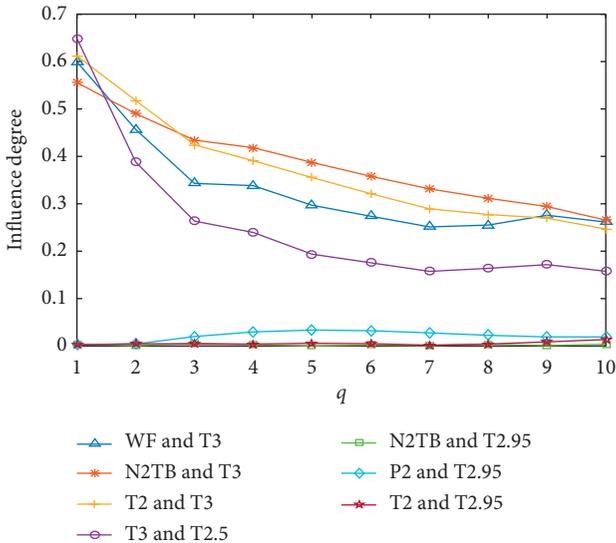


FIGURE 13: The influence degree of the 2-controlling-variables multifractal detrended partial cross-correlation for the aeroengine system.

coefficient for aeroengine system. It can be seen that the larger elements in the symmetrical matrix are located at row 3 or column 6, which denote $T3$ has a greater impact on the correlation between $N1$ and $N2$.

Concerning the influence degree $I(2, q)$ of the 2-controlling-variables MFDPC, we demonstrate 7 cases ($T3$ and WF , $T3$ and $N2TB$, $T3$ and $T2$, $T3$ and $T2.5$, $T2.95$ and $N2TB$, $T2.95$ and $P2$, and $T2.95$ and $T2$) for $q = 1, 2, \dots, 10$ in Figure 13. Larger influence degrees exist in the cases with the presence of $T3$ ($T3$ and WF , $T3$ and $N2TB$, $T3$ and $T2$, and $T3$ and $T2.5$), which is consistent with the 2-controlling-variables DPCC method, as seen in Figure 12.

For the aeroengine, the parameters $N1$ and $N2$ are chosen to indicate the engine thrust which depends on the throttle lever angle. Hence, the cross-correlation between them is strong. The temperature and pressure parameters are linked with many factors, including the compressor power, combustion efficiency, throttle lever angle, etc. Therefore, the dynamic interaction of these three groups makes the aeroengine function. These results estimate the influence of temperature and pressure parameters on the cross-correlation between $N1$ and $N2$.

4. Conclusion

In this paper, we propose the n th-order multifractal detrended partial cross-correlation analysis method and the n -controlling-variables multifractal detrended partial cross-correlation analysis method for understanding the interactions between two nonstationary time series. For comparing these new methods with classical measures, we introduce the influence degree function. We then apply the n -controlling-variables multifractal detrended partial cross-correlation analysis of stock markets and aeroengine performance parameters and measure the influence degree function of the partial cross-correlation in a dynamic system.

To understand the numerous real-world systems where the output signals exhibit complex cross-correlation, both cross-correlation and partial correlation are subjects of investigation. The information of n -variables MFDPC helps people to research information exchange in complex systems. This paper gives two examples, stock markets and aeroengine systems. For stock time series, our results indicate that, concerning closing index values, there is little information exchange between the Chinese stock markets and the American-European stock markets, whereas the SSEC, SZI, and HSI, by first-order MFDPC method and 2-controlling-variables MFDPC, show frequent and abundant information exchange in Chinese stock markets. For aeroengine performance parameters, our results show that there is some information exchange between the engine rotor system and the aeroengine parameters, such as the outlet temperature of the high-pressure compressor and the fuel flow.

We believe that the MFDPC method can be used to detect the intrinsic interactions among multiple dynamical systems, and therefore it can be widely applied to many research fields such as the aeroengine health monitoring systems and the investment portfolio where the covariance is employed to explore the interaction of assets income.

The multifractal detrended partial cross-correlation analysis is used to delete the possible indirect correlation, but it may also delete valuable information. This problem

required further investigation, both experimental and theoretical. Hence, the results of this paper should be considered as preliminary results on the multifractal detrended partial cross-correlation analysis. Therefore, we hope that this study will be extended to analyze the filtered information.

Data Availability

The stock market data used to support the findings of this study are available from the corresponding author upon request. The aeroengine data used to support the findings of this study have not been made available because of commercial secrets.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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