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*Research Article*

# Investigation of Unmeasured Parameters Estimation for Distributed Control Systems

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The problem of unmeasured parameters estimation for distributed control systems is studied in this paper. The Takagi–Sugeno fuzzy model which can appropriate any nonlinear systems is employed, and based on the model, an observer-based fuzzy $H\infty$ filter which has robustness against time-delay, external noise, and system uncertainties is designed. The sufficient condition for the existence of the desired filter is derived in terms of linear matrix inequalities (LMIs) solutions. Moreover, the underdetermined estimation problem in which the number of sensors available is typically less than the number of state variables to be estimated is specifically addressed. A systematic method is proposed to produce a model tuning parameter vector of appropriate dimension for the estimation of the filter, and the optimal transformation matrix is selected via iterative solution to minimize the estimated error. Finally, a simulation example for turbofan aeroengine is given to illustrate the effectiveness of the proposed method, and the estimated error is less than 2.5%.

## 1. Introduction

Along with the continuous development of computer and network technology, distributed control systems (DCSs) have gradually become a new trend and attract much attention [1, 2]. Compared with traditional point-to-point control systems, DCSs have advantages such as high reliability, reduced weight, low cost, and ease of maintenance [3, 4]. The DCSs provide the low-level processing function via intelligent unit and are more conductive to the implementation of complex control algorithms [5, 6]. However, the signal of DCSs is transmitted via bus network, which brings new challenges, such as networked-induced time-delay, packet dropout, and packet disordering.

In practice, many parameters are difficult to obtain due to the limitation of the system itself (for example, the sensors cannot be installed in poor working conditions) or cost constraints. Thus, the estimation of unmeasured system parameters is significantly important, since the estimated parameters can not only be used in the design of controller but also for online monitoring [7, 8]. For the estimation problem of unmeasured system parameters, there are two main methods: (1) high-precision mathematical model [9]; (2) filtering technique and its extension methods [10]. The first method is to establish the mathematical model based on the structure of system to solve the unmeasured parameters. However, the modeling process is often cumbersome and complicated. And for some systems, the structure is extremely complicated and cannot be accurately modeled. Moreover, since this method generally requires iterative calculation, the amount of calculation is large.

For the second method, the estimation of unmeasured parameters is based on the value of measurable parameters using the filtering technique. Because this method uses a recursive algorithm, it is easy to implement on computer and meet the requirements of accuracy and real-time. Among the filters, the $H\infty$ filter stands out because it has robust stability against external noise without priori knowledge of noise and precise mathematical model [11–13].

However, in some cases, the number of sensors available is typically less than the number of state variables to be estimated, which is an underdetermined estimation problem and cannot be solved directly. A common approach to address this shortcoming is to estimate a subset of the unknown parameters and assuming that others remain unchanged [14–16]. Although this approach enables online filter-based estimation, it will introduce error in the accuracy of overall estimation application. If any of the parameters which are assumed unchanged moves away from their nominal values, the estimates can no longer represent the true parameters. Litt [17] presented a novel approach based on singular value decomposition (SVD) that selects a model tuning parameter vector of low enough dimension. The model tuning parameter vector is constructed as a linear combination of all unmeasured parameters and the transformation matrix is generated by selecting the $k$ most significant terms of singular values, where $k$ is the number of available sensors. Similarly, the SVD is also used in [18] to determine which parts of the systems are observable if the whole system is unobservable. However, the unaccounted terms of singular values may also have great importance on the accuracy of overall estimation application even if they are small under some circumstances.

Furthermore, in [19], the transformation matrix is selected to minimize the theoretical mean squared estimation error at a steady-state open-loop linear design point. But the estimation results are affected by the parameter perturbation, and there are many limitations in application. Moreover, all the approaches above do not consider the impact of time-delay and uncertainties of the system. Thus, it has great significance and practical value to propose a novel approach for the underdetermined estimation problem.

Besides, due to the strong nonlinear fitting characteristics of complex nonlinear systems, the Takagi–Sugeno (T-S) fuzzy model has been widely used in the study of nonlinear systems since it was proposed [20–22]. Thus in this paper, an observer-based fuzzy $H\infty$ filter is constructed, and the unmeasured parameters estimation problem for T-S fuzzy distributed control systems with time-delay and parameters perturbation is studied.

The contributions of this paper can be concluded as follows: (i) a model tuning parameter vector of appropriate dimension is produced for the estimation of the filter; (ii) a systematic method is proposed for the selection of optimal transformation matrix to minimize the estimated error via iterative solution; (iii) the parameter perturbation is considered, so that the designed filter has a certain range of margins of disturbance.

The remainder of this paper is organized as follows. Section 2 introduces the modeling method of DCSs and the observer-based fuzzy $H\infty$ filter. The main results are presented in Section 3, including the analysis and synthesis of the filtering error system.Section 4 presents the approach of optimal transformation matrix selection. Numerical simulation results are shown in Section 5 and finally conclusion and a discussion of further application of the method are presented.

## 2. Problem Formulation

Consider the DCSs with time-delay, which can be described by a class of T-S fuzzy model as follows:

Plant rule $i$: If $f_1(t)$ is $\Phi_1^i$ and $\cdots$ $f_g(t)$ is $\Phi_g^i$, then

$$\begin{aligned}
\dot{x}(t) &= (\mathbf{A}_i + \Delta\mathbf{A}_i)\mathbf{x}(t) + (\mathbf{B}_i + \Delta\mathbf{B}_i)\mathbf{u}(t-\tau(t)) + \mathbf{L}_i\mathbf{h}(t) + (\mathbf{E}_{1i} + \Delta\mathbf{E}_{1i})\mathbf{w}(t),\\
\mathbf{y}(t) &= (\mathbf{C}_i + \Delta\mathbf{C}_i)\mathbf{x}(t) + (\mathbf{D}_i + \Delta\mathbf{D}_i)\mathbf{u}(t-\tau(t)) + \mathbf{M}_i\mathbf{h}(t) + (\mathbf{E}_{2i} + \Delta\mathbf{E}_{2i})\mathbf{w}(t),\\
\mathbf{z}(t) &= (\mathbf{F}_i + \Delta\mathbf{F}_i)\mathbf{x}(t) + (\mathbf{G}_i + \Delta\mathbf{G}_i)\mathbf{u}(t-\tau(t)) + \mathbf{N}_i\mathbf{h}(t)\\
\mathbf{x}(t) &= \phi(t),\\
&\quad t \in [-\tau_m, 0],
\end{aligned} \tag{1}$$

where $\Omega_j^i\,(i = 1,\ldots,r;\, j = 1,\ldots,g)$ denotes the fuzzy set, $r$ denotes the number of IF-THEN rules, and $f_j(t)$ denotes the premise variable.$\mathbf{x}(t) \in \mathbb{R}^n$ is the vector of state variables; $\mathbf{u}(t) \in \mathbb{R}^m$ is the vector of control inputs; $\mathbf{y}(t) \in \mathbb{R}^p$ is the vector of measured outputs; $\mathbf{z}(t) \in \mathbb{R}^l$ is the vector of unmeasured outputs and $\mathbf{w}(t)$ is noise signal which belongs to $L_2(0,\infty]$. $\mathbf{A}_i, \mathbf{B}_i, \mathbf{C}_i, \mathbf{D}_i, \mathbf{E}_{1i}, \mathbf{E}_{2i}, \mathbf{F}_i, \mathbf{G}_i, \mathbf{L}_i, \mathbf{M}_i, \mathbf{N}_i$ are matrices with appropriate dimension. $\Delta\mathbf{A}_i, \Delta\mathbf{B}_i, \Delta\mathbf{C}_i, \Delta\mathbf{D}_i, \Delta\mathbf{E}_{1i}, \Delta\mathbf{E}_{2i}, \Delta\mathbf{F}_i, \Delta\mathbf{G}_i$ are unknown matrices which represent the time-varying uncertainties of the system. $\mathbf{h}(t) \in \mathbb{R}^q$ is the health parameters of system, which represents the physical characteristics of each component.

*Remark 1.* The health parameters are considered in this paper, because the performance degradation of each component in the system is inevitable during the working process. And the system's performance is affected by the level of degradation, which is generally described in terms of unmeasured health parameters such as efficiencies and capacities related to each major module in most cases. If the health parameters move away from their nominal values, the shift in other performance variables will be induced. They may be treated as a set of biases and can be augmented to the system states.

It is assumed that both sensors and actuators are time-driven. The data has timestamp and is transmitted in a single-packet, and incorrect order of the data packet does not exist. Since time-delay is related to the bus load at a certain moment, it is assumed that the time-delay has an upper bound, which is $\tau(t) \le \tau_m$ (under the above assumption, the upper bound of time-delay can be estimated; see [23] in detail). As a consequence, $\tau(t)$ can be modeled as a finite state Markov stochastic process on a finite set $\Lambda = \{1, 2, \ldots, \tau_m\}$. The transition probability from $\tau(t) = i$ at time $t$ to $\tau(t) = j\,(j \ne i)$ at time $t + \Delta t$ is

$$\Pr(\tau(t+\Delta t) = j \mid \tau(t) = i) = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \ne j, \\ 1 + \pi_{ij}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{2}$$

where $\Delta t > 0$ and $\lim_{\Delta t \longrightarrow 0}(o(\Delta t)/\Delta t) = 0$. $\pi_{ij} \ge 0$ is the transition probability rates from $\tau(t) = i$ at time $t$ to $\tau(t) = j\,(j \ne i)$ at time $t + \Delta t$, and there is $\sum_{j=1, j\ne i}^{\tau_m} \pi_{ij} = -\pi_{ii}$.

The state feedback controller is applied in this paper, which is $\mathbf{u}(t) = \mathbf{K}_c\mathbf{x}(t)$. Then, by fuzzy blending, the global fuzzy augmented model can be obtained as follows:

$$\begin{cases} \dot{x}_h(t) = \sum_{i=1}^{r} h_i(f(t))\left[(\mathbf{A}_{hi} + \Delta\mathbf{A}_{hi})\mathbf{x}_h(t) + (\mathbf{B}_{hi} + \Delta\mathbf{B}_{hi})\mathbf{x}_h \right. \\ \quad \left. \cdot (t - \tau(t)) + (\mathbf{E}_{h1i} + \Delta\mathbf{E}_{h1i})\mathbf{w}(t)\right], \\ \mathbf{y}(t) = \sum_{i=1}^{r} h_i(f(t))\left[(\mathbf{C}_{hi} + \Delta\mathbf{C}_{hi})\mathbf{x}_h(t) + (\mathbf{D}_{hi} + \Delta\mathbf{D}_{hi})\mathbf{x}_h \right. \\ \quad \left. \cdot (t - \tau(t)) + (\mathbf{E}_{2i} + \Delta\mathbf{E}_{2i})\mathbf{w}(t)\right], \\ \mathbf{z}(t) = \sum_{i=1}^{r} h_i(f(t))\left[(\mathbf{F}_{hi} + \Delta\mathbf{F}_{hi})\mathbf{x}_h(t) + (\mathbf{G}_{hi} + \Delta\mathbf{G}_{hi}) \right. \\ \quad \left. \cdot \mathbf{x}_h(t - \tau(t))\right], \\ \mathbf{x}_h(t) = \begin{bmatrix} \phi(t) \\ 0 \end{bmatrix}, \\ t \in [-\tau_m, 0], \end{cases} \tag{3}$$

where

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{h}(t) \end{bmatrix}, \\ \mathbf{A}_{hi} &= \begin{bmatrix} \mathbf{A}_i & \mathbf{L}_i \\ 0 & \mathbf{I} \end{bmatrix}, \\ \mathbf{B}_{hi} &= \begin{bmatrix} \mathbf{B}_i\mathbf{K}_c & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta\mathbf{A}_{hi} &= \begin{bmatrix} \Delta\mathbf{A}_i & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta\mathbf{B}_{hi} &= \begin{bmatrix} \Delta\mathbf{B}_i\mathbf{K}_c & 0 \\ 0 & 0 \end{bmatrix}, \\ \mathbf{E}_{h1i} &= \begin{bmatrix} \mathbf{E}_{1i} \\ 0 \end{bmatrix}, \\ \mathbf{C}_{hi} &= \begin{bmatrix} \mathbf{C}_i & \mathbf{M}_i \end{bmatrix}, \\ \Delta\mathbf{C}_{hi} &= \begin{bmatrix} \Delta\mathbf{C}_i & 0 \end{bmatrix}, \\ \Delta\mathbf{E}_{h1i} &= \begin{bmatrix} \Delta\mathbf{E}_{1i} \\ 0 \end{bmatrix}, \\ \mathbf{D}_{hi} &= \begin{bmatrix} \mathbf{D}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ \Delta\mathbf{D}_{hi} &= \begin{bmatrix} \Delta\mathbf{D}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ \mathbf{E}_{h2i} &= \mathbf{E}_{2i}, \\ \Delta\mathbf{E}_{h2i} &= \Delta\mathbf{E}_{2i}, \\ \mathbf{F}_{hi} &= \begin{bmatrix} \mathbf{F}_i & \mathbf{N}_i \end{bmatrix}, \\ \mathbf{G}_{hi} &= \begin{bmatrix} \mathbf{G}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ \Delta\mathbf{F}_{hi} &= \begin{bmatrix} \Delta\mathbf{F}_i & 0 \end{bmatrix}, \\ \Delta\mathbf{G}_{hi} &= \begin{bmatrix} \Delta\mathbf{G}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ h_i(f(t)) &= \frac{\vartheta_i(f(t))}{\sum_{i=1}^{r}\vartheta_i(f(t))}, \\ \vartheta_i(f(t)) &= \prod_{j=1}^{2}\Omega_j^i(f(t)). \end{aligned} \tag{4}$$

For the underdetermined estimation problem, a low-dimensional model tuning parameter vector $\mathbf{q}(t)$ is produced to represent the information of high-dimensional health parameter vector $\mathbf{h}(t)$. The model tuning parameter vector $\mathbf{q}(t)$ is constructed as a linear combination of all health parameters, given by

$$\mathbf{q}(t) = \mathbf{V}^*\mathbf{h}(t), \tag{5}$$

where $\mathbf{h}(t) \in \mathbb{R}^q, \mathbf{q}(t) \in \mathbb{R}^k, k < q$, and $\mathbf{V}^*$ is a $k \times q$ transformation matrix, which is applied to construct the tuning parameter vector. The selection of optimal $\mathbf{V}^*$ is obtained in Section 4.

Thus, the estimation of the health parameters $\widehat{\mathbf{h}}$ can be obtained as

$$\widehat{\mathbf{h}} = \mathbf{V}^{*\dagger}\mathbf{q}, \tag{6}$$

where $\mathbf{V}^{*\dagger}$ is the pseudoinverse of $\mathbf{V}^*$.

The final augmented dynamic fuzzy model can be rewritten as

$$\begin{cases} \dot{x}_q(t) = \sum_{i=1}^{r} h_i(f(t))\left[\left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right)\mathbf{x}_q(t) + \left(\mathbf{B}_{qi} + \Delta\mathbf{B}_{qi}\right)\mathbf{x}_q(t-\tau(t)) + \left(\mathbf{E}_{q1i} + \Delta\mathbf{E}_{q1i}\right)\mathbf{w}(t)\right], \\ \mathbf{y}(t) = \sum_{i=1}^{r} h_i(f(t))\left[\left(\mathbf{C}_{qi} + \Delta\mathbf{C}_{qi}\right)\mathbf{x}_q(t) + \left(\mathbf{D}_{qi} + \Delta\mathbf{D}_{qi}\right)\mathbf{x}_q(t-\tau(t)) + \left(\mathbf{E}_{2i} + \Delta\mathbf{E}_{2i}\right)\mathbf{w}(t)\right], \\ \mathbf{z}(t) = \sum_{i=1}^{r} h_i(f(t))\left[\left(\mathbf{F}_{qi} + \Delta\mathbf{F}_{qi}\right)\mathbf{x}_q(t) + \left(\mathbf{G}_{qi} + \Delta\mathbf{G}_{qi}\right)\mathbf{x}_q(t-\tau(t))\right], \\ \mathbf{x}_q(t) = \begin{bmatrix} \phi(t) \\ 0 \end{bmatrix}, \\ t \in [-\tau_m, 0], \end{cases} \tag{7}$$

where

$$\begin{aligned} \mathbf{x}_q(t) &= \begin{bmatrix} \mathbf{x}(t) \\ \mathbf{q}(t) \end{bmatrix}, \\ \mathbf{A}_{qi} &= \begin{bmatrix} \mathbf{A}_i & \mathbf{L}_i\mathbf{V}^{*\dagger} \\ 0 & \mathbf{I} \end{bmatrix}, \\ \mathbf{E}_{q1i} &= \begin{bmatrix} \mathbf{E}_{1i} \\ 0 \end{bmatrix}, \\ \Delta\mathbf{A}_{qi} &= \begin{bmatrix} \Delta\mathbf{A}_i & 0 \\ 0 & 0 \end{bmatrix}, \\ \mathbf{B}_{qi} &= \begin{bmatrix} \mathbf{B}_i\mathbf{K}_c & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta\mathbf{B}_{qi} &= \begin{bmatrix} \Delta\mathbf{B}_i\mathbf{K}_c & 0 \\ 0 & 0 \end{bmatrix}, \\ \Delta\mathbf{E}_{q1i} &= \begin{bmatrix} \Delta\mathbf{E}_{1i} \\ 0 \end{bmatrix}, \\ \mathbf{C}_{qi} &= \begin{bmatrix} \mathbf{C}_i & \mathbf{M}_i\mathbf{V}^{*\dagger} \end{bmatrix}, \\ \Delta\mathbf{C}_{qi} &= \begin{bmatrix} \Delta\mathbf{C}_i & 0 \end{bmatrix}, \\ \mathbf{D}_{qi} &= \begin{bmatrix} \mathbf{D}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ \Delta\mathbf{D}_{qi} &= \begin{bmatrix} \Delta\mathbf{D}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ \mathbf{E}_{q2i} &= \mathbf{E}_{2i}, \\ \Delta\mathbf{E}_{q2i} &= \Delta\mathbf{E}_{2i}, \\ \mathbf{F}_{qi} &= \begin{bmatrix} \mathbf{F}_i & \mathbf{N}_i\mathbf{V}^{*\dagger} \end{bmatrix}, \\ \mathbf{G}_{qi} &= \begin{bmatrix} \mathbf{G}_i\mathbf{K}_c & 0 \end{bmatrix}, \\ \Delta\mathbf{F}_{qi} &= \begin{bmatrix} \Delta\mathbf{F}_i & 0 \end{bmatrix}, \\ \Delta\mathbf{G}_{qi} &= \begin{bmatrix} \Delta\mathbf{G}_i\mathbf{K}_c & 0 \end{bmatrix}. \end{aligned} \tag{8}$$

*Remark 2.* If the model tuning parameter vector is of appropriate dimension, the augmented system (7) is observable and the observer-based filter can be used to estimate the unknown health and performance parameters. And the key of the estimation is to find the optimal transformation matrix so that the low dimension tuning vector can represent as much of the information as possible. Moreover, different from the method in [17–19], the solution of the optimal transformation matrix is transformed into an optimization problem.

It is assumed that the uncertainties of the system can be described in the following form:

$$\begin{bmatrix} \Delta\mathbf{A}_{qi} & \Delta\mathbf{B}_{qi} & \Delta\mathbf{E}_{q1i} & \Delta\mathbf{F}_{qi}^T \\ \Delta\mathbf{C}_{qi} & \Delta\mathbf{D}_{qi} & \Delta\mathbf{E}_{q2i} & \Delta\mathbf{G}_{qi}^T \end{bmatrix} = \begin{bmatrix} \mathbf{U}_{1i} \\ \mathbf{U}_{2i} \end{bmatrix}\mathbf{S}_i\begin{bmatrix} \mathbf{V}_{1i} & \mathbf{V}_{2i} & \mathbf{V}_{3i} & \mathbf{V}_{4i} \end{bmatrix}, \tag{9}$$

where $\mathbf{U}_{1i}, \mathbf{U}_{2i}, \mathbf{V}_{1i}, \mathbf{V}_{2i}, \mathbf{V}_{3i}, \mathbf{V}_{4i}$ are known matrices and $\mathbf{S}_i$ is a time-varying unknown matrix which satisfies

$$\mathbf{S}_i^T\mathbf{S}_i \leq \mathbf{I}. \tag{10}$$

Then the following observer-based fuzzy filter is constructed:

$$\begin{aligned} \dot{\widehat{\mathbf{x}}}(t) &= \sum_{i=1}^{r} h_i(f(t))\left[\left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right)\widehat{\mathbf{x}}(t)\right. \\ &\quad \left. + \left(\mathbf{B}_{qi} + \Delta\mathbf{B}_{qi}\right)\widehat{\mathbf{x}}(t-\tau(t)) + \mathbf{K}_i(\mathbf{y}(t) - \widehat{\mathbf{y}}(t))\right], \\ \widehat{\mathbf{y}}(t) &= \sum_{i=1}^{r} h_i(f(t))\left[\left(\mathbf{C}_{qi} + \Delta\mathbf{C}_{qi}\right)\widehat{\mathbf{x}}(t) + \left(\mathbf{D}_{qi} + \Delta\mathbf{D}_{qi}\right)\widehat{\mathbf{x}}(t-\tau(t))\right], \\ \widehat{\mathbf{z}}(t) &= \sum_{i=1}^{r} h_i(f(t))\left[\left(\mathbf{F}_{qi} + \Delta\mathbf{F}_{qi}\right)\widehat{\mathbf{x}}(t) + \left(\mathbf{G}_{qi} + \Delta\mathbf{G}_{qi}\right)\widehat{\mathbf{x}}(t-\tau(t))\right], \end{aligned} \tag{11}$$

where $\hat{\mathbf{x}}(t)$ is the filter state, and $\hat{\mathbf{y}}(t)$ and $\hat{\mathbf{z}}(t)$ are the filter outputs. $\mathbf{K}_i$ is the filter parameter to be determined.

The estimation error of state is

$$\begin{aligned}\tilde{x}(t) &= \mathbf{x}_q(t) - \hat{x}(t)\\ &= \left(\mathbf{A}_{qi} - \mathbf{K}_i\mathbf{C}_{qi} + \Delta\mathbf{A}_{qi} - \mathbf{K}_i\Delta\mathbf{C}_{qi}\right)\tilde{x}(t)\\ &\quad + \left(\mathbf{B}_{qi} - \mathbf{K}_i\mathbf{D}_{qi} + \Delta\mathbf{B}_{qi} - \mathbf{K}_i\Delta\mathbf{D}_{qi}\right)\tilde{x}(t-\tau(t))\\ &\quad + \left(\mathbf{E}_{q1i} - \mathbf{K}_i\mathbf{E}_{2i} + \Delta\mathbf{E}_{q1i} - \mathbf{K}_i\Delta\mathbf{E}_{2i}\right)\mathbf{w}(t).\end{aligned} \tag{12}$$

Define $\boldsymbol{\xi}(t) = \left[\mathbf{x}_q^T \ \ \tilde{x}^T(t)\right]^T$ and $\mathbf{e}(t) = \mathbf{z}(t) - \hat{z}(t)$, and the filtering error system is given by

$$\begin{aligned}\dot{\boldsymbol{\xi}}(t) &= \sum_{i=1}^{r} h_i(f(t))\left[(\overline{A}_i + \Delta\overline{A}_i)\boldsymbol{\xi}(t) + (\overline{B}_i + \Delta\overline{B}_i)\boldsymbol{\xi}(t-\tau(t))\right.\\ &\quad \left. + (\overline{E}_i + \Delta\overline{E}_i)\mathbf{w}(t)\right],\\ \dot{\mathbf{e}}(t) &= \sum_{i=1}^{r} h_i(f(t))\left[(\overline{F}_i + \Delta\overline{F}_i)\boldsymbol{\xi}(t) + (\overline{G}_i + \Delta\overline{G}_i)\boldsymbol{\xi}(t-\tau(t))\right],\end{aligned} \tag{13}$$

where

$$\begin{aligned}\overline{\mathbf{A}}_i &= \begin{bmatrix}\mathbf{A}_{qi} & 0\\ 0 & \mathbf{A}_{qi} - \mathbf{K}_i\mathbf{C}_{qi}\end{bmatrix},\\ \overline{\mathbf{B}}_i &= \begin{bmatrix}\mathbf{B}_{qi} & 0\\ 0 & \mathbf{B}_{qi} - \mathbf{K}_i\mathbf{D}_{qi}\end{bmatrix},\\ \Delta\overline{\mathbf{A}}_i &= \begin{bmatrix}\Delta\mathbf{A}_{qi} & 0\\ 0 & \Delta\mathbf{A}_{qi} - \mathbf{K}_i\Delta\mathbf{C}_{qi}\end{bmatrix},\\ \overline{\mathbf{E}}_i &= \begin{bmatrix}\mathbf{E}_{q1i}\\ \mathbf{E}_{q1i} - \mathbf{K}_i\mathbf{E}_{2i}\end{bmatrix},\\ \Delta\overline{\mathbf{B}}_i &= \begin{bmatrix}\Delta\mathbf{B}_{qi} & 0\\ 0 & \Delta\mathbf{B}_{qi} - \mathbf{K}_i\Delta\mathbf{D}_{qi}\end{bmatrix},\\ \Delta\overline{\mathbf{E}}_i &= \begin{bmatrix}\Delta\mathbf{E}_{q1i}\\ \Delta\mathbf{E}_{q1i} - \mathbf{K}_i\Delta\mathbf{E}_{2i}\end{bmatrix},\\ \overline{\mathbf{F}}_i &= \begin{bmatrix}0 & \mathbf{F}_{qi}\end{bmatrix},\\ \Delta\overline{\mathbf{F}}_i &= \begin{bmatrix}0 & \Delta\mathbf{F}_{qi}\end{bmatrix},\\ \overline{\mathbf{G}}_i &= \begin{bmatrix}0 & \mathbf{G}_{qi}\end{bmatrix},\\ \Delta\overline{\mathbf{G}}_i &= \begin{bmatrix}0 & \Delta\mathbf{G}_{qi}\end{bmatrix}.\end{aligned} \tag{14}$$

Define the following parameters:

$$\begin{aligned}\overline{\mathbf{A}} &= \sum_{i=1}^{r} h_i(f(t))\overline{\mathbf{A}}_i,\\ \Delta\overline{\mathbf{A}} &= \sum_{i=1}^{r} h_i(f(t))\Delta\overline{\mathbf{A}}_i,\\ \overline{\mathbf{B}} &= \sum_{i=1}^{r} h_i(f(t))\overline{\mathbf{B}}_i,\\ \Delta\overline{\mathbf{B}} &= \sum_{i=1}^{r} h_i(f(t))\Delta\overline{\mathbf{B}}_i,\\ \overline{\mathbf{E}} &= \sum_{i=1}^{r} h_i(f(t))\overline{\mathbf{E}}_i,\\ \Delta\overline{\mathbf{E}} &= \sum_{i=1}^{r} h_i(f(t))\Delta\overline{\mathbf{E}}_i,\\ \overline{\mathbf{F}} &= \sum_{i=1}^{r} h_i(f(t))\overline{\mathbf{F}}_i,\\ \Delta\overline{\mathbf{F}} &= \sum_{i=1}^{r} h_i(f(t))\Delta\overline{\mathbf{F}}_i,\\ \overline{\mathbf{G}} &= \sum_{i=1}^{r} h_i(f(t))\overline{\mathbf{G}}_i,\\ \Delta\overline{\mathbf{G}} &= \sum_{i=1}^{r} h_i(f(t))\Delta\overline{\mathbf{G}}_i.\end{aligned} \tag{15}$$

The goal is to design a filter in form of (11) so that the filtering error system can meet the following requirements simultaneously:

(1) The filtering error system is asymptotically stable when $\mathbf{w}(t) = 0$

(2) Under the zero initial condition, the filtering error system satisfies

$$\|\mathbf{e}(t)\|_2 \le \gamma\|\mathbf{w}(t)\|_2, \tag{16}$$

for any nonzero $\mathbf{w}(t) \in L_2(0,\infty]$.

## 3. Main Results

In this section, the sufficient condition for the existence of the desired filter is derived in terms of LMIs solutions. Before proceeding with the study, the following Lemma is needed.

**Lemma 1** (see [24]). $\mathbf{D},\mathbf{E},\mathbf{F}$ *are real matrices with appropriate dimensions, and* $\mathbf{F}$ *is a time-varying unknown matrix which satisfies* $\mathbf{F}^T\mathbf{F} \le \mathbf{I}$. *Then for a scalar* $\varepsilon > 0$, *the following inequality*

$$\mathbf{DF}\quad \mathbf{E} + \mathbf{E}^T\mathbf{F}^T\mathbf{D}^T \le \varepsilon^{-1}\mathbf{D}\mathbf{D}^T + \varepsilon\mathbf{E}^T\mathbf{E}, \tag{17}$$

*always holds.*

**Theorem 1.** *If there exist positive matrices* $\mathbf{P} > 0, \mathbf{Q} > 0, \mathbf{R} > 0$, *and matrix* $\mathbf{W}$ *such that the following inequality holds:*

$$\overline{\Psi} = \begin{bmatrix} \Gamma & \mathbf{P}(\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}}) + \mathbf{W} & \mathbf{P}(\overline{\mathbf{E}} + \Delta\overline{\mathbf{E}}) & \tau_m\mathbf{W} & \tau_m(\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}})^T\mathbf{R} & (\overline{\mathbf{F}} + \Delta\overline{\mathbf{F}})^T \\ * & -\mathbf{Q} & 0 & 0 & \tau_m(\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}})^T\mathbf{R} & (\overline{\mathbf{G}} + \Delta\overline{\mathbf{G}})^T \\ * & * & -\gamma^2\mathbf{I} & 0 & \tau_m(\overline{\mathbf{E}} + \Delta\overline{\mathbf{E}})^T\mathbf{R} & 0 \\ * & * & * & -\tau_m\mathbf{R} & 0 & 0 \\ * & * & * & * & -\tau_m\mathbf{R} & 0 \\ * & * & * & * & * & -\mathbf{I} \end{bmatrix} < 0, \tag{18}$$

*where*

$$\Gamma = \mathbf{P}(\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}}) + (\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}})^T\mathbf{P} + \mathbf{Q} - \mathbf{W} - \mathbf{W}^T, \tag{19}$$

*then the filtering error system is asymptotically stable and the prescribed H∞ performance $\gamma$ can be guaranteed.*

*Proof.* Select the Lyapunov function as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{20}$$

where

$$\begin{aligned} V_1(t) &= \boldsymbol{\xi}^T(t)\mathbf{P}\boldsymbol{\xi}(t), \\ V_2(t) &= \int_{t-\tau(t)}^{t} \boldsymbol{\xi}^T(\alpha)\mathbf{Q}\boldsymbol{\xi}(\alpha)\mathrm{d}\alpha, \\ V_3(t) &= \int_{-\tau_m}^{0}\int_{t+\beta}^{t} \dot{\xi}^T(\alpha)\mathbf{R}\dot{\xi}(\alpha)\mathrm{d}\alpha\,\mathrm{d}\beta. \end{aligned} \tag{21}$$

When $w(t) = 0$, the derivation of $V(t)$ is

$$\begin{aligned} \dot{V}_1(t) &= 2\boldsymbol{\xi}^T(t)\mathbf{P}((\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}})\boldsymbol{\xi}(t) + (\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}})\boldsymbol{\xi}(t-\tau(t))), \\ \dot{V}_2(t) &\le \boldsymbol{\xi}^T(t)\mathbf{Q}\boldsymbol{\xi}(t) - \boldsymbol{\xi}^T(t-\tau(t))\mathbf{Q}\boldsymbol{\xi}(t-\tau(t)), \\ \dot{V}_3(t) &\le \tau_m\dot{\xi}^T(t)\mathbf{R}\dot{\xi}(t) - \int_{t-\tau(t)}^{t} \dot{\xi}^T(\alpha)\mathbf{R}\dot{\xi}(\alpha)\mathrm{d}\alpha. \end{aligned} \tag{22}$$

It is obvious that there is

$$\boldsymbol{\xi}(t-\tau(t)) = \boldsymbol{\xi}(t) - \int_{t-\tau(t)}^{t} \dot{\boldsymbol{\xi}}^T(\alpha)\mathrm{d}\alpha. \tag{23}$$

Thus, $V(t)$ can be further rewritten as

$$\begin{aligned} \dot{V}(t) \le{}& 2\boldsymbol{\xi}^T(t)\mathbf{P}((\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}})\boldsymbol{\xi}(t) + (\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}})\boldsymbol{\xi}(t-\tau(t))) + \boldsymbol{\xi}^T(t)\mathbf{Q}\boldsymbol{\xi}(t) - \boldsymbol{\xi}^T(t-\tau(t))\mathbf{Q}\boldsymbol{\xi}(t-\tau(t)) \\ &+ \tau_m\dot{\xi}^T(t)\mathbf{R}\dot{\xi}(t) - \int_{t-\tau(t)}^{t} \dot{\xi}^T(\alpha)\mathbf{R}\dot{\xi}(\alpha)\mathrm{d}\alpha + 2\boldsymbol{\xi}^T(t)\mathbf{W}\left(\boldsymbol{\xi}^T(t-\tau(t)) - \boldsymbol{\xi}^T(t) + \int_{t-\tau(t)}^{t} \dot{\xi}^T(\alpha)\mathrm{d}\alpha\right) \\ \le{}& \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \begin{bmatrix} \boldsymbol{\xi}(t) \\ \boldsymbol{\xi}(t-\tau(t)) \\ \dot{\xi}(\alpha) \end{bmatrix}^T \boldsymbol{\Psi}_0 \begin{bmatrix} \boldsymbol{\xi}(t) \\ \boldsymbol{\xi}(t-\tau(t)) \\ \dot{\xi}(\alpha) \end{bmatrix}, \end{aligned} \tag{24}$$

where

$$\boldsymbol{\Psi}_0 = \left(\begin{bmatrix} \Gamma & \mathbf{P}(\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}}) + \mathbf{W} & \tau_m\mathbf{W} \\ * & -\mathbf{Q} & 0 \\ * & * & -\tau_m\mathbf{R} \end{bmatrix} + \tau_m\begin{bmatrix} (\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}})^T \\ (\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}})^T \\ 0 \end{bmatrix}\mathbf{R}\begin{bmatrix} (\overline{\mathbf{A}} + \Delta\overline{\mathbf{A}})^T \\ (\overline{\mathbf{B}} + \Delta\overline{\mathbf{B}})^T \\ 0 \end{bmatrix}^T\right). \tag{25}$$

By the Schur complement, if the inequality (24) holds, it can be obtained that there is $\boldsymbol{\Psi}_0 < 0$. Thus, there is $\dot{V}(t) < 0$ and the filtering error system is asymptotically stable.

Secondly, define a new function

$$J = \int_0^T \left[\mathbf{e}^T(t)\mathbf{e}(t) - \gamma^2\mathbf{w}^T(t)\mathbf{w}(t)\right]\mathrm{d}t, \tag{26}$$

where $T$ is a scalar which satisfies $T > 0$.

Thus, under the zero initial condition, there is

$$
\begin{aligned}
J &= \int_0^T \left[\mathbf{e}^T(t)\mathbf{e}(t) - \gamma^2\mathbf{w}^T(t)\mathbf{w}(t) + \dot{V}(t)\right]\mathrm{d}t - V(T) \\
&\le \int_0^T \left[\mathbf{e}^T(t)\mathbf{e}(t) - \gamma^2\mathbf{w}^T(t)\mathbf{w}(t) + \dot{V}(t)\right]\mathrm{d}t \\
&\le \begin{bmatrix} \boldsymbol{\xi}(t) \\ \boldsymbol{\xi}(t-\tau(t)) \\ w(t) \\ \dot{\boldsymbol{\xi}}(\alpha) \end{bmatrix}^T \boldsymbol{\psi}_1 \begin{bmatrix} \boldsymbol{\xi}(t) \\ \boldsymbol{\xi}(t-\tau(t)) \\ w(t) \\ \dot{\boldsymbol{\xi}}(\alpha) \end{bmatrix},
\end{aligned} \tag{27}
$$

where

$$
\begin{aligned}
\boldsymbol{\psi}_1 = &\begin{bmatrix} \Gamma & \mathbf{P}(\overline{B}+\Delta\overline{B})+\mathbf{W} & \mathbf{P}(\overline{E}+\Delta\overline{E}) & \tau_m\mathbf{W} \\ * & -\mathbf{Q} & 0 & 0 \\ * & * & -\gamma^2\mathbf{I} & 0 \\ * & * & * & -\tau_m\mathbf{R} \end{bmatrix} \\
&+ \tau_m \begin{bmatrix} (\overline{A}+\Delta\overline{A})^T \\ (\overline{B}+\Delta\overline{B})^T \\ (\overline{E}+\Delta\overline{E})^T \\ 0 \end{bmatrix} \mathbf{R} \begin{bmatrix} (\overline{A}+\Delta\overline{A})^T \\ (\overline{B}+\Delta\overline{B})^T \\ (\overline{E}+\Delta\overline{E})^T \\ 0 \end{bmatrix}^T \\
&+ \begin{bmatrix} (\overline{F}+\Delta\overline{F})^T \\ (\overline{G}+\Delta\overline{G})^T \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} (\overline{F}+\Delta\overline{F})^T \\ (\overline{G}+\Delta\overline{G})^T \\ 0 \\ 0 \end{bmatrix}^T .
\end{aligned} \tag{28}
$$

According to the Schur complement, it follows that

$$
\boldsymbol{\psi}_1 < 0 \Leftrightarrow \overline{\boldsymbol{\psi}} < 0. \tag{29}
$$

Thus, if the inequality $\overline{\boldsymbol{\psi}} < 0$ holds, there is $J < 0$ and equation (16) holds for any nonzero $\mathbf{w}(t) \in L_2[0,\infty]$. The proof is completed.

**Theorem 2.** *For a given scalar $\gamma > 0$, the filtering problem is solvable under the conditions above with a H∞ performance level $\gamma$, if there exist positive matrices $\mathbf{P}_1, \mathbf{P}_2, \mathbf{Q}_1, \mathbf{Q}_2, \mathbf{R}_1, \mathbf{R}_2$, matrices $\mathbf{W}_1, \mathbf{W}_2, \mathbf{Z}_i$ and scalars $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3ij}, \varepsilon_{4i}, \varepsilon_{5i} > 0\,(1 \le i \le j \le r)$ such that the following inequalities hold:*

$$
\boldsymbol{\Omega}_{ii} < 0, \quad i = 1, 2, \ldots, r, \tag{30}
$$

$$
\boldsymbol{\Omega}_{ij} + \boldsymbol{\Omega}_{ji} < 0, \quad i < j, i, j = 1, 2, \ldots, r, \tag{31}
$$

*where*

$$
\begin{aligned}
\boldsymbol{\Theta}_{1i} &= \left[\,(\mathbf{P}_1\mathbf{U}_{i1})^T \;\; (\mathbf{P}_2\mathbf{U}_{i1})^T \;\; 0\;0\;0\;0\;0 \;\; (\tau_m\mathbf{R}_1\mathbf{U}_{1i})^T \;\; (\tau_m\mathbf{R}_2\mathbf{U}_{1i})^T \;\; 0\,\right]^T, \\
\boldsymbol{\Xi}_{1i} &= \left[\,\mathbf{V}_{1i}\;\;0\;\;\mathbf{V}_{2i}\;\;0\;\;\mathbf{V}_{3i}\;\;0\;0\;0\;0\;0\,\right], \\
\boldsymbol{\Theta}_{2i} &= \left[\,0\;\;(\mathbf{P}_2\mathbf{U}_{1i})^T\;\;0\;0\;0\;0\;0\;0\;\;(\tau_m\mathbf{R}_2\mathbf{U}_{1i})^T\;\;0\,\right]^T, \\
\boldsymbol{\Xi}_{2i} &= \left[\,0\;\;\mathbf{V}_{1i}\;\;0\;\;\mathbf{V}_{2i}\;\;0\;0\;0\;0\;0\;0\,\right], \\
\boldsymbol{\Theta}_{3ij} &= \left[\,0\;\;(\mathbf{P}_2\mathbf{K}_i\mathbf{U}_{2i})^T\;\;0\;0\;0\;0\;0\;0\;\;(\tau_m\mathbf{R}_2\mathbf{K}_i\mathbf{U}_{2i})^T\;\;0\,\right]^T, \\
\boldsymbol{\Xi}_{3i} &= \left[\,0\;\;\mathbf{V}_{1i}\;\;0\;\;\mathbf{V}_{2i}\;\;\mathbf{V}_{3i}\;\;0\;0\;0\;0\;0\,\right], \\
\boldsymbol{\Theta}_{4i} &= \left[\,0\;0\;0\;0\;0\;0\;0\;0\;0\;\;\mathbf{U}_{1i}^T\,\right]^T, \\
\boldsymbol{\Xi}_{4i} &= \left[\,0\;\;\mathbf{V}_{4i}\;\;0\;0\;0\;0\;0\;0\;0\;0\,\right], \\
\boldsymbol{\Theta}_{5i} &= \left[\,0\;0\;0\;0\;0\;0\;0\;0\;0\;\;\mathbf{U}_{2i}^T\,\right]^T, \\
\boldsymbol{\Xi}_{5i} &= \left[\,0\;0\;0\;\;\mathbf{V}_{4i}\;\;0\;0\;0\;0\;0\;0\,\right], \\
\Gamma_{11} &= \mathbf{P}_1\mathbf{A}_{qi} + \mathbf{A}_{qi}^T\mathbf{P}_1 + \mathbf{Q}_1 - \mathbf{W}_1 - \mathbf{W}_1^T, \\
\Gamma_{22} &= \mathbf{P}_2\mathbf{A}_{qi} - \mathbf{Z}_{1i}\mathbf{C}_{qi} + \mathbf{A}_{qi}^T\mathbf{P}_2 - \mathbf{C}_{qi}^T\mathbf{Z}_{1i}^T + \mathbf{Q}_2 - \mathbf{W}_2 - \mathbf{W}_2^T, \\
\Gamma_{13} &= \mathbf{P}_1\mathbf{B}_{qi} + \mathbf{W}_1, \\
\Gamma_{24} &= \mathbf{P}_2\mathbf{B}_{qi} - \mathbf{Z}_{1i}\mathbf{D}_{qi} + \mathbf{W}_2, \\
\Gamma_{25} &= \mathbf{P}_2\mathbf{E}_{q1i} - \mathbf{Z}_{1i}\mathbf{E}_{2i}, \\
\boldsymbol{\Omega}_{ij} &= \begin{bmatrix} \boldsymbol{\Omega}_{0ij} + \boldsymbol{\varepsilon}_{1i}\boldsymbol{\Xi}_{1i}^T\boldsymbol{\Xi}_{1i} + \varepsilon_{2i}\boldsymbol{\Xi}_{2i}^T\boldsymbol{\Xi}_{2i} - \varepsilon_{3ij}\boldsymbol{\Xi}_{3i}^T\boldsymbol{\Xi}_{3i} + \varepsilon_{4i}\boldsymbol{\Xi}_{4i}^T\boldsymbol{\Xi}_{4i} + \varepsilon_{5i}\boldsymbol{\Xi}_{5i}^T\boldsymbol{\Xi}_{5i} & \boldsymbol{\Theta}_{1i} & \boldsymbol{\Theta}_{2i} & \boldsymbol{\Theta}_{3ij} & \boldsymbol{\Theta}_{4i} & \boldsymbol{\Theta}_{5i} \\ * & -\varepsilon_{1i} & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_{1i2} & 0 & 0 & 0 \\ * & * & * & \varepsilon_{3ij} & 0 & 0 \\ * & * & * & * & -\varepsilon_{4i} & 0 \\ * & * & * & * & * & -\varepsilon_{5i} \end{bmatrix},
\end{aligned}
$$

$$\Omega_{0ij} = \begin{bmatrix} \Gamma_{11} & 0 & \Gamma_{13} & 0 & \mathbf{P}_1\mathbf{E}_{q1i} & \tau_m\mathbf{W}_1 & 0 & \tau_m\mathbf{A}_{qi}^T\mathbf{R}_1 & 0 & 0 \\ * & \Gamma_{22} & 0 & \Gamma_{24} & \Gamma_{25} & 0 & \tau_m\mathbf{W}_2 & 0 & \tau_m\left[\mathbf{A}_{qi}^T\mathbf{R}_2 - \mathbf{C}_{qi}^T\mathbf{Z}_{2i}\right] & \mathbf{F}_{qi}^T \\ * & * & -\mathbf{Q}_1 & 0 & 0 & 0 & 0 & \tau_m\mathbf{B}_{qi}^T\mathbf{R} & 0 & 0 \\ * & * & * & -\mathbf{Q}_2 & 0 & 0 & 0 & 0 & \tau_m\left[\mathbf{B}_{qi}^T\mathbf{R}_2 - \mathbf{D}_{qi}^T\mathbf{Z}_{2i}\right] & \mathbf{G}_{qi}^T \\ * & * & * & * & -\gamma^2\mathbf{I} & 0 & 0 & \tau_m\mathbf{E}_{q1i}^T\mathbf{R}_1 & \tau_m\left[\mathbf{E}_{q1i}^T\mathbf{R}_2 - \mathbf{E}_{2i}^T\mathbf{Z}_{2i}\right] & 0 \\ * & * & * & * & * & -\tau_m\mathbf{R}_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\tau_m\mathbf{R}_2 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\tau_m\mathbf{R}_1 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\tau_m\mathbf{R}_2 & 0 \\ * & * & * & * & * & * & * & * & * & -\mathbf{I} \end{bmatrix}, \tag{32}$$

*Moreover, the filter parameter can be solved by*

$$\mathbf{K}_i = \mathbf{P}_2^{-1}\mathbf{Z}_{1i}. \tag{33}$$

*Proof.* Let

$$\begin{aligned} \mathbf{P} &= \operatorname{diag}\{\mathbf{P}_1, \mathbf{P}_2\}, \\ \mathbf{Q} &= \operatorname{diag}\{\mathbf{Q}_1, \mathbf{Q}_2\}, \\ \mathbf{R} &= \operatorname{diag}\{\mathbf{R}_1, \mathbf{R}_2\}, \\ \mathbf{W} &= \operatorname{diag}\{\mathbf{W}_1, \mathbf{W}_2\}. \end{aligned} \tag{34}$$

Then $\overline{\boldsymbol{\psi}}$ can be rewritten as

$$\overline{\boldsymbol{\psi}} = \begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} & \Upsilon_{13} & \tau_m\mathbf{W} & \tau_m\Upsilon_{15} & (\overline{F} + \Delta\overline{F})^T \\ * & -\mathbf{Q} & 0 & 0 & \tau_m\Upsilon_{25} & (\overline{G} + \Delta\overline{G})^T \\ * & * & -\gamma^2\mathbf{I} & 0 & \tau_m\Upsilon_{35} & 0 \\ * & * & * & -\tau_m\mathbf{R} & 0 & 0 \\ * & * & * & * & -\tau_m\mathbf{R} & 0 \\ * & * & * & * & * & -\mathbf{I} \end{bmatrix}, \tag{35}$$

where

$$\begin{aligned} \Upsilon_{11} &= \operatorname{diag}\{\boldsymbol{\delta}_1, \boldsymbol{\delta}_2\}, \\ \boldsymbol{\delta}_1 &= \mathbf{P}_1\left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right) + \left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right)^T\mathbf{P}_1 + \mathbf{Q}_1 - \mathbf{W}_1 - \mathbf{W}_1^T, \\ \boldsymbol{\delta}_2 &= \mathbf{P}_2\left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right) - \mathbf{P}_2\mathbf{K}_i\left(\mathbf{C}_{qi} + \Delta\mathbf{C}_{qi}\right) + \left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right)^T \\ &\quad \cdot \mathbf{P}_2 - \left(\mathbf{C}_{qi} + \Delta\mathbf{C}_{qi}\right)^T\mathbf{K}_i^T\mathbf{P}_2 + \mathbf{Q}_2 - \mathbf{W}_2 - \mathbf{W}_2^T, \\ \Upsilon_{12} &= \operatorname{diag}\left\{\mathbf{P}_1\left(\mathbf{B}_{qi} + \Delta\mathbf{B}_{qi}\right) + \mathbf{W}_1, \mathbf{P}_2\left(\mathbf{B}_{qi} + \Delta\mathbf{B}_{qi}\right)\right. \\ &\quad \left. - \mathbf{P}_2\mathbf{K}_i\left(\mathbf{D}_{qi} + \Delta\mathbf{D}_{qi}\right) + \mathbf{W}_2\right\}, \\ \Upsilon_{13} &= \begin{bmatrix} \mathbf{P}_1\left(\mathbf{E}_{q1i} + \Delta\mathbf{E}_{q1i}\right) \\ \mathbf{P}_2\left(\mathbf{E}_{q1i} + \Delta\mathbf{E}_{q1i}\right) - \mathbf{P}_2\mathbf{K}_i\left(\mathbf{E}_{2i} + \Delta\mathbf{E}_{2i}\right) \end{bmatrix}, \\ \Upsilon_{15} &= \operatorname{diag}\left\{\left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right)^T\mathbf{R}_1, \left(\mathbf{A}_{qi} + \Delta\mathbf{A}_{qi}\right)^T\mathbf{R}_2 - \left(\mathbf{C}_{qi} + \Delta\mathbf{C}_{qi}\right)^T\mathbf{K}_i^T\mathbf{R}_2\right\}, \\ \Upsilon_{25} &= \operatorname{diag}\left\{\left(\mathbf{B}_{qi} + \Delta\mathbf{B}_{qi}\right)^T\mathbf{R}_1\left(\mathbf{B}_{qi} + \Delta\mathbf{B}_{qi}\right)^T\mathbf{R}_2 - \left(\mathbf{D}_{qi} + \Delta\mathbf{D}_{qi}\right)^T\mathbf{K}_i^T\mathbf{R}_2\right\}, \\ \Upsilon_{35} &= \left[\left(\mathbf{E}_{q1i} + \Delta\mathbf{E}_{q1i}\right)^T\mathbf{R}_1\left(\mathbf{E}_{q1i} + \Delta\mathbf{E}_{q1i}\right)^T\mathbf{R}_2 - \left(\mathbf{E}_{2i} + \Delta\mathbf{E}_{2i}\right)^T\mathbf{K}_i^T\mathbf{R}_2\right]. \end{aligned} \tag{36}$$

Then, for further analysis, based on equation (9) and Lemma 1, equation (35) can be decomposed into

$$\begin{aligned} \overline{\boldsymbol{\psi}} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(f(t))h_j(f(t))\left\{\boldsymbol{\Omega}_{0ij} + \boldsymbol{\Theta}_{1i}\mathbf{S}_i\boldsymbol{\Xi}_{1i} + \boldsymbol{\Xi}_{1i}^T\mathbf{S}_i^T\boldsymbol{\Theta}_{1i}^T\right. \\ &\quad + \boldsymbol{\Theta}_{2i}\mathbf{S}_i\boldsymbol{\Xi}_{2i} + \boldsymbol{\Xi}_{2i}^T\mathbf{S}_i^T\boldsymbol{\Theta}_{2i}^T - \boldsymbol{\Theta}_{3ij}\mathbf{S}_i\boldsymbol{\Xi}_{3i} - \boldsymbol{\Xi}_{3i}^T\mathbf{S}_i^T\boldsymbol{\Theta}_{3ij}^T \\ &\quad \left. + \boldsymbol{\Theta}_{4i}\mathbf{S}_i\boldsymbol{\Xi}_{4i} + \boldsymbol{\Xi}_{4i}^T\mathbf{S}_i^T\boldsymbol{\Theta}_{4i}^T + \boldsymbol{\Theta}_{5i}\mathbf{S}_i\boldsymbol{\Xi}_{5i} + \boldsymbol{\Xi}_{5i}^T\mathbf{S}_i^T\boldsymbol{\Theta}_{5i}^T\right\} \\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(f(t))h_j(f(t))\left\{\boldsymbol{\Omega}_{0ij} + \frac{1}{\varepsilon_{1i}}\boldsymbol{\Theta}_{1i}\boldsymbol{\Theta}_{1i}^T + \varepsilon_{1i}\boldsymbol{\Xi}_{1i}^T\boldsymbol{\Xi}_{1i}\right. \\ &\quad + \frac{1}{\varepsilon_{2i}}\boldsymbol{\Theta}_{2i}\boldsymbol{\Theta}_{2i}^T + \varepsilon_{2i}\boldsymbol{\Xi}_{2i}^T\boldsymbol{\Xi}_{2i} - \frac{1}{\varepsilon_{3ij}}\boldsymbol{\Theta}_{3ij}\boldsymbol{\Theta}_{3ij}^T - \varepsilon_{3ij}\boldsymbol{\Xi}_{3i}^T\boldsymbol{\Xi}_{3i} \\ &\quad \left. + \frac{1}{\varepsilon_{4i}}\boldsymbol{\Theta}_{4i}\boldsymbol{\Theta}_{4i}^T + \varepsilon_{4i}\boldsymbol{\Xi}_{4i}^T\boldsymbol{\Xi}_{4i} + \frac{1}{\varepsilon_{5i}}\boldsymbol{\Theta}_{5i}\boldsymbol{\Theta}_{5i}^T + \varepsilon_{5i}\boldsymbol{\Xi}_{5i}^T\boldsymbol{\Xi}_{5i}\right\}. \end{aligned} \tag{37}$$

Moreover, define

$$\begin{aligned} \mathbf{Z}_{1i} &= \mathbf{P}_2\mathbf{K}_i, \\ \mathbf{Z}_{2i} &= \mathbf{K}_i^T\mathbf{R}_2. \end{aligned} \tag{38}$$

By applying the Schur complement, equation (37) is equivalent to

$$\sum_{i=1}^{r} h_i^2(f(t))\boldsymbol{\Omega}_{ii} + \sum_{i<j}^{r} h_i(f(t))h_j(f(t))\left(\boldsymbol{\Omega}_{ij} + \boldsymbol{\Omega}_{ji}\right) < 0. \tag{39}$$

Thus, the inequalities (30) and (31) hold. The proof is completed.

## 4. Optimal Transformation Matrix Selection

It can readily be obtained that the estimation precision is directly affected by the estimation of the unknown states (health parameters). Thus, the key of the parameters estimation is to find an optimal transformation matrix $\mathbf{V}^*$, which can ensure that the low-dimensional tuning vector $\mathbf{q}$ can represent as much of the information of the health parameters as possible.

In this paper, the optimization objective is to find an optimal transformation matrix $\mathbf{V}^*$, which can ensure that the estimation error of the measured outputs $\rho = (\mathbf{y} - \widehat{y})^T (\mathbf{y} - \widehat{y})$ is minimum.

Thus, the optimization problem can be described as

$$\begin{aligned} &\min \quad \rho \\ &\text{s.t.} \quad \rho = (\mathbf{y} - \widehat{y})^T (\mathbf{y} - \widehat{y}). \end{aligned} \tag{40}$$

And the method based on iterative solution is used in this paper, and the process of optimal transformation matrix selection is shown in Figure 1:

(1) To begin with, the initial value of $\mathbf{V}^*$ is given randomly. And during the estimation process, the optimal $\mathbf{V}^*$ of the last moment is used as the initial value to reduce the quantity of calculation. In order to avoid the calculation converging to a poorly scaled result, the Frobenius norm of $\mathbf{V}^*$ must satisfy $\|V^*\|_F = 1$.

(2) With reference to equation (7), construct the reduced-order state-space model.

(3) Solve the parameters of the $H\infty$ filter.

(4) Calculate estimated error using equation (40).

(5) Determine whether the estimated error $\rho$ achieves convergence within a tolerance (user-specified):

    (1) If converged, skip step 6 and proceed directly to step 7.
    (2) If not converged, proceed to step 6.
    (3) If the number of iterations exceeds 100, skip step 6 and proceed directly to step 7 to meet the real-time requirements.

(6) Use the MATLAB *lsqnonlin* function to update $\mathbf{V}^*$, and the new value still needs to satisfy $\|V^*\|_F = 1$.

(7) Return the optimal value of $\mathbf{V}^*$, and ends.

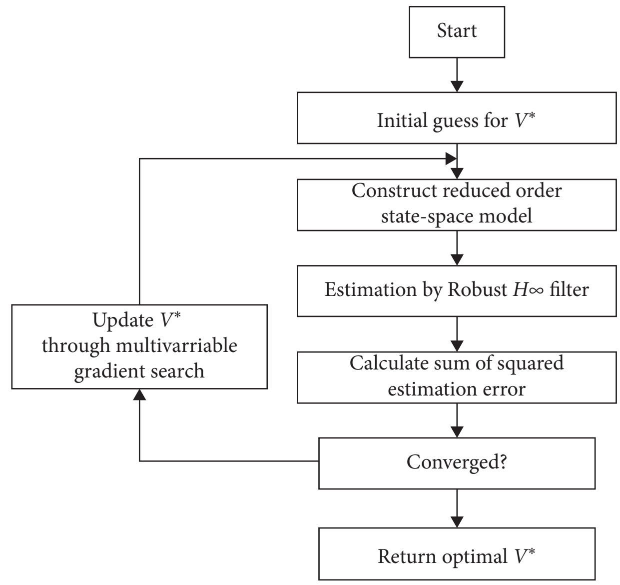


Figure 1: Flowchart of $\mathbf{V}^*$ iterative search.

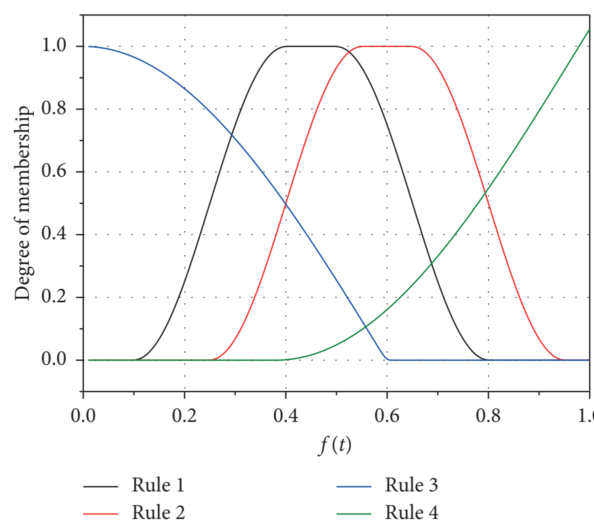


Figure 2: Initial membership function.

## 5. Simulation Example

Before proceeding with the simulation, the high-precision aero-thermodynamic mathematical model of aeroengine in full envelope is the basis of the simulation. It can not only build the small deviation dynamic state space models, but can also replace the real engine in simulation. The detailed modeling process is carried out by reference to [25, 26]. This paper takes a type of double-rotor turbofan engine as research object and uses modeling method based on component characteristics. The afterburner is not considered in this engine model. For greater adaptability, faster calculation speed, and stronger convergence, the self-tuning Broyden quasi-Newton method [27] is used to solve equilibrium equations in the component-level mathematical model. And the small deviation dynamic model is solved by fitting method [28–30].

Assuming that the aeroengine works in the maximum state and then based on the methods of flight envelop division in [31–33], a T-S fuzzy model is constructed and the initial membership function is shown in Figure 2. In this paper, the T-S fuzzy model has two state variables, four health parameters, two control inputs, five measured outputs, and three unmeasured outputs, all shown in Table 1.

Then the four T-S fuzzy rules with the selection of $H_f$ and Ma can be obtained as follows:

Rule 1. If $H_f$ is about 0 km and Ma is about 0, then

Table 1: Variables of aeroengine T-S fuzzy model.

| State variables | Health parameters | Control inputs | Measured outputs | Unmeasured outputs |
|---|---|---|---|---|
| $n_l$—low-pressure spool speed | Low-pressure compressor (LPC) efficiency | $wf$—fuel flow | $n_l$—low-pressure spool speed | $T_5$—LPT exit total temperature |
| $n_h$—high-pressure spool speed | LPC flow capacity | $A_8$—nozzle area | $n_h$—high-pressure spool speed | $F_n$—net thrust |
| | Low-pressure turbine (LPT) efficiency | | $P_{2b}$—LPC exit total pressure | $\Pi_T$—turbine pressure ratio |
| | LPT flow capacity | | $T_{2b}$—LPC exit total temperature | |
| | | | $T_3$—HPC exit total temperature | |

$$
\begin{aligned}
\mathbf{A}_1 &= \begin{bmatrix} -1.26 & 1.02 \\ -0.14 & -1.24 \end{bmatrix}, \\
\mathbf{B}_1 &= \begin{bmatrix} 0.35 & 0.54 \\ 0.36 & 0.32 \end{bmatrix}, \\
\mathbf{L}_1 &= \begin{bmatrix} -1.02 & 0.90 & -0.62 & 0.51 \\ -0.26 & -0.11 & 0.28 & 0.23 \end{bmatrix}, \\
\mathbf{E}_{11} &= \begin{bmatrix} -0.5 \\ 1.0 \end{bmatrix}, \\
\mathbf{C}_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.55 & -0.29 \\ 0.51 & -0.07 \\ 0.36 & 0.54 \end{bmatrix}, \\
\mathbf{D}_1 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.19 & -0.82 \\ 0.05 & -0.22 \\ 0.04 & -0.16 \end{bmatrix}, \\
\mathbf{M}_1 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.94 & -0.18 & 0.02 & -0.07 \\ 0.24 & -0.39 & 0.01 & -0.02 \\ 0.13 & -0.23 & -0.09 & -0.08 \end{bmatrix}, \\
\mathbf{E}_{21} &= \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ 0.1 \\ 0.15 \end{bmatrix}, \\
\mathbf{F}_1 &= \begin{bmatrix} 0.25 & -0.54 \\ -0.14 & 1.27 \\ 0.40 & 0.04 \end{bmatrix}, \\
\mathbf{G}_1 &= \begin{bmatrix} 0.45 & 0.09 \\ -0.10 & 0.39 \\ 0.24 & 0.40 \end{bmatrix}, \\
\mathbf{N}_1 &= \begin{bmatrix} -0.23 & -0.29 & 0.07 & -0.18 \\ -0.09 & 0.26 & -0.33 & -0.19 \\ 0.81 & -0.08 & 0.04 & -0.08 \end{bmatrix}.
\end{aligned}
\tag{41}
$$

Rule 2. If $H_f$ is about 4.8 km and Ma is about 1.24, then

$$
\begin{aligned}
\mathbf{A}_2 &= \begin{bmatrix} -2.66 & 1.10 \\ -0.61 & -1.39 \end{bmatrix}, \\
\mathbf{B}_2 &= \begin{bmatrix} 0.39 & 0.42 \\ 0.36 & 0.33 \end{bmatrix}, \\
\mathbf{L}_2 &= \begin{bmatrix} -0.86 & 0.87 & -0.64 & 0.52 \\ -0.28 & -0.10 & 0.27 & 0.23 \end{bmatrix}, \\
\mathbf{E}_{12} &= \begin{bmatrix} -0.3 \\ 0.1 \end{bmatrix}, \\
\mathbf{C}_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2.03 & -0.34 \\ 0.66 & -0.06 \\ 0.42 & 0.58 \end{bmatrix}, \\
\mathbf{D}_2 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.14 & -0.67 \\ 0.02 & -0.11 \\ 0.02 & -0.10 \end{bmatrix}, \\
\mathbf{M}_2 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.92 & -0.16 & 0.02 & -0.05 \\ 0.15 & -0.30 & 0 & -0.01 \\ 0.08 & -0.18 & -0.09 & -0.07 \end{bmatrix}, \\
\mathbf{E}_{22} &= \begin{bmatrix} 0 \\ 0 \\ 0.3 \\ 0.25 \\ 0.1 \end{bmatrix}, \\
\mathbf{F}_2 &= \begin{bmatrix} -0.34 & -0.52 \\ -0.06 & 1.26 \\ 2.10 & 0.12 \end{bmatrix}, \\
\mathbf{G}_2 &= \begin{bmatrix} 0.45 & 0.09 \\ -0.10 & 0.43 \\ 0.54 & -0.57 \end{bmatrix}, \\
\mathbf{N}_2 &= \begin{bmatrix} -0.31 & -0.22 & 0.08 & -0.17 \\ 0 & 0.19 & -0.34 & -0.19 \\ 0.94 & -0.18 & 0.09 & -0.19 \end{bmatrix}.
\end{aligned}
\tag{42}
$$

Rule 3. If $H_f$ is about 12.8 km and Ma is about 0.7, then

$$\mathbf{A}_3 = \begin{bmatrix} -0.31 & 0.19 \\ -0.05 & -0.28 \end{bmatrix},$$
$$\mathbf{B}_3 = \begin{bmatrix} 0.11 & 0.11 \\ 0.09 & 0.09 \end{bmatrix},$$
$$\mathbf{L}_3 = \begin{bmatrix} -0.24 & 0.21 & -0.16 & 0.13 \\ -0.07 & -0.02 & 0.07 & 0.06 \end{bmatrix},$$
$$\mathbf{E}_{13} = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix},$$
$$\mathbf{C}_3 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0.53 & -0.18 \\ 0.49 & -0.04 \\ 0.35 & 0.55 \end{bmatrix},$$
$$\mathbf{D}_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.19 & -0.84 \\ 0.05 & -0.21 \\ 0.04 & -0.15 \end{bmatrix},$$
$$\mathbf{M}_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.94 & -0.17 & 0.02 & -0.08 \\ 0.23 & -0.41 & 0.01 & -0.02 \\ 0.13 & -0.25 & -0.10 & -0.09 \end{bmatrix}, \tag{43}$$
$$\mathbf{E}_{23} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \\ 0.2 \\ 0.1 \end{bmatrix},$$
$$\mathbf{F}_3 = \begin{bmatrix} 0.22 & -0.31 \\ -0.10 & 1.04 \\ 0.59 & 0.12 \end{bmatrix},$$
$$\mathbf{G}_3 = \begin{bmatrix} 0.45 & 0.12 \\ -0.08 & 0.36 \\ 0.45 & -0.58 \end{bmatrix},$$
$$\mathbf{N}_3 = \begin{bmatrix} -0.25 & -0.25 & 0.07 & -0.20 \\ -0.07 & 0.21 & -0.34 & -0.19 \\ 1.05 & -0.15 & 0.07 & -0.17 \end{bmatrix}.$$

Rule 4. If $H_f$ is about 18.8 km and Ma is about 1.72, then

$$\mathbf{A}_4 = \begin{bmatrix} -0.63 & 0.24 \\ -0.13 & -0.29 \end{bmatrix},$$
$$\mathbf{B}_4 = \begin{bmatrix} 0.08 & 0.10 \\ 0.08 & 0.07 \end{bmatrix},$$
$$\mathbf{L}_4 = \begin{bmatrix} -0.20 & 0.19 & -0.14 & 0.11 \\ -0.06 & -0.02 & 0.06 & 0.05 \end{bmatrix},$$
$$\mathbf{E}_{14} = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix},$$
$$\mathbf{C}_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 2.29 & -0.36 \\ 0.73 & -0.07 \\ 0.46 & 0.58 \end{bmatrix},$$
$$\mathbf{D}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.14 & -0.66 \\ 0.03 & -0.12 \\ 0.03 & -0.11 \end{bmatrix},$$
$$\mathbf{M}_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.95 & -0.16 & 0.02 & -0.05 \\ 0.18 & -0.30 & 0 & -0.01 \\ 0.09 & -0.17 & -0.09 & -0.08 \end{bmatrix}, \tag{44}$$
$$\mathbf{E}_{24} = \begin{bmatrix} 0 \\ 0 \\ 0.2 \\ 0.1 \\ 0.1 \end{bmatrix},$$
$$\mathbf{F}_4 = \begin{bmatrix} -0.37 & -0.50 \\ -0.07 & 1.25 \\ 2.00 & 0.14 \end{bmatrix},$$
$$\mathbf{G}_4 = \begin{bmatrix} 0.46 & 0.05 \\ -0.11 & 0.47 \\ 0.61 & -0.43 \end{bmatrix},$$
$$\mathbf{N}_4 = \begin{bmatrix} -0.28 & -0.21 & 0.07 & -0.18 \\ -0.02 & 0.18 & -0.33 & -0.18 \\ 0.79 & -0.20 & 0.09 & -0.22 \end{bmatrix}.$$

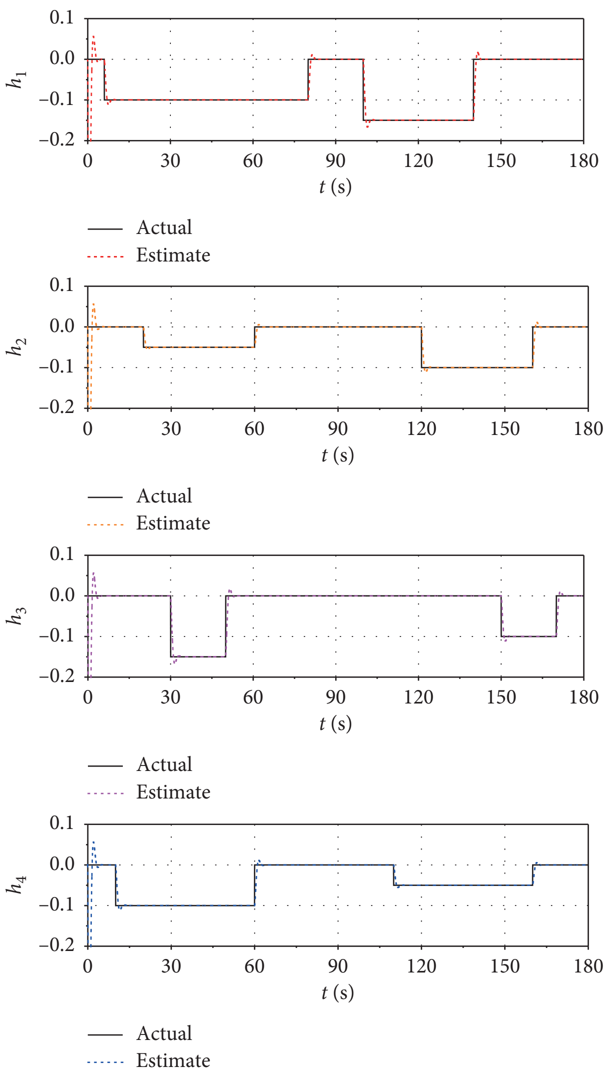


Figure 3: The estimation of health parameters for Type I.

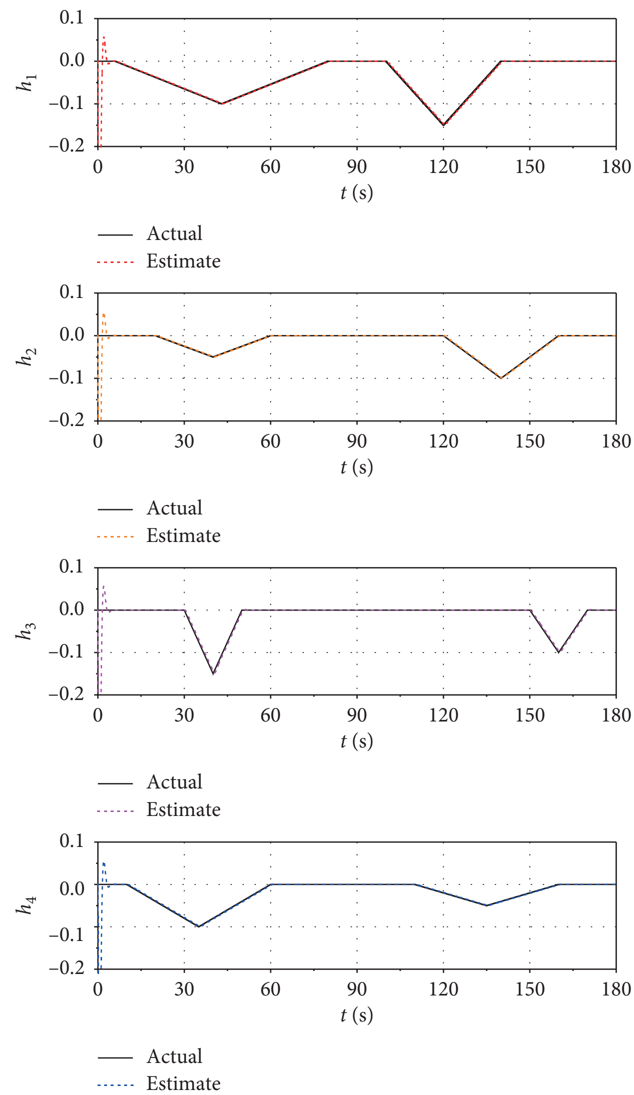


Figure 4: The estimation of health parameters for Type II.

Moreover, the uncertainties of the system are

$$\begin{aligned}
\mathbf{U}_{1i} &= \left[\, 0.2 \;\; -0.3 \;\; 0 \;\; 0 \;\; 0 \,\right]^T, \\
\mathbf{V}_{4i} &= \left[\, -0.1 \;\; 0.2 \;\; 0.15 \,\right], \\
\mathbf{U}_{2i} &= \left[\, 0.1 \;\; 0.3 \;\; -0.1 \;\; 0.2 \;\; 0.05 \,\right]^T, \\
\mathbf{V}_{3i} &= 0.1, \\
\mathbf{V}_{1i} &= \left[\, 0.1 \;\; 0.5 \;\; 0 \;\; 0 \;\; 0 \,\right], \\
\mathbf{V}_{2i} &= \left[\, 0.2 \;\; -0.1 \;\; 0 \;\; 0 \;\; 0 \,\right], \\
& (i = 1, 2, 3, 4).
\end{aligned} \tag{45}$$

The system sampling period is set as $T = 20\,\text{ms}$, and the external disturbance $w(t)$ is

$$w(t) = \frac{0.3\,\sin(0.8t)}{5\,\sin(6t) + 3\,\cos(8t)}. \tag{46}$$

The state space of time-delay is $\tau(t) \in \{1, 2, 3\}$, and the state transition matrix of $\tau(t)$ is

$$\mathbf{\Pi} = \begin{bmatrix} -0.6 & 0.4 & 0.2 \\ 0.3 & -0.5 & 0.2 \\ 0.1 & 0.4 & -0.5 \end{bmatrix}. \tag{47}$$

The working condition is chosen as $H = 5\,\text{km}$ and $\text{Ma} = 0.3$. The given value of $\gamma$ is $\gamma = 2$.

Two types of performance degradation are selected to simulate, which are slow varying and sudden change of health parameters. For the sudden change type (Type I), there is a step change for health parameters, and this type is used to represent the sudden damage, such as foreign objects damage. And the slow varying type (Type II) is defined as the linear variation of health parameters to represent the gradual performance degradation during using process. These two types can cover almost all the health parameters changes in practice, and they are simulated respectively to illustrate the effectiveness of the method proposed in this paper.

The results are shown in Figures 3–6. It can be seen that the method proposed in this paper can effectively simulate

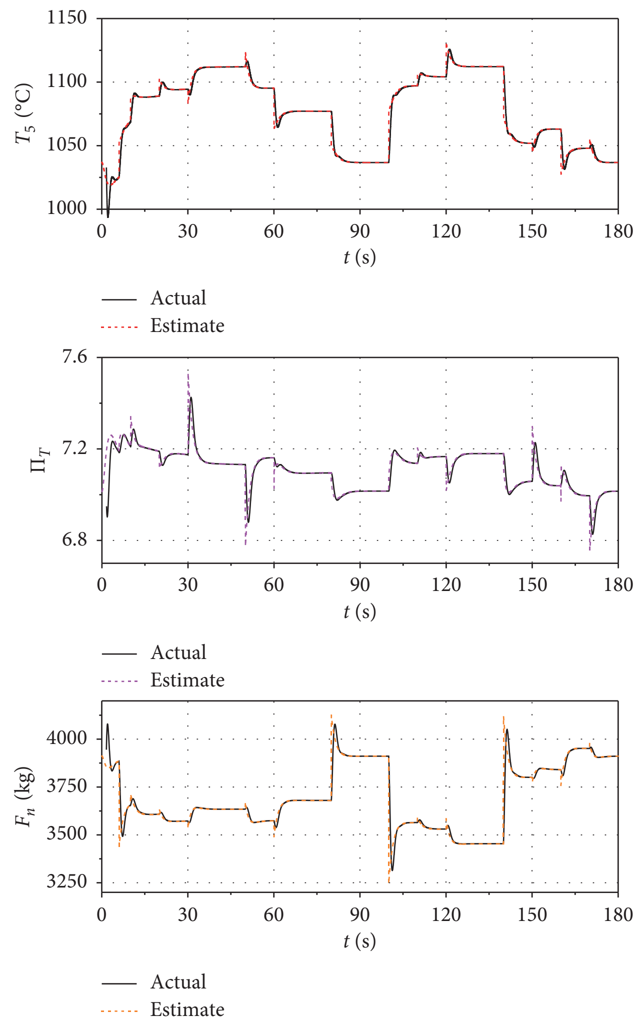


Figure 5: The estimation of performance parameter for Type I.

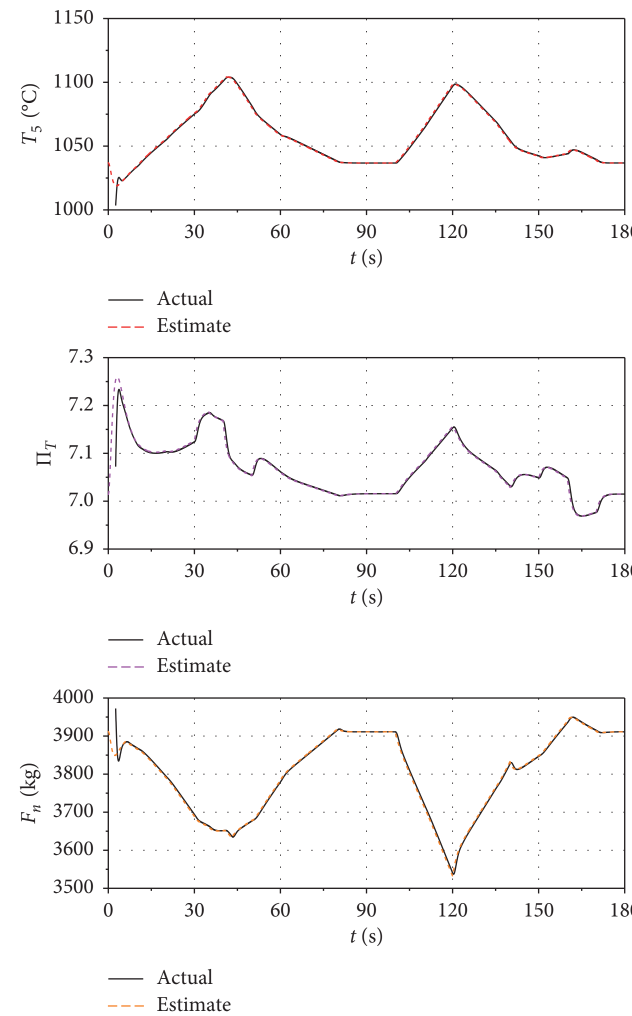


Figure 6: The estimation of performance parameter for Type II.

the health and performance parameters of the aeroengine. The estimated values can accurately track the changes as the health parameters shift.

By comparing the results of the two types of health parameters change, it can be concluded that the estimated values for slow varying type are more accurate. For the type of sudden change, it takes a while for the estimated values to stabilize and the overshoot is slightly larger.

Table 2 shows the average estimated error of the health and performance parameters. It can be seen that the estimated error is within 2.5% for all types. At the same time, the results are more accurate except for the beginning of simulation.

Furthermore, the results show that the method proposed in this paper can accurately estimate most system parameters of interest. On this basis, the method can also be used in the investigations of direct control, fault diagnosis, health management, and online monitoring. And for aeroengine, there are already some researches which indicate that the changes of health parameters can characterize specific faults, allowing for online monitoring and alerting [34–36].

Table 2: Estimated errors of health and performance parameters.

| Estimated error | Type I (%) | Type II (%) |
|---|---|---|
| $h_1$ | 0.46 | 0.40 |
| $h_2$ | 0.38 | 0.31 |
| $T_5$ | 1.27 | 0.98 |
| $F_n$ | 1.95 | 1.57 |
| $\Pi_T$ | 2.46 | 2.03 |

## 6. Conclusion

In this paper, a method is proposed for the underdetermined estimation problem of the distributed control systems. First, the T-S fuzzy model for DCSs is constructed and a model tuning parameter vector of appropriate dimension is produced. Then an observer-based fuzzy filter is designed and the sufficient condition for the existence of the designed filter is derived in terms of LMIs solutions. Besides, the method based on iterative solution is used to select the optimal transformation matrix to minimize the estimated error. Finally, the results of the simulation show that the proposed method can effectively estimate unmeasured parameters of the aeroengine and the estimated error is less than 2.5%.

## Data Availability

The data used to support the findings of this study are included within the article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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