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Research Article

A Novel Fast Convergence Control Scheme for a Class of 3D Chaotic Systems with Uncertain Parameters and External Disturbances

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This paper studies the control of a class of 3D chaotic systems with uncertain parameters and external disturbances. A new method which is referred as the analytical solution approach is firstly proposed for constructing Lyapunov function. Then, for suppressing the trajectories of the 3D chaotic system to its equilibrium point $0(0,0,0)$, a novel fast convergence controller containing parameter λ which determines the convergence rate of the system is presented. By using the designed Lyapunov function, the stability of the closed-loop system is proved via the Lyapunov stability theorem. Computer simulations are employed to a new chaotic system to illustrate the effectiveness of the theoretical results.

1. Introduction

Chaos is a very fascinating phenomenon which often appears in some nonlinear systems of physics, mathematics, psychology, engineering, and so on. A system is called as the chaotic system, provided that it exhibits chaos phenomenon. Due to its powerful potential applications in secure communications, biological systems, information processing, etc., the control of chaotic system has attracted extensive attention of scholars in recent 20 years, and many efficient approaches have been presented for controlling chaos, such as OGY control [1], impulsive control [2], fuzzy control [3, 4], sliding mode control [5, 6], adaptive control [7, 8], composite learning control [9, 10], and so on. Nowadays, many papers about controlling chaotic systems have been published. For instance, in [11], an adaptive control scheme is presented to realize the control and synchronization of the uncertain Liu chaotic dynamical system. In papers [12, 13], the control of the unified chaotic system is investigated by using the output feedback control strategy and the passivity-based control method, respectively. The authors in paper [14] propose an adaptive control approach for controlling the chaotic power systems via the passivity-based control method and finite-time stability theory. In paper [15], a conventional adaptive controller is presented for stabilizing the uncertain chaotic Zhang system via a Lyapunov-like approach. Paper [16] discusses the control of fractional-order nonlinear systems by the composite learning adaptive method. A fractional dynamic surface is introduced to avoid the explosion of complexity of the designed controller. The adaptive fuzzy backstepping control for a class of uncertain fractional-order nonlinear systems with unknown external disturbances is investigated in [17], where the stability is analyzed by using the Lyapunov function.

It is easy to see that these control schemes presented in papers [11–15] are only valid for one kind of chaotic system, which limits their range of application. In addition, the proposed methods do not consider the effect of uncertain parameters and external disturbances which exist widely in practical systems. Moreover, the controllers proposed in papers [11–17] do not contain the parameters that can change the convergence rate of the controlled system which means once the controller is given, the convergence rate of the controlled system is fixed. From the practical viewpoint of application, it is desired that the chaos control or chaos synchronization process can be accomplished quickly in a short time. Thus, how to design a controller such that the process of chaos control or synchronization can be completed quickly in a short time is an important issue.

Motivated by the above discussions, in this paper we investigate the control of a class of 3D chaotic systems with uncertain parameters and external disturbances. A novel fast convergence control scheme for controlling 3D chaotic system to its equilibrium point $0(0,0,0)$ is presented. Numerical simulations verify the effectiveness of the presented method.

The main contributions of this paper can be summarized into three aspects. First, a new method which is referred as the analytical solution approach is proposed. This technique has two advantages: (a) the rate of the convergence can be known by the designer; (b) a Lyapunov function for the controlled system can be easily constructed. Second, a new 3D chaotic model with fractional power terms of state x_i is proposed. Third, a novel controller for controlling 3D chaotic system to its equilibrium point $0(0,0,0)$ is presented. The proposed controller contains parameter λ which determines the convergence rate of the system. In general, the larger the value of $|\lambda|$, the faster the rate of convergence.

The rest of the paper is organized as follows. In the next section, the problem formulation and assumption are introduced. The main theoretical results are presented in Section 3. In Section 4, the control of a novel chaotic attractor and its simulation results are given. Concluding remarks are finally given Section 5.

2. The Problem Formulation

Consider the following chaotic systems with three inputs described by

$$\begin{cases} \dot{x}_1 = f_1(x) + \alpha_1^T g_1(x) + d_1 + u_1, \\ \dot{x}_2 = f_2(x) + \alpha_2^T g_2(x) + d_2 + u_2, \\ \dot{x}_3 = f_3(x) + \alpha_3^T g_3(x) + d_3 + u_3, \end{cases} \quad (1)$$

where $f_1(x) = a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}}$, $f_2(x) = a_{21}x_1^{m_{21}} + a_{22}x_2^{m_{22}} + a_{23}x_1^{m_{23}}x_2^{m_{24}} + a_{24}x_3^{m_{25}}$, $f_3(x)$ is a continuous function. $\alpha_1 = (\alpha_{11}, \alpha_{12}, \ldots, \alpha_{1m_1})^T$, $\alpha_2 = (\alpha_{21}, \alpha_{22}, \ldots, \alpha_{2m_2})^T$, and $\alpha_3 = (\alpha_{31}, \alpha_{32}, \ldots, \alpha_{3m_3})^T$ are unknown parameter vectors. $g_1(x) = (g_{11}(x), g_{11}(x), \ldots, g_{1m_1}(x))^T$, $g_2(x) = (g_{21}(x), g_{22}(x), \ldots, g_{2m_2}(x))^T$, and $g_3(x) = (g_{31}(x), g_{32}(x), \ldots, g_{3m_3}(x))^T$ are continuous functions, m_1, m_2, m_3 are positive integers. d_i is external disturbance, u_i is the controller, $i = 1,2,3$. In addition, $a_{11}, a_{12}(\neq 0)$, $a_{21}, a_{22}, a_{23}, a_{24}(\neq 0)$ are parameters. $m_{11}, m_{12}(= p_1/q_1)$, $m_{21}, m_{22}, m_{23}, m_{24}, m_{25}(= p_2/q_2)$ are positive constants. p_1, p_2, q_1, q_2 are positive odd numbers.

The control aim of this paper is to design some adaptive control laws, in the presence of the uncertain parameters and unknown disturbances, such that the states of system (1) are asymptotically stable for any given initial conditions, i.e., $\lim_{t\longrightarrow\infty} x_1 = \lim_{t\longrightarrow\infty} x_2 = \lim_{t\longrightarrow\infty} x_3 = 0$.

Before proceeding to the main results, we make the following Assumption 1.

Assumption 1. There exists a positive number $M(>0)$ such that $|d_i| \le M$, $i = 1,2,3$.

3. The Main Results

In order to obtain some adaptive controllers, we first consider the control problem of the following system:

$$\begin{cases} \dot{x}_1 = f_1(x), \\ \dot{x}_2 = f_2(x), \\ \dot{x}_3 = u_0, \end{cases} \quad (2)$$

where u_0 is the controller to be designed such that the equilibrium point $(0,0,0)$ of system (2) is asymptotically stable.

The purpose of considering system (2) firstly is that through designing controller u_0, one can get the analytical solution of system (2). By using the obtained analytical solution, we can easily construct the Lyapunov function which is helpful in proving Theorem 2.

Theorem 1. *The equilibrium point $(0,0,0)$ of system (2) is asymptotically stable if u_0 is chosen as*

$$\begin{aligned} u_0 = {} & \frac{-\left(\lambda^3 w_1/2\right) - \left(\lambda^3 + 3\lambda^2 - 3\lambda - 1\right)\left(w_2/2\right) - \left(\lambda^3 - 3\lambda^2 - 3\lambda + 1\right)\left(w_3/2\right)}{a_{12}m_{12}x_2^{m_{12}-1}a_{24}m_{25}x_1^{m_{25}-1}} \\ & - \frac{a_{12}m_{12}(m_{12}-1)x_2^{m_{12}-2}f_2(x)^2 + a_{11}m_{11}(m_{11}-1)f_1(x)^2}{a_{12}m_{12}x_2^{m_{12}-1}a_{24}m_{25}x_3^{m_{25}-1}} \\ & - \frac{a_{11}m_{11}x_1^{m_{11}-1}\left(a_{11}m_{11}x_1^{m_{11}-1}f_1(x) + a_{12}m_{12}x_2^{m_{12}-1}f_2(x)\right)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ & - \frac{\left(a_{21}m_{21}x_1^{m_{21}-1} + a_{23}m_{23}x_1^{m_{23}-1}x_2^{m_{24}}\right)f_1(x)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ & - \frac{\left(a_{22}m_{22}x_2^{m_{22}-1} + a_{23}m_{24}x_1^{m_{23}}x_2^{m_{24}-1}\right)f_2(x)}{a_{24}m_{25}x_3^{m_{25}-1}}, \end{aligned} \quad (3)$$

where $\lambda < 0$ *and*

$$w_1 = \begin{vmatrix} x_1 & 1 & 1 \\ a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}} & \lambda + 1 & \lambda - 1 \\ \rho & \lambda^2 + 2\lambda - 1 & \lambda^2 - 2\lambda - 1 \end{vmatrix},$$

$$w_2 = \begin{vmatrix} 1 & x_1 & 1 \\ \lambda & a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}} & \lambda - 1 \\ \lambda^2 & \rho & \lambda^2 - 2\lambda - 1 \end{vmatrix}, \quad (4)$$

$$w_3 = \begin{vmatrix} 1 & 1 & x_1 \\ \lambda & \lambda + 1 & a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}} \\ \lambda^2 & \lambda^2 + 2\lambda - 1 & \rho \end{vmatrix},$$

$$\rho = a_{11}m_{11}x_1^{m_{11}-1} f_1(x) + a_{12}m_{12}x_2^{m_{12}-1} f_2(x).$$

Proof. Theorem 1 will be proved by constructing the exact analytical solutions of system (2).

Let

$$x_1 = e^{\lambda t}(1 + \cos t + \sin t), \quad (5)$$

be a solution of system (2), where $\lambda < 0$. Obviously, we have $\lim_{t\longrightarrow\infty} x_2 = 0$.

Taking the derivative on both sides of equation (5), we have

$$\begin{aligned} \dot{x}_1 &= \lambda e^{\lambda t}(1 + \cos t + \sin t) + e^{\lambda t}(\cos t - \sin t) \\ &= e^{\lambda t}(\lambda + (\lambda + 1)\cos t + (\lambda - 1)\sin t). \end{aligned} \quad (6)$$

Since $\dot{x}_1 = f_1(x)$, we get

$$e^{\lambda t}(\lambda + (\lambda + 1)\cos t + (\lambda - 1)\sin t) = a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}}. \quad (7)$$

Then, we obtain

$$a_{12}x_2^{m_{12}} = e^{\lambda t}(\lambda + (\lambda + 1)\cos t + (\lambda - 1)\sin t) - a_{11}x_1^{m_{11}}. \quad (8)$$

Taking the derivative on both sides of the above equation, it yields

$$\begin{aligned} a_{12}m_{12}x_2^{m_{12}-1}\dot{x}_2 &= e^{\lambda t}\left(\lambda^2 + \left(\lambda^2 + 2\lambda - 1\right)\cos t + \left(\lambda^2 - 2\lambda - 1\right)\sin t\right) \\ &\quad - a_{11}m_{11}x_1^{m_{11}-1}\left(a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}}\right). \end{aligned} \quad (9)$$

i.e.,

$$\begin{aligned} e^{\lambda t}\left(\lambda^2 + \left(\lambda^2 + 2\lambda - 1\right)\cos t + \left(\lambda^2 - 2\lambda - 1\right)\sin t\right) &= a_{11}m_{11}x_1^{m_{11}-1} \\ \left(a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}}\right) + a_{12}m_{12}x_2^{m_{12}-1}&\left(a_{21}x_1^{m_{21}} + a_{22}x_2^{m_{22}} \right. \\ \left. + a_{23}x_1^{m_{23}}x_2^{m_{24}} + a_{24}x_3^{m_{25}}\right). \end{aligned} \quad (10)$$

According to (5), (7), and (10), we have

$$\begin{cases} e^{\lambda t}(1 + \cos t + \sin t) = x_1, \\ e^{\lambda t}(\lambda + (\lambda + 1)\cos t + (\lambda - 1)\sin t) = a_{11}x_1^{m_{11}} + a_{12}x_2^{m_{12}}, \\ e^{\lambda t}\left(\lambda^2 + \left(\lambda^2 + 2\lambda - 1\right)\cos t + \left(\lambda^2 - 2\lambda - 1\right)\sin t\right) = \rho. \end{cases} \quad (11)$$

By using Cramer's rule, the solution of system (11) is easily obtained as

$$\begin{cases} e^{\lambda t} = -\dfrac{w_1}{2}, \\ \cos t e^{\lambda t} = -\dfrac{w_2}{2}, \\ \sin t e^{\lambda t} = -\dfrac{w_3}{2}. \end{cases} \quad (12)$$

Calculating the derivative on both sides of equation (10), we derive

$$\begin{aligned} e^{\lambda t}&\left(\lambda^3 + \left(\lambda^3 + 3\lambda^2 - 3\lambda - 1\right)\cos t + \left(\lambda^3 - 3\lambda^2 - 3\lambda + 1\right)\sin t\right) = a_{11}m_{11}\left(m_{11} - 1\right)x_1^{m_{11}-2} f_1(x)^2 \\ &+ a_{11}m_{11}x_1^{m_{11}-1}\left(a_{11}m_{11}x_1^{m_{11}-1} f_1(x) + a_{12}m_{12}x_2^{m_{12}-1} f_2(x)\right) + a_{12}m_{12}\left(m_{12} - 1\right)x_2^{m_{12}-2} f_2(x)^2 \\ &+ a_{12}m_{12}x_2^{m_{12}-1}\left(\left(a_{21}m_{21}x_1^{m_{21}-1} + a_{23}m_{23}x_1^{m_{23}-1}x_2^{m_{24}}\right) f_1(x) + \left(a_{22}m_{22}x_2^{m_{22}-1} + a_{23}m_{24}x_1^{m_{23}}x_2^{m_{24}-1}\right) f_2(x) + a_{24}m_{25}x_3^{m_{25}-1}\dot{x}_3\right). \end{aligned} \quad (13)$$

Based on equation (13), we get

$$\dot{x}_3 = u_0, \tag{14}$$

where

$$\begin{aligned} u_0 &= \frac{e^{\lambda t}\left(\lambda^3 + \left(\lambda^3 + 3\lambda^2 - 3\lambda - 1\right)\cos t + \left(\lambda^3 - 3\lambda^2 - 3\lambda + 1\right)\sin t\right)}{a_{12}m_{12}x_2^{m_{12}-1}a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{a_{12}m_{12}(m_{12}-1)x_2^{m_{12}-2}f_2(x)^2 + a_{11}m_{11}(m_{11}-1)x_1^{m_{11}-2}f_1(x)^2}{a_{12}m_{12}x_2^{m_{12}-1}a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{a_{11}m_{11}x_1^{m_{11}-1}\left(a_{11}m_{11}x_1^{m_{11}-1}f_1(x) + a_{12}m_{12}x_2^{m_{12}-1}f_2(x)\right)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{\left(a_{21}m_{21}x_1^{m_{21}-1} + a_{23}m_{23}x_1^{m_{23}-1}x_2^{m_{24}}\right)f_1(x)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{\left(a_{22}m_{22}x_2^{m_{22}-1} + a_{23}m_{24}x_1^{m_{23}}x_2^{m_{24}-1}\right)f_2(x)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ &= \frac{-\left(\lambda^3 w_1/2\right) - \left(\lambda^3 + 3\lambda^2 - 3\lambda - 1\right)(w_2/2) - \left(\lambda^3 - 3\lambda^2 - 3\lambda + 1\right)(w_3/2)}{a_{12}m_{12}x_2^{m_{12}-1}a_{24}m_{25}x_1^{m_{25}-1}} \\ &\quad - \frac{a_{12}m_{12}(m_{12}-1)x_2^{m_{12}-2}f_2(x)^2 + a_{11}m_{11}(m_{11}-1)x_1^{m_{11}-2}f_1(x)^2}{a_{12}m_{12}x_2^{m_{12}-1}a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{a_{11}m_{11}x_1^{m_{11}-1}\left(a_{11}m_{11}x_1^{m_{11}-1}f_1(x) + a_{12}m_{12}x_2^{m_{12}-1}f_2(x)\right)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{\left(a_{21}m_{21}x_1^{m_{21}-1} + a_{23}m_{23}x_1^{m_{23}-1}x_2^{m_{24}}\right)f_1(x)}{a_{24}m_{25}x_3^{m_{25}-1}} \\ &\quad - \frac{\left(a_{22}m_{22}x_2^{m_{22}-1} + a_{23}m_{24}x_1^{m_{23}}x_2^{m_{24}-1}\right)f_2(x)}{a_{24}m_{25}x_3^{m_{25}-1}}. \end{aligned} \tag{15}$$

According to (14), we know that if u_0 is taken as (15), then equation (11) is the solution of system (2). From (11), it is obvious that the equilibrium point $(0, 0, 0)$ of system (2) is asymptotically stable. □

Remark 1. Theorem 1 is proved without using the Lyapunov stability theory. In fact, Theorem 1 can also be proved by using the Lyapunov stability theory. To show this, we choose the following function as the Lyapunov candidate:

$$V_0 = \frac{w_1^2}{|\lambda|} + \frac{w_2^2}{4} + \frac{w_3^2}{9}. \tag{16}$$

This is because V_0 can be rewritten as

$$\begin{aligned} V_0 &= 4\left(\frac{e^{2\lambda t}}{|\lambda|} + \left(\frac{\cos t e^{\lambda t}}{2}\right)^2 + \left(\frac{\sin t e^{\lambda t}}{3}\right)^2\right) \\ &= 4e^{2\lambda t}\left(\frac{1}{|\lambda|} + \left(\frac{\cos^2 t}{4}\right) + \left(\frac{\sin^2 t}{9}\right)\right). \end{aligned} \tag{17}$$

The derivative of V_0 along system (2) is

$$\begin{aligned}\dot{V}_0 &= 8\lambda e^{2\lambda t}\left(\frac{1}{|\lambda|}+\left(\frac{\cos^2 t}{4}\right)+\left(\frac{\sin^2 t}{9}\right)\right)+4e^{2\lambda t}\\ &\quad\cdot\left(\left(-\frac{\sin 2t}{4}\right)+\left(\frac{\sin 2t}{9}\right)\right)\\ &= 4e^{2\lambda t}\left(-2+2\lambda\left(\frac{\cos^2 t}{4}+\frac{\sin^2 t}{9}\right)-\frac{\sin 2t}{4}+\frac{\sin 2t}{9}\right).\end{aligned} \tag{18}$$

Since $\lambda < 0$, we have $\dot{V}_0 < 0$. According the Lyapunov stability theory, we derive $\lim_{t\longrightarrow\infty} w_1 = \lim_{t\longrightarrow\infty} w_2 = \lim_{t\longrightarrow\infty} w_3 = 0$ which implies that the equilibrium point $(0, 0, 0)$ of system (2) is asymptotically stable.

Remark 2. It is easy to see that the Lyapunov function V_0 is constructed according to the analytical solutions. So, in this paper, the new method of obtaining Lyapunov function is referred as the analytical solution approach. This new technique can be applied to other chaotic systems for designing Lyapunov function.

Remark 3. By (11), one can see that system (2) is globally exponentially stable at the origin and the rate of the convergence is determined by λ. The larger the value of $|\lambda|$, the faster the speed of convergence.

With the help of V_0 described by (17), we are now in a position to discuss the control of system (1).

Theorem 2. *Suppose that Assumption 1 holds. If controllers u_1, u_2, u_3 are taken as*

$$\begin{cases} u_1 = -\overline{\alpha}_1^T g_1(x) - M\operatorname{sign}\left(\dfrac{\partial V_0}{\partial x_1}\right),\\ u_2 = -\overline{\alpha}_2^T g_2(x) - M\operatorname{sign}\left(\dfrac{\partial V_0}{\partial x_2}\right),\\ u_3 = -f_3(x) + u_0 - \overline{\alpha}_3^T g_3(x) - M\operatorname{sign}\left(\dfrac{\partial V_0}{\partial x_3}\right),\end{cases} \tag{19}$$

with the update laws:

$$\begin{cases} \dot{\overline{\alpha}}_1 = \dfrac{\partial V_0}{\partial x_1} g_1(x),\\ \dot{\overline{\alpha}}_2 = \dfrac{\partial V_0}{\partial x_2} g_2(x),\\ \dot{\overline{\alpha}}_3 = \dfrac{\partial V_0}{\partial x_3} g_3(x),\end{cases} \tag{20}$$

Then, the equilibrium point $(0, 0, 0)$ of system (1) is asymptotically stable, where $\overline{\alpha}_1, \overline{\alpha}_2, \overline{\alpha}_3$ are the estimated values of $\alpha_1, \alpha_2, \alpha_3$, respectively.

Proof. Plugging (19) into system (1) yields

$$\begin{cases} \dot{x}_1 = f_1(x) + (\alpha_1 - \overline{\alpha}_1)^T g_1(x) + d_1 - M\operatorname{sign}\left(\dfrac{\partial V_0}{\partial x_1}\right),\\ \dot{x}_2 = f_2(x) + (\alpha_2 - \overline{\alpha}_2)^T g_2(x) + d_2 - M\operatorname{sign}\left(\dfrac{\partial V_0}{\partial x_2}\right),\\ \dot{x}_3 = u_0 + (\alpha_3 - \overline{\alpha}_3)^T g_3(x) + d_3 - M\operatorname{sign}\left(\dfrac{\partial V_0}{\partial x_3}\right).\end{cases} \tag{21}$$

Now, consider the following Lyapunov candidate:

$$\begin{aligned} V &= V_0 + \frac{1}{2}\Big((\alpha_1 - \overline{\alpha}_1)^T(\alpha_1 - \overline{\alpha}_1) + (\alpha_2 - \overline{\alpha}_2)^T(\alpha_2 - \overline{\alpha}_2)\\ &\quad + (\alpha_3 - \overline{\alpha}_3)^T(\alpha_3 - \overline{\alpha}_3)\Big).\end{aligned} \tag{22}$$

The derivative of (22) along system (21) is

$$\begin{aligned} \dot{V} &= \frac{\partial V_0}{\partial x_1}\dot{x}_1 + \frac{\partial V_0}{\partial x_2}\dot{x}_2 + \frac{\partial V_0}{\partial x_3}\dot{x}_3 - (\alpha_1 - \overline{\alpha}_1)^T\dot{\overline{\alpha}}_1 - (\alpha_2 - \overline{\alpha}_2)^T\dot{\overline{\alpha}}_2 - (\alpha_3 - \overline{\alpha}_3)^T\dot{\overline{\alpha}}_3\\ &= \frac{\partial V_0}{\partial x_1}\left(f_1(x) + (\alpha_1 - \overline{\alpha}_1)^T g_1(x) + d_1 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_1}\right)\right) + \frac{\partial V_0}{\partial x_2}\left(f_2(x) + (\alpha_2 - \overline{\alpha}_2)^T g_2(x) + d_2 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_2}\right)\right)\\ &\quad + \frac{\partial V_0}{\partial x_3}\left(u_0 + (\alpha_3 - \overline{\alpha}_3)^T g_3(x) + d_3 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_3}\right)\right) - (\alpha_1 - \overline{\alpha}_1)^T\dot{\overline{\alpha}}_1 - (\alpha_2 - \overline{\alpha}_2)^T\dot{\overline{\alpha}}_2 - (\alpha_3 - \overline{\alpha}_3)^T\dot{\overline{\alpha}}_3\\ &= \left(\frac{\partial V_0}{\partial x_1}f_1(x) + \frac{\partial V_0}{\partial x_2}f_2(x) + \frac{\partial V_0}{\partial x_3}u_0\right) + (\alpha_1 - \overline{\alpha}_1)^T\left(\frac{\partial V_0}{\partial x_1}g_1(x) - \dot{\overline{\alpha}}_1\right) + (\alpha_2 - \overline{\alpha}_2)^T\left(\frac{\partial V_0}{\partial x_2}g_2(x) - \dot{\overline{\alpha}}_2\right) + (\alpha_3 - \overline{\alpha}_3)^T\left(\frac{\partial V_0}{\partial x_3}g_3(x) - \dot{\overline{\alpha}}_3\right)\\ &\quad + \frac{\partial V_0}{\partial x_1}\left(d_1 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_1}\right)\right) + \frac{\partial V_0}{\partial x_2}\left(d_2 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_2}\right)\right) + \frac{\partial V_0}{\partial x_3}\left(d_3 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_3}\right)\right).\end{aligned} \tag{23}$$

By Assumption 1, we know that $|d_i| \le M$. Thus, we have

$$\frac{\partial V_0}{\partial x_1}\left(d_1 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_1}\right)\right) + \frac{\partial V_0}{\partial x_2}\left(d_2 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_2}\right)\right) + \frac{\partial V_0}{\partial x_3}\left(d_3 - M\operatorname{sign}\left(\frac{\partial V_0}{\partial x_3}\right)\right) \le 0. \tag{24}$$

According to update laws (20), we get

$$(\alpha_1 - \overline{\alpha}_1)^T\left(\frac{\partial V_0}{\partial x_1} g_1(x) - \dot{\overline{\alpha}}_1\right) + (\alpha_2 - \overline{\alpha}_2)^T\left(\frac{\partial V_0}{\partial x_2} g_2(x) - \dot{\overline{\alpha}}_2\right) + (\alpha_3 - \overline{\alpha}_3)^T\left(\frac{\partial V_0}{\partial x_3} g_3(x) - \dot{\overline{\alpha}}_3\right) = 0. \tag{25}$$

In view of Remark 1, we obtain

$$\frac{\partial V_0}{\partial x_1} f_1(x) + \frac{\partial V_0}{\partial x_2} f_2(x) + \frac{\partial V_0}{\partial x_3} u_0 < 0. \tag{26}$$

Substituting (24), (25), and (26) into (23), one derives that

$$\dot{V} < 0. \tag{27}$$

Based on the Lyapunov stability theory, we conclude $\lim_{t\longrightarrow\infty} w_1 = \lim_{t\longrightarrow\infty} w_2 = \lim_{t\longrightarrow\infty} w_3 = 0$, which implies that the equilibrium point $(0, 0, 0)$ of system (1) is asymptotically stable. □

Remark 4. Note that λ determines the speed of the convergence of system (2) and system (2) is the main part of system (1); therefore, λ determines the speed of the convergence of system (1). The simulation results show that the smaller the value of λ, the faster the speed of convergence.

4. The Control of an Novel Chaotic Attractor and its Simulation Results

Consider the following new three-dimensional chaotic system [18]:

$$\begin{cases} \dot{x}_1 = x_2 - a x_1 + b x_2 x_3, \\ \dot{x}_2 = c x_2 - x_1 x_3 + x_3, \\ \dot{x}_3 = d x_1 x_2 - h x_3, \end{cases} \tag{28}$$

where $(x_1, x_2, x_3)^T \in R^3$ is the state vector, and a, b, c, d and h are positive real constants. It is reported that this chaotic system has many interesting complex dynamical behaviors [18]. The new two-wing chaotic attractor of system (28) with $a = 3, b = 2.7, c = 4.7, d = 2$ and $h = 9$ is shown in Figure 1. In the following numerical process, we assume $a = 3, b = 2.7, c = 4.7, d = 2$, and $h = 9$. For simplicity, we suppose that b and d are unknown parameter.

If system (28) is affected by external disturbances, then system (28) with three inputs can be rewritten as

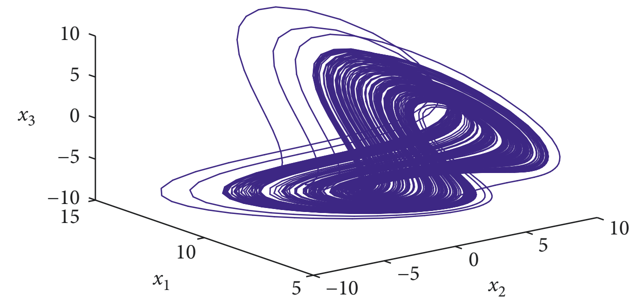

Figure 1: The chaos attractor of system (28) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

$$\begin{cases} \dot{x}_1 = x_2 - a x_1 + b x_2 x_3 + \sin(x_2 x_1) + u_1, \\ \dot{x}_2 = c x_2 + \theta x_1 x_3 + x_3 + \cos(x_2 x_3) + u_2, \\ \dot{x}_3 = d x_1 x_2 - h x_3 + 2\sin(x_1)\cos(x_3) + u_3, \end{cases} \tag{29}$$

where $\theta = -1$ and u_1, u_2, u_3 are three controllers.

Compared with system (1), we know that $(m_{11}, m_{12}, m_{21}, m_{22}, m_{23}, m_{24}, m_{25}) = (1, 1, 1, 1, 1, 1, 1). a_{11} = -a, a_{12} = 1, a_{21} = 0, a_{22} = c, a_{23} = 0, a_{24} = 1.$ $\alpha_{11} = b, \alpha_{21} = \theta, \alpha_{31} = d.$ $g_{11}(x) = x_2 x_3, g_{21}(x) = x_1 x_3, g_{31}(x) x_1 x_2, f_3 \quad (x) = -h x_3.$ The external disturbances are $d_1 = \sin(x_2 x_1), d_2 = \cos(x_2 x_3), d_3 = 2\sin(x_1)\cos(x_3)$.

The reason why we introduce θ is that in $f_2(x)$ of system (1) there is no term $x_2 x_3$. So, in order to use the results obtained in the above section, we assume θ is the unknown parameter.

Based on system (2), we derive the following auxiliary system:

$$\begin{cases} \dot{x}_1 = x_2 - 3x_1, \\ \dot{x}_2 = 4.7 x_2 + x_3, \\ \dot{x}_3 = u_0, \end{cases} \tag{30}$$

Now, for convenience of comparison, we consider two cases: $\lambda = -1$ and $\lambda = -10$.

Case 1. $\lambda = -1$.

According to Theorem 2, in this case, one can obtain

$$\begin{cases} w_1 = -10x_1 - \dfrac{37x_2}{5} - 2x_3, \\ w_2 = 6x_1 + 2.7x_2 + x_3, \\ w_3 = 2x_1 + 4.7x_2 + x_3, \\ u_0 = -\dfrac{1}{2}\left(\dfrac{2609x_2}{50} - 20x_1 + \dfrac{47x_3}{5}\right), \\ V_{01} = \left(10x_1 + \frac{37x_2}{5} + 2x_3\right)^2 + \dfrac{1}{4}\left(6x_1 + 2.7x_2 + x_3\right)^2 \\ \quad + \dfrac{1}{9}\left(2x_1 + 4.7x_2 + x_3\right)^2. \end{cases} \tag{31}$$

Therefore, the controllers can be taken as

$$\begin{cases} u_1 = -\bar{b}x_2x_3 - 2\operatorname{sign}\left(\dfrac{\partial V_{01}}{\partial x_1}\right), \\ u_2 = -\bar{\theta}x_1x_3 - 2\operatorname{sign}\left(\dfrac{\partial V_{01}}{\partial x_2}\right), \\ u_3 = 9x_3 + u_0 - \bar{d}x_1x_2 - 2\operatorname{sign}\left(\dfrac{\partial V_{01}}{\partial x_3}\right), \end{cases} \quad (32)$$

with the update laws:

$$\begin{cases} \dot{\bar{b}} = \dfrac{\partial V_{01}}{\partial x_1}x_2x_3, \\ \dot{\bar{\theta}} = \dfrac{\partial V_{01}}{\partial x_2}x_1x_3, \\ \dot{\bar{d}} = \dfrac{\partial V_{01}}{\partial x_3}x_2x_1, \end{cases} \quad (33)$$

where $\bar{b}, \bar{\theta}, \bar{d}$ are the estimated vales of b, θ, d, respectively.

Case 2. $\lambda = -10$.

Based on Theorem 2, in this case, we have

$$\begin{cases} w_1 = -100x_1 - \dfrac{217x_2}{5} - 2x_3, \\ w_2 = 42x_1 + 20.7x_2 + x_3, \\ w_3 = 56x_1 + 22.7x_2 + x_3, \\ u_0 = -350x_1 - 368.99x_2 - 31.7x_3, \\ V_{02} = \dfrac{\left(-100x_1 - (217x_2/5) - 2x_3\right)^2}{10} \\ \qquad + \dfrac{\left(42x_1 + 20.7x_2 + x_3\right)^2}{4} + \dfrac{\left(56x_1 + 22.7x_2 + x_3\right)^2}{9}. \end{cases} \quad (34)$$

Therefore, the controllers can be taken as

$$\begin{cases} u_1 = -\bar{b}x_2x_3 - 2\operatorname{sign}\left(\dfrac{\partial V_{02}}{\partial x_1}\right), \\ u_2 = -\bar{\theta}x_1x_3 - 2\operatorname{sign}\left(\dfrac{\partial V_{02}}{\partial x_2}\right), \\ u_3 = 9x_3 + u_0 - \bar{d}x_1x_2 - 2\operatorname{sign}\left(\dfrac{\partial V_{02}}{\partial x_3}\right), \end{cases} \quad (35)$$

with the update laws:

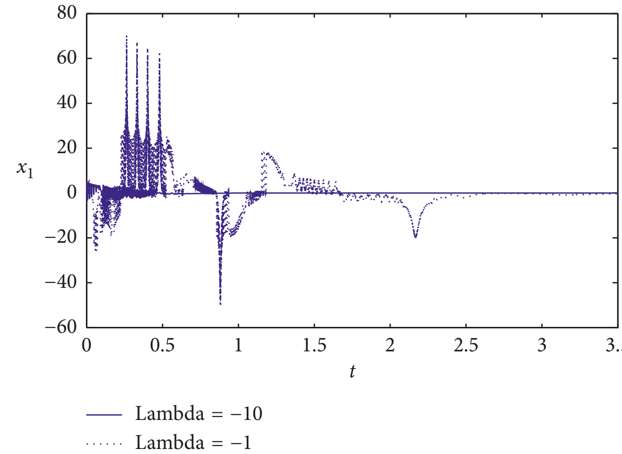

FIGURE 2: The time response of state x_1 of system (29) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

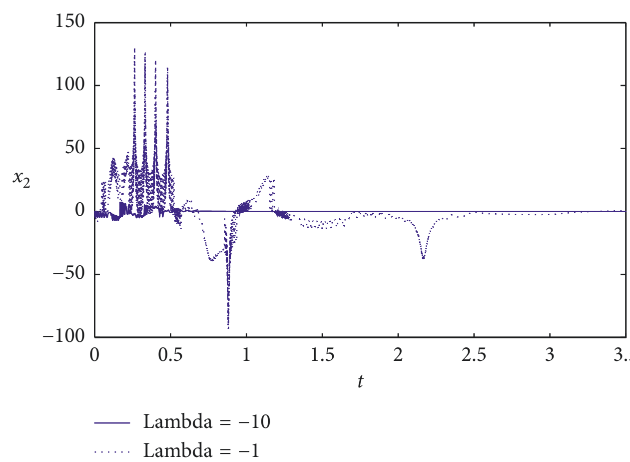

FIGURE 3: The time response of state x_2 of system (29) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

$$\begin{cases} \dot{\bar{b}} = \dfrac{\partial V_{02}}{\partial x_1}x_2x_3, \\ \dot{\bar{\theta}} = \dfrac{\partial V_{02}}{\partial x_2}x_1x_3, \\ \dot{\bar{d}} = \dfrac{\partial V_{02}}{\partial x_3}x_2x_1, \end{cases} \quad (36)$$

where $\bar{b}, \bar{\theta}, \bar{d}$ are the estimated vales of b, θ, d, respectively.

According to Theorem 2, we know that in the two cases, the equilibrium point $(0, 0, 0)$ of system (29) is asymptotically stable. The Figures 2–7 show the simulation results with $\lambda = -1$ and $\lambda = -10$. The initial values are taken as $(x_1(0), x_2(0), x_3(0)) = (5, 0, -4)$, $(\bar{b}(0), \bar{\theta}(0), \bar{d}(0)) = (1, 1, 1)$.

From Figures 2–4, it can be seen that when $\lambda = -1$, the states x_1, x_2, x_3 need about 2.5 s to reach the equilibrium point. However, for $\lambda = -10$, the states x_1, x_2, x_3 only need about 0.6 s to reach the equilibrium point, which means that the convergence time is greatly shortened compared

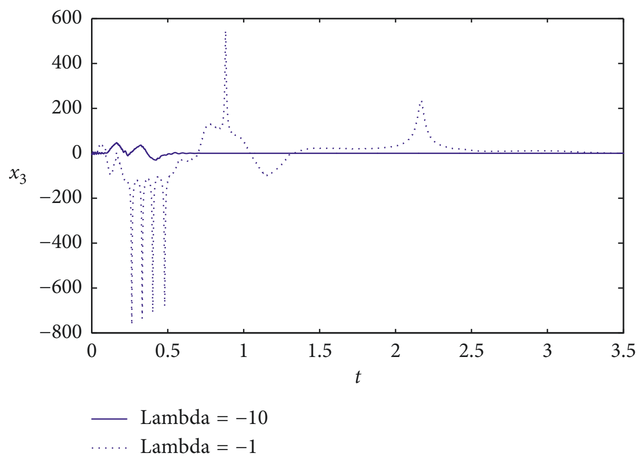

Figure 4: The time response of state x_3 of system (29) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

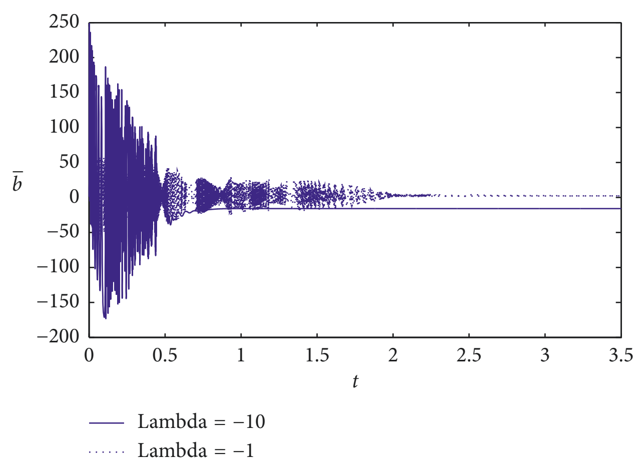

Figure 5: The time response of state $\overline{b}$ of system (29) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

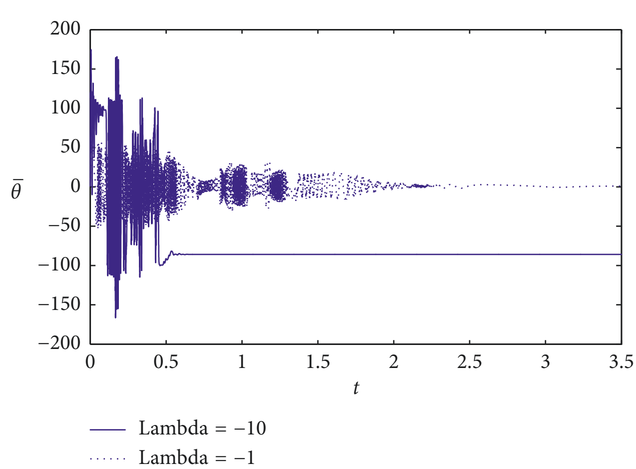

Figure 6: The time response of state $\overline{\theta}$ of system (29) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

with that of $\lambda = -1$. Therefore, one can conclude that the larger the value of $|\lambda|$, the faster the rate of convergence. In addition, the vibration amplitude of $\lambda = -10$ is smaller

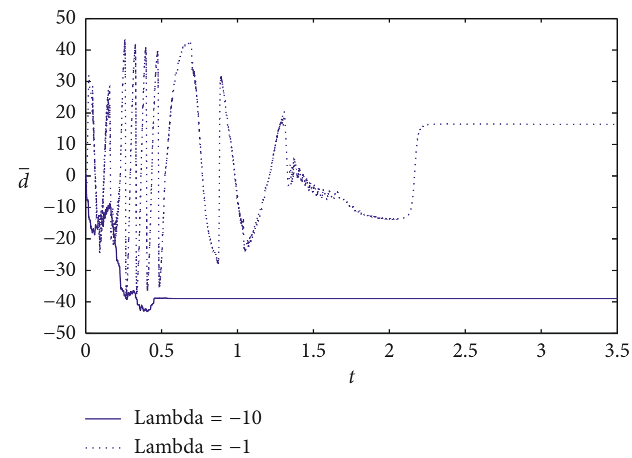

Figure 7: The time response of state $\overline{d}$ of system (29) with $x_1(0) = 5, x_2(0) = 0, x_3(0) = -4$.

than that of $\lambda = -1$ which implies that the vibration amplitude is decreased with the increase of the absolute value of λ.

5. Conclusions

The control of a class of 3D chaotic systems with uncertain parameters and external disturbances via an adaptive control method has been systematically investigated in this paper. Through designing the analytical solution of the chaotic system, a proper Lyapunov function can be easily obtained. This method of constructing Lyapunov function by using the analytical solution can be applied to other chaotic systems. By using the designed Lyapunov function, a novel fast convergence controller containing parameter λ which determines the convergence rate of the system is presented. In addition, as shown in Section 4, since the terms that are not included in functions $f_1(x), f_2(x), f_3(x)$ can be considered to contain uncertain parameters, the control scheme presented in this paper can be employed to any 3D continuous chaotic systems. Finally, the effectiveness of the theoretical results is illustrated by some numerical simulations.

In this paper, only the integer-order chaotic system, which is a special case of fractional-order system, has been considered. It is well known that compared with integer-order derivatives, the fractional derivatives has many distinguishing features such as the long-term memory and self-similarity which can be used for the description of memory and hereditary properties of various materials and processes. Thus, the control and synchronization of fractional-order chaotic systems via the analytical solution approach is an important issue. This issue will be our research focus in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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