Research Article

Hyperchaotic Oscillation in the Deformed Rikitake Two-Disc Dynamo System Induced by Memory Effect

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The fundamental dynamics of the deformed Rikitake two-disc dynamo system is explored in this paper. Memory effect on the dynamical behavior of the generator system is studied by introducing a quadratic flux-controlled memristor. Hyperchaotic oscillation in the deformed Rikitake two-disc coupled generator is therefore firstly found. Lyapunov exponents, bifurcation diagram, and phase portraits prove the abundant dynamic behavior consistently.

1. Introduction

Memristor is a passive element with nonlinearity and nonvolatility. According to the completeness of basic circuit components, in 1971, Chua first predicted the existence of the fourth circuit component, which describes the relationship between charge and flux [1]. In 2008, Hewlett-Packard Laboratory successfully fabricated the nanoscale memristor based on metal and metal oxides [2], which aroused great interest in the scientific and technological community. Recently, great progress has been made in the research of memristors, and the application of memristor also has become a hot focus. Because of the small size, low power consumption, nonlinearity, and non-volatility, the memristor can be applied in many areas such as nonlinear chaotic circuits [3–6], electronic engineering [7–9], artificial intelligence [10–12], and neural networks [13–15].

It is considered that the geomagnetic field is associated with the conductive outer core, which has been proved by some dynamical models. In 1958, T. Rikitake firstly proposed a Two-Disc Dynamo System (RTDDS) and observed this physical law [16]. RTDDS is a simple model to

demonstrate the polarity reversal of the earth's magnetic field, in which the current from each disc excites the coil of the other [17]. In fact, it is a chaotic system [18] and exhibits abundant dynamical behavior [19–21].

Recently, much attention has been paid to the RTDDS. A new attractor synthesis algorithm was applied to model the attractors in the Rikitake system [22]. By applying synchronization technique based on control theory, an active controller was designed for the synchronization of two identical RTDDS [19]. A method to stabilize asymptotically the nontrivial Lyapunov stable states of Rikitake two-disc dynamo dynamics was given in [23]. It was proven that the two non-hyperbolic equilibrium points of the Rikitake system are all stable for all positive parameters [24]. A simple realization of the symmetric Rikitake system was given in [25]. A reduced-order projective synchronization system was designed for the Rikitake system without any equilibria or with two non-hyperbolic equilibrium points in [26]. In [27], a 4-D hyperchaotic Rikitake dynamo system without any equilibria was proposed. In [28], a 5-D hyperchaotic Rikitake dynamo system with three positive Lyapunov exponents was proposed, which has a hidden attractor without any equilibrium point. From aforementioned references, the

hyperchaotic system of RTDDS was constructed by artificial numerical methods directly through feedback and other means.

With the continuous development of industrial automation, intelligent motors with control and learning capabilities have also been realized. In this paper, a deformed RTDDS is constructed by introducing an extra flux-controlled memristor. The motor can be controlled to avoid the chaotic region through the combination of memristors and memristor control parameters. After the memristor is added, the bifurcation point of the system is changed, and the original behavior state of the RTDDS is also changed. Therefore, the research in this paper provides an important idea for the control of the double-disk motor. This system is proven with abundant dynamical behaviors, including a line of equilibria and hyperchaos [29–33]. In Section 2, the brief introduction of the flux-controlled memristor is given, thereafter a new lossy RTDDS is given, and the deformed RTDDS is constructed by adding one extra flux-controlled memristor. Rich dynamics of the presented system are analyzed in Section 3. In Section 4, the analog circuit of the new memristive hyperchaotic system is implemented based on Multisim simulation. Some conclusions are finally drawn in Section 5.

2. Modeling of the Deformed Two-Disc Generator

2.1. Memristor Model. According to the relationship between voltage v and current of a flux-controlled memristor [1, 34, 35], choose a cubic nonlinearity to describe the q function [36–38], and then, $W(\varphi)$ is

$$\begin{cases} i = W(\varphi)y, \\ W(\varphi) = \alpha + 3\beta\varphi^2, \\ \dot{\varphi} = y, \end{cases} \quad (1)$$

where α and β are two positive constants. Figure 1 shows the voltage-current relationship of the memristor with $\alpha = 2$ and $\beta = 0.1$ under the frequency $f = 1$ Hz. The hysteresis curve agrees the inherent characteristics of the memristor.

In the following, the above memristor is applied to study the memory effect of the deformed RTDDS.

2.2. Description of the Deformed RTDDS. In 1958, Rikitake first proposed a two-disc generator, in which the dimensionless equations are [16, 18]

$$\begin{cases} \dot{x}_1 = -\mu x_1 + x_2 x_3, \\ \dot{x}_2 = -\mu x_2 + x_1 [x_3 - \mu(\sigma^2 - \sigma^{-2})], \\ \dot{x}_3 = 1 - x_1 x_2, \end{cases} \quad (2)$$

where x_1 and x_2 are dimensionless currents, x_3 is the angular velocity of the two discs, μ and σ are adjustable parameters.

In the chaotic double-disk generator, considering the wear rate of the double-disk generator, the model of the deformed double-disk coupled generator is constructed under electromechanical coupling. The new deformed RTDDS is proposed as

FIGURE 1: Pinched hysteresis loop of the memristor with $\alpha = 2$ and $\beta = 0.1$ ($f = 1$ Hz).

$$\begin{cases} \dot{x} = -ax + y(z + c), \\ \dot{y} = -by + x(z - c), \\ \dot{z} = dz - xy, \end{cases} \quad (3)$$

where a and b are the ratio of resistance to self-inductance of the two loops, which represent the dissipative performance of the generator, and are closely related to the working conditions of the generator. Here, c is the difference in angular velocity between two rotors of the coupled generators, d is the wear parameter, x and y represent the current across the two loops, and z is the angular velocity of the double discs. System (3) with $a = 2$, $b = 3$, $c = 5$, and $d = 0.75$ has a chaotic solution with Lyapunov exponents: $LE_1 = 0.2825$, $LE_2 = 0$, and $LE_3 = -4.4179$ under the initial condition $(0.1, 0.1, 0.1)$, as shown in Figure 2. System (3) shows rotational symmetry since it is recovered by the transformation $(x, y, z) \rightarrow (-x, -y, z)$.

2.3. Memristive Deformed RTDDS. In RTDDS, assume that the loop current of the first disc depends on the change of the current of the other disc, and the memory effect can be indicated by the memristor. A flux-controlled memristor is applied in RTDDS with $\alpha = 2$ and $\beta = 0.1$, and the memristive deformed RTDDS is

$$\begin{cases} \dot{x} = -ax + yz + kW(\varphi)y, \\ \dot{y} = -by + x(z - c), \\ \dot{z} = dz - xy, \\ \dot{\varphi} = y, \end{cases} \quad (4)$$

where a , b , c , and d are positive parameters, and k is a positive parameter representing the strength of the memristor. System (4) with $a = 2$, $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$ has a hyperchaotic attractor with Lyapunov exponents $LE_1 = 0.3784$, $LE_2 = 0.0218$, $LE_3 = 0.0000$, and $LE_4 = -4.6503$, two of which are positive indicating hyperchaos, as shown in Figure 3. The Kaplan–York dimension is $D_{KY} = 3.0861$. Poincaré map is a line representing chaos if the surface represents hyperchaos, and the Poincaré map of the system is shown in Figure 4; in this system, Poincaré map is surface, so it is hyperchaos. The invariance of system (4) under the transformation $(x, y, z, \varphi) \rightarrow (-x, -y, z, -\varphi)$ shows the

FIGURE 2: Phase portraits of system (3) with $a=2$, $b=3$, $c=5$, and $d=0.75$: (a) x - y plane, (b) x - z plane, and (c) y - z plane.

FIGURE 3: Phase portraits of system (4) with $a=2$, $b=3$, $c=5$, $d=0.75$, and $k=1$: (a) x - y plane, (b) y - z plane, and (c) y - φ plane.

FIGURE 4: Poincaré map of system (4) with $a=2$, $b=3$, $c=5$, $d=0.75$, and $k=1$: (a) x - y plane; (b) y - φ plane.

rotational symmetric structure. Therefore, newly introduced memristor transforms the system to be a hyperchaotic one.

3. Dynamical Behaviours of the Proposed Chaotic System

3.1. Line of Equilibria and Stability Analysis. The equilibrium points of system (4) can be derived by solving the following equations:

$$\begin{cases} -ax + yz + kW(\varphi)y = 0, \\ -by + x(z - c) = 0, \\ dz - xy = 0, \\ y = 0. \end{cases} \quad (5)$$

System (5) has a line of equilibria $[0, 0, 0, \varphi]$, where φ is a real variable. By linearizing system (5) at the equilibria, the Jacobian matrix can be obtained:

FIGURE 5: Lyapunov exponent spectra and bifurcation of system (4) with $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$, and a varies in $[2, 10]$: (a) Lyapunov exponent spectra: LE_1 is blue, LE_2 is red, and LE_3 is green; (b) bifurcation diagram.

$$J_o = \begin{bmatrix} -a & kW(\varphi) & 0 & 0 \\ -c & -b & 0 & 0 \\ 0 & 0 & d & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}. \quad (6)$$

According to equation (6), the characteristic equation can be obtained as

$$\lambda(\lambda - d)[\lambda^2 + (a + b)\lambda + ab + ckW(\varphi)] = 0. \quad (7)$$

So, the eigenvalues are

$$\begin{aligned} \lambda_1 &= 0, \\ \lambda_2 &= d, \\ \lambda_{3,4} &= \frac{-(a + b) \pm \sqrt{(a - b)^2 - 4ckW(\varphi)}}{2}. \end{aligned} \quad (8)$$

Therefore, the stability of the line equilibria is associated with the values of a , b , c , k , and $W(\varphi)$. From λ_2 and $\lambda_{3,4}$, it can be seen that when a , b , and d are positive, the line equilibria point is unstable, command $\Delta_k = (a - b)^2 - 4ckW(\varphi)$, where if $\Delta_k > 0$, and the line equilibria are unstable nodes or else if $\Delta_k < 0$, the line equilibria are unstable saddle foci. Specifically, when $a = 2$, $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$, the line equilibria are unstable saddle focus.

3.2. Dynamical Analysis. The dynamic behavior of system (4) will be further investigated with Lyapunov exponent spectra and bifurcation diagram.

When $b = 3$, $c = 5$, $d = 0.75$, $k = 1$, and a varies in $[2, 10]$, set the initial condition $IC = (0.1, 0.1, 0.1, 0.1)$, step size of a is 0.01, time step is 0.01 s, and running time is 300 s; the corresponding Lyapunov exponent spectra and bifurcation diagram are obtained as shown in Figure 5. As shown in Figure 5(a), when $a \in [0, 2.58]$, the largest Lyapunov exponent is positive, so system (4) is chaotic; when $a \in [2.58, 2.9]$, two of

the Lyapunov exponents are positive. Thus, system (4) is hyperchaotic; thereafter, system (4) enters into the periodic mode at $a = 2.9$. It remains periodic when $a \in [2.9, 3.42]$. When $a \in [3.42, 5.33] \cup [5.81, 6.4] \cup [7.24, 7.6]$, it stays in chaos; system (4) drops in the periodic state occasionally when $a \in [5.33, 5.81] \cup [6.4, 7.24] \cup [7.6, 10]$. Bifurcation diagram shown in Figure 5(b) agrees with the Lyapunov exponents. Specific periodic oscillations are shown Figure 6, and the detail information of Lyapunov exponents is shown in Table 1. We noticed that system (4) sometimes provides a symmetric oscillation and sometimes gives a symmetric pair of limit cycles.

The memristor bridges the loop current of two disks. Different phase portraits can be obtained under different initial values, as shown in Figure 7. A symmetric oscillation or a symmetric pair of limit cycles is found simultaneously.

Let $a = 2$, $b = 3$, $d = 0.75$, and $k = 1$, while c varies from 0 to 10, and the initial condition $IC = (0.1, 0.1, 0.1, 0.1)$, the Lyapunov exponent spectra, and the corresponding bifurcation diagram are shown in Figure 8. As shown in Figure 8, when $c \in [0, 1.9] \cup [4.6, 8] \cup [8.24, 10]$, system (4) is chaotic; when $c \in [2.7, 4.6]$, system (4) is hyperchaotic; and when $c \in [1.9, 2.64] \cup [8, 8.42]$, system (4) is periodic. Let $a = 2$, $b = 3$, $c = 5$, and $d = 0.75$, while k varies in $[0, 5]$, set the initial condition $IC = (0.1, 0.1, 0.1, 0.1)$, and the Lyapunov exponent spectra and the corresponding bifurcation diagram are shown in Figure 9, showing system (4) stays in chaos robustly. Usually, a memristive system shows multistability induced by the memory effect, however when $a = 2$, $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$, let the initial condition $IC = (0.1, 0.1, 0.1, \varphi)$, φ varies from -5 to 5, the Lyapunov exponent spectra and the corresponding bifurcation diagram are shown in Figure 10, showing system (4) stays in chaotic orbit.

4. Circuit Simulation Based on Multisim

In order to further observe the specific hyperchaotic oscillation behavior induced from the memory effect, a circuit

FIGURE 6: Various periodic oscillations in system (4) with $b=2$, $c=5$, $d=0.75$, and $k=1$ and (a) $a=3.2$, (b) $a=5.8$, (c) $a=7$, (d) $a=8.5$, (e) $a=9$, and (f) $a=9.8$, $IC=(0.1, 0.1, 0.1, 0.1)$ is blue, $(-0.1, 0.1, 0.1, 0.1)$ is red, and $(0.1, 0.1, -0.1, 0.1)$ is green.

TABLE 1: Periodic oscillations in system (4) under different parameters of a .

Cases	Parameter a	Lyapunov exponents	Solution type of system (4)
A	$a=3.2$	0.0002 -0.0665 -0.9399 -4.4432	Symmetric attractor
B	$a=5.8$	0.1214 0.0000 -0.3017 -7.8699	Symmetric attractor
C	$a=7$	0.0005 -0.1337 -0.5036 -8.6132	Asymmetric attractor
D	$a=8.5$	0.0000 -0.4863 -0.7792 -9.4844	Asymmetric attractor
E	$a=9$	0.0014 -0.5440 -0.8858 -9.8216	Asymmetric attractor
F	$a=9.8$	0.0006 -0.2361 -0.7574 -11.0571	Asymmetric attractor

FIGURE 7: Various periodic oscillations in system (4) with $a=7$, $b=3$, $c=5$, $d=0.75$, and $k=1$: (a) $IC=(0.1, 0.1, 0.1, 0.1)$ is blue, $(0.1, -0.1, 0.1, 0.1)$ is red, (b) $IC=(0.1, 5, 0.1, 0.1)$ is blue, $(0.1, -5, 0.1, 0.1)$ is red, and (c) $IC=(0.1, 10, 0.1, 0.1)$ is blue, $(0.1, -10, 0.1, 0.1)$ is red.

simulation is realized based on Multisim software [39]. In the circuit design, resistors, capacitors, operational amplifiers (OPA404AG), analog multipliers, and other elements are applied. The supply voltages for OPA404AG operational amplifiers (with saturated voltages $V_{\text{sat}} \approx \pm 13.5$ V) are ± 15 V. In fact, the variable z is beyond the normal operating range

of the device. Therefore, here, the variables are transformed by proportional compression, that is, $v_x = 10v_{x1}$, $v_y = 10v_{y1}$, $v_z = 10v_{z1}$, and $v_\phi = 10v_{\phi1}$, where v_{x1} , v_{y1} , v_{z1} , and $v_{\phi1}$ are the voltages on the integral capacitor, respectively. By time rescaling of system (4), the equations can be obtained as follows:

FIGURE 8: Lyapunov exponent spectra and bifurcation diagram of system (4) with $a = 2$, $b = 3$, $d = 0.75$, and $k = 1$, while c varies in $[1, 10]$: (a) Lyapunov exponent spectra: LE_1 is blue, LE_2 is red, and LE_3 is green; (b) bifurcation diagram.

FIGURE 9: Lyapunov exponent spectra and bifurcation diagram of system (4) with $a = 2$, $b = 3$, $c = 5$, and $d = 0.75$, while k varies in $[0, 5]$: (a) Lyapunov exponent spectra: LE_1 is blue, LE_2 is red, and LE_3 is green; (b) bifurcation diagram.

FIGURE 10: Lyapunov exponent spectra and bifurcation diagram of system (4) with $a = 2$, $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$, while ϕ varies in $[-5, 5]$: (a) Lyapunov exponent spectra: LE_1 is blue, LE_2 is red, and LE_3 is green; (b) bifurcation diagram.

FIGURE 11: The analog circuit of the deformed RTDDS.

FIGURE 12: Simulation results obtained from Multisim software: (a) attractor on the x - y plane, (b) attractor on the y - z plane, (c) attractor on the x - z plane, and (d) attractor on the y - φ plane.

FIGURE 13: Phase portraits of system (4) with memristor $W(\varphi) = -m + n|\varphi|$, $m = 2$, $n = 5$, and $a = 2$, $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$: (a) x - y plane, (b) y - z plane, and (c) y - φ plane.

$$(9) \quad \begin{cases} C_1 \dot{v}_x = -\frac{1}{R_{13}}v_x + \frac{1}{R_{12}}v_y v_z + \frac{1}{R_{11}}W(\varphi)v_y, \\ C_2 \dot{v}_y = -\frac{1}{R_{21}}v_y + \frac{1}{R_{22}}v_x v_z - \frac{1}{R_{23}}v_x, \\ C_3 \dot{v}_z = \frac{1}{R_{31}}v_z - \frac{1}{R_{32}}v_x v_y, \\ C_4 \dot{v}_\varphi = \frac{1}{R_{41}}v_y. \end{cases}$$

The corresponding analog circuit is shown in Figure 11. The circuit in Figure 11(a) represents the quadratic non-linear flux-controlled memristor, which consists of an integration circuit and a proportional circuit. Resistor R41, operational amplifier U2C, and capacitor C4 constitute an integral circuit, which integrates the voltage v across the memristor giving the magnetic flux through the memristor.

In Figure 11, the capacitances and resistances are $C_1 = C_2 = C_3 = C_4 = 1$ nF, $R_{12} = R_{23} = R_{32} = R_{41} = 10$ k Ω , $R_{13} = 50$ k Ω , $R_{21} = 33.3$ k Ω , $R_{22} = 20$ k Ω , $R_{31} = 133.3$ k Ω , $R_{42} = R_{44} = 30$ k Ω , and $R_{14} = R_{15} = R_{24} = R_{25} = R_{33} = R_{34} = R_{43} = R_{11} = 100$ k Ω . Figure 12 gives the phase trajectories shown in the oscilloscopes.

In order to verify the memory effect in RTDDS, a new flux-controlled memristor $W(\varphi) = -m + n|\varphi|$, with $m = 2$ and $n = 5$, is introduced. System (4) has a hyperchaotic attractor with Lyapunov exponents $LE_1 = 0.3225$, $LE_2 = 0.0554$, $LE_3 = 0.0000$, and $LE_4 = -4.6309$ when $a = 2$, $b = 3$, $c = 5$, $d = 0.75$, and $k = 1$, two of which are positive indicating hyperchaos, as shown in Figure 13.

5. Conclusions and Discussion

The deformed Rikitake two-disc dynamo system possesses rich dynamics including chaos, hyperchaos, and different periodic oscillations. A memristive deformed RTDDS was constructed for observing the memory effect. Consequently, an analog circuit based on the flux-controlled memristor was designed for further verification. Circuit simulation agrees with the theoretical analysis and numerical simulation. By the memristor model built the control circuit, through memristor and matching parameters of the memristor, the RTDDS can be controlled to avoid the chaotic region and realize the smooth operation. This research provides a meaningful reference for motor design and control.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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