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*Research Article*

# Guaranteed Cost Formation Tracking Control for Swarm Systems with Intermittent Communications

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The current paper studies guaranteed cost time-varying formation tracking design and analysis problems of high-order swarm systems subject to intermittent communications. Different from the existing work of the time-varying formation control, the time-varying formation tracking can be achieved while certain performance can be guaranteed, and the impacts of the intermittent communications and switching topologies are considered. First, a new intermittent time-varying formation tracking control protocol with a global performance index is proposed, where not only the formation regulation performances but also the control energy expenditures are involved. The codesign of the gain matrix with the performance index is achieved to compromise the formation regulation performances against control energy expenditures, and the guaranteed cost is determined to restrain the upper bound of the performance index. Then, guaranteed cost time-varying formation tracking design and analysis criteria are given, where the matrix variable of the linear matrix inequality conditions is used to design the gain matrix and to determine the guaranteed cost. Finally, a simulation example is provided to illustrate the effectiveness of the theoretical results.

## 1. Introduction

As one of the most important topics of the distributed cooperative control of swarm systems, formation control has aroused many attentions from researchers in recent years [1–7]. Distributed formation control means to design formation control protocol using only local information such that a team of autonomous agents forms and maintains the expected geometrical shape. Recently, due to the rapid development of the consensus theory, many scholars investigated the formation control problem via consensus-based approaches [8–14]. The core idea of the consensus-based formation control is to drive the agents to the desired states such that they can keep the specified difference from the virtual agreement states, which can be determined by the consensus control. Formation can be categorized into leaderless formation and formation tracking according to the different communication topology structure. For the leaderless one, each agent plays equal roles to determine the formation shape cooperatively. However, for the formation tracking, the followers should form the expected formation and track the leader, which determines the swarm property of the whole swarm systems.

A basic problem of the consensus-based formation control is the time-invariant formation control, where the expected formation shape is fixed. In this case, the relative position between any two agents will not change when the formation is formed. Time-invariant formation control/formation tracking can be regarded as the directed extension of the consensus/consensus tracking, and it was widely investigated in recent years [15–19]. However, due to the complicated task requirements and task updates, time-varying formation tracking is often needed in many applications, such as cooperative attack task, obstacle avoidance, and resource exploration. Compared with the time-invariant formation, time-varying formation tracking is more challenging since it should consider the impacts of the derivate of the formation function and the formation changes in time. For second-order swarm systems with switching topologies, the time-varying formation tracking control was studied [20]. Wang et al. [21] investigated the robust time-varying formation design problems for second-order swarm systems with external disturbances where a new distributed extended state observer was constructed to compensate the influence of the disturbances. For general high-order swarm systems, time-varying formation tracking control and adaptive time-varying formation control were addressed [22, 23], respectively.

In many practical applications, the communication among agents may not always be continuous due to some communication faults including congestion of communication channels, packet losses, and sensing device failures. These communication faults can be modeled as intermittent communication where each agent can exchange its information to its neighbors over connected communication time units, but the interaction among agents will disappear in disconnected communication time units. Consensus of second-order swarm systems with intermittent communications was studied in [24]. Considering the impact of time delays and intermittent communications, Fattahi and Afshar [25] investigated the adaptive consensus control problem for high-order swarm systems. Sun and Wang [26] addressed the consensus problem for high-order swarm systems with Lipschitz nonlinear dynamics and intermittent communications where an interesting sampling time unit approach was proposed to transfer the intermittent consensus control problems to asymptotical stability problems of the swarm systems with input delays. In [24–26], the communication topology was fixed over connected communication time units. However, many swarm systems suffer switching topologies due to the changes of communication channels among agents as shown in [27–29] where the intermittent communications were not considered. It should be noted that it is difficult to deal with the intermittent communications and the switching topologies over connected communication time units simultaneously, and they were not considered in time-varying formation tracking problems, which form one of the motivations of the current paper.

Note that the abovementioned works only studied the formation achievement strategy without considering the performance constraints. However, it makes much sense to investigate the time-varying formation tracking control method with optimal/suboptimal performance indexes since there exist many resource limitations, and the control protocol should be optimized in real-world applications. In this sense, it is challenging to design a proper formation control protocol such that the time-varying formation tracking can be achieved, while the associated performance is guaranteed. For consensus control of swarm systems, there are some interesting works that address the optimal/suboptimal control problems. In [30], the optimal consensus algorithm was proposed with global performance indexes for first-order swarm systems where it was shown that the complete graph is needed to realize the global optimization of consensus. To relax the topology from the complete graph to the connected undirected graph, guaranteed cost consensus was achieved with different conditions in [31–33]. However, there are rare works that consider the time-varying formation tracking with guaranteed cost performance analysis, and it is interesting to investigate how the formation variance affects the guaranteed cost of swarm systems, which also motivates the study of the current paper.

Guaranteed cost time-varying formation tracking problems for high-order swarm systems with intermittent communications and switching topologies are addressed in the current paper. First, an intermittent time-varying formation tracking control protocol containing the switching topologies is constructed with the corresponding performance index. Then, the dynamics of the whole closed-loop system is decomposed into two parts by a nonsingular transformation and an orthonormal transformation successively, which can convert the formation tracking problem of the swarm system to the asymptotical stability problem of the reduced-order system. The stability of the reduced-order system is analyzed, respectively, in the connected communication time units and the disconnected communication time units to obtain the exponential condition of the asymptotical stability. Sufficient conditions of guaranteed cost time-varying formation tracking design and analysis are derived in the form of linear matrix inequality, and the guaranteed cost is determined to restrain the upper bound of the performance index.

Compared with the relative works about the formation control, the contributions of the current paper are twofold. First, the guaranteed cost time-varying formation tracking control problem is addressed, which can ensure that swarm systems can not only achieve the time-varying formation tracking but also satisfy the guaranteed cost constraints; that is, the compromise design between the formation regulation performance and the control energy expenditure should be realized with respect to the proposed performance index, and the guaranteed cost is determined to describe the upper bound of the performance index. However, the results on the time-varying formation control in [20–23] did not consider the impact of the guaranteed cost constraints. Second, the communication constraints of both intermittent communications and switching topologies are introduced into the design process of the guaranteed cost time-varying formation tracking, which contains two challenging problems. The first one is that the swarm stability of the whole swarm systems should be analyzed in the connected communication time units and the disconnected communication time units, respectively. In this case, the stability analysis methods in [20–23] are invalid, and the divergent property of the whole system is tackled in the disconnected communication time units by proposing the new method. The second one is that the impact of the intermittent communications should be considered for the guaranteed cost performance analysis. In this sense, the performance index becomes a piecewise continuous integral function and is difficult to be addressed when deriving the main results of the current paper.

The rest of the current paper is shown in the following sections orderly. Section 2 formulates the problem model where the basic concepts of the communication topology are introduced and the formation tracking control protocol is proposed. In Section 3, guaranteed cost time-varying formation tracking design and analysis criteria are given, respectively, and the guaranteed cost is determined. Section 4 presents a numerical simulation to illustrate the correctness of the proposed theorems. In Section 5, the main results of the current paper are summarized. Throughout the whole paper, $\mathbb{N}$ and $\mathbb{N}^{+}$ are used to stand for the natural numbers and positive natural numbers, respectively. $\mathbb{R}$ represents the real matrices with proper dimensions. $\otimes$ is the Kronecker product, and $*$ is the symmetric terms in the matrix. The positive definite and symmetric matrix is denoted by $P^{T} = P > 0$.

## 2. Problem Description

*2.1. Model the Switching Communication Topology.* Each communication topology in the switching topology set, $G_a = \{\mathcal{G}_a^1, \mathcal{G}_a^2, \ldots, \mathcal{G}_a^j\}$ $(j \geq 2)$ is modeled as the directed graph $\mathcal{G}_a$, where the node set is represented by $\mathcal{G} = \{n_1, n_2, \ldots, n_N\}$, and the edge set is denoted by $\mathcal{E} = \{(n_m, n_k)\colon n_m, n_k \in \mathcal{G}\}$. Let $\kappa(t)\colon [0, +\infty) \longrightarrow \{1, 2, \ldots, j\}$ be the switching signal; then, the edge weighting $a_{mk}^{\kappa(t)}$ is positive if the edge $(n_k, n_m), n_m, n_k \in \mathcal{G}$, exists from $n_k$ to $n_m$, and $a_{mk}^{\kappa(t)}$ is zero otherwise. Define the neighboring set and the indegree of the node $n_m$ as $N_m^{\kappa(t)} = \{n_k \in \mathcal{G}\colon (n_k, n_m) \in \mathcal{E}\}$ and $d_{mm}^{\kappa(t)} = \deg(n_m) = \sum_{k \in N_m^{\kappa(t)}} a_{mk}^{\kappa(t)}$, respectively. The Laplacian matrix is defined as $L^{\kappa(t)} = [l_{mk}^{\kappa(t)}]_{N \times N}$ with $l_{mk}^{\kappa(t)} = d_{mm}^{\kappa(t)} - a_{mk}^{\kappa(t)}$. Note that the switching instants $t^{s_i}$ $(s_i \in \mathbb{N})$ of the switching topology set should satisfy $t^{s_i+1} - t^{s_i} > T_d > 0$ with $T_d$ the dwell time. It is assumed that there is no self-loop for all nodes. A directed path from node $n_m$ to node $n_k$ is a sequence of edges in the form of $\{(n_m, n_i), (n_i, n_j), \ldots, (n_s, n_k)\}$. The directed graph is said to have a spanning tree if a directed path exists from the root node to any other nodes. To address the formation tracking problems, it is supposed that each communication topology in the switching topology set has a spanning tree with the leader locating at the root node. More details for the basic concept of graph theory can be found in [34].

*2.2. Design the Intermittent Formation Tracking Protocol.* Consider a group of $N$ agents, where agent 1 is the leader, and the other $N-1$ agents are the homogenous followers. The dynamics model of the leader-follower swarm system is described as follows:

$$\begin{cases} \dot{x}_1(t) = Ax_1(t), \\ \dot{x}_m(t) = Ax_m(t) + Bu_m(t), \end{cases} \tag{1}$$

where $m \in \{2, 3, \ldots, N\}$, $A \in \mathbb{R}^{p \times p}$, $B \in \mathbb{R}^{p \times q}$, $x_m(t)$ is the state, and $u_m(t)$ is the control signal of agent $m$. Notice that the leader lies on the root node of the spanning tree and receives no information from followers. It is supposed that the leader's control input is zero, and the communication between neighboring followers is undirected.

Since the swarm system (1) is subjected to the intermittent communications and the switching topologies, their effects should be analyzed before moving on. It is assumed that a sequence of uniformly bounded time units $[t_{2i}, t_{2i+2}) = [t_{2i}, t_{2i+1}) \cup [t_{2i+1}, t_{2i+2})$ $(i \in \mathbb{N})$ exists such that $t_{2i} = t_{2i}^1 < t_{2i}^2 < \cdots < t_{2i}^{s_i} = t_{2i+1} < t_{2i+1} + \sigma_i = t_{2i+2}$, where $s_i$ and $\sigma_i$ are integers. Let $t_0 = 0$ and $0 < \theta_{\min} \leq \theta_i = t_{2i+2} - t_{2i} \leq \theta_{\max}$. Define the communication failure rate as $\varepsilon_i = (t_{2i+2} - t_{2i+1})/(t_{2i+2} - t_{2i})$ with $0 < \varepsilon_i \leq \varepsilon_{\max} < 1$. In general, one can find that the communication channel between neighboring agents is smooth, and the communication topology may be switched in time units $[t_{2i}, t_{2i+1})$, but all the communication channels will be disappeared in time units $[t_{2i+1}, t_{2i+2})$. Hence, the communication is intermittent for the swarm system (1). Moreover, it should be pointed out that $0 < T_d \leq t_{2i}^{s_i} - t_{2i}^{s_{i-1}} \leq \overline{T}_d$, where $\overline{T}_d$ is bounded and $T_d$ is called the minimum dwell time, and the communication topology switches at time instant $t_{2i}^{s_i}$.

Let a piecewise continuous differentiable function $h_m(t)$ $(m = 2, 3, \ldots, N)$ represent the expected time-varying formation structure formed by the followers; then, a new guaranteed cost formation tracking control protocol with the associated performance index is proposed as

$$\begin{cases} u_m(t) = \begin{cases} K_l \displaystyle\sum_{k \in N_m^{\kappa(t)}, k \neq 1} a_{mk}^{\kappa(t)} (x_k(t) - h_k(t) - x_m(t) + h_m(t)) \\ + K_l a_{m1}^{\kappa(t)} (x_1(t) - x_m(t) + h_m(t)), & t \in [t_{2i}, t_{2i+1}), \\ 0, & t \in [t_{2i+1}, t_{2i+2}), \end{cases} \\ J_c = \displaystyle\sum_{i=0}^{+\infty} \left( \int_{t_{2i}}^{t_{2i+1}} (J_h(t) + J_u(t)) \mathrm{d}t + \int_{t_{2i+1}}^{t_{2i+2}} J_h(t) \mathrm{d}t \right), \end{cases} \tag{2}$$

where $m \in \{2, 3, \ldots, N\}$,

$$\begin{aligned} J_h(t) &= \sum_{m=1}^{N} \sum_{k \in N_m^{\kappa(t)}} a_{mk}^{\delta(t)} (x_k(t) - h_k(t) - x_m(t) + h_m(t))^T Q (x_k(t) - h_k(t) - x_m(t) + h_m(t)), \\ J_u(t) &= \sum_{m=2}^{N} u_m^T(t) R u_m(t). \end{aligned} \tag{3}$$

$i \in \mathbb{N}$, $Q = Q^T > 0$, $R = R^T > 0$, and $K_l$ is denoting the control gain matrix. $J_c$ is the performance index representing the total of the formation regulation performance and the control energy expenditure.

For swarm systems subjected to intermittent communications and switching topologies, the definition of the guaranteed cost formation tracking control is given as follows.

*Definition 1.* For any given bounded initial states $x_m(0) - h_m(0)$ $(m = 2, 3, \cdots, N)$, the swarm system (1) is said to be guaranteed cost formation tracking achievable if there exists a gain matrix $K_l$ such that $\lim_{t \longrightarrow +\infty} (x_m(t) - h_m(t) - x_1(t)) = 0$ $(m = 2, 3, \cdots, N)$ and $J_c \leq C_{\text{ost}}$, where $C_{\text{ost}}$ is called the guaranteed cost.

The current paper mainly focuses on the guaranteed cost formation tracking design problems, in which the gain matrix is designed, and the guaranteed cost is determined. Moreover, for the given gain matrix, the guaranteed cost formation tracking analysis criterion is derived.

*Remark 1.* Protocol (2) consists of two parts. The first one is the intermittent control input, which is constructed by the state and formation errors between neighboring followers and those between the leader and the followers over the time units $[t_{2i}, t_{2i+1})$ and is set as zero in the time units $[t_{2i+1}, t_{2i+2})$, $i \in \mathbb{N}$. With the intermittent control input and the switching neighboring sets and edge weightings, protocol (2) is piecewise continuous, which will lead to the piecewise continuous right hand sides of the closed-loop system in the system stability analysis and is challenging to be dealt with. The second one is the performance index, which describes the total cost of the guaranteed cost formation tracking design. The weighting matrices $Q$ and $R$ represent the proportion of the formation regulation performance and the control energy expenditure in the performance index, respectively, which will be taken into consideration in the gain matrix design. Note that the performance index $J_c$ is a piecewise continuous integral function due to the intermittent control input. Moreover, different from the guaranteed cost consensus, the guaranteed cost formation tracking can drive the swarm system to form an expected formation structure, while the guaranteed cost can be satisfied. Note that the expected formation structure can be time-varying and can be designed as much as required if it can satisfy the formation feasibility condition as shown in Theorem 1 in the following content.

## 3. Main Results

In this section, first, the formation tracking problem of the swarm system (1) is converted to the asymptotical stability problem of a reduced-order subsystem via nonsingular transformation. Then, guaranteed cost formation tracking design and analysis criteria are derived, and the guaranteed cost is determined to show the upper bound of the performance index.

Denote $\varphi_m(t) = x_m(t) - h_m(t)$, and one can obtain from (1) and (2) that

$$\dot{\varphi}_m(t) = \begin{cases} A(\varphi_m(t) + h_m(t)) + BK_l \sum_{k \in N_m^{\kappa(t)}} a_{mk}^{\kappa(t)} (\varphi_k(t) - \varphi_m(t)) & t \in [t_{2i}, t_{2i+1}), \\ +BK_l a_{m1}^{\kappa(t)} (x_1(t) - \varphi_m(t)) - \dot{h}_m(t), & \\ A(\varphi_m(t) + h_m(t)) - \dot{h}_m(t), & t \in [t_{2i+1}, t_{2i+2}). \end{cases} \quad (4)$$

Since no formation is required to be formed by the leader, one can define the auxiliary variable $h_1(t) \equiv 0$ and set $\varphi_1(t) = x_1(t) - h_1(t)$. Let $\varphi(t) = [\varphi_1^T(t), \varphi_2^T(t), \cdots, \varphi_N^T(t)]^T$, $x(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]^T$, and $h(t) = [h_1^T(t), h_2^T(t), \ldots, h_N^T(t)]^T$, then equation (4) can be represented by the compact form as

$$\dot{\varphi}(t) = \begin{cases} (I_N \otimes A)(\varphi(t) + h(t)) - \left(L^{\kappa(t)} \otimes BK_l\right)\varphi(t) - \left(I_N \otimes I_p\right)\dot{h}(t), & t \in [t_{2i}, t_{2i+1}), \\ (I_N \otimes A)(\varphi(t) + h(t)) - \left(I_N \otimes I_p\right)\dot{h}(t), & t \in [t_{2i+1}, t_{2i+2}), \end{cases} \quad (5)$$

where the structure of $L^{\kappa(t)}$ is shown as follows:

$$L^{\kappa(t)} = \begin{bmatrix} 0 & 0 \\ L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)} & -\Gamma_l^{\kappa(t)} \end{bmatrix},$$
$$\Lambda_l^{\kappa(t)} = \operatorname{diag}\{a_{21}^{\kappa(t)}, a_{31}^{\kappa(t)}, \dots, a_{N1}^{\kappa(t)}\}, \tag{6}$$
$$\Gamma_l^{\kappa(t)} = \left[a_{21}^{\kappa(t)}, a_{31}^{\kappa(t)}, \dots, a_{N1}^{\kappa(t)}\right]^T,$$

and $L_f^{\kappa(t)}$ represents the Laplacian matrix of followers.

In the sequel, by nonsingular transformation, the closed-loop system (5) will be decomposed into two subsystems. First, define the following nonsingular matrix:

$$U^{\kappa(t)} = \begin{bmatrix} 1 & 0 \\ \mathbf{1}_{N-1} & I_{N-1} \end{bmatrix}, \tag{7}$$

such that

$$\left(\left(U^{\kappa(t)}\right)^{-1} \otimes I_p\right)\varphi(t) = \left[x_1^T(t), \widetilde{\varphi}_2^T(t), \dots, \widetilde{\varphi}_N^T(t)\right]^T, \tag{8}$$

with $\widetilde{\varphi}_m(t) = \varphi_m(t) - x_1(t)$, $m = 2, 3, \dots, N$, and

$$\left(U^{\kappa(t)}\right)^{-1} L^{\kappa(t)} U^{\kappa(t)} = \begin{bmatrix} 0 & 0 \\ 0 & L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)} \end{bmatrix}, \tag{9}$$

where the fact $\Lambda_l^{\kappa(t)}\mathbf{1}_{N-1} = \Gamma_l^{\kappa(t)}$ is utilized.

Then, the block $L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)}$ is diagonalized. For each communication topology in the switching topology set, since there at least exists a spanning tree with the leader locating at the root node and the communication channels among followers are undirected and connected, the eigenvalue 0 of $L_f^{\kappa(t)}$ is simple and the block $L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)}$ is positive definite and symmetric. In this sense, there exists an orthonormal matrix $W^{\kappa(t)}$ such that

$$\left(W^{\kappa(t)}\right)^T\left(L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)}\right)W^{\kappa(t)} = D_f^{\kappa(t)} = \operatorname{diag}\{\lambda_2^{\kappa(t)}, \lambda_3^{\kappa(t)}, \dots, \lambda_N^{\kappa(t)}\}, \tag{10}$$

where $\lambda_m^{\kappa(t)}$ $(m = 2, 3, \cdots, N)$ are the eigenvalues of $L_f^{\kappa(t)}$ with the order $0 < \lambda_2^{\kappa(t)} \le \lambda_3^{\kappa(t)} \le \cdots \le \lambda_N^{\kappa(t)}$. Denote $\widetilde{\varphi}(t) = [\widetilde{\varphi}_2^T(t), \widetilde{\varphi}_3^T(t), \dots, \widetilde{\varphi}_N^T(t)]^T$ and $((W^{\kappa(t)})^T \otimes I_p)\widetilde{\varphi}(t) = \phi(t) = [\phi_2^T(t), \phi_3^T(t), \dots, \phi_N^T(t)]^T$; then, equation (5)fd5 is converted to the following two subdynamics:

$$\dot{x}_1(t) = Ax_1(t), \tag{11}$$

$$\dot{\phi}(t) = \begin{cases} (I_{N-1} \otimes A)\left(\phi(t) + \left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes I_p\right)h(t)\right) \\ -\left(D_f^{\kappa(t)} \otimes BK_l\right)\phi(t) - \left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes I_p\right)\dot{h}(t), & t \in [t_{2i}, t_{2i+1}), \\ (I_{N-1} \otimes A)\left(\phi(t) + \left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes I_p\right)h(t)\right) \\ -\left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes I_p\right)\dot{h}(t), & t \in [t_{2i+1}, t_{2i+2}). \end{cases} \tag{12}$$

Because $U^{\kappa(t)}$ is nonsingular and $W^{\kappa(t)}$ is orthonormal, one can derive that the swarm system (1) with control protocol (2) achieves the time-varying formation tracking if subdynamics (12) is asymptotical stable; that is, $\lim_{t \longrightarrow +\infty}\phi(t) = 0$.

Let $\lambda_{\min} = \min\{\lambda_2^s : \forall s \in \{1, 2, \cdots, j\}\}$ and $\lambda_{\max} = \max\{\lambda_N^s : \forall s \in \{1, 2, \dots, j\}\}$; then, the following theorem gives the guaranteed cost formation tracking criterion for the swarm system (1) with protocol (2).

**Theorem 1.** *Swarm system (1) is guaranteed cost formation tracking achievable by protocol (2) with* $K_l = \lambda_{min}^{-1}\gamma B^T P^{-1}/2$, *if* $\mu(1 - \varepsilon_{max}) > \varpi\varepsilon_{max}e^{\varpi\varepsilon_{max}\theta_{max}}$, $\dot{h}_m(t) = Ah_m(t)$, $m = 2, 3, \dots, N$, *and there exist* $\gamma > 0$ *and* $P = P^T > 0$ *such that*

$$\Theta_\varpi = \begin{bmatrix} AP + PA^T - \varpi P & 2\lambda_{\max}PQ \\ * & -2\lambda_{\max}Q \end{bmatrix} < 0,$$
$$\Theta_\mu = \begin{bmatrix} AP + PA^T + \mu P - \gamma BB^T & 2\lambda_{\max}PQ & \lambda_{\max}\lambda_{\min}^{-1}\gamma BR/2 \\ * & -2\lambda_{\max}Q & 0 \\ * & * & -R \end{bmatrix} < 0. \tag{13}$$

*In this case, the guaranteed cost is*

$$C_{\text{ost}} = (x(0) - h(0))^T\left(\begin{bmatrix} N-1 & -\mathbf{1}_{N-1}^T \\ -\mathbf{1}_{N-1} & I_{N-1} \end{bmatrix} \otimes P^{-1}\right)(x(0) - h(0)). \tag{14}$$

*Proof.* Construct Lyapunov functional candidate as follows:

$$V(t) = \phi^T(t)\left(I_{N-1} \otimes P^{-1}\right)\phi(t). \quad (15)$$

For $t \in [t_{2i}, t_{2i+1})$, $i \in \mathbb{N}$, taking the time derivate of $V(t)$ with respect to the trajectories of equation (12) gives

$$\begin{aligned} V(t) = {} & \phi^T(t)\left(I_{N-1} \otimes \left(P^{-1}A + A^T P^{-1}\right) - D_f^{\kappa(t)} \otimes \left(P^{-1}BK_u + K_u^T B^T P^{-1}\right)\right)\phi(t) \\ & + 2\phi^T(t)\left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes P^{-1}A\right)h(t) \\ & - 2\phi^T(t)\left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes P^{-1}\right)\dot{h}(t). \end{aligned} \quad (16)$$

Define an auxiliary variable

$$\Xi(t) = \left[\left(Ah_2(t) - \dot{h}_2(t)\right)^T, \left(Ah_3(t) - \dot{h}_3(t)\right)^T, \cdots, \left(Ah_N(t) - \dot{h}_N(t)\right)^T\right]^T. \quad (17)$$

Then, one can find that $\Xi(t) = 0$, if $\dot{h}_m(t) = Ah_m(t)$, $m = 2, 3, \cdots, N$. Based on the above fact, it can be obtained that

$$V(t) = \phi^T(t)\left(I_{N-1} \otimes \left(P^{-1}A + A^T P^{-1}\right) - D_f^{\kappa(t)} \otimes \left(P^{-1}BK_u + K_u^T B^T P^{-1}\right)\right)\phi(t). \quad (18)$$

Let $K_u = \lambda_{\min}^{-1}\gamma B^T P^{-1}/2$; then, one can show that

$$\dot{V}(t) + \mu V(t) = \sum_{m=2}^{N} \phi_i^T(t)\left(P^{-1}A + A^T P^{-1} + \mu P^{-1} - \lambda_m^{\kappa(t)}\lambda_{\min}^{-1}\gamma P^{-1}BB^T P^{-1}\right)\phi_i(t). \quad (19)$$

It can be derived by pre- and postmultiplying $AP + PA^T + \mu P - \gamma BB^T < 0$ with $P^{-1}$ that

$$P^{-1}A + A^T P^{-1} + \mu P^{-1} - \gamma P^{-1}BB^T P^{-1} < 0. \quad (20)$$

Since $\lambda_m^{\kappa(t)}\lambda_{\min}^{-1} \geq 1$, $m = 2, 3, \cdots, N$, one can deduce that

$$\dot{V}(t) < -\mu V(t). \quad (21)$$

For $t \in [t_{2i+1}, t_{2i+2})$, $i \in \mathbb{N}$, taking the time derivate of $V(t)$ along equation (12) yields

$$\begin{aligned} V(t) = {} & \phi^T(t)\left(I_{N-1} \otimes \left(P^{-1}A + A^T P^{-1}\right)\right)\phi(t) \\ & + 2\phi^T(t)\left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes P^{-1}A\right)h(t) \\ & - 2\phi^T(t)\left(\left(W^{\kappa(t)}\right)^T [0, I_{N-1}]\left(U^{\kappa(t)}\right)^{-1} \otimes P^{-1}\right)\dot{h}(t). \end{aligned} \quad (22)$$

Due to $\dot{h}_m(t) = Ah_m(t)$, $m = 2, 3, \cdots, N$, one can derive by similar analysis that

$$\dot{V}(t) - \varpi V(t) = \sum_{m=2}^{N} \phi_i^T(t)\left(P^{-1}A + A^T P^{-1} - \varpi P^{-1}\right)\phi_i(t). \quad (23)$$

Note that $P^{-1}A + A^T P^{-1} - \varpi P^{-1} < 0$ can be obtained by pre- and postmultiplying $AP + PA^T - \varpi P < 0$ with $P^{-1}$; then, it holds that

$$\dot{V}(t) < \varpi V(t). \quad (24)$$

For $t \in [t_0, t_2)$, i.e., $i = 0$, one has

$$V(t_2) < e^{\varpi(t_2 - t_1)}V(t_1) < e^{\varpi(t_2 - t_1)}e^{-\mu(t_1 - t_0)}V(t_0) = e^{-(\mu - (\mu + \varpi)\varepsilon_0)\theta_0}V(0). \quad (25)$$

In virtue of $\mu(1 - \varepsilon_{\max}) > \varpi\varepsilon_{\max}e^{\varpi\varepsilon_{\max}\theta_{\max}}$ and the fact that $e^{\varpi\varepsilon_{\max}\theta_{\max}} > 1$, it can be deduced that $-(\mu - (\mu + \varpi)\varepsilon_0)\theta_0 < 0$. Then, one can obtain for $\forall i \in \mathbb{N}$ that

$$V(t_{r+1}) < V(0)e^{-\sum_{d=0}^{r}(\mu - (\mu + \varpi)\varepsilon_d)\theta_d}. \quad (26)$$

Hence, one can show that for $\forall t > 0$, there exists a $\nu \in \mathbb{N}^+$ such that $t_{2k} < t \le t_{2k+2}$. In this case, one has

$$V(t) \le e^{\varpi\theta_{\max}} V(t_\nu) \le e^{\varpi\theta_{\max}} V(0) e^{-\sum_{s=0}^{\nu-1}\varphi_s} \le e^{\varpi\theta_{\max}} V(0) e^{-\nu(\mu-(\mu+\varpi)\varepsilon_{\max})\theta_{\min}} \le e^{\varpi\theta_{\max}} V(0) e^{-(((\mu-(\mu+\varpi)\varepsilon_{\max})\theta_{\min})/\theta_{\max})t}. \quad (27)$$

From (27), it can be concluded that subdynamics (12) is asymptotical stable; that is, $\lim_{t\longrightarrow+\infty}\phi(t) = 0$. Therefore, one can find that the formation tracking can be achieved for the swarm system (1) with protocol (2).

In the next content, the guaranteed cost formation tracking achievability is discussed with the performance index $J_c$. From (2), one can deduce that

$$\begin{aligned} J_h(t) &= 2\widetilde{\varphi}^T(t)\left(\left(L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)}\right)\otimes Q\right)\widetilde{\varphi}(t), \\ J_u(t) &= \widetilde{\varphi}^T(t)\left(\left(L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)}\right)^2 \otimes K_l^T R K_l\right)\widetilde{\varphi}(t). \end{aligned} \quad (28)$$

Due to $\phi(t) = ((W^{\kappa(t)})^T \otimes I_p)\widetilde{\varphi}(t)$ and $K_u = \lambda_{\min}^{-1}\gamma B^T P^{-1}/2$, one can obtain that

$$J_c \le \sum_{i=0}^{+\infty}\sum_{m=2}^{N}\int_{t_{2i}}^{t_{2i+1}} \frac{1}{4}\lambda_{\max}^2\lambda_{\min}^{-2}\gamma^2\phi_m^T(t)\left(P^{-1}BRB^TP^{-1}\right)\phi_m(t)\mathrm{d}t + \sum_{i=0}^{+\infty}\sum_{m=2}^{N}\int_{t_{2i}}^{t_{2i+2}} 2\overline{\lambda}_{\max}\phi_m^T(t)Q\phi_m(t)\mathrm{d}t. \quad (29)$$

According to (19), (23), and (29), it can be derived that

$$\begin{aligned} J_c \le{}& \sum_{i=0}^{+\infty}\sum_{m=2}^{N}\int_{t_{2i}}^{t_{2i+1}} \frac{1}{4}\lambda_{\max}^2\lambda_{\min}^{-2}\gamma^2\phi_m^T(t)\left(P^{-1}BRB^TP^{-1}\right)\phi_m(t)\mathrm{d}t \\ &+ \sum_{i=0}^{+\infty}\sum_{m=2}^{N}\int_{t_{2i}}^{t_{2i+1}} \phi_m^T(t)\left(P^{-1}A + A^TP^{-1} + \mu P^{-1} - \lambda_m^{\kappa(t)}\lambda_{\min}^{-1}\gamma P^{-1}BB^TP^{-1}\right)\phi_m(t)\mathrm{d}t \\ &+ \sum_{i=0}^{+\infty}\sum_{m=2}^{N}\int_{t_{2i+1}}^{t_{2i+2}} \phi_m^T(t)\left(P^{-1}A + A^TP^{-1} - \varpi P^{-1}\right)\phi_m(t) \\ &+ \sum_{i=0}^{+\infty}\sum_{m=2}^{N}\int_{t_{2i}}^{t_{2i+2}} 2\lambda_{\max}\phi_m^T(t)Q\phi_m(t)\mathrm{d}t \\ &- \sum_{i=0}^{+\infty}\left(\int_{t_{2i}}^{t_{2i+1}}(\dot{V}(t) + \mu V(t))\mathrm{d}t + \int_{t_{2i+1}}^{t_{2i+2}}(\dot{V}(t) - \varpi V(t))\mathrm{d}t\right). \end{aligned} \quad (30)$$

By $\lambda_m^{\kappa(t)}\lambda_{\min}^{-1} \ge 1$, $\Theta_\varpi < 0$, $\Theta_\mu < 0$, and Schur complement, it holds as $i \longrightarrow +\infty$ that

$$J_c \le V(0) - \sum_{i=0}^{+\infty}\left(\int_{t_{2i}}^{t_{2i+1}}\mu V(t)\mathrm{d}t - \int_{t_{2i+1}}^{t_{2i+2}}\varpi V(t)\mathrm{d}t\right). \quad (31)$$

Utilizing the mean value theorem of integrals gives

$$\begin{aligned} J_c &\le V(0) - \sum_{i=0}^{+\infty}\left(\mu V(t_{2i+1})(t_{2i+1} - t_{2i}) - \varpi V(t_{2i+2})(t_{2i+2} - t_{2i+1})\right) \\ &= V(0) - \sum_{i=0}^{+\infty}\left(\mu(1-\varepsilon_i)V(t_{2i+1}) - \varpi\varepsilon_i V(t_{2i+2})\right)\theta_i \\ &\le V(0) - \sum_{i=0}^{+\infty}\left(\mu(1-\varepsilon_i) - \varpi\varepsilon_i e^{\varpi\varepsilon_i\theta_i}\right)V(t_{2i+1})\theta_i. \end{aligned} \quad (32)$$

By $\mu(1-\varepsilon_{\max}) > \varpi\varepsilon_{\max}e^{\varpi\varepsilon_{\max}\theta_{\max}}$, one can deduce that

$$J_c \le V(0) = \phi^T(0)\left(I_{N-1}\otimes P^{-1}\right)\phi(0). \quad (33)$$

Since $\phi(0) = ((W^{\kappa(t)})^T \otimes I_p)\widetilde{\varphi}(0)$, one can find that

$$\phi^T(0)\left(I_{N-1}\otimes P^{-1}\right)\phi(0) = \widetilde{\varphi}^T(0)\left(W^{\kappa(t)}\left(W^{\kappa(t)}\right)^T \otimes P^{-1}\right)\widetilde{\varphi}(0). \quad (34)$$

Then, due to $\widetilde{\varphi}(0) = ([0, I_{N-1}](U^{\kappa(t)})^{-1} \otimes I_p)\varphi(0)$, it follows from (33) that

$$C_{\mathrm{ost}} = V(0) = \varphi^T(0)\left(\begin{bmatrix} N-1 & -\mathbf{1}_{N-1}^T \\ -\mathbf{1}_{N-1} & I_{N-1}\end{bmatrix}\otimes P^{-1}\right)\varphi(0). \quad (35)$$

This completes the proof of Theorem 1. □

*Remark 2.* Note that the condition $\dot{h}_m(t) = Ah_m(t)$ in Theorem 1 is called the formation feasibility condition, which indicates whether an expected formation is feasible or not to be achieved by swarm systems. It should be pointed out that not all formation can be achieved due to the dynamic constraint of the agent. For time-varying formation, the formation function derivate $\dot{h}_m(t)$ affects the feasibility of the formation, whose constraint is shown in the condition $\dot{h}_m(t) = Ah_m(t)$. It can be found that the condition is associated with the dynamic matrix $A$ of each agent. However, if $\dot{h}_m(t) \equiv 0$, which means that the formation is time-invariant, then the formation feasibility becomes $Ah_m = 0$, which can be found in [19].

*Remark 3.* Due to the jointed effect of the intermittent communication and the switching topology, the right hand side of the closed-loop system becomes piecewise continuous. Besides, from the proof of Theorem 1, it can be found that the system stability analysis is divided into two parts due to the jointed effect of the intermittent communication and the switching topology. On the one hand, for time units $[t_{2i}, t_{2i+1})$, $i \in \mathbb{N}$, it can be concluded that the Lyapunov functional candidate is decreased exponentially by a rate faster than $\mu$. On the other hand, the value of the Lyapunov functional candidate may be increased along a rate less than $\varpi$ in time units $[t_{2i+1}, t_{2i+2})$. By combining these two aspects of the stability analysis, it can be shown that the Lyapunov functional candidate converges with the rate $(\mu - (\mu + \varpi)\varepsilon_{\max})\theta_{\min}/\theta_{\max}$ exponentially according to the condition $\mu(1 - \varepsilon_{\max}) > \varpi\varepsilon_{\max}e^{\varpi\varepsilon_{\max}\theta_{\max}}$. Note that if the guaranteed cost performance is not considered, then the condition $\mu(1 - \varepsilon_{\max}) > \varpi\varepsilon_{\max}$ can guarantee the stability of subdynamics (12). The condition $\mu(1 - \varepsilon_{\max}) > \varpi\varepsilon_{\max}e^{\varpi\varepsilon_{\max}\theta_{\max}}$ ensures that the performance index $J_c$ can be upper bounded by the guaranteed cost $C_{ost}$. Generally speaking, the condition $\mu(1 - \varepsilon_{\max}) > \varpi\varepsilon_{\max}e^{\varpi\varepsilon_{\max}\theta_{\max}}$ can always guarantee $\mu(1 - \varepsilon_{\max}) > \varpi\varepsilon_{\max}$ since $e^{\varpi\varepsilon_{\max}\theta_{\max}} > 1$ for positive $\varpi$, $\varepsilon_{\max}$, and $\theta_{\max}$.

Theorem 1 provides the criterion of the guaranteed cost formation tracking design where the gain matrix $K_l$ is determined. However, if $K_l$ is given, then it is interesting to analyze whether $K_l$ is feasible to solve the guaranteed cost formation tracking problems. Set $\overline{P} = P^{-1}$ and use the convex property of linear matrix inequalities, then the following theorem gives the sufficient conditions of the guaranteed cost formation tracking analysis for given $K_l$.

**Theorem 2.** *For any given $K_l$, the swarm system (1) with protocol (2) achieves guaranteed cost formation tracking if $K_u = \lambda_{\min}^{-1}\gamma B^T P^{-1}/2$ $\dot{h}_m(t) = Ah_m(t)$, $m = 2, 3, \cdots, N$, and there exists a matrix $\overline{P} = \overline{P}^T > 0$ such that*

$$
\begin{gathered}
\overline{P}A + A^T\overline{P} - \varpi\overline{P} + 2\lambda_{\max}Q < 0,\\
\begin{bmatrix} \overline{P}A + A^T\overline{P} + \mu\overline{P} - \lambda_{\min}\left(\overline{P}BK_l + K_l^T B^T\overline{P}\right) & 2\lambda_{\min}Q & \lambda_{\min}K_l^T R \\ * & -2\lambda_{\min}Q & 0 \\ * & * & -R \end{bmatrix} < 0,\\
\begin{bmatrix} \overline{P}A + A^T\overline{P} + \mu\overline{P} - \lambda_{\max}\left(\overline{P}BK_l + K_l^T B^T\overline{P}\right) & 2\lambda_{\max}Q & \lambda_{\max}K_l^T R \\ * & -2\lambda_{\max}Q & 0 \\ * & * & -R \end{bmatrix} < 0.
\end{gathered} \tag{36}
$$

*In this case, the guaranteed cost is*

$$
C_{\text{ost}} = (x(0) - h(0))^T\left(\begin{bmatrix} N-1 & -\mathbf{1}_{N-1}^T \\ -\mathbf{1}_{N-1} & I_{N-1} \end{bmatrix} \otimes P^{-1}\right)(x(0) - h(0)). \tag{37}
$$

*Remark 4.* The formation design in [20–23] only took care about how to design a proper gain matrix such that the expected formation can be achieved, but they did not consider the guaranteed cost performance when designing the formation control protocol. In contrast, the current paper constructs a performance index to describe the total cost, where the weighting matrices between the formation regulation performance and the control energy expenditure are denoted by $Q$ and $R$. In this case, weighting matrices $Q$ and $R$ are introduced into the design procedure of the gain matrix, which can assure that not only the formation tracking can be achieved but also the performance index can be constrained by the guaranteed cost. By adjusting the relative value of $Q$ and $R$, the compromise design between the formation regulation performance and the control energy expenditure can be achieved. Moreover, the guaranteed costs obtained in Theorems 1 and 2 are associated with the initial states and formations and the interaction matrix. Note that the initial states and formations are often available in applications and the interaction matrix is related to a time-invariant star graph, which can be obtained when the number of agents is determined. Furthermore, in the gain matrix design, the eigenvalues $\lambda_{\min}$ and $\lambda_{\max}$ are needed, which is difficult to be calculated. Fortunately, $\lambda_{\min}$ can be obtained via the method in [35], and $\lambda_{\max}$ can be estimated by Gersgorin's disc theorem in [36].

*Remark 5.* Swarm system with the leaderless structure describes the dynamics of each agent, where each agent plays the equal role of the collaborative behavior. However, the swarm system with the leader-following structure describes the dynamics of the leader with no control input and that of the follower. Different from the formation design of leaderless swarm systems, the guaranteed cost formation tracking problem of leader-follower swarm systems owns two interesting features. First, although the communication topology among followers is undirected and connected, the Laplacian matrix of the whole system is asymmetric due to the existence of the leader. In this sense, a nonsingular transformation and an orthonormal transformation are adopted successively to diagonalize the block $L_f^{\kappa(t)} + \Lambda_l^{\kappa(t)}$ of the Laplacian matrix such that the dynamics of the closed-loop system can be linearly decoupled to solve the guaranteed cost formation tracking problem. Second, the guaranteed cost is associated with the Laplacian matrix $\begin{bmatrix} N-1 & -\mathbf{1}_{N-1}^T \\ -\mathbf{1}_{N-1} & I_{N-1} \end{bmatrix}$ of a star graph with the leader locating at the center, which indicates that the global interaction mechanism of the whole swarm system is determined by the leader for the guaranteed cost formation tracking problem. Besides, the formation tracking movement is fully determined by the state response leader.

## 4. Numerical Simulation

In this section, a simulation is given to illustrate the effectiveness of the proposed guaranteed cost time-varying formation tracking design method in the above sections.

The third-order swarm system considered in the simulation is composed with one leader labeled by 1 and five followers labeled from 2 to 6 whose dynamics is modeled as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.5 & -1 & 0.5 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}. \tag{38}$$

The switching topology set of the swarm system is shown in Figure 1, where the topology is switched among topologies $\mathcal{G}_a^1$, $\mathcal{G}_a^2$, $\mathcal{G}_a^3$, and $\mathcal{G}_a^4$ with the dwell time $T_d = 0.3$ s in the connected communication time units $t \in [0.6i, 0.6i + 0.51)$ s, $i \in \mathbb{N}$, and the communication among all agents is interrupted in the disconnected communication time units $t \in [0.6i + 0.51, 0.6(i+1))$ s. In this case, the maximum communication failure rate is $\varepsilon_{\max} = 0.15$. The initial states of the whole swarm system are given as follows:

$$\begin{aligned} x_1(0) &= [3.5, -2.7, 1.5]^T, \\ x_2(0) &= [3.5, 5.2, -1.3]^T, \\ x_3(0) &= [4.2, -2.5, 2.3]^T, \\ x_4(0) &= [-3.1, -2.5, 4.6]^T, \\ x_5(0) &= [-2.1, 4.8, -1.5]^T, \\ x_6(0) &= [-6.2, 2.4, -3.5]^T. \end{aligned} \tag{39}$$

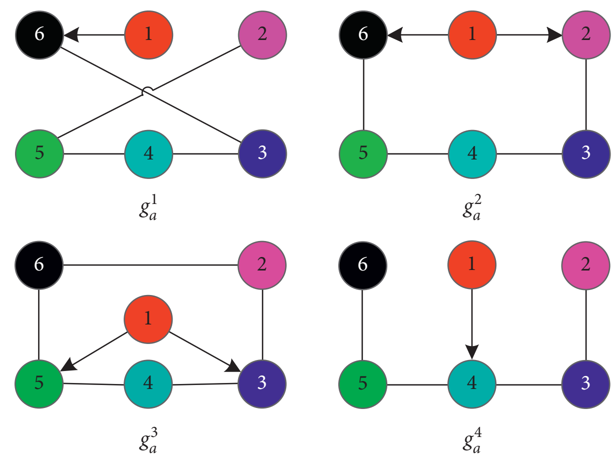



Figure 1: Switching topology set $G_a$.

The expected time-varying formation function is chosen as follows:

$$h_m(t) = \begin{bmatrix} \sin\left(t + \frac{2(m-1)\pi}{5}\right) \\ \cos\left(t + \frac{2(m-1)\pi}{5}\right) \\ -\sin\left(t + \frac{2(m-1)\pi}{5}\right) \end{bmatrix}, \quad m = 2, 3, \ldots, 6. \tag{40}$$

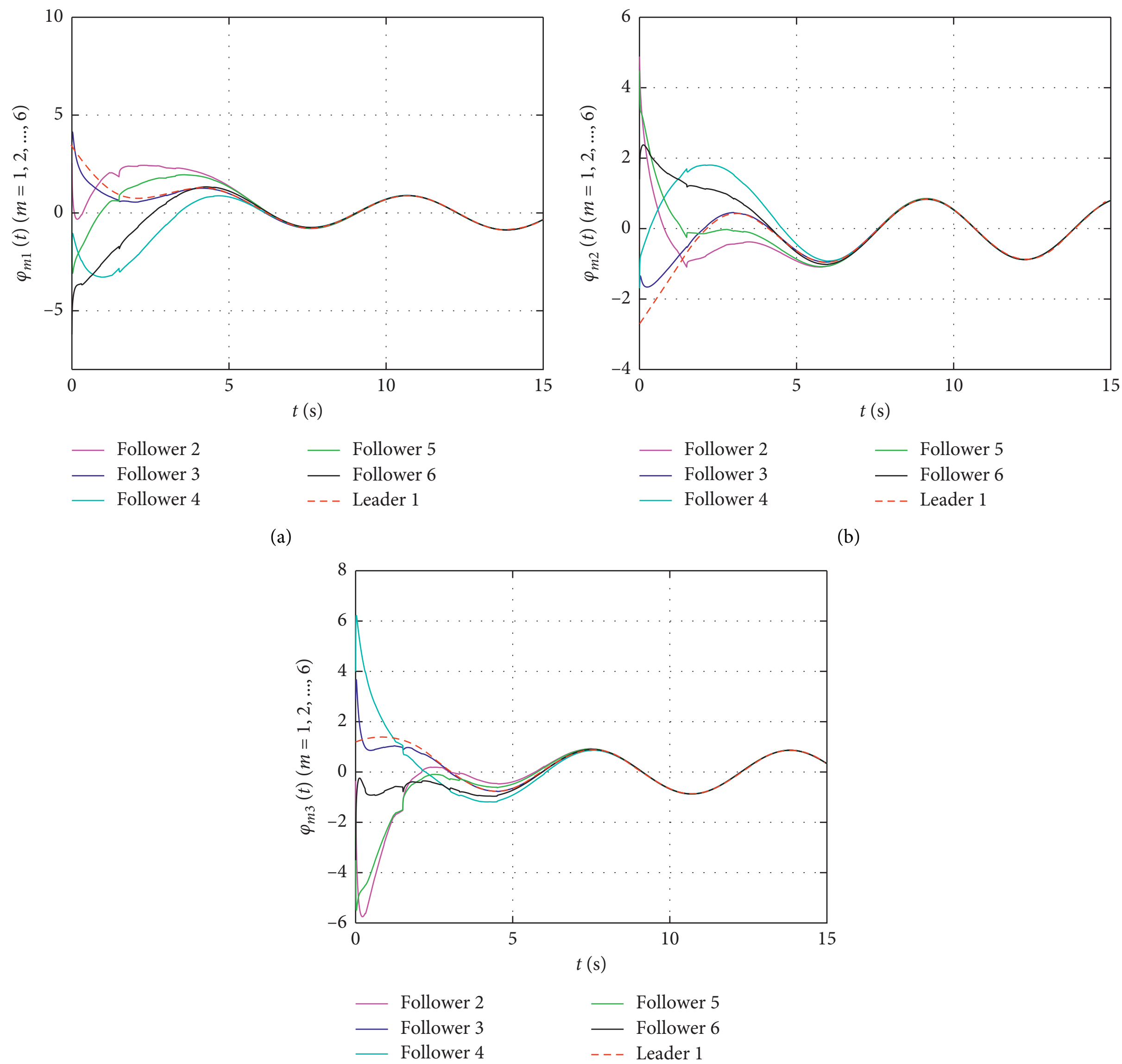
According to the above form of $h_m(t)$, it can be found that five followers should shape into a regular pentagon and keep rotating around its center. Meanwhile, the conditions $\dot{h}_m(t) = Ah_m(t)$, $(m = 2, 3, \cdots, 6)$ are satisfied. Set $\mu = 0.9$, $\varpi = 5$, $R = 0.1$, and $Q = \text{diag}\{0.3, 0.1, 0.2\}$. By Theorem 1, it can be calculated by the FEASP solver in MATLAB that $\gamma = 0.0072$ and

$$P = \begin{bmatrix} 0.1002 & -0.0646 & 0.0442 \\ -0.0646 & 0.0508 & -0.0335 \\ 0.0442 & -0.0335 & 0.0570 \end{bmatrix}. \tag{41}$$

In this case, the guaranteed cost is determined as $C_{\text{ost}} = 11171.4191$, and the gain matrix is design as

$$K_l = (7.7145, 13.5458, 4.3332). \tag{42}$$

Figure 2 depicts the error trajectory between the state and formation of each follower and the leader within 15 s, where the trajectories of followers are full curves with different colors and that of the leader is a red imaginary line. One can see from Figure 2 that $\varphi_m(t)$ $(m = 2, 3, \cdots, 6)$ of five followers converge to the same value which equals to $\varphi_1(t)$ of the leader, which means that the error state $\varphi_m(t)$ of five followers achieve consensus and track to that of the leader.

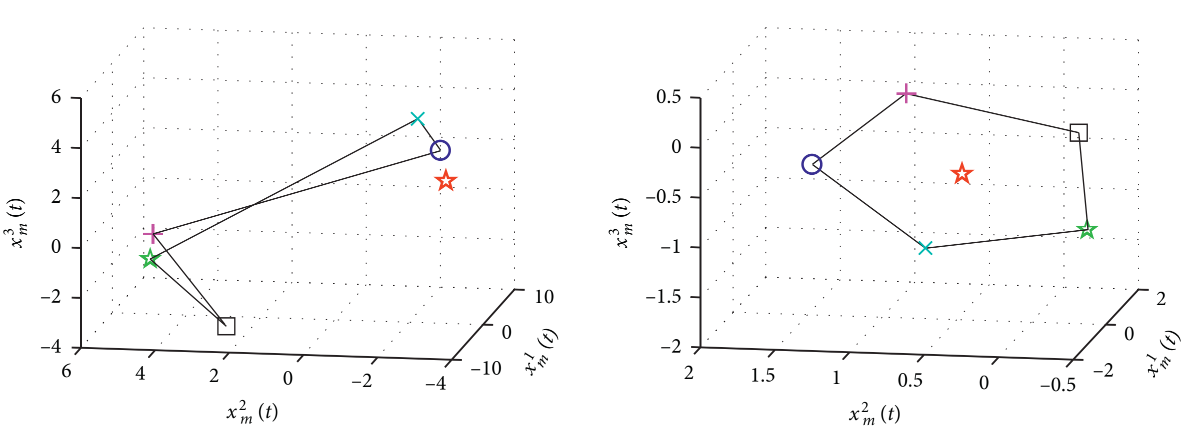



(a)

(b)

(c)

FIGURE 2: Curves of $\varphi_m(t)$ $(m = 1, 2, \ldots, 6)$. (a) $\varphi_{m1}(t)$. (b) $\varphi_{m2}(t)$. (c) $\varphi_{m3}(t)$.




(a)

(b)

FIGURE 3: Continued.

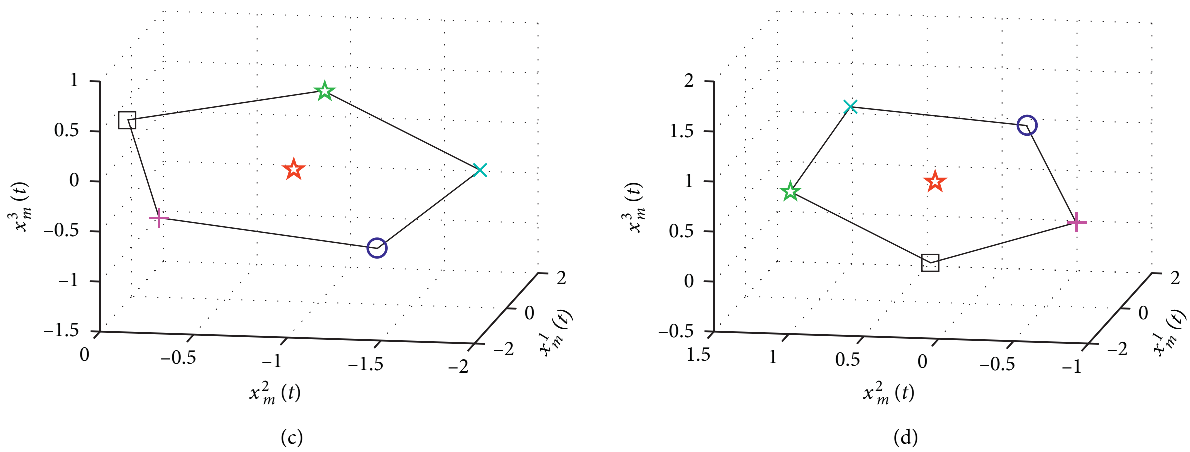



FIGURE 3: State of five followers and the leader at different moments. (a) $t = 0$ s. (b) $t = 10$ s. (c) $t = 12$ s. (d) $t = 14$ s.

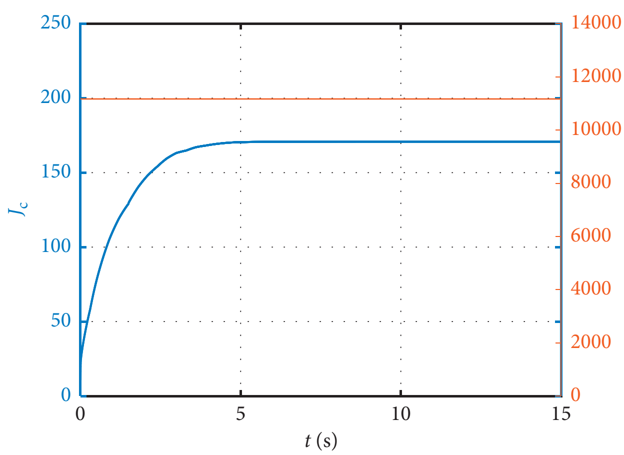



FIGURE 4: Curves of the performance index and the guaranteed cost.

The state snapshots of five followers and the leader are shown in Figure 3, where the state of the leader is described as the red pentacle and those of the five followers are depicted as pink pluses, blue circles, bluish x-marks, green pentacles, and black squares, orderly. From Figures 3(a)-3(b), it can be found that the formation of five followers is achieved with the geometrical shape of the regular pentagon, and the state of the leader locates at the center of the regular pentagon. From Figures 3(b)–3(d), one can see that the formation of five followers keeps rotating around the leader; that is, the time-varying formation tracking is achieved.

Figure 4 describes the curves of the performance index and the guaranteed cost, respectively. It can be shown that the value of the performance index increases to a finite value that is less than the guaranteed cost, i.e., $J_c \leq C_{\text{ost}}$.

From the simulation results in Figures 2–4, it can be concluded that the swarm system (1) with intermittent communications and switching topologies is guaranteed cost time-varying formation tracking achievable by protocol (2).

## 5. Conclusions

Guaranteed cost time-varying formation tracking design and analysis problems were studied for the swarm system with intermittent communications and switching topologies. An intermittent guaranteed cost formation tracking control protocol was constructed, which consisted of an intermittent control input and a performance index. It was shown that by designing the gain matrix of the control protocol, the time-varying formation tracking was achieved, while the certain performance was satisfied, where the upper bound of the performance index was restrained by determining the guaranteed cost. By adjusting the weighting matrices of the performance index, the compromised design between the control energy expenditure and the formation regulation performance was achieved. Sufficient conditions of the guaranteed cost time-varying formation design and analysis were given, and the guaranteed cost was determined. It was proven that if the formation and the communication failure rate satisfy the corresponding conditions in Theorem 1, then the high-order swarm system with intermittent communications and switching topologies can achieve the guaranteed cost time-varying formation tracking by designing the gain matrix of the formation control protocol. The further works will extend the main results of this paper from the switching connected topologies to the jointly switching topologies, and the communication among followers can be directed.

## Data Availability

The data used to support this study are included within this article.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

## Authors' Contributions

Purui Zhang and Xiaoqian Chen were involved in conceptualization; Purui Zhang and Xiaogang Yang were involved in methodology; Purui Zhang was involved in validation, formal analysis, investigation, and writing original draft preparation; Xiaoqian Chen and Xiaogang Yang were involved in writing the review and editing and funding acquisition; Xiaoqian Chen was involved in supervision and project administration. All authors have read and agreed to the published version of the manuscript.

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