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Research Article

Partial-State-Constrained Adaptive Intelligent Tracking Control of Nonlinear Nonstrict-Feedback Systems with Unmodeled Dynamics and Its Application

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In this paper, an adaptive intelligent control scheme is presented to investigate the problem of adaptive tracking control for a class of nonstrict-feedback nonlinear systems with constrained states and unmodeled dynamics. By approximating the unknown nonlinear uncertainties, utilizing Barrier Lyapunov functions (BLFs), and designing a dynamic signal to deal with the constrained states and the unmodeled dynamics, respectively, an adaptive neural network (NN) controller is developed in the frame of the backstepping design. In order to simplify the design process, the nonstrict-feedback form is treated by using the special properties of Gaussian functions. The proposed adaptive control scheme ensures that all variables involved in the closed-loop system are bounded, the corresponding state constraints are not violated. Meanwhile, the tracking error converges to a small neighborhood of the origin. In the end, the proposed intelligent design algorithm is applied to one-link manipulator to demonstrate the effectiveness of the obtained method.

1. Introduction

Over the past few decades, nonlinear control systems, which can be employed to model numerous applications such as biological systems, chemical processes, and aerospace vehicles, have aroused a wide range of concerns among researchers. In this area, adaptive control of nonlinear systems with uncertainties is a very active research subject. The backstepping method, as we all know, has been proposed in [1] as an effective method to solve the adaptive control problem of nonlinear strict-feedback systems with mismatched uncertainties. With the rapid development of adaptive control theory, the backstepping method has been widely used in the control design of different complex nonlinear systems such as interconnected large-scale systems, MIMO systems, and unmodeled dynamic systems [2–4].

On the other hand, unmodeled dynamics are common phenomenon in practical applications, which are mainly caused by modeling errors and external disturbances. The existence of unmodeled dynamics usually degrades control performance or leads to the instability for a control system, and thus dealing with unmodeled dynamics has drawn considerable attention from many scholars in the control field. In recent years, adaptive control of uncertain nonlinear systems with unmodeled dynamics has become a research hot spot, and many related achievements have been reported [5–9]. To just name a few, a robust controller is designed in [5] for a class of nonlinear systems with unmodeled dynamics by using the method of adaptive backstepping and introducing a dynamic signal; and an adaptive output feedback controller is designed in [6] for a class of stochastic nonlinear systems with output unmodeled dynamics by using a stochastic small-gain theorem. Obviously, all the above-mentioned references are nonlinear strict-feedback systems with unmodeled dynamics, and there are few results on nonlinear nonstrict-feedback systems with unmodeled dynamics at present. Therefore, how to deal with the nonstrict-feedback form is one of the hardest issues in the research field of nonlinear systems.

It should be pointed out that none of the above related adaptive results can be used to deal with completely unknown nonlinearities of control systems. To solve this problem, some elegant intelligent adaptive control algorithms are proposed by using NN or fuzzy logic systems [10–32]. Up to now, great progress has been made in the area of adaptive intelligent control for uncertain nonlinear systems with unmodeled dynamics, and a large number of valuable results are presented in [33–44]. For example, several adaptive intelligent control schemes are proposed in [33, 44] for several classes of nonlinear systems made up of unmodeled dynamics and strict-feedback form by using fuzzy logic and NN compensators, respectively. A suitable learning controller is proposed in [39] to overcome the disadvantages caused by parameter uncertainties and unmodeled dynamics for a class of multi-input and multi-output nonlinear systems. Meanwhile, the corresponding results have been obtained for interconnected nonlinear systems with unmodeled dynamics in [40]. However, the problem of constraints inevitably appears in various systems. The research in the field of handling complicated constraints has been paid more and more attention by researchers, and by utilizing the BLFs or the nonlinear mappings (NMs) to deal with state constraints or output constraints, a series of significant results have been obtained [45–48]. However, there exist a few intelligent control algorithms for nonstrict-feedback nonlinear systems with unmodeled dynamics to deal with the state constraints until now.

Motivated by the above research situation, this paper proposes an adaptive tracking control strategy for a class of nonstrict-feedback nonlinear systems with unmodeled dynamics and state constraints. The unknown functions are estimated by NN, then a dynamic signal is designed to handle the dynamic uncertainties to ensure that the considered system can be controlled effectively. Meanwhile, by using the BLFs to handle the state constraints, the proposed adaptive control approach can guarantee the boundedness of all the signals in the closed-loop system, and the tracking error converges to a small neighborhood of the origin. The main contributions of the proposed method are summarized as follows: (1) Compared with the variable partition technique in [36], this paper uses the essential property of Gaussian functions to deal with the nonstrict-feedback form, so that the controller design process is relatively simpler. (2) Barrier functions are applied in the design process to constrain state variables into the specified regions, despite the presence of unmodeled dynamics at the front end of the studied system. (3) This paper adopts a dynamic signal to handle the dynamic uncertainties to ensure the considered system can be controlled effectively so that the conservative assumption about unmodeled dynamics in [5] is not used.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation. In this paper, we study a class of nonstrict-feedback nonlinear systems with unmodeled dynamics as follows:

$$\begin{cases} \dot{\chi} = p(\chi, \xi), \\ \dot{\xi}_i = f_i(\bar{\xi}_i)\xi_{i+1} + g_i(\xi) + \mathrm{O}_i(\xi, \chi, t), \\ i = 1, \ldots, n-1, \\ \dot{\xi}_n = f_n(\bar{\xi}_n)u + g_n(\xi) + \mathrm{O}_n(\xi, \chi, t), \\ \eta = \xi_1, \end{cases} \quad (1)$$

where $\xi = \bar{\xi}_n = [\xi_1, \xi_2, \ldots, \xi_n]^T$ represents the state vector, and u and η depict the system input and output, severally; partial states are constrained in the compact sets; i.e., ξ_i is required to remain in the sets $|\xi_i| < k_{c_i}$ with k_{c_i} being positive constants, $i = 1, 2, \ldots, n$; $\chi \in R^{n_0}$ are the unmodeled dynamics, $\mathrm{O}_i(\cdot)$, $(i = 1, 2 \ldots, n)$ represent nonlinear disturbances, and $g_i(\cdot)$ and $f_i(\cdot)$, $(i = 1, 2 \ldots, n)$ are smooth uncertain functions with $g_i(0) = 0$. It is assumed that $\mathrm{O}_i(\cdot)$ and $p(\cdot)$ are indeterminate continuous Lipschitz functions.

Remark 1. Plant (1) has a nonstrict-feedback structure, where the diffusion terms $g_i(\cdot)$, $(i = 1, 2 \ldots, n)$ are the functions of $\xi = [\xi_1, \ldots, \xi_i]^T$, which is different from the strict-feedback structure in [49] and the semistrict-feedback structure in [50], because the functions $g_i(\cdot)$ are relevant to all stats of ξ.

We will establish an adaptive intelligent controller for system [1] so that the output η can track a given trajectory η_d, the corresponding state constraints are not violated, and all the reference signals of the closed-loop system are bounded. Therefore, we give the following assumptions.

Assumption 1 (see [51]). For system (1), there is an unknown constant $b_m > 0$ satisfying

$$0 < b_m \leq \left| f_i(\bar{\xi}_i) \right| < \infty. \quad (2)$$

Assumption 2 (see [51]). $\eta_d(t)$ is a reference signal, its up to nth-order are bounded and smooth. There exists a positive constant d such that $|\eta_d(t)| \leq d < k_{c_1}$.

Assumption 3 (see [5]). For $i = 1, \ldots, n$ the function $\mathrm{O}_i(\cdot)$ in (1) satisfies the following inequalities:

$$\left| \mathrm{O}_i(\xi, \chi, t) \right| \leq \varphi_{i1}\left(\left| \bar{\xi}_i \right| \right) + \varphi_{i2}(|\chi|), \quad (3)$$

where $\varphi_{i1}(\cdot)$ and $\varphi_{i2}(\cdot)$ are unknown nonnegative increasing smooth functions with $\varphi_{i2}(0) = 0$.

Assumption 4 (see [5]). For $\dot{\chi} = p(\chi, \xi)$ in (1), there is a Lyapunov function $V(\chi)$ such that

$$\omega_1(|\chi|) \le V(\chi) \le \omega_2(|\chi|), \tag{4}$$

$$\frac{\partial V(\chi)}{\partial \chi} p(\chi,\xi) \le -k_0 V(\chi) + \gamma(|\xi_1|) + d_0, \tag{5}$$

where (ω_1, ω_2) and γ represent class K_∞-functions, and $k_0 > 0$ and $d_0 > 0$ are known scalars.

Remark 2. Assumption 1 implies that the unknown functions $f_i^k(\chi_i)$ are strictly positive or negative. Further, let us assume that $0 < b_m \le |f_i(\bar{\xi}_i)| < \infty$ for generality. Assumption 2 is required in many literatures on the tracking control problem such as [36, 40], since we need to figure out its time derivatives up to n th in the design process. There exists the similar assumption in [5], where $\varphi_{i1}(\cdot)$ and $\varphi_{i2}(\cdot)$ are assumed to be available. However, Assumption 3 does not need this restriction and is thus more relaxed. Assumption 4 is the key condition to ensure the stability of the unmodeled dynamics in (1).

2.2. Preliminaries. To facilitate the design and analysis, the following Lemmas are given.

Lemma 1 (see [51]). *For $\dot{\chi} = p(\chi,\xi)$, there is a Lyapunov function V that satisfies (4) and (5), then for any values $\bar{k}$ in $(0, k_0)$, functions $(\bar{\gamma}(\xi_1) \ge \bar{\gamma}(|\xi_1|))$ and initial value $\chi_0 = \chi_0(0)$, there is a limited time $T_0 = T_0(\bar{k}, v_0, \chi_0)$, $B(t) \ge 0$ for all $t > 0$ and a signal is represented by*

$$\dot{v} = -\bar{k}v + \bar{\gamma}(\xi_1(t)) + d_0, \quad v(0) = v_0, \tag{6}$$

such that $B(t) = 0$ for all $t \ge T_0$

$$V(\chi(t)) \le v(t) + B(t). \tag{7}$$

The solutions are specified for $\forall t > 0$. We can select $\bar{\gamma}(s) = s^2\gamma_0(s^2)$, where $\bar{\gamma}(\cdot) > 0$ is a smooth function. Now, (7) makes

$$\dot{v} = -\bar{k}v + \xi_1^2\gamma_0(|\xi_1^2|) + d_0, \quad v(0) = v_0, \tag{8}$$

where γ_0 is a nonnegative smooth function.

Lemma 2 (see [52]). *For any $\zeta \in R$ and $\omega_n > 0$, the following relation holds: $0 \le |\zeta| - \zeta\tanh(\zeta/\varepsilon) \le \delta\omega_0$, $\delta = 0.2785$.*

Lemma 3 (see [53]). *For $\forall (x, y) \in R^n$, it can be obtained that $(xy \le (\varepsilon^p/p)|x|^p + (1/q\varepsilon^q)|y|^q)$ where $\varepsilon > 0$, $p > 1$, $q > 1$, and $(p-1)(q-1) = 1$.*

Lemma 4 (see [54]). *Assume that Ω_{x_1} is defined as $\Omega_{x_1} := \{x_1 \| x_1| < 0.8814v\}$, and the inequality $[1 - \tan h^2(x_1/r)] \le 0$ is satisfied for any $x_1 \notin \Omega_{x_1}$.*

In this paper, the smooth function $g(X)$: $R^n \longrightarrow R$ is estimated by radial basis functions (RBF) NN $g_{nn}(X)$. The RBF NN can be written as

$$g_{nn}(X) = E^T H(X), \tag{9}$$

where $E = [e_1, \ldots, e_l]^T \in R^l$ with $l > 1$ is weight vector, $X \in R^q$ is input vector, and $H(X) = [h_1(X), \ldots, h_l(X)]^T$ is the basis function vector of the Gaussian function. $h_i(X)$ can be expressed as

$$h_i(X) = \exp\left[-\frac{(X-\gamma_i)^T(X-\gamma_i)}{y^2}\right], \quad i = 1, \ldots, l, \tag{10}$$

where $\gamma_i = [\gamma_{i1}, \ldots, \gamma_{iq}]^T$ is the center of the receptive field, and y is the width of the Gaussian function. The RBF NN (9) with sufficiently large node number l can approximate any continuous function $g(X)$ over a compact set $\Omega_X \in R^q$ to arbitrarily accuracy $\varepsilon > 0$ as

$$g(X) = E^{*T}H(X) + \delta(X), \quad \forall X \in \Omega_X \in R^q, \tag{11}$$

where E^* is the desired weight vector and chosen as $E^* = \operatorname{argmin}_{E\in\overline{R}^l}\{\sup_{X\in\Omega_X}|g(X) - E^T H(X)|\}$ and $\delta(X)$ denotes the approximation error for $\delta(X) < \varepsilon$.

Lemma 5 (see [54]). *Let $\bar{\xi}_q = [\xi_1, \ldots, \xi_q]^T$ and $H(\bar{\xi}_q) = [H_1(\bar{\xi}_q), \ldots, H_l(\bar{\xi}_q)]^T$ be the basis function vector of the RBF NN. Now, for $(\forall k, q \in N^+)$ and $k \le q$, we have $\|H(\bar{\xi}_q)\|^2 \le \|H(\bar{\xi}_k)\|^2$.*

3. Main Result

For system (1), this part gives the concrete design process of the controller through the backstepping algorithm. The adaptive neural backstepping design requires n steps. The virtual control input in step i is designed as $\alpha_i (i = 1, \ldots, n-1)$, and the real controller u is added in step n to form a stabilized closed-loop system. They are represented separately as

$$\alpha_i = -c_i x_i - \frac{1}{2a_i^2}\frac{x_i}{(k_{b_i}^2 - x_i^2)}\widehat{\theta}_i H_i^T(X_i)H_i(X_i), \quad 1 \le i \le n-1, \tag{12}$$

$$u = -c_n x_n - \frac{1}{2a_n^2}\frac{x_n}{(k_{b_n}^2 - x_n^2)}\widehat{\theta}_n H_n^T(X_n)H_n(X_n), \tag{13}$$

where the design parameters are $c_i > 0$ and $a_i > 0$, $X_i = [\bar{\xi}_i^T, \bar{\widehat{\theta}}_i^T, \bar{\eta}_d^{(i)T}, v]^T$ with $\bar{\xi}_i = [\xi_i, \xi_2, \ldots, \xi_i]^T$, $\bar{\widehat{\theta}}_i = [\widehat{\theta}_1, \widehat{\theta}_2, \ldots, \widehat{\theta}_i]^T$, $\bar{\eta}_d^{(i)} = [\eta_d, \dot{\eta}_d, \ldots, \eta_d^{(i)}]^T$ with $\eta_d^{(i)} = (d^i\eta_d/dt^i)$, and the uncertain parameter θ_i is estimated to be $\widehat{\theta}_i$. The transformations of coordinates are selected as follows:

$$x_i = \xi_i - \alpha_{i-1}, \quad i = 1, 2, \ldots, n, \tag{14}$$

where $\alpha_0 = \eta_d(t)$.

The adaption laws are expressed as

$$\dot{\widehat{\theta}}_i = \frac{\kappa_i}{2a_i^2}\left(\frac{x_i}{k_{b_i}^2 - x_i^2}\right)^2 H_i^T(X_i)H(X_i) - \mu_i\widehat{\theta}_i, \quad i = 1, 2, \ldots, n, \tag{15}$$

where the design parameters are $\kappa_i > 0$ and $\mu_i > 0$.

For clarity, let us abbreviate the functions $f_i(\overline{\xi}_i)$ to f_i and set $\mathrm{O}_i(\xi, \chi, t) = \mathrm{O}_i$ and $H_i(X_i) = H_i$.

Now, let us start the design process.

Step 1. Based on $(x_1 = \xi_1 - \eta_d)$, one has

$$\dot{x}_1 = f_1\xi_2 + g_1(\xi) + \mathrm{O}_1 - \dot{\eta}_d. \tag{16}$$

Next, choose the following Lyapunov function:

$$V_1 = \frac{1}{2}\log\left(\frac{k_{b_1}^2}{k_{b_1}^2 - x_1^2}\right) + \frac{1}{\kappa_0}\nu + \frac{b_m}{2\kappa_1}\widetilde{\theta}_1^2, \tag{17}$$

where $\log(\vartheta)$ stands for the natural logarithm of ϑ, $\widetilde{\theta}_1 = \theta_1 - \widehat{\theta}_1$ denotes the parameter error, and κ_0 and κ_1 are positive constants. In the set Ω_{x_1}, V_1 is continuous.

Thus, according to Assumption 2, the derivative of V_1 along with (8) leads to

$$\begin{aligned}\dot{V}_1 \leq{}& \frac{x_1}{k_{b_1}^2 - x_1^2}\left(f_1\xi_2 + g_1(\xi) - \dot{\eta}_d\right) + \left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\varphi_{11}(\xi_1) \\ &+ \left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\varphi_{12}(|\chi|) + \frac{1}{\kappa_0}\left(\xi_1^2\gamma_0\left(\xi_1^2\right) + d_0\right) \\ &- \frac{\overline{k}}{\kappa_0}\nu - \frac{b_m}{\kappa_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1.\end{aligned} \tag{18}$$

Now, we conduct $|x_1/(k_{b_1}^2 - x_1^2)|\varphi_{11}(|\xi_1|)$ and $|x_1/(k_{b_1}^2 - x_1^2)|\varphi_{12}(|\chi|)$ in (18). According to Lemma 2, the following inequality holds:

$$\begin{aligned}\left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\varphi_{11}\left(\left|\xi_1\right|\right) \leq{}& \lambda_1' + \frac{x_1}{k_{b_1}^2 - x_1^2}\varphi_{11}\left(\left|\xi_1\right|\right)\tan h \\ &\cdot\left(\frac{\left(x_1/\left(k_{b_1}^2 - x_1^2\right)\right)\varphi_{11}\left(\left|\xi_1\right|\right)}{\lambda_1}\right) \\ ={}& \frac{x_1}{k_{b_1}^2 - x_1^2}\widehat{\varphi}_{11}(\xi_1) + \lambda_1',\end{aligned} \tag{19}$$

where $\lambda_1' = 0.2785\lambda_1$ and $\widehat{\varphi}_{11}(\xi_1) = \varphi_{11}(|\xi_1|)\tanh(x_1\varphi_{11}(|\xi_1|)/\lambda_1(k_{b_1}^2 - x_1^2))$ is a smooth function.

The same method in [32] is repeated

$$\begin{aligned}\left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\varphi_{12}(|\chi|) \leq{}& \left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\overline{\varphi}_{12}(\nu) + \frac{1}{4}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 \\ &+ d_1(t) \leq \frac{x_1}{k_{b_1}^2 - x_1^2}\overline{\varphi}_{12}(\nu)\tan h \\ &\cdot\left(\frac{\left(x_1/\left(k_{b_1}^2 - x_1^2\right)\right)\overline{\varphi}_{12}(\nu)}{\ell_1}\right) \\ &+ \ell_1' + \frac{1}{4}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 + d_1(t) \\ ={}& \frac{x_1}{k_{b_1}^2 - x_1^2}\widehat{\varphi}_{12}(\xi_1, \nu) + \ell_1' \\ &+ \frac{1}{4}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 + d_1(t),\end{aligned} \tag{20}$$

where $\ell_1' = 0.2785\ell_1, \overline{\varphi}_{12}(\nu) = \varphi_{12}\circ\omega_1^{-1}(2\nu), d_1(t) = (\varphi_{12}\circ\omega_1^{-1}(2B(t)))^2$, and $\widehat{\varphi}_{12}(\xi_1,\nu) = \overline{\varphi}_{12}\tanh(x_1\overline{\varphi}_{12}(\nu)/\ell_1(k_{b_1}^2 - x_1^2))$.

Combing (18), (19), and (20), it yields

$$\begin{aligned}\dot{V} \leq{}& \frac{x_1}{k_{b_1}^2 - x_1^2}\Bigg(f_1\xi_2 + g_1(\xi) - \dot{\eta}_d + \widehat{\varphi}_{11}(\xi_1) \\ &+ \widehat{\varphi}_{12}(\xi_1, \nu) + \frac{1}{4}\frac{x_1}{k_{b_1}^2 - x_1^2}\Bigg) + \frac{d_0}{\kappa_0} - \frac{\overline{k}}{\kappa_0}\nu + \lambda_1' + \ell_1' + d_1 \\ &- \frac{b_m}{\kappa_1}\widetilde{\theta}\dot{\widehat{\theta}} + x_1\left(\frac{\xi_1^2\gamma\left(\xi_1^2\right)}{x_1\kappa_0}\right),\end{aligned} \tag{21}$$

where $|d_1(t)| \leq d_1$.

For $x_1 = 0$, $(1/\kappa_0 x_1)\xi_1^2\gamma(\xi_1^2)$ in (21) is discontinuous, and the NN cannot be directly modeled, so we introduce a hyperbolic tangent function $\tan h^2(x_1/r)$ and (21) becomes

$$\begin{aligned}\dot{V} \leq{}& \frac{x_1}{k_{b_1}^2 - x_1^2}\left(f_1\xi_2 + \overline{g}_1(\chi_1)\right) - \frac{1}{2}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 - \frac{b_m}{\kappa_1}\widetilde{\theta}\dot{\widehat{\theta}} \\ &+ \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} - \frac{\overline{k}}{\kappa_0}\nu + \frac{d_0}{\kappa_0} + \lambda_1' + \ell_1' + d_1,\end{aligned} \tag{22}$$

where r is the positive constant, and the unknown nonlinear function $\widehat{g}_1(X_1)$ is expressed as

$$\widehat{g}_1(X_1) = \frac{1}{2}\frac{x_1}{k_{b_1}^2 - x_1^2} - \eta_d + \frac{1}{4}\frac{x_1}{k_{b_1}^2 - x_1^2} + g_1(\xi) + \widehat{\varphi}_{11}(\xi_1) + \widehat{\varphi}_{12}(\xi_1, \nu) + \frac{2x_1}{k_{b_1}^2 - x_1^2}\tan h^2\left(\frac{x_1}{r}\right)\frac{\xi_1^2\gamma_0(\xi_1^2)}{\kappa_0}. \quad (23)$$

For $\forall\varepsilon_1 > 0$, $\widehat{g}_1(X_1)$ is approximated to NN $E_1^T H_1(X_1)$, such that

$$\widehat{g}_1(X_1) = E_1^{*T} H_1(X_1) + \delta_1(X_1), \quad |\delta_1(X_1)| \le \varepsilon_1, \quad (24)$$

where $\delta_1(X_1)$ is the error of this model and $X_1 = [\xi^T, \eta_d, \dot{\eta}_d, \nu]^T$.

Based on Lemma 3 and Lemma 5, the following inequality holds:

$$\begin{aligned}\frac{x_1}{k_{b_1}^2 - x_1^2}\widehat{g}_1(X_1) &= \frac{x_1}{k_{b_1}^2 - x_1^2}\left(E_1^{*T} H_1(X_1) + \delta_1(X_1)\right),\\ &\le \left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\left(\|E_1^*\|\|H_1(X_1)\| + \varepsilon_1\right)\\ &\le \left|\frac{x_1}{k_{b_1}^2 - x_1^2}\right|\left(\|E_1^*\|\|H_1(P_1)\| + \varepsilon_1\right)\\ &\le \frac{1}{2a_1^2}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 b_m\theta_1 H_1^T(P_1)H_1(P_1)\\ &\quad + \frac{a_1^2}{2} + \frac{1}{2}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 + \frac{\varepsilon_1^2}{2},\end{aligned} \quad (25)$$

where $\theta_1 = (\|E_1^*\|^2/b_m)$ and $P_1 = [\xi_1, \eta_d, \dot{\eta}_d, \nu]^T$.

Therefore, substitute (25) into (22) to get

$$\begin{aligned}\dot{V}_1 &\le \frac{x_1}{k_{b_1}^2 - x_1^2} f_1 x_2 + \frac{x_1}{k_{b_1}^2 - x_1^2} f_1\alpha_1\\ &\quad + \frac{1}{2a_1^2}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 b_m\theta_1 \mathrm{H}_1^T H_1 + \frac{d_0}{\kappa_0} + \lambda_1'\\ &\quad + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma(\xi_1^2)}{\kappa_0} - \frac{k}{\kappa_0}\nu + \ell_1' + d_1\\ &\quad + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} - \frac{b_m}{\kappa_1}\widetilde{\theta}_1\dot{\widehat{\theta}}_1,\end{aligned} \quad (26)$$

where $x_2 = \xi_2 - \alpha_1$.

Next, using Assumption 1 and designing a virtual control signal α_1 in (12) when $i = 1$,

(26) becomes

$$\begin{aligned}\dot{V}_1 &\le \frac{x_1}{k_{b_1}^2 - x_1^2} f_1 x_2 - c_1 b_m \frac{x_1^2}{k_{b_1}^2 - x_1^2}\\ &\quad + \frac{b_m}{\kappa_1}\widetilde{\theta}_1\left(\frac{\kappa_1}{2a_1^2}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 H_1^T H_1 - \dot{\widehat{\theta}}_1\right)\\ &\quad + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0(\xi_1^2)}{\kappa_0} - \frac{k}{\kappa_0}\nu + \frac{d_0}{\kappa_0} + \lambda_1'\\ &\quad + \ell_1' + d_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2}.\end{aligned} \quad (27)$$

Then, we choose an adaptive law $\dot{\widehat{\theta}}_1$ from (15), when $i = 1$. $\dot{\widehat{\theta}}_1$ can be expressed as

$$\dot{\widehat{\theta}}_1 = \frac{\kappa_1}{2a_1^2}\left(\frac{x_1}{k_{b_1}^2 - x_1^2}\right)^2 \mathrm{H}_1^T H_1 - \mu_1\widehat{\theta}_1, \quad (28)$$

where $\kappa_1 > 0$ and $\mu_1 > 0$ are constants.

Substituting (28) into (27), it yields

$$\begin{aligned}\dot{V}_1 &\le \frac{x_1}{k_{b_1}^2 - x_1^2} f_1 x_2 - c_1 b_m \frac{x_1^2}{k_{b_1}^2 - x_1^2}\\ &\quad + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0(\xi_1^2)}{\kappa_0} - \frac{k}{\kappa_0} r + \frac{d_0}{\kappa_0} + \lambda_1' + \ell_1'\\ &\quad + d_1 + \frac{a_1^2}{2} + \frac{\varepsilon_1^2}{2} + \frac{b_m\mu_1}{\kappa_1}\widetilde{\theta}_1\widehat{\theta}_1.\end{aligned} \quad (29)$$

Under the action of

$$\frac{\mu_1 b_m}{\kappa_1}\widetilde{\theta}_1\widehat{\theta}_1 = \frac{\mu_1 b_m}{\kappa_1}\widetilde{\theta}_1\left(\theta_1 - \widetilde{\theta}_1\right) \le -\frac{\mu_1 b_m}{2\kappa_1}\widetilde{\theta}_1^2 + \frac{\mu_1 b_m}{2\kappa_1}\theta_1^2, \quad (30)$$

we get

$$\begin{aligned}\dot{V}_1 &\le \frac{x_1}{k_{b_1}^2 - x_1^2} f_1 x_2 + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0(\xi_1^2)}{\kappa_0} - \frac{k}{\kappa_0}\nu\\ &\quad + \frac{d_0}{\kappa_0} + B_1 - k_1\frac{x_1^2}{k_{b_1}^2 - x_1^2} - \frac{\mu_1 b_m}{2\kappa_1}\widetilde{\theta}_1^2,\end{aligned} \quad (31)$$

where $k_1 = c_1 b_m > 0$ and $B_1 = (\mu_1 b_m/2\kappa_1)\theta_1^2 + \lambda_1' + \ell_1' + d_1 + (a_1^2/2) + (\varepsilon_1^2/2)$. Step i $(2 \le i \le n-1)$: Let $x_i = \xi_i - \alpha_{i-1}$, then one has

$$\dot{x}_i = f_i\xi_{i+1} + g_i(\xi) + \mathrm{O}_i - \dot{\alpha}_{i-1}, \quad (32)$$

where

$$\dot{\alpha}_{i-1} = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j} g_j(\xi) + \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}\mathrm{O}_j + \Xi_{i-1}, \quad (33)$$

with

$$\Xi_{i-1}=\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}f_j\xi_{j+1}+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\widehat{\theta}_j}\dot{\widehat{\theta}}_j+\sum_{j=0}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\eta_d^{(j)}}\eta_d^{(j+1)}+\frac{\partial\alpha_{i-1}}{\partial v}\dot{v}. \tag{34}$$

Then, construct a Lyapunov function V_i as

$$V_i=V_{i-1}+\frac{1}{2}\log\left(\frac{k_{b_i}^2}{k_{b_i}^2-x_i^2}\right)+\frac{b_m}{2\kappa_i}\widetilde{\theta}_i^2, \tag{35}$$

where $\kappa_i>0$ is a design parameter and $\widetilde{\theta}_i=\theta_i-\widehat{\theta}_i$ is the error. In the set Ω_{x_i}, $(k_{b_i}^2/(k_{b_i}^2-x_i^2))$ is continuous.

Thus, the derivative of V_i is

$$\dot{V}_i=\dot{V}_{i-1}+\frac{x_i}{k_{b_i}^2-x_i^2}\left(f_i\xi_{i+1}+g_i(x)-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}g_j(\xi)+\overline{\mathrm{O}}_i-\Xi_{i-1}\right)-\frac{b_m}{\kappa_i}\widetilde{\theta}_i\dot{\widehat{\theta}}_i, \tag{36}$$

where $(\overline{\mathrm{O}}_i=\mathrm{O}_i-\sum_{j=1}^{i-1}(\partial\alpha_{i-1}/\partial\xi_j)\mathrm{O}_j)$ and similar to the method in the Step 1, we can obtain

$$\begin{aligned}\dot{V}_{i-1}\le{}&\frac{x_{i-1}}{k_{b_{i-1}}^2-x_{i-1}^2}f_{i-1}x_i-\sum_{j=1}^{i-1}k_j\frac{x_j^2}{k_{b_{j-1}}^2-x_{j-1}^2}\\&-\sum_{j=1}^{i-1}\frac{\mu_jb_m}{2\kappa_j}\widetilde{\theta}_j^2+\frac{d_0}{\kappa_0}+\left(1-2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0(\xi_1^2)}{\kappa_0}\\&-\frac{k}{\kappa_0}v+\sum_{j=1}^{i-1}B_j,\end{aligned} \tag{37}$$

where $k_j=c_jb_m>0$, $d_j(t)\le d_j$ and $(B_j=(1/2)a_j^2+(1/2)\varepsilon_j^2+(\mu_jb_m/2\kappa_j)\theta_j^2+\lambda_j'+\ell_j'+d_j)$. $(j=1,2,\ldots,i-1.)$.

By using Assumption 3 and the absolute value inequality, it yields

$$\begin{aligned}\left|\frac{x_i}{k_{b_i}^2-x_i^2}\overline{\mathrm{O}}_i\right|\le{}&\left|\frac{x_i}{k_{b_i}^2-x_i^2}\right|\left(\varphi_{i1}\left(\left|\overline{\xi}_i\right|\right)+\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right|\varphi_{j1}\left(\left|\overline{\xi}_j\right|\right)\right)\\&+\left|\frac{x_i}{k_{b_i}^2-x_i^2}\right|\left(\varphi_{i2}(|\chi|)+\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right|\varphi_{j2}(|\chi|)\right).\end{aligned} \tag{38}$$

Using the same method as (19) and (20), we can get

$$\begin{aligned}&\left|\frac{x_i}{k_{b_i}^2-x_i^2}\right|\left(\varphi_{i1}\left(\left|\overline{\xi}_i\right|\right)+\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right|\varphi_{j1}\left(\left|\overline{\xi}_j\right|\right)\right)\\&\le\frac{x_i}{k_{b_i}^2-x_i^2}\widehat{\varphi}_{i1}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)+\lambda_i',\end{aligned} \tag{39}$$

$$\begin{aligned}&\left|\frac{x_i}{k_{b_i}^2-x_i^2}\right|\left(\varphi_{i2}(|\chi|)+\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right|\varphi_{j2}(|\chi|)\right)\\&\le\frac{1}{4}\left(\frac{x_i}{k_{b_i}^2-x_i^2}\right)^2\left[1+\sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right)^2\right]\\&\quad+\frac{x_i}{k_{b_i}^2-x_i^2}\widehat{\varphi}_{i2}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)+\ell_i'+d_i,\end{aligned} \tag{40}$$

where

$$\begin{aligned}\widehat{\varphi}_{i1}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)={}&\left(\varphi_{i1}+\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}\varphi_{j1}\right)\\&\times\tanh\left(x_i\left(\frac{\varphi_{i1}+\sum_{j=1}^{i-1}\left|(\partial\alpha_{i-1}/\partial\xi_j)\right|\varphi_{j1}}{\lambda_i\left(k_{b_i}^2-x_i^2\right)}\right)\right),\\\lambda_i'={}&0.2785\lambda_i,\ \widehat{\varphi}_{i2}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)\\={}&\overline{\varphi}_{i2}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)\tanh\left(\frac{x_i\overline{\varphi}_{i2}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)}{\ell_i\left(k_{b_i}^2-x_i^2\right)}\right),\\\overline{\varphi}_{i2}\left(\overline{\xi}_i,\overline{\widehat{\theta}}_{i-1},v\right)={}&\varphi_{i2}\circ\omega_1^{-1}(2v)+\sum_{j=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial\xi_j}\right|\varphi_{j2}\circ\omega_1^{-1}(2v),\\\ell_i'={}&0.2785\ell_i,\end{aligned} \tag{41}$$

$d_i(t)=\sum_{j=1}^{i}(\varphi_{j2}\circ\omega_1^{-1}(2D(t)))^2$ and $d_i(t)\ge0$ for $\forall t\ge0$.

Then, substitute (36)–(39) into (35) to get

$$\dot{V}_i \leq -\sum_{j=1}^{i-1}\frac{k_j x_j^2}{k_{b_j}^2 - x_j^2} - \sum_{j=1}^{i-1}\frac{\mu_j b_m}{2\kappa_j}\tilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu + \frac{x_i}{k_{b_i}^2 - x_i^2}\left(f_i\xi_{i+1} + \widehat{g}_i(\chi_i)\right) - \frac{1}{2}\left(\frac{x_i}{k_{b_i}^2 - x_i^2}\right)^2 + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} - \frac{b_m}{\kappa_i}\tilde{\theta}_i\dot{\hat{\theta}}_i + \sum_{j=1}^{i-1}B_j + \frac{d_0}{\kappa_0} + \lambda_i' + \ell_i' + d_i, \quad (42)$$

where

$$\widehat{g}_i(X_i) = \frac{1}{2}\frac{x_i}{k_{b_i}^2 - x_i^2} + \frac{x_{i-1}}{k_{b_{i-1}}^2 - x_{i-1}^2}f_{i-1} + \frac{1}{4}\frac{x_i}{k_{b_i}^2 - x_i^2}\left[1 + \sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{j-1}}{\partial\xi_j}\right)^2\right] - \Xi_{i-1} + \widehat{\varphi}_{i1}\left(\overline{\xi}_i, \overline{\hat{\theta}}_{i-1}, \nu\right) + \widehat{\varphi}_{i2}\left(\overline{\xi}_i, \overline{\hat{\theta}}_{i-1}, \nu\right) + \frac{x_i}{k_{b_i}^2 - x_i^2}g_i(x) - \frac{x_i}{k_{b_i}^2 - x_i^2}\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\xi_j}g_l(\xi). \quad (43)$$

For $\forall\varepsilon_i > 0$, the unknown smooth function $\widehat{g}_i(X_i)$ is estimated by the RBF NN $E_i^T H_i(X_i)$ and we have

$$\widehat{g}_i(X_i) = E_i^{*T}H_i(X) + \delta_i(X_i), \quad \left|\delta(X_i)\right| \leq \varepsilon_i, \quad (44)$$

where $X_i = [\xi^T, \overline{\eta}_d^T, \overline{\hat{\theta}}_i^T, \nu]^T$, $\overline{\eta}_d = [\eta_d, \eta_d', \ldots, \eta_d^{(i)}]^T$ and $\hat{\theta}_i = [\tilde{\theta}_i, \ldots, \tilde{\theta}_i]^T$.

Thus, it yields

$$\begin{aligned}\frac{x_i}{k_{b_i}^2 - x_i^2}\widehat{g}_i(X_i) &= \frac{x_i}{k_{b_i}^2 - x_i^2}\left(E_i^{*T}H_i(X_i) + \delta_i(X_i)\right),\\ &\leq \left|\frac{x_i}{k_{b_i}^2 - x_i^2}\right|\left(\|E_i^*\|\|H_i(X_i)\| + \varepsilon_i\right)\\ &\leq \left|\frac{x_i}{k_{b_i}^2 - x_i^2}\right|\left(\|E_i^*\|\|H_i(P_i)\| + \varepsilon_i\right)\\ &\leq \frac{b_m}{2a_i^2}\left(\frac{x_i}{k_{b_i}^2 - x_i^2}\right)^2\theta_i H_i^T(P_i)H_i(P_i) + \frac{1}{2}a_i^2 + \frac{1}{2}\left(\frac{x_i}{k_{b_i}^2 - x_i^2}\right)^2 + \frac{1}{2}\varepsilon_i^2,\end{aligned} \quad (45)$$

where a_i is the design parameter, $P_i = [\overline{\xi}_i^T, \overline{\eta}_d^T, \overline{\hat{\theta}}_i^T, \nu]^T$ and $\theta_i = (\|E_i^*\|^2/b_m)$.

Therefore, substitute (45) into (42) to get

$$\dot{V}_i \leq -\sum_{j=1}^{i-1}\frac{k_j x_j^2}{k_{b_i}^2 - x_j^2} - \sum_{j=1}^{i-1}\frac{\mu_j b_m}{2\kappa_j}\tilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu + \frac{x_i}{k_{b_i}^2 - x_i^2}f_i x_{i+1} + \frac{x_i}{k_{b_i}^2 - x_i^2}f_i\alpha_i + \frac{b_m}{2a_i^2}\left(\frac{x_i}{k_{b_i}^2 - x_i^2}\right)^2\theta_i H_i^T H_i + \frac{d_0}{\kappa_0} + \lambda_i' + \ell_i' + d_i + \frac{1}{2}a_i^2 + \frac{1}{2}\varepsilon_i^2 + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\cdot\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} - \frac{b_m}{\kappa_i}\tilde{\theta}_i\dot{\hat{\theta}}_i + \sum_{j=1}^{i-1}B_j, \quad (46)$$

where $x_{i+1} = \xi_{i+1} - \alpha_i$.

Next, designing a virtual control signal α_i in (12) and an adaptive law $\hat{\theta}_i$ from (15), then, using the same method as (27)–(31), we can get

$$\dot{V}_i \leq -\sum_{j=1}^{i-1}\frac{k_j x_j^2}{k_{b_i}^2 - x_j^2} - \sum_{j=1}^{i-1}\frac{\mu_j b_m}{2\kappa_j}\tilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu + \frac{x_i}{k_{b_i}^2 - x_i^2}f_i x_{i+1} + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} + \sum_{j=1}^{i-1}B_j + \frac{d_0}{\kappa_0}, \quad (47)$$

where $k_j = c_j b_m > 0$ and $(B_j = (1/2)a_j^2 + (1/2)\varepsilon_j^2 + (\mu_j b_m/2\kappa_j)\theta_j^2 + \lambda_j' + \ell_j' + d_j)$. Step n: In this step, we design the real controller u from (13), so the derivative of $\dot{x}_n$ is

$$\dot{x}_n = f_n u + g_n(\xi) + \mathrm{O}_n - \dot{\alpha}_{n-1}, \quad (48)$$

where $\dot{\alpha}_{n-1}$ is specified in (33) when $i = n$.

Then, choose the Lyapunov function as

$$V_n = V_{n-1} + \frac{1}{2}\log\frac{k_{b_n}^2}{k_{b_n}^2 - x_n^2} + \frac{b_m}{2\kappa_n}\tilde{\theta}_n^2. \quad (49)$$

Then, the dynamic equation of V_n is

$$\dot{V}_n \leq -\sum_{j=1}^{n-1}\frac{k_j x_j^2}{k_{b_j}^2 - x_j^2} - \sum_{j=1}^{n-1}\frac{\mu_j b_m}{2\kappa_j}\tilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu + \frac{d_0}{\kappa_0} + \sum_{j=1}^{n-1}B_j + \frac{x_n}{k_{b_j}^2 - x_n^2}\left(f_n u + g_n(\xi) - \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\xi_j}g_j(\xi) - \Xi_{n-1} + \frac{x_{n-1}}{k_{b_{n-1}}^2 - x_{n-1}^2}f_{n-1}\right) + \left|\frac{x_n}{k_{b_n}^2 - x_n^2}\overline{\mathrm{O}}_n\right| - \frac{b_m}{\kappa_n}\tilde{\theta}_n\dot{\hat{\theta}}_n + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0}, \quad (50)$$

where $\overline{\mathrm{O}}_n = \mathrm{O}_n - \sum_{j=1}^{n-1}(\partial\alpha_{n-1}/\partial\xi_j)\mathrm{O}_j$ and Ξ_{n-1} has been defined in (38) with $i = n$.

Using the same method as (38) to (40), we can get

$$\left|\frac{x_n}{k_{b_n}^2 - x_n^2}\overline{\text{O}}_n\right| \leq \frac{x_n}{k_{b_n}^2 - x_n^2}\widehat{\varphi}_{n1}\left(\overline{\xi}_n, \overline{\overline{\theta}}_{n-1}, \nu\right) + \frac{x_n}{k_{b_n}^2 - x_n^2}\widehat{\varphi}_{n2}\left(\overline{\xi}_n, \overline{\overline{\theta}}_{n-1}, \nu\right) + \frac{1}{4}\left(\frac{x_n}{k_{b_n}^2 - x_n^2}\right)^2 \cdot \left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial \xi_j}\right)^2\right] + \lambda_n' + \ell_n' + d_n(t), \tag{51}$$

where $\widehat{\varphi}_{n1}(\overline{\xi}_n, \overline{\overline{\theta}}_{n-1}, \nu)$ and $\widehat{\varphi}_{n2}(\overline{\xi}_n, \overline{\overline{\theta}}_{n-1}, \nu)$ are defined in (47) and (48), respectively.

In view of (51), (50) is written as

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n-1}\frac{k_j x_j^2}{k_{b_j}^2 - x_j^2} - \sum_{j=1}^{n-1}\frac{\mu_j b_m}{2\kappa_j}\widetilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu + \frac{d_0}{\kappa_0} + \sum_{j=1}^{n-1}B_j \\ & + \left(1 - 2\tan h^2\left(\frac{x_1}{r\left(k_{b_1}^2 - x_1^2\right)}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} \\ & - \frac{1}{2}\left(\frac{x_n}{k_{b_n}^2 - x_n^2}\right)^2 + \frac{x_n}{k_{b_n}^2 - x_n^2}\left(f_n u + \widehat{g}_n(X_n)\right) \\ & - \frac{b_m}{\kappa_n}\widetilde{\theta}_n\dot{\widehat{\theta}}_n + \lambda_n' + \ell_n' + d_n,\end{aligned} \tag{52}$$

where $d_n(t) \leq d_n$ and

$$\begin{aligned}\widehat{g}_n(X_n) = & \frac{1}{2}\frac{x_n}{k_{b_n}^2 - x_n^2} + g_n(\xi) - \sum_{j=1}^{n-1}\frac{\partial \alpha_{n-1}}{\partial \xi_j}g_j(\xi) - \Xi_{n-1} \\ & + \widehat{\varphi}_{n1}\left(\overline{\xi}_n, \overline{\overline{\theta}}_{n-1}, \nu\right) \\ & + \frac{1}{4}\left(\frac{x_n}{k_{b_n}^2 - x_n^2}\right)^2\left[1 + \sum_{j=1}^{n-1}\left(\frac{\partial \alpha_{n-1}}{\partial \xi_j}\right)^2\right] \\ & + \frac{x_{n-1}}{k_{b_{n-1}}^2 - x_{n-1}^2}f_{n-1} + \widehat{\varphi}_{n2}\left(\overline{\xi}_n, \overline{\overline{\theta}}_{n-1}, \nu\right).\end{aligned} \tag{53}$$

For $\forall \varepsilon_n > 0$, the unknown smooth function $\widehat{g}_n(X_n)$ is estimated of the RBF NN $E_n^T H_n(X_n)$ and we have

$$\widehat{g}_n(X_n) = E_n^{*T} H_n(X_n) + \delta_n(X_n), \quad \left|\delta(X_n)\right| \leq \varepsilon_n, \tag{54}$$

where $\delta_i(X_n)$ is the estimated error and $\varepsilon_n > 0$ denotes a given constant.

Just like (45), one has

$$\frac{x_n}{k_{b_n}^2 - x_n^2}\widehat{g}_n(X_n) \leq \frac{b_m}{2a_n^2}\left(\frac{x_n}{k_{b_n}^2 - x_n^2}\right)^2 \theta_n H_n^T H_n + \frac{1}{2}a_n^2 + \frac{1}{2}\left(\frac{x_n}{k_{b_n}^2 - x_n^2}\right)^2 + \frac{1}{2}\varepsilon_n^2, \tag{55}$$

where the unknown constant $\theta_n = (\|E_n^*\|^2/b_m)$.

By combining (52) with (55), it yields

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n-1}\frac{k_j x_j^2}{k_{b_j}^2 - x_j^2} + \frac{x_n}{k_{b_n}^2 - x_n^2}\left(f_n u + \frac{b_m}{2a_n^2}\frac{x_n}{k_{b_n}^2 - x_n^2}\theta_n H_n^T H_n\right) \\ & + \frac{1}{2}a_n^2 + \frac{1}{2}\varepsilon_n^2 \\ & + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} - \frac{b_m}{\kappa_n}\widetilde{\theta}_n\dot{\widehat{\theta}}_n + \sum_{j=1}^{n-1}B_j \\ & + \frac{d_0}{\kappa_0} + \lambda_n' + \ell_n' + d_n - \sum_{j=1}^{n-1}\frac{\mu_j b_m}{2\kappa_j}\widetilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu.\end{aligned} \tag{56}$$

Now, we are ready to design the actual controller u and adaptive law $\widehat{\theta}_n$, which are given as

$$\begin{aligned} u &= -c_n x_n - \frac{1}{2a_n^2}\frac{x_n}{k_{b_n}^2 - x_n^2}\widehat{\theta}_n H_n^T(X_n) H_n(X_n), \\ \dot{\widehat{\theta}}_n &= \frac{\kappa_n}{2a_n^2}\left(\frac{x_n}{k_{b_n}^2 - x_n^2}\right)^2 H_n^T H_n - \mu_n\widehat{\theta}_n,\end{aligned} \tag{57}$$

where the positive parameters are $c_n, a_n, \kappa_n, \mu_n$.

Then, following the same derivations as those used in Step 1, one has

$$\begin{aligned}\dot{V}_n \leq & -\sum_{j=1}^{n}\frac{k_j x_j^2}{k_{b_j}^2 - x_j^2} - \sum_{j=1}^{n}\frac{\mu_j b_m}{2\kappa_j}\widetilde{\theta}_j^2 - \frac{\overline{k}}{\kappa_0}\nu \\ & + \left(1 - 2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma_0\left(\xi_1^2\right)}{\kappa_0} + \sum_{j=1}^{n}B_j + \frac{d_0}{\kappa_0},\end{aligned} \tag{58}$$

where $k_j = c_j b_m > 0$ and $B_j = (1/2)a_j^2 + (1/2)\varepsilon_j^2 + (\mu_j b_m/2\kappa_j)\theta_j^2 + \lambda_j' + \ell_j' + d_j$, $(j = 1, 2, \ldots, n.)$.

So this backstepping control design process is complete. The main result is summarized in the next section.

4. Stability Analysis

Theorem 1. *Consider the closed-loop system with Assumptions 1– 4, which is composed of plant (1), the virtual control* *inputs α_i (12), real controller u (13), and adaptive laws (15), where RBF NN $E_i^T H_i(X_i)$ is employed to estimate the uncertain nonlinear function $\widehat{g}_i(X_i)$ with bounded errors $\delta_i\,(1\le i\le n)$. Suppose that the design parameters c_i, a_i and μ_i are appropriately chosen to satisfy $k_{c_{i+1}}>\overline{\alpha}_i+k_{b_{i+1}}$ with $\overline{\alpha}_i=\max|\alpha_i(\overline{x}_i,\widehat{\theta}_j,y_d,y_d^{(i)},j=1,\ldots,i)|$, then the final closed-loop system is SGUUB, and the tracking error converges to a small field around the origin with the partial-state constraints being valid.*

Proof. From (17), (35), and (49), we gain

$$V_n=\frac{1}{2}\sum_{j=1}^{n}\log\left(\frac{k_{b_j}^2}{k_{b_j}^2-x_j^2}\right)+\frac{1}{k_0}\nu+\frac{1}{2}\sum_{j=1}^{n}\frac{b_m}{\kappa_j}\widetilde{\theta}_j^2. \tag{59}$$

It is a fact that $\log(k_{b_j}^2/(k_{b_j}^2-x_j^2))<(x_j^2/(k_{b_j}^2-x_j^2))$ in the interval $|x_j|<k_{b_j}$. Then, (58) becomes

$$V_n\le\frac{1}{2}\sum_{j=1}^{n}\frac{x_j^2}{k_{b_j}^2-x_j^2}+\frac{1}{k_0}\nu+\frac{1}{2}\sum_{j=1}^{n}\frac{b_m}{\kappa_j}\widetilde{\theta}_j^2. \tag{60}$$

Using (60), inequality (58) can be represented as

$$\dot{V}_n\le-r_0V+\mu_0+\left(1-2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma(\xi_1^2)}{\kappa_0}, \tag{61}$$

where $r_0=\min\{2c_ib_m,\mu_i,\overline{k}\colon 1\le i\le n\}$ and $\mu_0=\sum_{j=1}^{n}B_j+(\mu_0/\kappa_0)$.

Obviously, we know from (61) that the first item on the right of the inequation must be negative, the second item is a positive constant, and the positive or negative of the last item depends on the size of x_1. So we are going to consider the closed-loop system in two different cases, and the results are as follows. □

Case 1. $x_1\in\Omega_{x_1}=\{x_1||x_1|<0.8814r\}$, r is a positive constant in (22). Because of the coordinate transformation (14), we can see that ξ_1 is bounded because x_1 is constructed to be bounded, and the reference signal η_d is also bounded. From (61), it gets

$$\dot{V}_n\le-r_0V+b_0, \tag{62}$$

where $b_0=\mu_0+k_0$.

Besides, (61) satisfies

$$0\le V_n\le\left(V(0)-\frac{b_0}{r_0}\right)e^{-r_0t}+\frac{b_0}{r_0}. \tag{63}$$

Case 2. $x_1\notin\Omega_{x_1}$. By applying Lemma 4 and $(\xi_1^2\gamma_0(\xi_1^2)/\kappa_0)\ge 0$, we have

$$\left(1-2\tan h^2\left(\frac{x_1}{r}\right)\right)\frac{\xi_1^2\gamma(\xi_1^2)}{\kappa_0}\le 0. \tag{64}$$

So, (61) is reduced to

$$\dot{V}_n\le-r_0V+\mu_0. \tag{65}$$

Then, using (65) we can get

$$0\le V_n(t)\le\left(V(0)-\frac{\mu_0}{r_0}\right)e^{-r_0t}+\frac{\mu_0}{r_0}. \tag{66}$$

Next, from (63) and (66), we have

$$0\le V_n(t)\le V(0)+\frac{b_0}{r_0},\quad t>0. \tag{67}$$

Then, it can be concluded from the above inequality and (59) that $\log k_{b_j}^2/(k_{b_j}^2-x_j^2)$ and $\widetilde{\theta}_j$ are bounded. Since θ_j is bounded and $\widehat{\theta}_j=\widetilde{\theta}_j+\theta_j$, $\widehat{\theta}_j$ must be bounded.

From $\xi_1=x_1+\eta_d$ and $|\eta_d|\le d$, we can obtain $|\xi_1|\le|x_1|+|\eta_d|<k_{b_1}+d$. Let $k_{b_1}=k_{c_1}-d$ and then, $|\xi_1|<k_{c_1}$. Apparently, $\alpha_1(\cdot)$ is a function of $\widehat{\theta}_1,\xi_1,x_1$ and $\dot{\eta}_d$. Because of the boundedness of $\widehat{\theta}_1,\xi_1,x_1$ and $\dot{\eta}_d$, $\alpha_1(\cdot)$ is bounded and satisfies $|\alpha_1(\cdot)|\le\overline{\alpha}_1$. Then, $|\xi_2|\le|\alpha_1|+|x_1|\le\overline{\alpha}_1+k_{b_2}$.This implies that $|\xi_2|<k_{c_2}$ if $k_{b_2}=k_{c_2}-\overline{\alpha}_1$. Similarly, it can in turn be proven that $|\xi_{i+1}|<k_{c_{i+1}}$, $i=1,\ldots,n-1$ as long as $k_{b_{i+1}}=k_{c_{i+1}}-\overline{\alpha}_i$. From the definition in (13), we can see that u is a function of $\widehat{\theta}_n$, ξ_n and $\eta_d,\dot{\eta}_d,\ldots,\eta_d^{(n)}$. Owing to the boundedness of $\widehat{\theta}_n$, ξ_n and $\eta_d,\dot{\eta}_d,\ldots,\eta_d^{(n)}$, the controller u is bounded.

In both cases, we can conclude that all the reference signals of the closed-loop system are bounded. In addition, combining (63) and (66), we can see that the tracking error finally converges to a small region of the origin, and the system partial states are not violated.

From (67), it is easy to obtain

$$\log\left(\frac{k_{b_1}^2}{k_{b_1}^2-x_1^2}\right)\le 2\left(V(0)-\frac{\mu_0}{r_0}\right)e^{-r_0t}+\frac{2\mu_0}{r_0}. \tag{68}$$

We take exponentials on both sides of the above inequality; it has

$$\frac{k_{b_1}^2}{k_{b_1}^2-x_1^2}\le e^{2\left(V(0)-(\mu_0/r_0)\right)e^{-r_0t}+(2\mu_0/r_0)}. \tag{69}$$

It is straightforward to get $(|x_1|\le k_{b_1}\sqrt{1-e^{-2(V(0)-(\mu_0/r_0))e^{-r_0t}-(2\mu_0/r_0)}}=\Delta)$. If $V_n(0)=\mu_0/r_0$, then it holds $|x_1|\le k_{b_1}\sqrt{1-e^{(-2\mu_0/r_0)}}=\Delta$. If $V_n(0)\ne(\mu_0/r_0)$, it can be concluded that, given any $\Delta>k_{b_1}\sqrt{1-e^{(-2\mu_0/r_0)}}$, there exists T such that for any $t>T$, it has $|x_1|\le\Delta$. As $t\longrightarrow\infty$, $|x_1|\le k_{b_1}\sqrt{1-e^{(-2\mu_0/r_0)}}$. This implies that $|x_1|\le k_{b_1}\sqrt{1-e^{(-2\mu_0/r_0)}}$. We can see that x_1 can be made arbitrarily small by selecting the design parameters appropriately.

This completes the proof.

5. Simulation Example

Example 1. In order to test the applicability of the proposed control method, the following one-link manipulator with motor dynamics and disturbances is considered:

$$D\ddot{q}+B\dot{q}+N\sin(q)=\tau+\tau_d, M_m\dot{\tau}+H_m\tau=u-K_m\dot{q}, \tag{70}$$

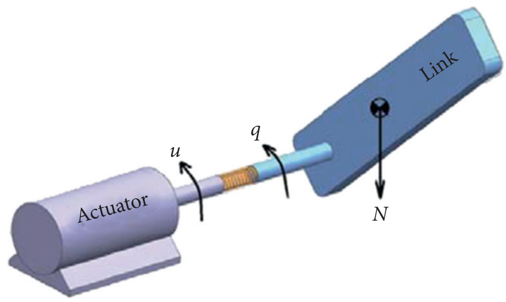

Figure 1: Sketch of one-link manipulator.

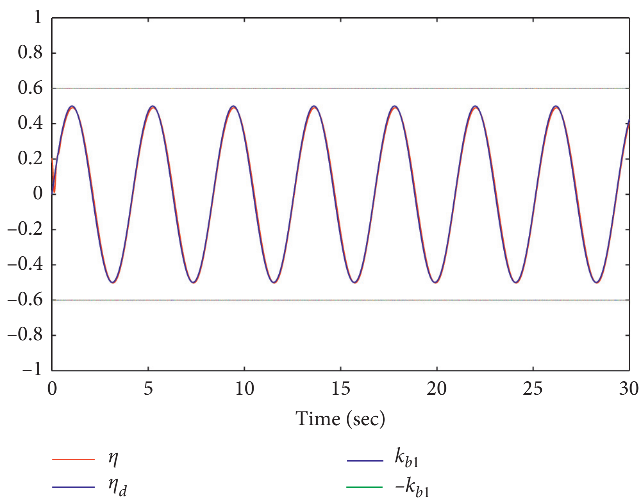

Figure 2: The output η and reference signal η_d.

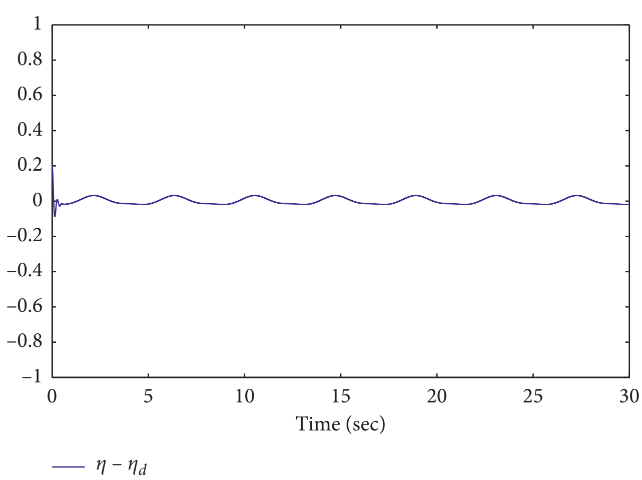

Figure 3: The tracking error $\eta - \eta_d$.

where q, $\dot{q}$, and $\ddot{q}$ are the link angular position, velocity, and acceleration. τ is the torque, $\tau_d = q^2 \cos(\dot{q}\tau)$ denotes the current disturbance, and u is the control input representing the voltage. Take these parameters as $D = 1\,\mathrm{kgm}^2, B = 1\,\mathrm{Nm}, M_m = 0.1\,H, H_m = 1.0\,\Omega,$ and $K_m = 0.2\,(\mathrm{Nm/A})$. Moreover, the sketch of the one-link manipulator is given in Figure 1.

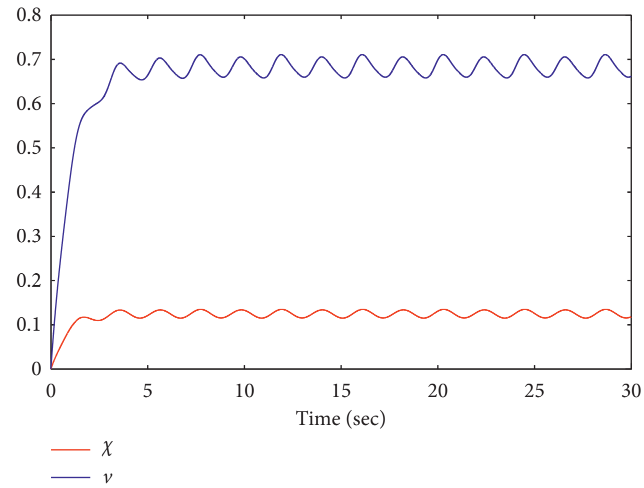

Figure 4: The unmodeled dynamics χ and v.

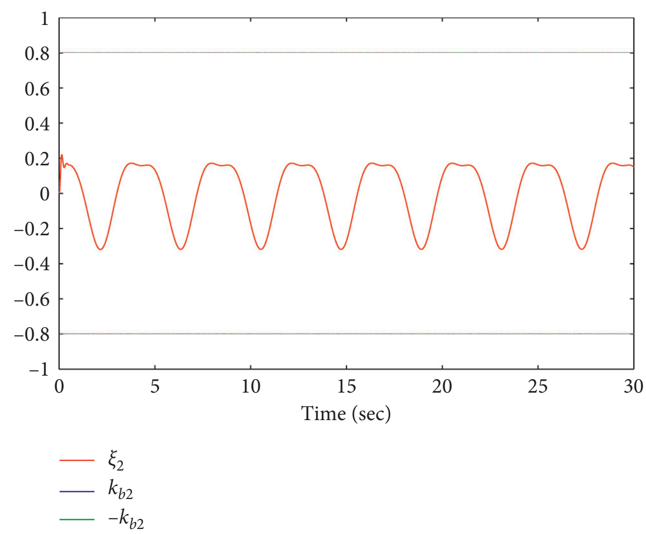

Figure 5: The state variable ξ_2.

Let $\xi_1 = q, \xi_2 = \dot{q}$, and $\xi_3 = \tau$. Thus, (70) can be translated into a nonlinear system as follows:

$$\begin{cases} \dot{\chi} = -1.25\chi + 0.25\xi_1^2 + 0.125, \\ \dot{\xi}_1 = 3\xi_2 + g_1(\xi) + \mathrm{O}_1, \\ \dot{\xi}_2 = 1.2\xi_3 + g_2(\xi) + \mathrm{O}_2, \\ \dot{\xi}_3 = 8u + g_3(\xi) + \mathrm{O}_3, \\ \eta = \xi_1, \end{cases} \tag{71}$$

where $g_1(\cdot) = 2\cos^2(\xi_1) - 1.75, g_2(\cdot) = \xi_1 \quad \xi_2 - 2\sin(\xi_1)$, $g_3(\cdot) = -6.25\,\xi_3$,
$\mathrm{O}_1 = \chi\sin(\xi_2)\mathrm{O}_2 = 0.2\chi\xi_1\cos(\xi_1), \mathrm{O}_3 = -2\chi\cos(\xi_1\xi_2)$.

The aim is to impel the output η of the system (70) to follow the reference trajectory $\eta_d = 0.5\sin(1.5t)$. We can easily check that Assumptions 1– 3 hold. Moreover, to prove that Assumption 4 is correct, $V_\chi(\chi) = 2\chi^2$ is chosen, then

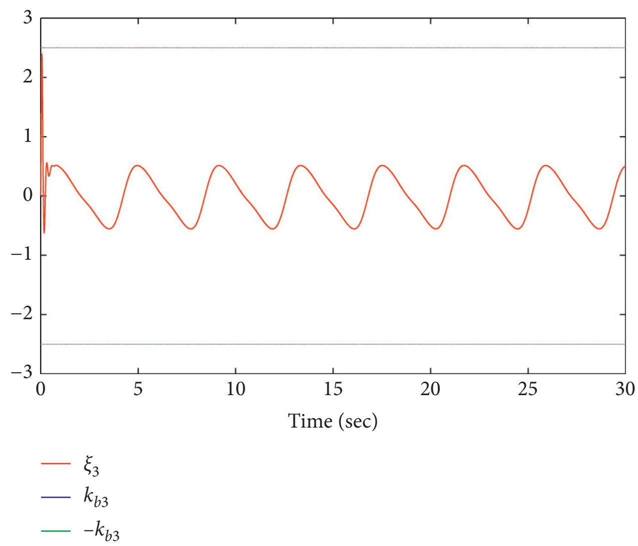

FIGURE 6: The state variable ξ_3.

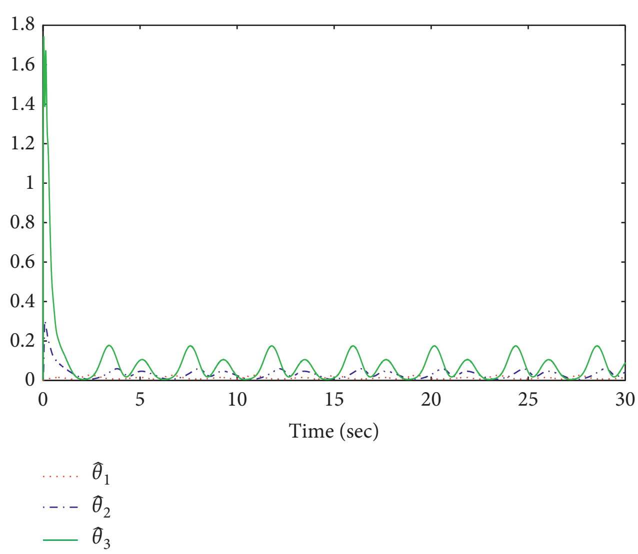

FIGURE 7: The adaptive parameters $\widehat{\theta}_1$, $\widehat{\theta}_2$, and $\widehat{\theta}_3$.

$$\begin{aligned} \dot{V}_\chi(\chi) &= 4\chi\left(-1.25\chi + 0.25\xi_1^2 + 0.25\right), \\ &\le -5\chi^2 + \frac{1}{4\tau}\chi^2 + \tau\xi_1^4 + \frac{\tau}{4} + \frac{\chi^2}{\tau}, \end{aligned} \quad (72)$$

When $\tau = 2.5$, we have

$$\dot{V}_\chi(\chi) \le -4.5\chi^2 + 2.5\xi_1^2 + 0.625. \quad (73)$$

Let us select $v_1(|\chi|) = 1.5\chi^2, v_2(|\chi|) = 4\chi^2, k_0 = 1.5, d_0 = 0.625$, and $\gamma(|\xi_1|) = 2.5\xi_1^4$. When $\bar{k} = 1 \in (0, k_0)$, a dynamical signal is given as

$$\dot{v} = -v + 2.5\xi_1^4 + 0.625. \quad (74)$$

Then, the virtual control input α_1, α_2, and real controller u are expressed as

$$\begin{aligned} \alpha_1 &= -c_1 x_1 - \frac{1}{2a_1^2}\frac{x_1}{\left(k_{b_1}^2 - x_1^2\right)}\widehat{\theta}_1 H_1^T H_1, \\ \alpha_2 &= -c_2 x_2 - \frac{1}{2a_2^2}\frac{x_2}{\left(k_{b_2}^2 - x_2^2\right)}\widehat{\theta}_2 H_2^T H_2, \\ u &= -c_3 x_3 - \frac{1}{2a_3^2}\frac{x_3}{\left(k_{b_3}^2 - x_3^2\right)}\widehat{\theta}_3 H_3^T H_3, \end{aligned} \quad (75)$$

and the adaptive laws are expressed as

$$\dot{\widehat{\theta}}_i = \frac{\kappa_i}{2a_i^2}\left(\frac{x_i}{k_{b_i}^2 - x_i^2}\right)^2 H_i^T H_i - \mu_i\widehat{\theta}_i, \quad i = 1, 2, 3, \quad (76)$$

with $x_1 = \xi_1 - \eta_d, x_2 = \xi_2 - \alpha_1$ and $x_3 = \xi_3 - \alpha_2$.

The design parameters are chosen as $[\xi_1(0), \xi_2(0), \xi_3(0)]^T = [0.0, 0.0, 0.0]^T, [\widehat{\theta}_1(0), \widehat{\theta}_2(0), \widehat{\theta}_3(0)]^T = [0.0, 0.0, 0.0]^T$, $c_1 = 10, c_2 = 15, c_3 = 20, a_1 = 1, a_2 = 1, a_3 = 15, \mu_1 = 2, \mu_2 = 3, \mu_3 = 5, \kappa_1 = 10, \kappa_2 = 15, \kappa_3 = 20$, respectively. The states are constrained in. $|\xi_1| < 0.6, |\xi_2| < 0.8, |\xi_3| < 2.5$.

Figure 2 shows the tracking result of the output trajectory η and the reference signal η_d. Figure 3 shows the tracking error of the closed-loop system, which obviously achieves a good tracking performance. The response of the unmodeled dynamics χ and v is shown in Figure 4. Figures 5 and 6 show the state variables ξ_2 and ξ_3, which show that the system states keep within their bounds. The performance of the adaptive laws is shown in Figure 7. All simulation results show that the proposed control algorithm is effective and applicable.

6. Conclusions

The paper has investigated the problem of adaptive tracking control for a class of nonstrict-feedback nonlinear systems with partial-state-constraints and unmodeled dynamics. An adaptive neural tracking control method has been presented by using the adaptive backstepping technique. Based on the inherent properties of Gaussian functions, and the universal approximation ability of RBF NN, a new method is proposed to deal with the nonstrict-feedback form of the considered nonlinear system. The proposed control algorithm can guarantee the boundedness of all the resulting closed-loop signals, the tracking error to converge to a small neighborhood of the origin, and the corresponding state constraints are not violated. Finally, the effectiveness and practicability of the obtained result are shown by a practical simulation example.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The author declares that there are no conflicts of interest.

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