Hindawi
Complexity
Volume 2020, Article ID 8841137, 13 pages
https://doi.org/10.1155/2020/8841137

Research Article

Novel Robust Stability Criteria of Uncertain Systems with Interval Time-Varying Delay Based on Time-Delay Segmentation Method and Multiple Integrals Functional

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Received 18 September 2020; Revised 27 October 2020; Accepted 18 November 2020; Published 10 December 2020

Academic Editor: Guang Li

Interval time-varying delay is common in control process, e.g., automatic robot control system, and its stability analysis is of great significance to ensure the reliable control of industrial processes. In order to improve the conservation of the existing robust stability analysis method, this paper considers a class of linear systems with norm-bounded uncertainty and interval time-varying delay as the research object. Less conservative robust stability criterion is put forward based on augmented Lyapunov-Krasovskii (L-K) functional method and reciprocally convex combination. Firstly, the delay interval is partitioned into multiple equidistant subintervals, and a new Lyapunov-Krasovskii functional comprising quadruple-integral term is introduced for each subinterval. Secondly, a novel delay-dependent stability criterion in terms of linear matrix inequalities (LMIs) is given by less conservative Wirtinger-based integral inequality approach. Three numerical comparative examples are given to verify the superiority of the proposed approach in reducing the conservation of conclusion. For the first example about closed-loop control systems with interval time-varying delays, the proposed robust stability criterion could get MADB (Maximum Allowable Delay Bound) about 0.3 more than the best results in the previous literature; and, for two other uncertain systems with interval time-varying delays, the MADB results obtained by the proposed method are better than those in the previous literature by about 0.045 and 0.054, respectively. All the example results obtained in this paper clearly show that our approach is better than other existing methods.

1. Introduction

Many dynamic model systems in the real world contain very significant time delays in the transmission of data and materials, in automatic robot control system, the acquisition and transmission of sensor signals, and the calculation of controller and the drive of brake may lead to time delay. In many kinds of time-delay types, the interval time-varying delay is more representative. The lower bound of its time delay is not necessarily zero, and the time delay is within a changing interval. It is common in practical application of engineering, especially in chemical reactors, internal combustion engines, and network control [1, 2]. Consequently, the stability analysis about interval time-delay systems has attracted wide attention in these years.

Generally, aiming to analyze the stability of time-delay system, the most common method is to construct an appropriate LK functional (Lyapunov–Krasovskii functional, LKF) in time domain and combine it with linear matrix inequalities (LMIs). In general, the free weight matrix method, the time-delay segmentation method, the integral inequality method, the interactive convex combination method, and so forth are used to analyze its stability. Augmented functional method [3–5] can make full use of the system's time-delay information to reduce the conservativeness of conclusion, but the introduction of matrix variables inevitably burdens the theoretical analysis and engineering calculation. Zhang et al. and Shen et al. [6, 7] obtain a conservative less stable stability criterion for linear systems with time-varying delays by constructing LKF with triple integral functional terms and optimize the stability conditions of time-delay systems. The integral inequality method has the characteristics of simple form and few matrix variables, which can promote the stability analysis of time-delay systems. Gu [8] first introduced Jensen's inequality into the stability analysis of time-delay systems, and then Ramakrishnan [9, 10], Zhang [11], and Gouaisbaut [12] further promoted Jensen's inequality, resulting in different and novel forms. In various forms, we have obtained effective conclusions of different conservation. As an innovative method, the interactive convex combination method [13, 14] can solve the stability problems of systems with interactive convex combination. Wu et al. [15] studied the issue of robust stability analysis for a sort of uncertain neutral system with mixed time-varying delays, and a novel discrete and neutral delay-dependent stability criterion based on linear matrix inequalities was given, which could greatly reduce the complexity of theoretical derivation and computation. Li et al. [16] deal with a set of positive functions combined with inverse convex weighting parameters by interactive convex combination technique instead of directly ignoring the term and deduce the stability criterion for uncertain neutral systems with mixed time delay. This criterion reduces the number of relevant decision variables while ensuring the conservativeness and avoids the complexity of numerical calculation.

Farnam et al. [17] studied the robust stability problem for a class of linear systems with time-varying delays. By constructing LKF with more time-delay information, the stability condition of LMIs is obtained by means of interactive convex combination definition technique. Finally, the numbers are used to demonstrate that the given stability conditions are less conservative in computational efficiency. Ding et al. [18] construct an augmented functional with specific time-delay information based on the idea of time-delayed partitioning. The free-weight matrix inequality is used to define the cross terms generated by the functional derivatives, and a lower-conservative stability criterion is obtained. Senthilraj et al. [19] introduced a novel method to study the robust stability problem of an interval-delayed neural network system by using a nonuniform time-delay segmentation method and the integral inequality definition technique. Delay-dependent stability conditions for ensuring the stability of the system are obtained. Cheng et al. [20] studied a time-delay-related state feedback control problem for a class of time-varying delay continuous systems via improved interactive convex combination techniques and Wirtinger-based integral inequalities, and new stability conditions and state feedback control are obtained. Zhang et al. [21] proposed a robust stability criterion for a class of linear systems with time-varying delays by using the Wirtinger-based integral inequality and the interactive convex combination lemma to effectively define the cross terms emerging in the LKF derivatives. The conclusions obtained are superior in terms of stability analysis. Chang et al. studied the control problem with time-varying norm bounded uncertainties and discrete-time nonlinear systems with parametric uncertainties; LMI are used to obtain the sufficient conditions for robust stabilization [22, 23].

On the basis of the above research results, for the sake of further revealing the relationship between the asymptotical stability of uncertain systems with interval time-varying delay and the constructed LKF and then to lower the conservatism caused by dealing with the functional derivatives, this paper attempts to study the robust stability problem of uncertain systems with interval time-varying delay by constructing a novel LKF and realizing less conservative integral inequalities. The main contributions of this paper include the following:

In this paper, the robust stability criterion is proposed based on the time-delay segmentation method. Specifically, the time-delay interval is divided into N equal parts. Then, a new LKF with quadruple integral term is constructed for different subintervals.

The constructed LKF is augmented with single integral terms and multiple integrals terms, which can make more connections among different vectors and then eliminate the redundant conservatism arising from estimating the interval time-varying delay. Moreover, in addition to the single integral, the double integral, and the triple integral, the quadruple integral is used as a term to construct the integral functional, which would make full use of more information about the upper and lower bounds of the time delay existing in the systems.

The Wirtinger-based integral inequality and interactive convex combination technique are used to give conclusion in the form of LMIs without any extra parameters.

$\mathbb{R}^n$denotes n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $*$ denotes symmetric terms in symmetric matrices. $\boldsymbol{I}$ denotes the identity matrix with proper dimensions. $\mathbf{M}=\mathbf{M}^T>0$ denotes that $\mathbf{M}$is symmetric matrix. e_i denotes block input matrix with proper dimensions; for instance, $e_6^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

2. Problem Description

The uncertain linear systems with interval time-varying delay are as follows:

$$\begin{cases} \dot{x}(t) = (A+\Delta A(t))x(t) + (B+\Delta B(t))x(t-h(t)), \\ x(t) = \varphi(t), \quad t \in [-h_M, 0], \end{cases} \tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector of the system, A and B are system matrices with appropriate dimensions, $h(t)$ is time-varying delay satisfying $0\le h_m\le h(t)\le h_M$, and $\Delta A(t)$ and $\Delta B(t)$ are unknown matrices with time-varying structure uncertainty. When $\Delta A(t)$ and $\Delta B(t)$ have norm bounded uncertainty, they can be described as follows:

$$\begin{bmatrix} \Delta A(t) & \Delta B(t) \end{bmatrix} = DF(t)\begin{bmatrix} E_a & E_b \end{bmatrix}, \tag{2}$$

where D, E_a, and E_b are known matrices with appropriate dimensions, while $F(t)$ is an uncertain matrix with measurable elements satisfying $F(t)^T F(t)\le I, \forall t$, in which I represents the unit matrix of the appropriate dimension. When $F(t)=0$, system (1) becomes a nominal system.

In this paper, assuming that N is a positive integer greater than zero, $h_i\,(i = 1, 2, \ldots, N+1)$ are scalars, and the time-delay interval $[h_m, h_M]$ can be averaged as follows:

$$h_m = h_1 < h_2 < h_3 < \cdots < h_N < h_{N+1} = h_M, \quad (3)$$

where $h_m = h_1$; $h_M = h_{N+1}$; then h_Δ represents the length of subinterval $[h_i, h_{i+1}]$; namely, $h_\Delta = h_{i+1} - h_i = (h_M - h_m)/N$.

To facilitate the proof of stability criteria, the following lemmas are summarized as follows:

Lemma 1 (see [12]). *Assuming any positive definite matrix $M = M^T > 0$, scalar $h > 0$, and continuous vector functions $x(t)\colon [0, h] \longrightarrow \mathbb{R}^n$, the following inequality is established:*

$$-h\int_{t-h}^{t}\dot{x}^T(s)M\dot{x}(s)\mathrm{d}s \le -[x(t)-x(t-h)]^T \cdot M[x(t)-x(t-h)] - 3\Theta^T M\Theta, \quad (4)$$

where $\Theta = x(t) + x(t-h) - (2/h)\int_{t-h}^{t}x(s)ds$.

Lemma 2 (see [17]). *Assuming any positive definite matrix $M = M^T > 0$, scalar $h > 0$, and continuous vector functions $x(t)\colon [0, h] \longrightarrow \mathbb{R}^n$, the following inequality is established:*

$$\begin{aligned}
-h\int_{t-h}^{t}x^T(s)Mx(s)\mathrm{d}s &\le -\int_{t-h}^{t}x^T(s)\mathrm{d}sM\int_{t-h}^{t}x(s)\mathrm{d}s \\
&\quad -\left(h^2/2\right)\int_{-h}^{0}\int_{t+\beta}^{t}x^T(s)Mx(s)\mathrm{d}s\mathrm{d}\beta \\
&\le -\int_{-h}^{0}\int_{t+\beta}^{t}x^T(s)\mathrm{d}s\mathrm{d}\beta M\int_{-h}^{0}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta, \\
&\quad -\left(h^3/6\right)\int_{-h}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t}x^T(s)Mx(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \\
&\le -\int_{-h}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t}x^T(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta M\int_{-h}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t}x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta.
\end{aligned} \quad (5)$$

Lemma 3 (see [17]). *Assuming any positive definite matrix $M = M^T > 0$, scalars $0 \le \alpha, \varepsilon \le 1$, $\alpha = ((h(t) - h_i)/(h_{i+1} - h_i))$, $\varepsilon = (((h(t))^2 - h_i^2)/(h_{i+1}^2 - h_i^2))$, $h_i \le h(t) \le h_{i+1}$, and vector functions $x(t)\colon [0, h] \longrightarrow \mathbb{R}^n$, the following inequality is established:*

$$\begin{aligned}
-\left(h_{i+1}-h_i\right)\int_{t-h_{i+1}}^{t-h_i}x^T(s)Mx(s)\mathrm{d}s &\le -\zeta^T(t)\left(e_7Me_7^T + e_6Me_6^T\right)\zeta(t) \\
&\quad -\alpha\zeta^T(t)e_7Me_7^T\zeta(t) - (1-\alpha)\zeta^T(t)e_6Me_6^T\zeta(t), \\
&\quad -\left(\left(h_{i+1}^2 - h_i^2\right)/2\right)\int_{-h_{i+1}}^{-h_i}\int_{t+\beta}^{t}x^T(s)Mx(s)\mathrm{d}s\mathrm{d}\beta \\
&\le -\zeta^T(t)\left(e_{10}Me_{10}^T + e_9Me_9^T\right)\zeta(t) \\
&\quad -\varepsilon\zeta^T(t)e_{10}Me_{10}^T\zeta(t) - (1-\varepsilon)\zeta^T(t)e_9Me_9^T\zeta(t),
\end{aligned} \quad (6)$$

where

$$\begin{aligned}
\zeta^T(t) = \Bigg[&x(t)x(t-h(t))x(t-h_i)x(t-h_{i+1})\int_{t-h_i}^{t}x(s)\mathrm{d}s\int_{t-h(t)}^{t-h_i}x(s)\mathrm{d}s\int_{t-h_{i+1}}^{t-h(t)}x(s)\mathrm{d}s\int_{-h_i}^{0}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta \\
&\cdot\int_{-h(t)}^{-h_i}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta\int_{-h_{i+1}}^{-h(t)}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta\Bigg].
\end{aligned} \quad (7)$$

3. Main Results

In this section, the stability of the system is discussed in two steps. First, the stability criterion of the nominal system is given, and then the stability of the uncertain system is analyzed. The nominal system of system (1) is as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t-h(t)), \\ x(t) = \varphi(t), \quad t \in [-h_M, 0]. \end{cases} \tag{8}$$

For nominal systems (8), a new quadruple integral term L-K functional containing more time-delay information is constructed in each subinterval. The following conclusions are obtained by combining Lemmas 1–3.

Theorem 1. *For given scalars h_m, h_M, and λ_1, λ_2 ($\lambda_1 > \lambda_2$), it is asymptotically stable for the nominal system (8), if there exist positive definite symmetric matrices $P_i\,(i = 1, 2, 3, 4, 5), Q_1, Q_2, U_1, U_2, X_j, R_j\,(j = 1, 2, 3, 4)$, such that the following linear matrix inequalities (LMIs) hold:*

$$\Phi = \left(\Phi_{i,j}\right)_{10\times 10} < 0, \tag{9}$$

where

$$\begin{aligned}
\Phi_{11} &= P_1A + A^TP_1 + Q_1 + h_i^2X_1 + h_i^2A^TX_2A - X_2 + h_\Delta^2X_3 + h_\Delta^2A^TX_4A + \left(\frac{h_i^4}{4}\right)R_1 - h_i^2R_2 \\
&\quad + \left(\frac{h_i^4}{4}\right)A^TR_2A + \left(\frac{\left(h_{i+1}^2 - h_i^2\right)^2}{4}\right)R_3 - 2h_\Delta^2R_4 + \left(\frac{\left(h_{i+1}^2 - h_i^2\right)^2}{4}\right)A^TR_4A - \left(\frac{h_i^4}{4}\right)U_1 + \left(\frac{h_i^6}{36}\right)A^TU_1A \\
&\quad + \left(\frac{\left(h_{i+1}^2 - h_i^2\right)^2}{4}\right)U_2 - \left(\frac{\left(h_{i+1}^3 - h_i^3\right)^2}{36}\right)A^TU_2A, \\
\Phi_{12} &= P_1B + h_i^2A^TX_2B + h_\Delta^2A^TX_4B + \left(\frac{h_i^4}{4}\right)A^TR_2B \\
&\quad + \left(\frac{\left(h_{i+1}^2 - h_i^2\right)^2}{4}\right)A^TR_4B + \left(\frac{h_i^6}{36}\right)A^TU_1B + \left(\frac{\left(h_{i+1}^3 - h_i^3\right)^2}{36}\right)A^TU_2B, \\
\Phi_{13} &= X, \Phi_{14} = 0, \Phi_{15} = P_2 + h_iR_2, \Phi_{16} = \Phi_{17} = h_\Delta R_4, \\
\Phi_{18} &= h_iP_4 + \left(h_i^2/2\right)U_1, \Phi_{19} = \Phi_{110} = h_\Delta P_5 + \left(\frac{\left(h_{i+1}^2 - h_i^2\right)}{2}\right)U_2, \\
\Phi_{22} &= h_i^2B^TX_2B - 2X_4 + h_\Delta^2B^TX_4B + \left(\frac{h_i^4}{4}\right)B^TR_2B + \left(\frac{h_i^6}{36}\right)B^TU_1B \\
&\quad + \left(\frac{\left(h_{i+1}^2 - h_i^2\right)^2}{4}\right)B^TR_4B + \left(\frac{\left(h_{i+1}^3 - h_i^3\right)^2}{36}\right)B^TU_2B, \\
\Phi_{23} &= \Phi_{24} = X_4, \Phi_{25} = \Phi_{26} = \Phi_{27} = \Phi_{28} = \Phi_{29} = \Phi_{210} = 0, \\
\Phi_{33} &= -Q_1 + Q_2 - X_2 - X_4, \Phi_{34} = 0, \Phi_{35} = -P_2, \Phi_{36} = \Phi_{37} = P_3, \\
\Phi_{38} &= \Phi_{39} = \Phi_{310} = 0, \Phi_{44} = -Q_2 - X_4, \Phi_{45} = 0, \Phi_{46} = \Phi_{47} = -P_3, \\
\Phi_{48} &= \Phi_{49} = \Phi_{410} = 0, \Phi_{55} = -X_1 - R_2, \Phi_{56} = \Phi_{57} = 0, \Phi_{58} = -P_4, \\
\Phi_{59} &= \Phi_{510} = 0, \Phi_{66} = -X_3 - R_4, \Phi_{67} = \Phi_{68} = 0, \Phi_{69} = \Phi_{610} = -P_5, \\
\Phi_{n} &= -X_3 - R_4, \Phi_{78} = 0, \Phi_{79} = \Phi_{710} = -P_5, \Phi_{88} = -R_1 - U_1, \\
\Phi_{89} &= \Phi_{810} = 0, \Phi_{99} = -R_3 - U_2, \Phi_{910} = U_2, \Phi_{1010} = -R_3 - U_2, \\
h_\Delta &= h_{i+1} = h_i = \frac{(h_M - h_m)}{N}, h_i = h_1 + \frac{(i - 1(h_M - h_m))}{N}.
\end{aligned} \tag{10}$$

Proof. For the sake of simplicity, Theorem 1 holds when $h(t) \in [h_2, h_3]$ first; and then Theorem 1 is generalized to be established when $h(t) \in [h_i + h_{i+1}]$ $(i = 1, 3, \ldots, N)$.

When $h(t) \in [h_2, h_3]$, the L-K functional is constructed as follows:

$$V_2(x(t)) = V_{21}(x(t)) + V_{22}(x(t)) + V_{23}(x(t)) + V_{24}(x(t)) + V_{25}(x(t)), \tag{11}$$

where

$$\begin{aligned}
V_{21}(x(t)) &= x^T(t)P_1x(t) + \int_{t-h_2}^{t} x^T(s)\mathrm{d}sP_2\int_{t-h_2}^{t} x(s)\mathrm{d}s \\
&+ \int_{t-h_3}^{t-h_2} x^T(s)\mathrm{d}sP_3\int_{t-h_3}^{t-h_2} x(s)\mathrm{d}s + \int_{-h_2}^{0}\int_{t+\beta}^{t} x^T(s)\mathrm{d}s\mathrm{d}\beta P_4\int_{-h_2}^{0}\int_{t+\beta}^{t} x(s)\mathrm{d}s\mathrm{d}\beta \\
&+ \int_{-h_3}^{-h_2}\int_{t+\beta}^{t} x^T(s)\mathrm{d}s\mathrm{d}\beta P_5\int_{-h_3}^{-h_2}\int_{t+\beta}^{t} x(s)\mathrm{d}s\mathrm{d}\beta, \\
V_{22}(x(t)) &= \int_{t-h_2}^{t} x^T(s)Q_1x(s)\mathrm{d}s + \int_{t-h_3}^{t-h_2} x^T(s)Q_2x(s)\mathrm{d}s, \\
V_{23}(x(t)) &= h_2\int_{-h_2}^{0}\int_{t+\beta}^{t} x^T(s)X_1x(s)\mathrm{d}s\mathrm{d}\beta + h_2\int_{-h_2}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)X_2\dot{x}(s)\mathrm{d}s\mathrm{d}\beta \\
&+ (h_3 - h_2)\int_{-h_3}^{-h_2}\int_{t+\beta}^{t} x^T(s)X_3x(s)\mathrm{d}s\mathrm{d}\beta + (h_3 - h_2)\int_{-h_3}^{-h_2}\int_{t+\beta}^{t} \dot{x}^T(s)X_4\dot{x}(s)\mathrm{d}s\mathrm{d}\beta, \\
V_{24}(x(t)) &= \left(\frac{h_2^2}{2}\right)\int_{-h_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} x^T(s)R_1x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \\
&+ \left(\frac{h_2^2}{2}\right)\int_{-h_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \\
&+ \left(\frac{(h_3^2 - h_2^2)}{2}\right)\int_{-h_3}^{-h_2}\int_{\beta}^{0}\int_{t+\lambda}^{t} x^T(s)R_3x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \\
&+ \left(\frac{(h_3^2 - h_2^2)}{2}\right)\int_{-h_3}^{-h_2}\int_{\beta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_4\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta, \\
V_{25}(x(t)) &= \left(\frac{h_2^3}{6}\right)\int_{-h_2}^{0}\int_{\beta}^{0}\int_{\lambda}^{0}\int_{t+\varphi}^{t} \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\varphi\mathrm{d}\lambda\mathrm{d}\beta \\
&+ \left(\frac{(h_3^3 - h_2^3)}{6}\right)\int_{-h_3}^{-h_2}\int_{\beta}^{0}\int_{\lambda}^{0}\int_{t+\varphi}^{t} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\varphi\mathrm{d}\lambda\mathrm{d}\beta.
\end{aligned} \tag{12}$$

The derivative of L-K functional $V(t)$ along the nominal system (8) is calculated as follows:

$$\dot{V}_2(t) = \dot{V}_{21}(t) + \dot{V}_{22}(t) + \dot{V}_{23}(t) + \dot{V}_{24}(t) + \dot{V}_{25}(t), \tag{13}$$

where

$$
\begin{aligned}
\dot{V}_{21}(t) = {} & 2x^T(t)A^TP_1x(t) + 2x^T(t-h(t))B^TP_1x(t) + 2x^T(t)P_2\int_{t-h_2}^{t}x(s)\mathrm{d}s \\
& - 2x^T(t-h_2)P_2\int_{t-h_2}^{t}x(s)\mathrm{d}s + 2x^T(t-h_2)P_3\int_{t-h_3}^{t-h_2}x(s)\mathrm{d}s \\
& - 2x^T(t-h_3)P_3\int_{t-h_3}^{t-h_2}x(s)\mathrm{d}s - 2\int_{t-h_2}^{t}x^T(s)\mathrm{d}sP_4\int_{-h_2}^{0}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta \\
& + 2h_2x^T(t)P_4\int_{-h_2}^{0}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta + 2(h_3-h_2)x^T(t)P_5\int_{-h_3}^{-h_2}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta \\
& - 2\int_{t-h_3}^{t-h_2}x^T(s)\mathrm{d}sP_5\int_{-h_3}^{-h_2}\int_{t+\beta}^{t}x(s)\mathrm{d}s\mathrm{d}\beta, \\
\dot{V}_{22}(t) = {} & x^T(t)Q_1x(t) - x^T(t-h_2)Q_1x(t-h_2) + x^T(t-h_2)Q_2x(t-h_2) - x^T(t-h_3)Q_2x(t-h_3), \\
\dot{V}_{23}(t) = {} & h_2^2x^T(t)X_1x(t) - h_2\int_{t-h_2}^{t}x^T(s)X_1x(s)\mathrm{d}s - h_2\int_{t-h_2}^{t}\dot{x}^T(s)X_2\dot{x}(s)\mathrm{d}s \\
& + h_2^2\dot{x}^T(t)X_2\dot{x}(t) + (h_3-h_2)^2x^T(t)X_3x(t) + (h_3-h_2)^2\dot{x}^T(t)X_4\dot{x}(t) \\
& - (h_3-h_2)\int_{t-h_3}^{t-h_2}x^T(s)X_3x(s)\mathrm{d}s - (h_3-h_2)\int_{t-h_3}^{t-h_2}\dot{x}^T(s)X_4\dot{x}(s)\mathrm{d}s, \\
\dot{V}_{24}(t) = {} & \left(\frac{h_2^4}{4}\right)x^T(t)R_1x(t) - \left(\frac{h_2^2}{2}\right)\int_{-h_2}^{0}\int_{t+\beta}^{t}x^T(s)R_1x(s)\mathrm{d}s\mathrm{d}\beta \\
& + \left(\frac{h_2^4}{4}\right)\dot{x}^T(t)R_2\dot{x}(t) - \left(\frac{h_2^2}{2}\right)\int_{-h_2}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\beta \\
& + \left(\frac{(h_3^2-h_2^2)^2}{4}\right)x^T(t)R_3x(t) - \left(\frac{(h_3^2-h_2^2)}{2}\right)\int_{-h_3}^{-h_2}\int_{t+\beta}^{t}x^T(s)R_3x(s)\mathrm{d}s\mathrm{d}\beta \\
& + \left(\frac{(h_3^2-h_2^2)^2}{4}\right)\dot{x}^T(t)R_4\dot{x}(t) - \left(\frac{(h_3^2-h_2^2)}{2}\right)\int_{-h_3}^{-h_2}\int_{t+\beta}^{t}\dot{x}^T(s)R_4\dot{x}(s)\mathrm{d}s\mathrm{d}\beta, \\
\dot{V}_{25}(t) = {} & \left(\frac{h_2^6}{36}\right)\dot{x}^T(t)U_1\dot{x}(t) + \left(\frac{(h_3^3-h_2^3)^2}{36}\right)\dot{x}^T(t)U_2\dot{x}(t) \\
& - \left(\frac{h_2^3}{6}\right)\int_{-h_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta - \left(\frac{(h_3^3-h_2^3)}{6}\right)\int_{-h_3}^{-h_2}\int_{\beta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta.
\end{aligned}
\tag{14}
$$

From Lemmas 1 and 2, we can obtain the following:

$$-h_2 \int_{t-h_2}^{t} x^T(s) X_1 x(s) \mathrm{d}s \le -\zeta^T(t) e_5 X_1 e_5^T \zeta(t), \tag{15}$$

$$\begin{aligned} -h_2 \int_{t-h_2}^{t} \dot{x}^T(s) X_2 \dot{x}(s) \mathrm{d}s \le & -\zeta^T(t)(e_1 - e_3) X_2 \left(e_1^T - e_3^T\right)\zeta(t) - \\ & \cdot 3\zeta^T(t)\left(e_1 + e_3 - \left(\frac{2}{h_2}\right)e_5\right) X_2 \left(e_1^T + e_3^T - \left(\frac{2}{h_2}\right)e_5^T\right)\zeta(t), \end{aligned} \tag{16}$$

where $\zeta(t)$ is consistent with $i = 2$ in Lemma 3.

From Lemma 3, we can obtain the following:

$$\begin{aligned} -(h_3 - h_2) \int_{t-h_3}^{t-h_2} x^T(s) X_3 x(s) \mathrm{d}s \le & -\zeta^T(t) e_7 X_3 e_7^T \zeta(t) \\ & -\zeta^T(t) e_6 X_3 e_6^T \zeta(t) - \alpha\zeta^T(t) e_7 X_3 e_7^T \zeta(t) - (1-\alpha)\zeta^T(t) e_6 X_3 e_6^T \zeta(t). \end{aligned} \tag{17}$$

Similarly, according to Lemma 3, we can obtain the following:

$$\begin{aligned} -(h_3 - h_2) \int_{t-h_3}^{t-h_2} \dot{x}^T(s) X_4 \dot{x}(s) \mathrm{d}s \le & -\zeta^T(t)(e_2 - e_4) X_4 \left(e_2^T - e_4^T\right)\zeta(t) \\ & -\zeta^T(t)(e_3 - e_2) X_4 \left(e_3^T - e_2^T\right)\zeta(t) - \alpha\zeta^T(t)(e_2 - e_4) X_4 \left(e_2^T - e_4^T\right)\zeta(t) \\ & -(1-\alpha)\zeta^T(t)(e_3 - e_2) X_4 \left(e_3^T - e_2^T\right)\zeta(t), \end{aligned} \tag{18}$$

$$-\left(\frac{h_2^2}{2}\right) \int_{-h_2}^{0} \int_{t+\beta}^{t} x^T(s) R_1 x(s) \mathrm{d}s\mathrm{d}\beta \le -\zeta^T(t) e_8 R_1 e_8^T \zeta(t), \tag{19}$$

$$-\left(\frac{h_2^2}{2}\right) \int_{-h_2}^{0} \int_{t+\beta}^{t} \dot{x}^T(s) R_2 \dot{x}(s) \mathrm{d}s\mathrm{d}\beta \le -\zeta^T(t)(h_2 e_1 - e_5) R_2 \left(h_2 e_1^T - e_5^T\right)\zeta(t), \tag{20}$$

$$\begin{aligned} -\left(\frac{\left(h_3^2 - h_2^2\right)}{2}\right) \int_{-h_3}^{-h_2} \int_{t+\beta}^{t} x^T(s) R_3 x(s) \mathrm{d}s\mathrm{d}\beta \le & -\zeta^T(t) e_{10} R_3 e_{10}^T \zeta(t) \\ & -\zeta^T(t) e_9 R_3 e_9^T \zeta(t) - \varepsilon\zeta^T(t) e_{10} R_3 e_{10}^T \zeta(t) - (1-\varepsilon)\zeta^T(t) e_9 R_3 e_9^T \zeta(t), \end{aligned} \tag{21}$$

$$\begin{aligned} -\left(\frac{\left(h_3^2 - h_2^2\right)}{2}\right) \int_{-h_3}^{-h_2} \int_{t+\beta}^{t} \dot{x}^T(s) R_4 \dot{x}(s) \mathrm{d}s\mathrm{d}\beta \le & \\ -\zeta^T(t)\left((h_3 - h_2)e_1 - e_7\right) R_4 \left((h_3 - h_2)e_1^T - e_7^T\right)\zeta(t) & \\ -\zeta^T(t)\left((h_3 - h_2)e_1 - e_6\right) R_4 \left((h_3 - h_2)e_1^T - e_6^T\right)\zeta(t) & \\ -\varepsilon\zeta^T(t)\left((h_3 - h_2)e_1 - e_7\right) R_4 \left((h_3 - h_2)e_1^T - e_7^T\right)\zeta(t) & \\ -(1-\varepsilon)\zeta^T(t)\left((h_3 - h_2)e_1 - e_6\right) R_4 \left((h_3 - h_2)e_1^T - e_6^T\right)\zeta(t), & \end{aligned} \tag{22}$$

$$-\left(\frac{h_2^3}{6}\right)\int_{-h_2}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \leq -\zeta^T(t)\left(\left(\frac{h_2^2}{2}\right)e_1 - e_8\right)U_1\left(\left(\frac{h_2^2}{2}\right)e_1^T - e_8^T\right)\zeta(t), \tag{23}$$

$$-\left(\frac{\left(h_3^3 - h_2^3\right)}{6}\right)\int_{-h_3}^{-h_2}\int_{\beta}^{0}\int_{t+\lambda}^{t}\dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \leq -\zeta^T(t)\left(\left(\frac{\left(h_3^2 - h_2^2\right)}{2}\right)e_1 - e_9 - e_{10}\right)U_2\left(\left(\frac{\left(h_3^2 - h_2^2\right)}{2}\right)e_1^T - e_9^T - e_{10}^T\right)\zeta(t). \tag{24}$$

Substituting (15) ~ (24) into (13), $\dot{V}_2(x(t))$ can be expressed as follows:

$$\dot{V}_2(x(t)) \leq \zeta^T(t)\left[\alpha\Gamma_1 + (1-\alpha)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_2\right]\zeta(t), \tag{25}$$

where

$$\begin{aligned}
\Gamma_{i1} &= \left(\frac{\Phi}{2}\right) - e_7X_3e_7^T - (e_2 - e_4)X_4\left(e_2^T - e_4^T\right),\\
\Gamma_{i2} &= \left(\frac{\Phi}{2}\right) - e_6X_3e_6^T - (e_3 - e_2)X_4\left(e_3^T - e_2^T\right),\\
\Gamma_{i3} &= \left(\frac{\Phi}{2}\right) - e_{10}R_3e_{10}^T - ((h_{i+1} - h_i)e_1 - e_7)R_4\left((h_{i+1} - h_i)e_1^T - e_7^T\right),\\
\Gamma_{i4} &= \left(\frac{\Phi}{2}\right) - e_9R_3e_9^T - ((h_{i+1} - h_i)e_1 - e_6)R_4\left((h_{i+1} - h_i)e_1^T - e_6^T\right).
\end{aligned} \tag{26}$$

For $0 \leq \alpha, \varepsilon \leq 1$, according to convex combination technique, the following inequality is established:

$$\begin{aligned}
&\alpha\left(\Gamma_1 + \lambda_1 I\right) + (1-\alpha)\left(\Gamma_2 + \lambda_1 I\right) < 0,\\
&\varepsilon\left(\Gamma_3 + \lambda_2 I\right) + (1-\varepsilon)\left(\Gamma_4 + \lambda_2 I\right) < 0.
\end{aligned} \tag{27}$$

Namely,

$$\alpha\Gamma_1 + (1-\alpha)\Gamma_2 < -\lambda_1 I, \tag{28}$$

$$\varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 < \lambda_2 I. \tag{29}$$

As a result of $\lambda_1 > \lambda_2$, combining (28) and (29), the following formula is available:

$$\alpha\Gamma_1 + (1-\alpha)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 < \left(\lambda_2 - \lambda_1\right)I < 0. \tag{30}$$

If $\alpha\Gamma_1 + (1-\alpha)\Gamma_2 + \varepsilon\Gamma_3 + (1-\varepsilon)\Gamma_4 < 0$, according to L-K stability theorem, there exists a sufficient small positive number δ_2 for $\dot{V}_2(t) < -\delta_2\|x(t)\|^2$ to hold, and then the nominal system (8) is asymptotically stable.

Without losing generality, when $h(t) \in [h_i, h_{i+1}]$ $(i = 1, 3, \ldots, N)$, the L-K function is constructed as follows:

$$\begin{aligned}
V_i(x(t)) = {} & V_{i1}(x(t)) + V_{i2}(x(t)) + V_{i3}(x(t))\\
& + V_{i4}(x(t)) + V_{i5}(x(t)),
\end{aligned} \tag{31}$$

where

$$
\begin{aligned}
V_{i1}(x(t)) &= x^T(t)P_1x(t) + \int_{t-h_i}^{t} x^T(s)\mathrm{d}sP_2\int_{t-h_i}^{t} x(s)\mathrm{d}s \\
&\quad + \int_{t-h_{i+1}}^{t-h_i} x^T(s)\mathrm{d}sP_3\int_{t-h_{i+1}}^{t-h_i} x(s)\mathrm{d}s + \int_{-h_i}^{0}\int_{t+\beta}^{t} x^T(s)\mathrm{d}s\mathrm{d}\beta P_4\int_{-h_i}^{0}\int_{t+\beta}^{t} x(s)\mathrm{d}s\mathrm{d}\beta \\
&\quad + \int_{-h_{i+1}}^{-h_i}\int_{t+\beta}^{t} x^T(s)\mathrm{d}s\mathrm{d}\beta P_5\int_{-h_{i+1}}^{-h_i}\int_{t+\beta}^{t} x(s)\mathrm{d}s\mathrm{d}\beta, \\
V_{i2}(x(t)) &= \int_{t+\beta}^{t} x^T(s)Q_1x(s)\mathrm{d}s + \int_{t-h_{i+1}}^{h_i} x^T(s)Q_2x(s)\mathrm{d}s, \\
V_{i3}(x(t)) &= h_i\int_{-h_i}^{0}\int_{t+\beta}^{t} x^T(s)X_1x(s)\mathrm{d}s\mathrm{d}\beta + h_i\int_{-h_i}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)X_2\dot{x}(s)\mathrm{d}s\mathrm{d}\beta \\
&\quad + (h_{i+1}-h_i)\int_{-h_{i+1}}^{-h_i}\int_{t+\beta}^{t} x^T(s)X_3x(s)\mathrm{d}s\mathrm{d}\beta + (h_{i+1}-h_i)\int_{-h_{i+1}}^{-h_i}\int_{t+\beta}^{t} \dot{x}^T(s)X_4\dot{x}(s)\mathrm{d}s\mathrm{d}\beta, \\
V_{i4}(x(t)) &= \left(\frac{h_i^2}{2}\right)\int_{-h_i}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} x^T(s)R_1x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta + \left(\frac{h_i^2}{2}\right)\int_{-h_i}^{0}\int_{\beta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta \\
&\quad + \left(\frac{\left(h_{i+1}^2-h_i^2\right)}{2}\right)\int_{-h_{i+1}}^{-h_i}\int_{\beta}^{0}\int_{t+\lambda}^{t} x^T(s)R_3x(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta + \left(\frac{\left(h_{i+1}^2-h_i^2\right)}{2}\right)\int_{-h_{i+1}}^{-h_i}\int_{\beta}^{0}\int_{t+\lambda}^{t} \dot{x}^T(s)R_4\dot{x}(s)\mathrm{d}s\mathrm{d}\lambda\mathrm{d}\beta, \\
V_{i5}(x(t)) &= \left(\frac{h_i^3}{6}\right)\int_{-h_i}^{0}\int_{\beta}^{0}\int_{\lambda}^{0}\int_{t+\varphi}^{t} \dot{x}^T(s)U_1\dot{x}(s)\mathrm{d}s\mathrm{d}\varphi\mathrm{d}\lambda\mathrm{d}\beta + \left(\frac{\left(h_{i+1}^3-h_i^3\right)}{6}\right)\int_{-h_{i+1}}^{-h_i}\int_{\beta}^{0}\int_{\lambda}^{0}\int_{t+\varphi}^{t} \dot{x}^T(s)U_2\dot{x}(s)\mathrm{d}s\mathrm{d}\varphi\mathrm{d}\lambda\mathrm{d}\beta,
\end{aligned}
\tag{32}
$$

where the definition of $\zeta(t)$ is the same as that in Lemma 3. $P_i\,(i = 1,2,3,4,5)$, Q_1, Q_2, Q_3, U_1, U_2, X_j, and $R_j\,(j = 1,2,3,4)$ are the matrices defined in the same formula (9). The same method is available. The following conclusions can be reached by the same method:

$$
\dot{V}_i(x(t)) \le \zeta^T(t)\left[\alpha\Gamma_{i1} + (1-\alpha)\Gamma_{i2} + \varepsilon\Gamma_{i3} + (1-\varepsilon)\Gamma_{i4}\right]\zeta(t), \tag{33}
$$

where

$$
\begin{aligned}
\Gamma_{i1} &= \frac{\Phi}{2} - e_7X_3e_7^T - (e_2-e_4)X_4\left(e_2^T-e_4^T\right), \\
\Gamma_{i2} &= \frac{\Phi}{2} - e_6X_3e_6^T - (e_3-e_2)X_4\left(e_3^T-e_2^T\right), \\
\Gamma_{i3} &= \frac{\Phi}{2} - e_{10}R_3e_{10}^T - ((h_{i+1}-h_i)e_1-e_7)R_4\left((h_{i+1}-h_i)e_1^T-e_7^T\right), \\
\Gamma_{i4} &= \frac{\Phi}{2} - e_9R_3e_9^T - ((h_{i+1}-h_i)e_1-e_6)R_4\left((h_{i+1}-h_i)e_1^T-e_6^T\right).
\end{aligned}
\tag{34}
$$

In the same way, it is known that there exists a sufficient small positive number δ_i to make $\dot{V}_t(t) < -\delta\|x(t)\|^2$ hold, and then the nominal system (8) is asymptotically stable.

The combination of (25) and (33) is equivalent to (9). This fulfills the proof.

Remark 1. Firstly, different from [15], in which the delay range was divided into two equidistant subintervals, new LKF comprising quadruple-integral term and quadratic forms of double-integral term was constructed. In this paper, for each subinterval, the time-delay interval is divided into N equal parts by using the method of time-delay partitioning. A new LKF with four integral terms is designed for each partitioned interval, and the quadratic form of double integral is introduced, such as $\iint x^T(s)\mathrm{d}s\mathrm{d}\beta M \iint x(s)\mathrm{d}s\mathrm{d}\beta$. Although the double integral functional term $\int_{-h}^{0}\int_{t+\beta}^{t} x(s)\mathrm{d}s\mathrm{d}\beta$ is also used in [1, 10], it is not introduced into the definition of augmented vector. Secondly, the triple integral functional term integrand used in the new LKF contains the state vector x, and the lower bound information of the delay interval is introduced. Thanks to the coexistence of the quadratic integral functional term and the quadratic term $\int x^T(s)\mathrm{d}s\mathrm{d}\beta$, the conservativeness of the stability conclusion is significantly reduced.

Remark 2. In formula (9), the new stability criterion does not involve redundant free-weight matrices but skillfully uses Wirtinger-based integral inequality to define the cross terms generated by LKF derivatives and uses a few free matrices to represent the relationship between the relevant terms. Therefore, the complexity of theoretical derivation and computation is reduced, and the conservatism of conclusions is reduced.

Remark 3. For a given scalar μ and time-delay rate $\dot{h}(t)$ satisfying $0 < \dot{h}(t) \leq \mu$, substituting the functional term $\int_{t-h(t)}^{t} x^T(s)Q_3x(s)\mathrm{d}s$ into the LKF constructed, the stability criterion containing the time-delay rate μ can be obtained according to the proof process of Theorem 1. The form is shown in Theorem 2.

Theorem 2. *For the scalars h_m, h_M, and μ, λ_1,λ_2 ($\lambda_1 > \lambda_2$), it is asymptotically stable for the nominal system (8), if there exist positive definite symmetric matrices $P_i\,(i = 1,2,3,4,5)$, Q_1, Q_2, Q_3, U_1, U_2, X_j, and $R_j\,(j = 1,2,3,4)$, such that the following LMIs hold:*

$$\widetilde{\Phi} = \left(\widetilde{\Phi}_{i,j}\right)_{10\times 10} < 0, \tag{35}$$

where $\widetilde{\Phi}_{11} = \Phi_{11} + Q_3$ and $\widetilde{\Phi}_{22} = \Phi_{22} - \mu Q_3$; other items in $\widetilde{\Phi}$ are defined the same as in Φ Theorem 1.

Next, the robust stability of uncertain systems with interval time-varying delays (1) is considered.

Theorem 3. *For the scalars $0 < h_m < h_M$ and μ, λ_1,λ_2 ($\lambda_1 > \lambda_2$), it is asymptotically stable for the uncertain system (1), if there exist positive definite symmetric matrices $P_i\,(i = 1,2,3,4,5)$, Q_1, Q_2, Q_3, U_1, U_2, X_j, and $R_j\,(j = 1,2,3,4)$, the scalar $\delta > 0$, and the free matrices with suitable dimension T_1, T_2, such that the following LMIs hold:*

$$\begin{bmatrix} \widetilde{\Phi} & \Gamma_1 D & \delta\Gamma_2^T \\ * & -\delta I & 0 \\ * & * & -\delta I \end{bmatrix} < 0, \tag{36}$$

where

$$\begin{aligned} \Gamma_1 &= \begin{bmatrix} T_1^T & 0 & 0 & 0 & 0 & 0 & T_2^T \end{bmatrix}, \\ \Gamma_2 &= \begin{bmatrix} E_a & 0 & E_b & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{37}$$

Proof. For the uncertain system (1), A and B in equation (9) are replaced by $A + \Delta A$ and $B + \Delta B$, respectively. According to the proof of Theorem 1, the asymptotic stability of system (1) is obtained. This fulfills the proof.

4. Numerical Examples

The following three numerical examples are used to compare the results of the existing literature with the method proposed in this paper. MADB (Maximum Allowable Delay Bound) is defined as the upper bound of the maximum allowable delay to ensure the stability of the system, and it is the most common criterion to compare the conservativeness of the stability conclusions of time-delay systems.

Example 1. First consider the following closed-loop control systems with interval time-varying delays:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 0 & 0 \\ -1 & 1 \end{bmatrix} x(t - h(t)). \tag{38}$$

For given h_m, according to (35) in Theorem 2 and (9) in Theorem 1, Tables 1 and 2 give corresponding MADB from two aspects, $\mu = 0.3$ and $\mu = any$, respectively. It can be clearly seen from Tables 1 and 2 that the method proposed in this paper is obviously better than the conclusion in the existing literature.

To verify the validity of the results, given $\mu = 0.3$, $h_m = 1$, and $h_M = 3.0796$ and given initial condition $\mathbf{x}(t) = \begin{bmatrix} 2 & -2 \end{bmatrix}^T$, the state response curve of $\mathbf{x}(t)$ is shown in Figure 1. It can be seen that the state trajectory of the abovementioned system can quickly reach a stable state under the action of the obtained MADB, which further verifies the correctness of the proposed stability criterion.

Example 2. Uncertain systems with interval time-varying delays are considered:

$$\begin{aligned} \dot{x}(t) = &\begin{bmatrix} -2 + \lambda_1 & 0 \\ 0 & -1 + \lambda_1 \end{bmatrix} x(t) \\ &+ \begin{bmatrix} -1 + \lambda_3 & 0 \\ -1 & -1 + \lambda_4 \end{bmatrix} x(t - h(t)), \end{aligned} \tag{39}$$

where λ_1, λ_2, λ_3, and λ_4 are unknown parameters satisfying $|\lambda_1| \leq 1.6$, $|\lambda_2| \leq 0.05$, $|\lambda_3| \leq 0.1$, $|\lambda_4| \leq 0.3$.

For given h_m, according to (36) in Theorem 3, Table 3 gives corresponding MADB in the simulation. From the comparison results, it can be seen that, for this example, this method improves the conclusions of the existing literature.

Given the initial condition $\mathbf{x}(t) = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}^T$, the state response curve of $\mathbf{x}(t)$ is shown in Figure 2 when the constant of time-varying delay $h(t)$ is 1.4723. When $h(t)$ takes variable $1.51 + 0.51\sin t$, the state response curve of $\mathbf{x}(t)$ is shown in Figure 3. It can be seen that $\mathbf{x}(t)$ can quickly reach a stable state under the action of the nonlinear disturbance and the obtained MADB, thus verifying the correctness of the proposed stability criterion.

Table 1: In Example 1, MADB is simulated to be obtained for different h_m and different methods.

μ	Method	$h_m = 1$	$h_m = 2$	$h_m = 3$
0.3	Literature [24]	2.4042	2.5870	3.4766
	Literature [18]	2.4328	2.6322	——
	Literature [11] ($N=2$)	2.5278	3.0744	3.9136
	Literature [11] ($N=3$)	2.7368	3.4836	4.2857
	Literature [25]	**3.16**	3.50	4.32
	Theorem 2	3.0796	**3.9064**	**4.4152**

Table 2: In Example 1, when $\mu = any$, MADB is simulated to be obtained for different h_m and different methods.

Method	$h_m = 0.3$	$h_m = 0.5$	$h_m = 0.8$
Literature [13] ($N=2$)	1.1677	1.3078	1.5333
Literature [13] ($N=4$)	1.2043	1.3429	1.5633
Literature [18]	1.3531	1.4663	1.6592
Literature [26]	1.4347	1.5336	1.7140
Literature [14]	1.6837	1.8120	2.0209
Literature [27]	1.78	1.81	1.90
Theorem 1	**1.9236**	**2.1384**	**2.2473**

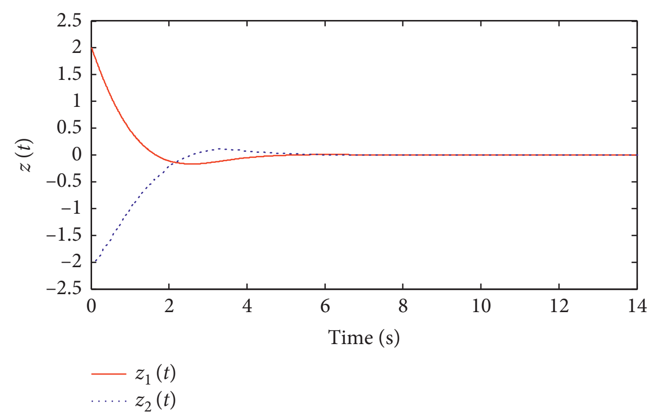

Figure 1: State response curve of $\mathbf{x}(t)$ when $h_M = 3.0796$.

Example 3. Consider another uncertain system with interval time-varying delays. The system parameters are as follows:

$$A = \begin{bmatrix} -0.4 & 0 \\ 0 & -1 \end{bmatrix},$$
$$B = \begin{bmatrix} -0.9 & 0 \\ -1 & -0.7 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$E_{a=} = E_b = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}. \quad (40)$$

Table 3: In Example 2, MADB is simulated to be obtained for different h_m and different methods.

Method	$h_m = 0.2$	$h_m = 0.4$	$h_m = 0.6$
Literature [10] ($N=2$)	1.1337	1.1703	1.2123
Literature [28] ($N=2$)	1.1783	1.2123	1.2527
Literature [28] ($N=4$)	1.1871	1.2246	1.2686
Literature [11] ($N=2$)	1.3369	1.3571	1.3817
Literature [11] ($N=3$)	1.3809	1.4003	1.4216
Theorem 3	**1.4241**	**1.4413**	**1.4723**

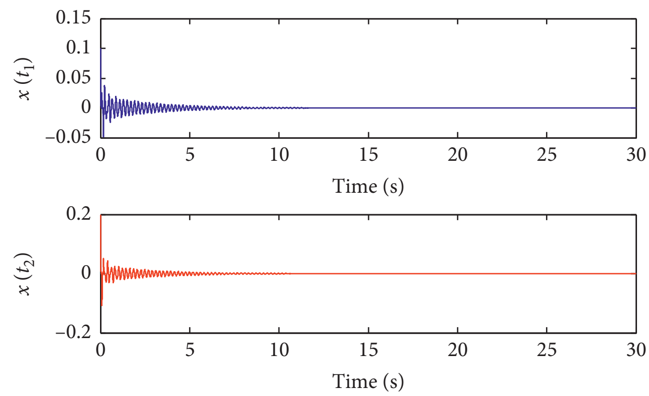

Figure 2: State response curve of $\mathbf{x}(t)$ when $h(t) = 1.4723$.

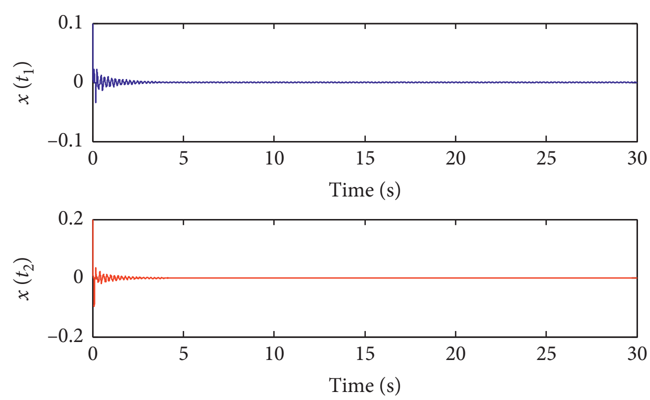

Figure 3: State response curve of $\mathbf{x}(t)$ when $h(t) = 1.51 + 0.51 \sin t$.

Table 4: In Example 3, when $\mu = any$, MADB is simulated to be obtained for different h_m and different methods.

Method	$h_m = 0$	$h_m = 0.4$	$h_m = 0.8$
Literature [9]	1.0571	1.1385	1.2392
Literature [10] ($N=2$)	1.1030	1.1703	1.2594
Literature [11] ($N=2$)	1.3213	1.3571	1.4102
Literature [11] ($N=3$)	1.3634	1.4003	1.4445
Theorem 3	**1.4127**	**1.4594**	**1.4987**

Similarly, according to (36) in Theorem 3, for given h_m and $\mu = \text{any}$, Table 4 gives corresponding MADB in the simulation. As can be seen from Table 4, the robust stability theorem proposed in this paper enlarges the upper bound of the maximum allowable delay to guarantee the stability of the system. It has lower conservatism.

5. Conclusion

In this paper, we study the robust stability of a class of uncertain systems with interval time-varying delays. A new stability criterion based on LMI is proposed by constructing a new LKF containing a generalized term of quadruple integral. In order to improve the computational efficiency and simplify the conclusion, the criterion avoids the use of model transformation and free weight matrix definition techniques. Instead, Wirtinger-based integral inequalities and interactive convex combination techniques with tighter definition techniques are adopted, which make full use of the lower bound information of the delay and obtain a lower conservative conclusion. Finally, numerical simulations show that the proposed criterion enlarges the upper bound of the maximum delay allowed to guarantee the stability of the system and is more competitive than the existing methods.

However, the new stability criterion proposed in this paper mainly focuses on a class of linear systems with norm-bounded uncertainty and interval time-varying delay. How to get the similar conclusion for nonlinear system is another interesting topic and the next work for us; and some related researches are hopeful to supply reference to us [29, 30].

Data Availability

Three numerical examples are used to compare the results of the existing literature with the method proposed in this paper.

Conflicts of Interest

The authors declare that there are no conflicts of interest with respect to the research, authorship, and/or publication of this article.

Acknowledgments

This research was partially supported by the Young Scientists Fund of the National Natural Science Foundation of China (Grant no. 61503391), China Postdoctoral Fund under Grant no. 2017T100770, the Key Laboratory Fund under Grant no. 6142003190204, and the Special Scientific Research Program of Department of Education of Shaanxi Province (Grant no. 20JK0728).

References

[1] A. Farnam and R. Mahboobi Esfanjani, "Improved stabilization method for networked control systems with variable transmission delays and packet dropout," *ISA Transactions*, vol. 53, no. 6, pp. 1746–1753, 2014.

[2] W. Chen, S. Xu, Y. Li, and Z. Zhang, "Stability analysis of neutral systems with mixed interval time-varying delays and nonlinear disturbances," *Journal of the Franklin Institute*, vol. 357, no. 6, pp. 3721–3740, 2020.

[3] J. Zhang, C. Peng, and M. Zheng, "Improved results for linear discrete-time systems with an interval time-varying input delay," *International Journal of Systems Science*, vol. 47, no. 2, pp. 492–499, 2015.

[4] R. Mohajerpoor, L. Shanmugam, H. Abdi, R. Rakkiyappan, S. Nahavandi, and P. Shi, "New delay range-dependent stability criteria for interval time-varying delay systems via wirtinger-based inequalities," *International Journal of Robust and Nonlinear Control*, vol. 28, no. 2, pp. 661–677, 2018.

[5] P. Singkibud and K. Mukdasai, "On robust stability for uncertain neutral systems with non-differentiable interval time-varying discrete delay and nonlinear perturbations," *Asian-European Journal of Mathematics*, vol. 11, no. 1, pp. 2253–2261, 2018.

[6] B. L. Zhang, L. H. Cheng, K. J. Pan, and X. M. Zhang, "Reducing conservatism of stability criteria for linear systems with time-varying delay using an improved triple-integral inequality," *Applied Mathematics & Computation*, vol. 380, 2020.

[7] C. Shen, Y. Li, X. Zhu, and W. Duan, "Improved stability criteria for linear systems with two additive time-varying delays via a novel Lyapunov functional," *Journal of Computational and Applied Mathematics*, vol. 363, pp. 312–324, 2020.

[8] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhäuser, Basel, Switzerland, 2003.

[9] K. Ramakrishnan and G. Ray, "Delay-dependent robust stability criteria for linear uncertain systems with interval time varying delay," in *TENCON 2009-2009 IEEE Region 10 Conference*, IEEE, Singapore, January 2009.

[10] K. Ramakrishnan and G. Ray, "Robust stability criteria for uncertain linear systems with interval time-varying delay," *Journal of Control Theory and Applications*, vol. 9, no. 4, pp. 559–566, 2011.

[11] H. X. Zhang, J. J. Hui, X. Zhou, and G. L. Li, "New robust stability criteria for uncertain systems with interval time-varying delay based on delay-partitioning approach," *Control and Decision*, vol. 29, no. 5, pp. 907–912, 2014.

[12] F. Gouaisbaut and A. Seuret, "Wirtinger-based integral inequality: application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, 2013.

[13] X.-L. Zhu, G.-H. Wang, and G. H. Yang, "New stability criteria for continuous-time systems with interval time-varying delay," *IET Control Theory & Applications*, vol. 4, no. 6, pp. 1101–1107, 2010.

[14] J. An, Z. Li, and X. Wang, "A novel approach to delay-fractional-dependent stability criterion for linear systems with interval delay," *ISA Transactions*, vol. 53, no. 2, pp. 210–219, 2014.

[15] Y. B. Wu, H. X. Zhang, X. X. Hu, J. J. Hui, and G. L. Li, "Novel robust stability condition for uncertain neutral systems with mixed time-varying delays," *Advances in Mechanical Engineering*, vol. 9, no. 10, pp. 1–11, 2017.

[16] Y. B. Li and X. Q. Xue, "Stability of uncertain neutral system with mixed time delays based on reciprocally convex combination approach," *Control and Decision*, vol. 31, no. 6, pp. 1105–1110, 2016.

[17] A. Farnam and R. Mahboobi Esfanjani, "Improved linear matrix inequality approach to stability analysis of linear systems with interval time-varying delays," *Journal of Computational and Applied Mathematics*, vol. 294, pp. 49–56, 2016.

[18] L. Ding, Y. He, M. Wu, and Z. Zhang, "A novel delay partitioning method for stability analysis of interval time-varying delay systems," *Journal of the Franklin Institute*, vol. 354, no. 2, pp. 1209–1219, 2017.

[19] S. Senthilraj, R. Raja, Q. Zhu, R. Samidurai, and Z. Yao, "New delay-interval-dependent stability criteria for static neural

networks with time-varying delays," *Neurocomputing*, vol. 186, pp. 1–7, 2016.

[20] J. Cheng, H. Wang, S. Chen, Z. Liu, and J. Yang, "Robust delay-derivative-dependent state-feedback control for a class of continuous-time system with time-varying delays," *Neurocomputing*, vol. 173, pp. 827–834, 2016.

[21] C.-K. Zhang, Y. He, L. Jiang, M. Wu, and H.-B. Zeng, "Stability analysis of systems with time-varying delay via relaxed integral inequalities," *Systems & Control Letters*, vol. 92, pp. 52–61, 2016.

[22] X.-H. Chang, J. Xiong, and J. H. Park, "Fuzzy robust dynamic output feedback control of nonlinear systems with linear fractional parametric uncertainties," *Applied Mathematics and Computation*, vol. 291, pp. 213–225, 2016.

[23] X. H. Chang, L. Zhang, and J. H. Park, "Robust static output feedback hinf control for uncertain fuzzy systems," *Fuzzy Sets and Systems*, vol. 273, pp. 87–104, 2015.

[24] L. V. Hien and H. Trinh, "An enhanced stability criterion for time-delay systems via a new bounding technique," *Journal of the Franklin Institute*, vol. 352, no. 10, pp. 4407–4422, 2015.

[25] W. Qian, Y. Gao, Y. Chen, and J. Yang, "The stability analysis of time-varying delayed systems based on new augmented vector method," *Journal of the Franklin Institute*, vol. 356, no. 3, pp. 1268–1286, 2019.

[26] O. M. Kwon, M. J. Park, J. H. Park, and S. M. Lee, "Enhancement on stability criteria for linear systems with interval time-varying delays," *International Journal of Control, Automation and Systems*, vol. 14, no. 1, pp. 12–20, 2016.

[27] P. G. Park, W. I. Lee, and S. Y. Lee, "Improved stability criteria for linear systems with interval time-varying delays: generalized zero equalities approach," *Applied Mathematics and Computation*, vol. 292, pp. 336–348, 2017.

[28] J.-J. Hui, H.-X. Zhang, X.-Y. Kong, and X. Zhou, "On improved delay-dependent robust stability criteria for uncertain systems with interval time-varying delay," *International Journal of Automation and Computing*, vol. 12, no. 1, pp. 102–108, 2015.

[29] L. Ma, G. Zong, X. Zhao, and X. Huo, "Observed-based adaptive finite-time tracking control for a class of nonstrict-feedback nonlinear systems with input saturation," *Journal of the Franklin Institute*, vol. 357, no. 16, pp. 11518–11544, 2020.

[30] L. Ma, N. Xu, X. Huo, and X. Zhao, "Adaptive finite-time output-feedback control design for switched pure-feedback nonlinear systems with average dwell time," *Nonlinear Analysis: Hybrid Systems*, vol. 37, Article ID 100908, 2020.