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Research Article

Observer-Based Adaptive Fuzzy Predefined Performance Control of a Class of Nonlinear Pure-Feedback Systems with Input Delay

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This paper presents a problem of observer-based adaptive fuzzy predefined performance control of a class of nonlinear pure-feedback systems with input delay and unknown control direction. Compared with the existing research, a novel predefined performance controller is proposed, which relaxes the assumption that the initial error is known. In addition, it is difficult to design the controllers due to input delay and nonaffine properties of the pure-feedback systems, which can be simplified by Pade approximation. Moreover, dynamic surface control and Nussbaum functions are applied to overcome the calculation explosion caused by repeated differentiations and unknown control direction, respectively. Based on the above methods, an adaptive fuzzy predefined performance controller is proposed, and it is proved that all the signals of a closed-loop system are semiglobally uniformly ultimately bounded (SGUUB). The tracking errors converge within a predefined range, while the observer estimation errors converge within a small zero region. Finally, the simulation results demonstrate the effectiveness of the proposed control system.

1. Introduction

In the past years, the adaptive nonlinear systems based on the backstepping method has matured increasingly and received widespread attention in [1, 2]. At an earlier time, there was an unmodeled nonlinear problem in the above system, which greatly limited the application of this technology. To solve the above problems, fuzzy logic systems (FLSs) and neural networks (NNs) were applied extensively to approximate unknown nonlinear function in [3–7]. However, the characteristic of the backstepping method is a class of recursive design procedures coupled with Lyapunov function candidates; hence, the repeated differentiation of virtual controller leads to the complexity explosion problem. Afterwards, the dynamic surface control (DSC) technology was integrated into the backstepping method to solve this problem in [8–10]. In addition, since the unmeasurable state in the application has a great restriction, the state observer was employed to estimate the unmeasured state in [11–15]. Among them, an equivalent output injection sliding mode observer was proposed in [12], which could estimate the status of each follower and its neighbor. And high gain observer was used to estimate the position, course, and speed of the vessel in [13]. In recent years, observer-based adaptive fuzzy control with the DSC technology was investigated in [16–18].

It is well known that different from strict-feedback systems, pure-feedback systems have nonaffine structure of the variables, which presents more challenges to the controller design. Fortunately, the mean value theorem was proposed to solve the variables coupling problem of nonaffine structures in [19, 20]. Moreover, pure-feedback systems usually have the problem of unknown input control direction, which could be solved by Nussbaum functions in [21–24]. In addition, the input and output of the control systems have many restrictions, such as input saturation, dead zone, and input delay in [25–33]. It is worth mentioning that an adaptive predictor incorporated with a high-order neural network observer was proposed to obtain the predictions of the future system states in pure-feedback systems in [28], which were applied in the control design to avoid the input delay and nonlinearities. Subsequently, the input delay was solved by Pade approximation technique and intermediate variables in the strict-feedback systems in [29–31], which simplified the controller design. However, to the best of the author's knowledge, the combination of input delay and pure-feedback systems was rarely considered. Therefore, the controller design of pure-feedback systems with input delay is complicated, which needs to be further developed.

On the other side, the predefined control performance is better able to achieve the desired performance, such as overshoot, convergence rate, and convergence accuracy. Therefore, the prescribed performance control was proposed, which can satisfy preset transient and steady-state tracking performance in [34–36]. In particular, an adjustable finite-time prescribed performance function with fast convergence speed was adopted in [37, 38], which ensures real-time adjustment of controller parameters cased by the tracking error. Although the research of the prescribed performance control method is approaching maturity, there was still limitation of unknown initial values. Fortunately, a predefined performance function with time-varying design parameters was proposed to reduce the impact of unknown initial tracking error in [39]. However, it is not applied to the unknown nonlinear pure-feedback systems. In summary, the existing predefined performance control methods are insufficient to deal with a class of nonlinear pure-feedback systems with input delay. Therefore, the controller design for the above conditions needs to be developed.

Based on the above discussion, this article presents a method for observer-based adaptive fuzzy predefined performance control of a class of nonlinear pure-feedback systems with unknown control direction and input delay. State observer and FLSs are proposed to solve the problem of approximate unmeasurable state and unknown nonlinear functions, respectively. Compared with the existing literature, the main contributions of this paper are as follows:

(1) In the existing literatures [34, 37], the initial values in predefined performance control are assumed to be known. In order to relax that assumption, a novel predefined performance control method is proposed, which is a variable-parameter scheme independent of the initial error. Therefore, the restriction of the unknown initial error in the predefined performance control is solved.

(2) Compared with [36], the input delay is introduced into the pure-feedback systems. It is difficult to design the controllers due to input delay and non-affine properties of the pure-feedback systems, which can be simplified by Pade approximation and mean value theorem, respectively.

(3) By combining DSC technology and backstepping method, the issue of complexity explosion caused by repeated differentiations of some intermediate variables is eliminated. And the Nussbaum functions are proposed to solve unknown control direction.

The framework of this article is as follows. In Section 2, preliminaries and problem formulation are presented. In Section 3, an observer-based adaptive fuzzy predefined performance controller is designed of a class of nonlinear pure-feedback systems with unknown control direction and input delay, and the stability analysis is given. In Section 4, an example simulation is given to verify the feasibility of the proposed method. Finally, the Section 5 is the conclusions and the prospect of the future work.

Notations: $\mathfrak{R}$ denotes the set of real numbers, $\mathfrak{R}^i$ denotes the i-dimensional vector space, and R_+ is the set of all nonnegative real numbers. $\|\cdot\|$ indicates the Euclidean norm of vectors or matrix. For a matrix X, X^T indicates its transpose and X^{-1} indicates its inverse. For a matrix Q, $\lambda_{\min}(Q)$ stands for the smallest eigenvalue of Q and $\lambda_{\max}(Q)$ stands for the largest eigenvalue of Q.

2. Preliminaries and Problem Formulation

2.1. System Descriptions and Assumptions. A class of nonlinear pure-feedback systems with input delay is considered as

$$\begin{cases} \dot{x}_i = f_i(\underline{x}_i, x_{i+1}) + d_i(t), \quad 1 \le i \le n-1, \\ \dot{x}_n = f_n(\underline{x}_n, u(t-\delta)) + d_n(t), \\ y = x_1, \end{cases} \tag{1}$$

where $\underline{x}_i = [x_1, \ldots, x_i]^T \in \mathfrak{R}^i$ are the state vector, $y \in \mathfrak{R}$ is the output, $f_i(\underline{x}_i, x_{i+1}), f_n(\underline{x}_n, u(t-\delta))$ are unknown smooth functions, $d_i(t)$ means unknown and bounded external disturbance inputs, and δ denotes the input delay, which is an small unknown positive constant caused by network delay. Moreover, the output y is measurable.

Because of the coupling between states x_{i+1} and $u(t-\delta)$ in smooth functions $f_i(\underline{x}_i, x_{i+1})$ and $f_n(\underline{x}_n, u(t-\delta))$, which makes the desired control objectives difficult to design, the mean value theorem is used as

$$\begin{aligned} f_i(\underline{x}_i, x_{i+1}) &= f_i(\underline{x}_i, 0) + g_i(\underline{x}_i, x_{i+1})x_{i+1}, \\ f_n(\underline{x}_n, u(t-\delta)) &= f_n(\underline{x}_n, 0) + g_n(\underline{x}_n, u(t-\delta))u(t-\delta), \end{aligned} \tag{2}$$

where $g_i(\underline{x}_i, x_{i+1}) = \partial f_i(\underline{x}_i, x_{i+1})/\partial x_{i+1}|_{x_{i+1}^0}$ and $g_n(\underline{x}_n, u(t-\delta)) = \partial f_n(\underline{x}_n, u(t-\delta))/\partial u(t-\delta)|_{u_{(t-\delta)}^0}$, x_{i+1}^0 is certain point between zero and x_{i+1}, and $u_{(t-\delta)}^0$ is certain point between zero and $u(t-\delta)$. Let $f_i(\underline{x}_i, 0) = f_i(\underline{x}_i)$, $f_n(\underline{x}_n, 0) = f_n(\underline{x}_n)$, $g_i(\underline{x}_i, x_{i+1}) = g_i$, and $g_n(\underline{x}_n, u(t-\delta)) = g_n$.

Substituting (2) into system (1), one can obtain as

$$\begin{cases} \dot{x}_i = f_i(\underline{x}_i) + g_i x_{i+1} + d_i(t), \\ \dot{x}_n = f_n(\underline{x}_n) + g_n u(t-\delta) + d_n(t), \\ y = x_1. \end{cases} \tag{3}$$

To get the actual control input u by removing the effect of input delay δ, the Pade approximation method and the delay theorem of Laplace transform can be used, which solve the analysis complexity problem caused by time delay, and it follows that

$$\ell\{u(t-\delta)\} = e^{-\delta s}\ell(u(t)) \approx \frac{1-\delta s/2}{1+\delta s/2}\ell\{u(t)\}, \tag{4}$$

where s represents the Laplace variable and $\ell\{u(t)\}$ is the Laplace transform of $u(t)$.

Define the intermediate variable x_{n+1} as

$$\frac{1-\delta s/2}{1+\delta s/2}\ell\{u(t)\} = \ell\{x_{n+1}\} - \ell\{u(t)\}. \tag{5}$$

By transforming formula (5), one can be given as

$$2\ell\{u(t)\} = \ell\{x_{n+1}(t)\} + \frac{\delta s}{2}\ell\{u(t)\}. \tag{6}$$

By taking the inverse Laplace transform,

$$\dot{x}_{n+1} = 2\beta u - \beta x_{n+1}, \tag{7}$$

where $\beta = 2/\delta$ is a variable.

Remark 1. Pade approximation has been used in [31]. In this article, since the Pade approximation is applied to solve a class of small time delay problems, $e^{-\delta s}$ is approximately equal to $1-\delta s/2/(1+\delta s/2)$ when the time delay is very small. And the intermediate variable x_{n+1} is not a real variable of system (1), which can be viewed as an error variable. And this has been verified in the simulation in [31].

By using the above methods, (3) can be further written as

$$\begin{cases} \dot{x}_i = f_i(\underline{x}_i) + g_i x_{i+1} + d_i(t), \\ \dot{x}_n = f_n(\underline{x}_n) + g_n x_{n+1} - g_n u + d_n(t), \\ \dot{x}_{n+1} = -\beta x_{n+1} + 2\beta u, \\ y = x_1. \end{cases} \tag{8}$$

Assumption 1 (see [9]). The expected signal y_d and its derivatives $\dot{y}_d$ and $\ddot{y}_d$ are all known and bounded, which is $\Upsilon = \{y_d, \dot{y}_d, \ddot{y}_d\colon y_d^2 + \dot{y}_d^2 + \ddot{y}_d^2 \le \overline{Y}\}$, where $\overline{Y}$ is a positive constant.

Assumption 2 (see [20]). The sign of g_i is unknown, but g_i has the same sign and its public super bound is known, which is $0 < |g_i| < g^*$.

Assumption 3. The disturbance d_i is bounded to a positive constant d_i^*, that is, $|d_i| \le d_i^*$.

Assumption 4 (see [27]). There is a known constant s_i that satisfies $f_i(\underline{x}_i) - f_i(\widehat{\underline{x}}_i) \le s_i\|\underline{x}_i - \widehat{\underline{x}}_i\|$, where $\widehat{\underline{x}}_i = [\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_i]^T$ is the estimate of $\underline{x}_i = [x_1, x_2, \ldots, x_i]^T$, and $\|X\|$ represents the 2 norm of the vector X.

2.2. Fuzzy Logic Systems. Because the nonlinear function is unknown, FLSs is proposed. Build FLSs with the if-then rules.

R^q: if x_1 is F_1^q and x_2 is F_2^q and … and x_n is F_n^q. Then, y is $B^q, q = 1, 2, \ldots, a$. Here, $x = [x_1, \ldots, x_n]^T$ and y are the FLS input and output, respectively. Fuzzy sets F_i^q and B^q, associated with the fuzzy functions $\mu_{F_i^q}(x_i)$ and $\mu_{B^q}(y)$, respectively. a is the rules number. Thus, FLS can be calculated by formula

$$y(x(t)) = \frac{\sum_{q=1}^{a} \widetilde{y}_q \prod_{i=1}^{n} \mu_{F_i^q}(x_i)}{\sum_{q=1}^{a}\left(\prod_{i=1}^{n} \mu_{F_i^q}(x_i)\right)}, \tag{9}$$

where $\widetilde{y}_q = \max_{y\in R}\mu_{B^q}(y)$.

Let $\varphi_q = \prod_{i=1}^{n}\mu_{F_i^q}(x_i)/\sum_{q=1}^{a}(\prod_{i=1}^{n}\mu_{F_i^q}(x_i))$ and denote $\theta = [\widetilde{y}_1, \widetilde{y}_2, \ldots, \widetilde{y}_a]^T = [\theta_1, \theta_2, \ldots, \theta_a]^T$ and $\varphi^T(x) = [\varphi_1(x), \ldots, \varphi_a(x)]$; then, FLS can be rewritten as $y(x) = \theta^T\varphi(x)$.

Lemma 1 (see [40]). Let $f(x)$ be a continuous function defined on a compact set Ω. Then, for any constant $\varepsilon > 0$, there exists an FLS such as

$$\sup_{x\in\Omega}\left|f(x) - \theta^T\varphi(x)\right| \le \varepsilon. \tag{10}$$

Define the idealized parameter vector θ_i^* as

(i) $\theta_1^* = \arg\min_{\theta_1\in\Omega_1}[\sup_{x_1\in U_1}|\widehat{f}_1(x_1|t\theta_1) - f_1(x_1)|]$

(ii) $\theta_i^* = \arg\min_{\theta_i\in\Omega_i}[\sup_{\hat{\underline{x}}_i\in U_i}|\widehat{f}_i(\hat{\underline{x}}_i|t\theta_i) - f_i(\hat{\underline{x}}_i)|]$, $(i = 2, \ldots, n)$

Here, Ω_1, Ω_i, U_1, and U_i are compact for θ_1, θ_i, x_1, and $\hat{\underline{x}}_i$, respectively.

By Lemma 1, the nonlinear functions can be approximated by the following FLSs:

$$\begin{aligned} \widehat{f}_1(x_1 \mid \theta_1) &= \theta_1^T\varphi_1(x_1), \\ \widehat{f}_i(\hat{\underline{x}}_i \mid \widehat{\theta}_i) &= \widehat{\theta}_i^T\varphi_i(\hat{\underline{x}}_i), \quad i = 2, \ldots, n. \end{aligned} \tag{11}$$

The fuzzy minimum approximation errors can be defined as $\varepsilon_1(x_1) = f_1(x_1) - \widehat{f}_1(x_1|\theta_1^*)$ and $\varepsilon_i(\hat{x}_i) = f_i(\hat{x}_i) - \widehat{f}_i(\hat{\underline{x}}_i \mid \theta_i^*)$, where $\hat{\underline{x}}_i$ are the estimation of the state $\underline{x}_i$.

Assumption 5. The approximation error ε_i is bounded, and there is a constant ε_i^* that satisfies $|\varepsilon_i| \le \varepsilon_i^*$.

From (11), system (8) can be expressed as

$$\begin{cases} \dot{x}_i = g_i x_{i+1} + \theta_i^{*T}\varphi_i(\hat{\underline{x}}_i) + \varepsilon_i(\hat{\underline{x}}_i) + d_i + \Delta f_i, \\ \dot{x}_n = g_n x_{n+1} - g_n u + \theta_n^{*T}\varphi_n(\hat{\underline{x}}_n) + \varepsilon_n(\hat{\underline{x}}_n) + d_n + \Delta f_n, \\ \dot{x}_{n+1} = -\beta x_{n+1} + 2\beta u, \\ y = x_1, \end{cases} \tag{12}$$

where $\Delta f_i = f_i(\underline{x}_i) - f_i(\hat{\underline{x}}_i)$, $i = 2, \ldots, n$.

Rewriting (12) in the following formula,

$$\begin{cases} \dot{x} = Ax + \overline{F}^* + \varepsilon + d + \Delta f + E_n u, \\ y = E_0^T x, \end{cases} \tag{13}$$

where $x = [x_1, x_2, \ldots, x_n, x_{n+1}]^T$, $A = \begin{bmatrix} 0 & g_1 & 0 & \ldots & 0 & 0 \\ 0 & 0 & g_2 & \ldots & 0 & 0 \\ \vdots \\ 0 & 0 & 0 & \ldots & g_{n-1} & 0 \\ 0 & 0 & 0 & \ldots & 0 & g_n \\ 0 & 0 & 0 & \ldots & 0 & -\beta \end{bmatrix}$, $\varepsilon = [\varepsilon_1(x_1), \varepsilon_2(\underline{\widehat{x}}_2), \ldots, \varepsilon_n(\underline{\widehat{x}}_n), 0]^T$, $\overline{F}^* = [\theta_1^{*T}\varphi_1(x_1), \theta_2^{*T}\varphi_2(\underline{\widehat{x}}_2), \ldots, \theta_n^{*T}\varphi_n(\underline{\widehat{x}}_n), 0]^T$, $d = [d_1, d_2, \ldots, d_n, 0]^T$, $\Delta f = [0, \Delta f_2, \ldots, \Delta f_n, 0]^T$, $E_0^T = [1, 0, \ldots, 0]$, and $E_n^T = [0, \ldots, 0, -g_n, 2\beta]$.

2.3. Nussbaum-Type Function. A continuous function $N(\mu)$ is called the Nussbaum function if it has the following properties:

$$\begin{aligned} &\lim_{m\longrightarrow\infty} \sup \frac{1}{m} \int_0^m N(\mu)\mathrm{d}\mu = \infty, \\ &\lim_{m\longrightarrow\infty} \inf \frac{1}{m} \int_0^m N(\mu)\mathrm{d}\mu = -\infty, \end{aligned} \tag{14}$$

where m is the integral upper boundary. For instance, the frequently used continuous Nussbaum-type functions contain $\mu^2\cos(\mu), \mu^2\sin(\mu), e^{\mu^2}\cos(\mu)$, and so on. In this work, the continuous Nussbaum-type function $N(\mu) = \mu^2\cos(\mu)$ is utilized.

Lemma 2 (see [41, 42]). Smooth functions $V(\cdot)$ and $\mu(\cdot)$ are defined on $[0, t_f)$, where $V(t) \geq 0$ $(\forall\in [0, t_f))$ and $N(\mu)$ is a Nussbaum-type function. If the following inequality holds

$$V(t) \leq c \pm \int_0^t \sum_{l=1}^n (gN(\mu_l) + 1)\dot{\mu}_l \mathrm{d}\tau, \tag{15}$$

where g is a nonzero constant and c represents appropriate constant, then $V(t)$, $\mu(t)$, and $\int_0^t \sum_{l=1}^n (gN(\mu_l) + 1)\dot{\mu}_l d\tau$ must be bounded on $[0, t_f)$.

Remark 2. The parameters g_i and g_n are time-varying parameters and their signs are unknown. If the control direction changes rapidly, it is difficult to effectively guarantee the stability of the closed-loop system under the self-adaptive condition. Therefore, compared with the control direction which is assumed to be known, Assumption2 is more flexible in the application. In addition, similar to [20], this paper uses the Nussbaum functions to solve the control direction problem, which relaxes the prior knowledge.

2.4. Fuzzy State Observer Design. To estimate the unmeasurable states of the system, the corresponding fuzzy observer is designed as

$$\begin{cases} \dot{\widehat{x}}_i = g_i\widehat{x}_{i+1} + \theta_i^T\varphi_i(\underline{\widehat{x}}_i) + k_i(y - \widehat{x}_1), \\ \dot{\widehat{x}}_n = g_n\widehat{x}_{n+1} - g_n u + \theta_n^T\varphi_n(\underline{\widehat{x}}_n) + k_n(y - \widehat{x}_1), \\ \dot{\widehat{x}}_{n+1} = -\beta\widehat{x}_{n+1} + 2\beta u, \\ \widehat{y} = \widehat{x}_1. \end{cases} \tag{16}$$

Rewriting (16) in the following formula,

$$\begin{cases} \dot{\widehat{x}} = A_0\widehat{x} + Ky + \overline{F} + E_n u, \\ \widehat{y} = E_0^T\widehat{x}, \end{cases} \tag{17}$$

where $\widehat{x} = [\widehat{x}_1, \widehat{x}_2, \ldots, \widehat{x}_n, \widehat{x}_{n+1}]^T$, $A_0 = \begin{bmatrix} -k_1 & g_1 & 0 & \ldots & 0 & 0 & 0 \\ -k_2 & 0 & g_2 & \ldots & 0 & 0 & 0 \\ \vdots \\ -k_{n-1} & 0 & 0 & \ldots & 0 & g_{(n-1)} & 0 \\ -k_n & 0 & 0 & \ldots & 0 & 0 & g_n \\ 0 & 0 & 0 & \ldots & 0 & 0 & -\beta \end{bmatrix}$, $K = [k_1, \ldots, k_n, 0]^T$, and $\overline{F} = [\theta_1^T\varphi_1(x_1), \theta_2^T\varphi_2(\underline{\widehat{x}}_2), \ldots, \theta_n^T\varphi_n(\underline{\widehat{x}}_n), 0]^T$.

The observer gain matrix K is given such that A_0 is a Hurwitz matrix. Therefore, for any chosen positive definite matrix $Q = Q^T > 0$, there is a positive definite matrix $P = P^T > 0$ that satisfies

$$A_0^T P + PA_0 = -Q. \tag{18}$$

The observer errors can be obtained as

$$e = x - \widehat{x} = [e_1, \ldots, e_n]^T. \tag{19}$$

From (12), (16), and (17), the observer error is

$$\dot{e} = A_0 e + d + \varepsilon + \Delta f + \widetilde{\Theta}, \tag{20}$$

where $\widetilde{\theta}_i = \theta_i^* - \theta_i$ and $\widetilde{\Theta} = [\widetilde{\theta}_1^T\varphi_1(x_1), \ldots, \widetilde{\theta}_n^T\varphi_n(\underline{\widehat{x}}_n)]^T$.

Consider the Lyapunov function candidate as

$$V_0 = e^T Pe. \tag{21}$$

The time derivative of V_0 with (20) is

$$\begin{aligned} \dot{V}_0 &= \dot{e}^T Pe + e^T P\dot{e}, \\ &= \left(e^T A_0^T + d^T + \varepsilon^T + \Delta f^T + \widetilde{\Theta}^T\right)Pe \\ &\quad + e^T P\left[A_0 e + d + \varepsilon + \Delta f + \widetilde{\Theta}\right], \\ &= e^T\left[A_0^T P + PA_0\right]e + 2e^T P[d + \varepsilon + \Delta f + \widetilde{\Theta}], \\ &= -e^T Qe + 2e^T P[d + \varepsilon + \Delta f + \widetilde{\Theta}]. \end{aligned} \tag{22}$$

By using Young's inequality and Assumptions 3–5, the inequalities can be obtained as

$$2e^T P\varepsilon + 2e^T Pd \leq 2\|e\|^2 + \|P\|^2\left\|\varepsilon^*\right\|^2 + \|P\|^2\left\|d^*\right\|^2, \tag{23}$$

$$2e^T P\widetilde{\Theta} \leq \|e\|^2 + \|P\|^2 \sum_{l=1}^n \widetilde{\theta}_l^T\widetilde{\theta}_l, \tag{24}$$

$$2e^{T}P\Delta f \leq \|e\|^{2} + \|P\|^{2}\|\Delta f\|^{2} \leq \|e\|^{2} + \|P\|^{2}\left(\sum_{j=2}^{n} L_{j}^{2}\|e\|^{2}\right) = r_{0}\|e\|^{2}, \tag{25}$$

where $r_0 = 1 + \|P\|^2 \sum_{j=2}^{n} L_j^2$.

Substituting (23)–(25) into (22) yields

$$\begin{aligned} \dot{V}_0 \leq & -e^{T}Qe + (r_0 + 3)\|e\|^2 + \|P\|^2\left\|\varepsilon^*\right\|^2 \\ & + \|P\|^2\left\|d^*\right\|^2 + \|P\|^2 \sum_{l=1}^{n} \tilde{\theta}_l^{T}\tilde{\theta}_l. \end{aligned} \tag{26}$$

2.5. Tracking Error Transformation. A new variable parameter independent of the initial error is proposed, which satisfies the expected variable-parameter scheme tracking performance constraints. Then, the tracking error is defined as $E_1 = y - y_d$. Predefined performance control constraint will be obtained as inequality holds for all $t \geq 0$:

$$-\iota_d(t)\rho(t) < E_1(t) < \iota_u(t)\rho(t), \tag{27}$$

where smooth function $\iota_d(t)$ and $\iota_u(t)$ satisfies the follow properties. (1) $\iota_d(t) > 0$, $\iota_u(t) > 0$ and strictly decreasing. (2) $\lim_{t\longrightarrow 0} \iota_d(t) = +\infty$; $\lim_{t\longrightarrow\infty}\iota_d(t) = C_1, C_1 \in R_+$; $\lim_{t\longrightarrow 0}\iota_u(t) = +\infty$; and $\lim_{t\longrightarrow\infty}\iota_u(t) = C_2, C_2 \in R_+$.

In this article, $\iota_d(t)$ and $\iota_u(t)$ can be chosen as

$$\begin{cases} \dot{\iota}_d(t) = -\lambda_d\iota_d(t) + h_d, \\ \dot{\iota}_u(t) = -\lambda_u\iota_u(t) + h_u, \end{cases} \tag{28}$$

where λ_d, λ_u, h_d, and h_u are positive constants.

And $\rho(t)$ is an appropriate performance boundary function and is defined as $\rho(t) = (\rho(0) - \rho_\infty)e^{-\lambda t} + \rho_\infty$, which satisfies the following. (1) $\rho(t)$ is positive and strictly decreasing. (2) $\lim_{t\longrightarrow\infty}\rho(t) = \rho_\infty > 0$.

Integrating equality (28) over $[0, t)$, we have

$$\iota_d(t) = \left[\iota_d(0) - \frac{h_d}{\lambda_d}\right]e^{-\lambda_d t} + \frac{h_d}{\lambda_d}. \tag{29}$$

In the same way,

$$\iota_u(t) = \left[\iota_u(0) - \frac{h_u}{\lambda_u}\right]e^{-\lambda_u t} + \frac{h_u}{\lambda_u}. \tag{30}$$

By the above analysis, $\iota_d(t)$ and $\iota_u(t)$ converge exponentially to constants h_d/λ_d and h_u/λ_u, and the convergence rate can be improved by adding λ_d and λ_u. When $\iota_d(t)$ and $\iota_u(t)$ converge to constant values, inequality constraint (27) is degenerated as follows:

$$-\frac{h_d}{\lambda_d}\rho(t) < E_1(t) < \frac{h_u}{\lambda_u}\rho(t). \tag{31}$$

According to (31), when the system is stable, the upper bound of the steady-state error is $\max\{(h_d/\lambda_d), (h_u/h_u)\}\rho(\infty)$, and the error convergence speed and the maximum overpass can be adjusted by the coefficient λ_d, λ_u, h_d, h_u, and $\rho(t)$.

The inequality constraint is transformed into equality constant, and the error transformation function $\phi(z, \iota_d, \iota_u)$ is defined as

$$E_1(t) = \rho(t)\phi(z, \iota_d, \iota_u), \tag{32}$$

where z is transform error, and the continuous function $\phi(z, \iota_{\text{down}}, \iota_{\text{up}})$ satisfies the following properties. (1) $\phi(z, \iota_d, \iota_u)$ is smooth and strictly increasing; (2) $-\iota_d(t) < \phi(z, \iota_d, \iota_u) < \iota_u(t)$; (3) $\lim_{z\longrightarrow-\infty}\phi(z, \iota_d, \iota_u) = -\iota_d(t)$; $\lim_{z\longrightarrow+\infty}\phi(z, \iota_d, \iota_u) = \iota_u(t)$.

By the properties of function $\phi(z, \iota_d, \iota_u)$, the inverse transformation is

$$z_1 = \phi^{-1}\left[\frac{E_1(t)}{\rho(t)}, \iota_d, \iota_u\right]. \tag{33}$$

The error transformation function is defined as

$$\phi[z, \iota_d, \iota_u] = \frac{\iota_u e^{z} - \iota_d e^{-z}}{e^{z} + e^{-z}}. \tag{34}$$

Therefore, by (32), the boundless of the error transformation function z_1 results in the prescribed performance of the tracking error E_1.

Differentiating (33), it yields

$$\dot{z}_1(t) = \frac{\partial\phi^{-1}}{\partial(E_1/\rho)}\frac{\partial(E_1/\rho)}{\partial t} = \frac{\partial\phi^{-1}}{\partial(E_1/\rho)}\left(\dot{E}_1 - \frac{E_1\dot{\rho}}{\rho}\right). \tag{35}$$

Combining (35) and (12), one can obtain as

$$\dot{z}_1 = \varpi\left(g_1(t)x_2 + \theta_1^{*T}\varphi_1(x_1) + \varepsilon_1(x_1) + d_1\right) + D, \tag{36}$$

where $\varpi = (\partial\phi^{-1})/(\partial(E_1/\rho))1/\rho > 0$ and $D = (-\partial\phi^{-1})/(\partial(E_1/\rho))1/\rho(\dot{y}_d + E_1\dot{\rho}/\rho)$.

By derivation,

$$\begin{aligned} \dot{z}_1 &= \frac{1}{2}\log\left(\frac{E_1}{\rho} + \iota_u\right) - \frac{1}{2}\log\left(\iota_d - \frac{E_1}{\rho}\right), \\ \varpi &= \frac{1}{2\rho}\left[\frac{1}{E_1/\rho + \iota_d} - \frac{1}{E_1/\rho - \iota_u}\right]. \end{aligned} \tag{37}$$

Remark 3. Note that the functions ι_d and ι_u are set to solve the problem of unknown initial error $E(0)$. In the predefined performance control of [34], the initial error is assumed to be known, but in the actual control, the exact value of the initial error is often not obtained, which limits the use of the predefined performance control. The reason why the initial values of $\iota_d(t)$ and $\iota_u(t)$ are set to infinity is mainly to guarantee $E(0) \in (-\iota_d, \iota_u)$; however, $\iota_d(0)$ and $\iota_u(0)$ just need to be set to a sufficiently large constant in the actual design. The facts justify this treatment.

3. Control Design and Stability Analysis

In this section, by utilizing adaptive backstepping technique and Lyapunov stability theorem, an observer-based adaptive fuzzy predefined performance decentralized control approach is developed.

The coordinate changes are as follows:

$$\begin{cases} E_1 = y - y_d, \\ E_i = \hat{x}_i - \overline{\alpha}_i \,(i = 2, \ldots, n-1), \\ E_n = \hat{x}_n - \overline{\alpha}_n + \hat{x}_{n+1}\dfrac{g_n}{\beta}, \\ \xi_i = \overline{\alpha}_i - \alpha_{i-1}, \end{cases} \tag{38}$$

where α_i is the virtual control law and $\overline{\alpha}_i$ and ξ_i are the filter signal and filter error of the first-order filter, respectively. Define a first-order filter as

$$\zeta_i \dot{\overline{\alpha}}_i + \overline{\alpha}_i = \alpha_{i-1}, \quad \overline{\alpha}_i(0) = \alpha_{i-1}(0), \quad i = 2, \ldots, n, \tag{39}$$

where ζ_i is the design constant.

Step 1. By substituting (38) into (36), one can obtain as

$$\dot{z}_1 = \varpi\left(g_1\left(E_2 + \xi_2 + \alpha_1 + e_2\right) + \theta_1^* \varphi_1(x_1) + \varepsilon_1(x_1) + d_1\right) + D. \tag{40}$$

Consider the Lyapunov function candidate as

$$V_1 = V_0 + \frac{1}{2}z_1^2 + \frac{1}{2\upsilon_1}\tilde{\theta}_1^{\,T}\tilde{\theta}_1, \tag{41}$$

where $\upsilon_1 > 0$ is a design constant. The time derivative of V_1 and substitute (40) and (26) into (41) as

$$\begin{aligned} \dot{V}_1 &= \dot{V}_0 + z_1\dot{z}_1 + \frac{1}{\upsilon_1}\tilde{\theta}_1^{\,T}\dot{\tilde{\theta}}_1 \leq -e^T Q e + (r_0 + 3)\|e\|^2 \\ &\quad + \|P\|^2\left\|\varepsilon^*\right\|^2 + \|P\|^2\left\|d^*\right\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^{\,T}\tilde{\theta}_l + \theta_1^*\varphi_1(x_1) \\ &\quad + z_1\left[\varpi\left(g_1\left(E_2 + \xi_2 + \alpha_1 + e_2\right) + \varepsilon_1(x_1) + d_1\right) + D\right] \\ &\quad + \frac{1}{\upsilon_1}\tilde{\theta}_1^{\,T}\dot{\tilde{\theta}}_1. \end{aligned} \tag{42}$$

By using Young's inequality,

$$z_1\varpi g_1\left(E_2 + \xi_2\right) \leq \varpi^2 z_1^2 + \frac{1}{2}g_1^{*\,2}\left(E_2^2 + \xi_2^2\right), \tag{43}$$

$$z_1\varpi g_1 e_2 \leq \frac{1}{2}\varpi^2 z_1^2 + \frac{1}{2}g_1^{*\,2}\|e\|^2, \tag{44}$$

$$z_1\varpi\varepsilon_1 \leq \frac{1}{2}\varpi^2 z_1^2 + \frac{1}{2}\varepsilon_1^2, \tag{45}$$

$$z_1\varpi d_1 \leq \frac{1}{2}\varpi^2 z_1^2 + \frac{1}{2}d_1^2. \tag{46}$$

From (43)–(46), one can obtain as

$$\begin{aligned} \dot{V}_1 &\leq -\left(\lambda_{\min}(Q) - r_0 - 3 - \frac{1}{2}g_1^{*\,2}\right)\|e\|^2 + \|P\|^2\left\|\varepsilon^*\right\|^2 \\ &\quad + \|P\|^2\left\|d^*\right\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^{\,T}\tilde{\theta}_l \\ &\quad + z_1\left[\varpi\left(\frac{5}{2}\varpi z_1 + g_1\alpha_1 + \theta_1^T\varphi_1(x_1)\right) + D\right] \\ &\quad + \frac{1}{2}\varepsilon_1^{*\,2} + \frac{1}{2}d_1^{*\,2} + \frac{1}{2}g_1^{*\,2}\left(E_2^2 + \xi_2^2\right) \\ &\quad + \frac{1}{\upsilon_1}\tilde{\theta}_1^{\,T}\left(\upsilon_1\varphi_1(x_1)z_1\varpi - \dot{\theta}_1\right). \end{aligned} \tag{47}$$

Select the virtual control function α_1 and the adaptive laws θ_1 as

$$\alpha_1 = N(\mu_1)\left[\omega_1 z_1 + \frac{5}{2}\varpi z_1 + \theta_1^T\varphi_1(x_1) + \frac{D}{\varpi}\right], \tag{48}$$

$$\dot{\mu}_1 = z_1\varpi\left[\omega_1 z_1 + \frac{5}{2}\varpi z_1 + \theta_1^T\varphi_1(x_1) + \frac{D}{\varpi}\right], \tag{49}$$

$$\dot{\theta}_1 = \upsilon_1\varphi_1(x_1)z_1\varpi - \sigma\theta_1, \tag{50}$$

where $\omega_1 > 0$ and $\sigma > 0$ are design constants.

Substituting (48)–(50) into (47) yields

$$\begin{aligned} \dot{V}_1 &\leq -r_1\|e\|^2 + \|P\|^2\left\|\varepsilon^*\right\|^2 + \|P\|^2\left\|d^*\right\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^{\,T}\tilde{\theta}_l \\ &\quad - \omega_1\varpi z_1^2 + \frac{\sigma}{\upsilon_1}\tilde{\theta}_1^{\,T}\theta_1 + \frac{1}{2}g_1^{*\,2}\left(E_2^2 + \xi_2^2\right) \\ &\quad + \left(gN(\mu_1) + 1\right)\dot{\mu}_1 + \frac{1}{2}\varepsilon_1^{*\,2} + \frac{1}{2}d_1^{*\,2}, \end{aligned} \tag{51}$$

where $r_1 = \lambda_{\min}(Q) - r_0 - 3 - (1/2)g_1^{*\,2}$.

By using Young's inequality,

$$\frac{\sigma}{\upsilon_1}\tilde{\theta}_1^T\theta_1 = \frac{\sigma}{\upsilon_1}\tilde{\theta}_1^T\left(-\tilde{\theta}_1 + \theta_1^*\right) \leq -\frac{\sigma}{2\upsilon_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{\sigma}{2\upsilon_1}\theta_1^{*\,T}\theta_1^*. \tag{52}$$

Substitute (52) into (51) as follows:

$$\begin{aligned} \dot{V}_1 &\leq -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^{\,T}\tilde{\theta}_l - \omega_1\varpi z_1^2 - \frac{\sigma}{2\upsilon_1}\tilde{\theta}_1^T\tilde{\theta}_1 \\ &\quad + \frac{1}{2}g_1^{*\,2}\left(E_2^2 + \xi_2^2\right) + \left(gN(\mu_1) + 1\right)\dot{\mu}_1 + M_1, \end{aligned} \tag{53}$$

where $M_1 = \|P\|^2\|\varepsilon^*\|^2 + \|P\|^2\|d^*\|^2 + (1/2)\varepsilon_1^{*\,2} + (1/2)d_1^{*\,2} + (\sigma/2\upsilon_1)\theta_1^{*\,T}\theta_1^*$.

Step 2 (i ($i = 2, \ldots, n-1$)). The time derivative of E_i is

$$\begin{aligned}\dot{E}_i &= \dot{\hat{x}}_i - \dot{\bar{\alpha}}_i = g_i\hat{x}_{i+1} + \theta_i^{*T}\varphi_i(\hat{\underline{x}}_i) + \varepsilon_i + k_ie_1 - \dot{\bar{\alpha}}_i \\ &= g_i(E_{i+1} + \xi_{i+1} + \alpha_i) + \theta_i^{*T}\varphi_i(\hat{\underline{x}}_i) + \varepsilon_i + k_ie_1 - \dot{\bar{\alpha}}_i.\end{aligned} \tag{54}$$

Consider the Lyapunov function candidate as

$$V_i = V_{i-1} + \frac{1}{2}E_i^2 + \frac{1}{2v_i}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\xi_i^2, \tag{55}$$

where $v_i > 0$ is a design constant. The derivative of V_i with time is

$$\begin{aligned}\dot{V}_i &= \dot{V}_{i-1} + E_i\dot{E}_i + \frac{1}{v_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i + \xi_i\dot{\xi}_i \le -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^T\tilde{\theta}_l \\ &- \omega_1\varpi z_1^2 - \sum_{l=1}^{i-1}\frac{\sigma}{2v_l}\tilde{\theta}_l^T\tilde{\theta}_l + \frac{1}{2}\sum_{l=1}^{i-1}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) \\ &+ M_{i-1} - \sum_{l=2}^{i-1}\omega_lE_l^2 + E_i\left[g_i(E_{i+1} + \xi_{i+1} + \alpha_i)\right. \\ &\left. + \theta_i^{*T}\varphi_i(\hat{\underline{x}}_i) + \varepsilon_i + k_ie_1 - \dot{\bar{\alpha}}_i\right] + \frac{1}{v_i}\tilde{\theta}_i^T\dot{\tilde{\theta}}_i + \xi_i\dot{\xi}_i \\ &+ \sum_{l=1}^{i-1}(gN(\mu_l) + 1)\dot{\mu}_l,\end{aligned} \tag{56}$$

where $M_{i-1} = M_{i-2} + (1/2)\varepsilon_{i-1}^{*2} + (\sigma/2v_{i-1})\theta_{i-1}^{*T}\theta_{i-1}^*$.

By using Young's inequality,

$$E_ig_i(E_{i+1} + \xi_{i+1}) \le E_i^2 + \frac{1}{2}g_i^{*2}\left(E_{i+1}^2 + \xi_{i+1}^2\right), \tag{57}$$

$$E_i\varepsilon_i \le \frac{1}{2}E_i^2 + \frac{1}{2}\varepsilon_i^{*2}. \tag{58}$$

From (57) and (58), one can obtain as

$$\begin{aligned}\dot{V}_i &\le -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^T\tilde{\theta}_l - \omega_1\varpi z_1^2 - \sum_{l=1}^{i-1}\frac{\sigma}{2v_l}\tilde{\theta}_l^T\tilde{\theta}_l \\ &+ \frac{1}{2}\sum_{l=1}^{i-1}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) + M_{i-1} - \sum_{l=2}^{i-1}\omega_lE_l^2 \\ &+ E_i\left[g_i\alpha_i + \theta_i^T\varphi_i(\hat{\underline{x}}_i) + \frac{3}{2}E_i + k_ie_1 - \dot{\bar{\alpha}}_i\right] \\ &+ \frac{1}{2}g_i^{*2}\left(E_{i+1}^2 + \xi_{i+1}^2\right) + \frac{1}{2}\varepsilon_i^{*2} + \frac{1}{v_i}\tilde{\theta}_i^T\left(v_iE_i\varphi_i(\hat{\underline{x}}_i) - \dot{\theta}_i\right) \\ &+ \xi_i\dot{\xi}_i + \sum_{l=1}^{i-1}(gN(\mu_l) + 1)\dot{\mu}_l.\end{aligned} \tag{59}$$

Select the virtual control function α_i and the adaptive laws θ_i as

$$\alpha_i = N(\mu_i)\left[\omega_iE_i + \frac{3}{2}E_i + \theta_i^T\varphi_i(\hat{\underline{x}}_i) + k_ie_1 - \dot{\bar{\alpha}}_i\right], \tag{60}$$

$$\dot{\mu}_i = E_i\left[\omega_iE_i + \frac{3}{2}E_i + \theta_i^T\varphi_i(\hat{\underline{x}}_i) + k_ie_1 - \dot{\bar{\alpha}}_i\right], \tag{61}$$

$$\dot{\theta}_i = v_iE_i\varphi_i(\hat{\underline{x}}_i) - \sigma\theta_i. \tag{62}$$

Substituting (60)–(62) into (59) yields

$$\begin{aligned}\dot{V}_i &\le -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^T\tilde{\theta}_l - \omega_1\varpi z_1^2 - \sum_{l=1}^{i-1}\frac{\sigma}{2v_l}\tilde{\theta}_l^T\tilde{\theta}_l \\ &+ \frac{1}{2}\sum_{l=1}^{i}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) + M_i + \sum_{l=1}^{i}\xi_l\dot{\xi}_l - \sum_{l=2}^{i}\omega_lE_l^2 \\ &+ \sum_{l=1}^{i}(gN(\mu_l) + 1)\dot{\mu}_l + M_{i-1} + \frac{1}{2}\varepsilon_i^{*2} + \frac{\sigma}{v_i}\tilde{\theta}_i^T\theta_i.\end{aligned} \tag{63}$$

According to the definition of α_1, α_2, and α_i, one can get $\dot{\alpha}_i = \eta_i(E_1, \ldots, E_i, E_{i+1}, \theta_1, \ldots, \theta_i, y_d, \dot{y}_d, \ddot{y}_d, \xi_2, \ldots, \xi_i)$, where η_i is a continuous function. Given that any ψ, $C_i = \left\{(E_1, \ldots, E_i, E_{i+1}, \theta_1, \ldots, \theta_i, y_d, \dot{y}_d, \ddot{y}_d, \xi_2, \ldots, \xi_i)^T : \vartheta < \psi\right\}$ is a prefixed compact set, where the compact set C_i can be made larger as needed. Therefore, the maximum value of the continuous function η_i is B_{i+1} on $C_i * \Delta_0$ based on Assumption 1 of the compact set Υ and the compact set C_i. It is obvious that

$$\begin{aligned}\dot{\xi}_i &= -\frac{\xi_i}{\chi_i} + \eta_{i-1}(E_1, \ldots, E_i, \theta_1, \ldots, \theta_{i-1}, y_d, \dot{y}_d, \ddot{y}_d, \xi_2, \ldots, \xi_{i-1}) \\ &\le -\frac{\xi_i}{\chi_i} + B_i.\end{aligned} \tag{64}$$

Utilising Young's inequality,

$$\frac{\sigma}{v_i}\tilde{\theta}_i^T\theta_i = \frac{\sigma}{v_i}\tilde{\theta}_i^T\left(-\tilde{\theta}_i + \theta_i^*\right) \le -\frac{\sigma}{2v_i}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{\sigma}{2v_i}\theta_i^{*T}\theta_i^*, \tag{65}$$

$$\xi_i\dot{\xi}_i \le \xi_i\left[-\frac{\xi_i}{\chi_i} + B_i\right] \le -\frac{\xi_i^2}{\chi_i} + \xi_iB_i \le -\left[\frac{1}{\chi_i} - \frac{B_i^2}{2\pi_i}\right]\xi_i^2 + \frac{\pi_i}{2}, \tag{66}$$

where π_i is the design constant.

Substitute (65) and (66) into (63) as follows:

$$\begin{aligned}\dot{V}_i &\le -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^T\tilde{\theta}_l - \omega_1\varpi z_1^2 - \sum_{l=1}^{i}\frac{\sigma}{2v_l}\tilde{\theta}_l^T\tilde{\theta}_l \\ &+ \frac{1}{2}\sum_{l=1}^{i}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) - \sum_{l=1}^{i}\left[\frac{1}{\chi_i} - \frac{B_i^2}{2\pi_i}\right]\xi_i^2 \\ &- \sum_{l=2}^{i}\omega_lE_l^2 + \sum_{l=1}^{i}(gN(\mu_l) + 1)\dot{\mu}_l + M_i,\end{aligned} \tag{67}$$

where $M_i = M_{i-1} + (1/2)\varepsilon_i^{*2} + (\sigma/2v_i)\theta_i^{*T}\theta_i^* + (\pi_i/2)$.

Step 3 n. The time derivative of E_n is

$$\begin{aligned}\dot{E}_n &= \dot{\hat{x}}_n - \dot{\bar{\alpha}}_n + \dot{\hat{x}}_{n+1}\frac{g_n}{\beta} = g_n(\hat{x}_{n+1} - u) + \theta_n^{*T}\varphi_n(\underline{\hat{x}}_n) \\ &\quad + \varepsilon_n + k_n e_1 - \dot{\bar{\alpha}}_n + \frac{g_n}{\beta}(-\beta\hat{x}_{n+1} + 2\beta u) \\ &= g_n u + \theta_n^{*T}\varphi_n(\underline{\hat{x}}_n) + \varepsilon_n + k_n e_1 - \dot{\bar{\alpha}}_n.\end{aligned} \tag{68}$$

Consider the Lyapunov function candidate as

$$V_n = V_{n-1} + \frac{1}{2}E_n^2 + \frac{1}{2v_n}\tilde{\theta}_n^T\tilde{\theta}_n + \frac{1}{2}\xi_n^2, \tag{69}$$

where $v_n > 0$ is a design constant. The derivative of V_n with time along with (68) is

$$\begin{aligned}\dot{V}_n &= \dot{V}_{n-1} + E_n\dot{E}_n + \frac{1}{v_n}\tilde{\theta}_n^T\dot{\tilde{\theta}}_n + \xi_n\dot{\xi}_n \\ &= \dot{V}_{n-1} + E_n\left[g_n u + \theta_n^{*T}\varphi_n(\underline{\hat{x}}_n) + \varepsilon_n + k_n e_1 - \dot{\bar{\alpha}}_n\right] \\ &\quad + \frac{1}{v_n}\tilde{\theta}_n^T\dot{\tilde{\theta}}_n + \xi_n\dot{\xi}_n.\end{aligned} \tag{70}$$

By using Young's inequality,

$$E_n\varepsilon_n \le \frac{1}{2}E_n^2 + \frac{1}{2}\varepsilon_n^{*2}. \tag{71}$$

From (71), one can obtain

$$\begin{aligned}\dot{V}_n &\le \dot{V}_{n-1} + E_n\left[g_n u + \theta_n^T\varphi_n(\underline{\hat{x}}_n) + \frac{1}{2}E_n + k_n e_1 - \dot{\bar{\alpha}}_n\right] \\ &\quad + \frac{1}{2}\varepsilon_n^{*2} + \frac{1}{v_n}\tilde{\theta}_n^T\left(v_n E_n\varphi_n(\underline{\hat{x}}_n) - \dot{\theta}_n\right) + \xi_n\dot{\xi}_n.\end{aligned} \tag{72}$$

Select the actual control input u and the adaptive laws θ_n as

$$u = N(\mu_n)\left[\omega_n E_n + \frac{1}{2}E_n + \theta_n^T\varphi_n(\underline{\hat{x}}_n) + k_n e_1 - \dot{\bar{\alpha}}_n\right], \tag{73}$$

$$\dot{\mu}_n = E_n\left[\omega_n E_n + \frac{1}{2}E_n + \theta_n^T\varphi_n(\underline{\hat{x}}_n) + k_n e_1 - \dot{\bar{\alpha}}_n\right], \tag{74}$$

$$\dot{\theta}_n = v_n E_n\varphi_n(\underline{\hat{x}}_n) - \sigma\theta_n. \tag{75}$$

Substituting (73)–(75) into (72) yields

$$\begin{aligned}\dot{V}_n &\le -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^T\tilde{\theta}_l - \omega_1\varpi z_1^2 - \sum_{l=1}^{n-1}\frac{\sigma}{2v_l}\tilde{\theta}_l^T\tilde{\theta}_l \\ &\quad + \frac{1}{2}\sum_{l=1}^{n-1}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) + \sum_{l=1}^{n}\xi_l\dot{\xi}_l - \sum_{l=2}^{n}\omega_l E_l^2 \\ &\quad + \sum_{l=1}^{n}(gN(\mu_l) + 1)\dot{\mu}_l + M_{n-1} + \frac{1}{2}\varepsilon_n^{*2} + \frac{\sigma}{v_n}\tilde{\theta}_n^T\theta_n.\end{aligned} \tag{76}$$

Combining Young's inequality and inequality (64), one can obtain

$$\frac{\sigma}{v_n}\tilde{\theta}_n^T\theta_n = \frac{\sigma}{v_n}\tilde{\theta}_n^T\left(-\tilde{\theta}_n + \theta_n^*\right) \le -\frac{\sigma}{2v_n}\tilde{\theta}_n^T\tilde{\theta}_n + \frac{\sigma}{2v_n}\theta_n^{*T}\theta_n^*, \tag{77}$$

$$\xi_n\dot{\xi}_n \le -\frac{\xi_n^2}{\chi_n} + \xi_n B_n \le -\left[\frac{1}{\chi_n} - \frac{B_n^2}{2\pi_n}\right]\xi_n^2 + \frac{\pi_n}{2}. \tag{78}$$

Substitute (77) and (78) into (76) as follows

$$\begin{aligned}\dot{V}_n &\le -r_1\|e\|^2 + \|P\|^2\sum_{l=1}^{n}\tilde{\theta}_l^T\tilde{\theta}_l - \omega_1\varpi z_1^2 - \sum_{l=1}^{n}\frac{\sigma}{2v_l}\tilde{\theta}_l^T\tilde{\theta}_l \\ &\quad + \frac{1}{2}\sum_{l=1}^{n-1}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) + M_n - \sum_{l=1}^{n}\left[\frac{1}{\chi_n} - \frac{B_n^2}{2\pi_n}\right]\xi_n^2 \\ &\quad - \sum_{l=2}^{n}\omega_l E_l^2 + \sum_{l=1}^{n}(gN(\mu_l) + 1)\dot{\mu}_l,\end{aligned} \tag{79}$$

where $M_n = M_{n-1} + (1/2)\varepsilon_n^{*2} + (\pi_n/2) + (\sigma/2v_l)\theta_l^{*T}\theta_l^*$.The following theorem is summarized by the above controller design and stability analysis.

Theorem 1. *Consider a class of nonlinear pure-feedback systems (1), with uncertain functions, unmeasurable states, and input delay, and the state observer is designed as (16). Under Assumptions1–5, the adaptive laws are designed as (48), (60), and (73). The virtual control functions are chosen as (50) and (62). And the actual control input function (75) can guarantee all the signals in the closed-loop system are SGUUB. Meanwhile, the tracking error does not deviate from the prescribed performance bound (31), and the observer errors converge within a small zero region.*

Proof. Let

$$\bar{c} = \min\left\{\frac{2r_1}{\lambda_{\min}(P)}, 2v_l\left(\frac{\sigma}{2v_l} - \|P\|^2\right), 2\varpi\omega_1, 2\omega_k, 2\left[\frac{1}{\chi_n} - \frac{B_n^2}{2\pi_n}\right]\right\},$$
$$k = 2,\ldots,n; l = 1,2,\ldots,n, \tag{80}$$

where $\bar{c}$ will be positive by choosing appropriate parameters. □

Then, (79) can be finally expressed as

$$\dot{V}_n \le -\bar{c}V_n + \sum_{l=1}^{n}(gN(\mu_l) + 1)\dot{\mu}_l + \frac{1}{2}\sum_{l=1}^{n-1}g_l^{*2}\left(E_{l+1}^2 + \xi_{l+1}^2\right) + M_n, \tag{81}$$

where $M_n = \|P\|^2\|\varepsilon^*\|^2 + \|P\|^2\|d^*\|^2 + \sum_{l=1}^{n}(1/2)\varepsilon_l^{*2} + \sum_{l=i}^{n}(\pi_l/2) + \sum_{l=1}^{n}(\sigma/2v_l)\theta_l^{*T}\theta_l^*$ is bounded.

Multiplying both sides of (81) by $e^{-\bar{c}t}$ generates

$$\begin{aligned}\frac{\mathrm{d}\left(V_n e^{-\bar{c}t}\right)}{\mathrm{d}t} \le & \sum_{l=1}^{n}\left(gN(\mu_l)+1\right)\dot{\mu}_l e^{-\bar{c}t} + M_n e^{-\bar{c}t} \\ & + \frac{1}{2}\sum_{l=1}^{n-1} g_l^{*2}\left(E_{l+1}^2+\xi_{l+1}^2\right)e^{-\bar{c}t}.\end{aligned} \tag{82}$$

Integrating the above inequality over $[0, t_f)$ and then multiplying both sides by $e^{-\bar{c}t}$ yields

$$\begin{aligned} V_n \le & V(0)e^{-\bar{c}t} + \frac{M_{n+1}}{\bar{c}}\left(1-e^{-\bar{c}t}\right) \\ & + e^{-\bar{c}t}\int_0^t \sum_{l=1}^{n}\left(gN(\mu_l)+1\right)\dot{\mu}_l e^{-\bar{c}\tau}\mathrm{d}\tau \\ & + e^{-\bar{c}t}\int_0^t \frac{1}{2}\sum_{l=1}^{n-1} g_l^{*2}\left(E_{l+1}^2+\xi_{l+1}^2\right)e^{-\bar{c}\tau}\mathrm{d}\tau. \end{aligned} \tag{83}$$

Based on Lemma 2, V_n and μ can be proved to be bounded. In addition, since E_{l+1} and ξ_{l+1} are semiglobal and ultimately uniformly bounded, $e^{-\bar{c}t}\int_0^t 1/2\sum_{l=1}^{n-1} g_l^{*2}(E_{l+1}^2+\xi_{l+1}^2)e^{-\bar{c}\tau}\mathrm{d}\tau$ is bounded. Therefore, the stability of the whole closed-loop system is demonstrated. Furthermore, since z_1 can be proved to be uniformly bounded, the prescribed performance tracking control is achieved.

4. Simulation Example

Consider a class of nonlinear pure-feedback systems with unavailable states, unknown control direction, and input delay:

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2) + d_1(t), \\ \dot{x}_2 = f_2(\underline{x}_2, u(t-\delta)) + d_2(t), \\ y = x_1, \end{cases} \tag{84}$$

where x_1 and x_2 are the system states and u and y are the system input and output, respectively. The smooth functions are used as $f_1(x_1, x_2) = x_1 + x_1x_2$ and $f(\underline{x}_2, u(t-\delta)) = (2+\sin(x_1x_2))(1+u(t-\delta))$, and the external disturbances in this simulation are given as $d_1(t) = 0.02\cos(t)$ and $d_2(t) = 0.03\sin(t)$. The input delay is chosen as $\delta = 0.01$, and the reference signal is given as $y_d = \sin(t)$.

The parameters in control functions and adaptive laws are given as $\omega_1 = 0.1$, $\omega_2 = 40$, $\upsilon_1 = \upsilon_2 = 5$, and $\sigma_1 = \sigma_2 = 20$. Parameter in a first-order filter is $\zeta_2 = 0.05$. As for the state observer, the observer gains are selected as $K = [k_1, k_2]^T = [10, 100]^T$. In predefined performance controls, $\iota_u(0) = \iota_d(0) = 30$, $\lambda_d = \lambda_u = 0.8$, $\lambda = 2$, $h_u = h_d = 0.4$, and $\rho_\infty = 0.4$. The initial value of the system states are $x_1(0) = 0.3$ and $x_2(0) = 0.5$, and the initial values for the other parameters are zero.

The simulation results are shown in Figures 1–7, where the red and blue lines represent the approach proposed in this article, and the black line represents the removal of the variable-parameter predefined performance. Figure 1 shows the output trajectories of y and the expected output signal y_d. The output tracking error and the prescribed performance boundaries are shown in Figures 2, and Figures 3 and 5 illustrate the trajectories of system states x_1 and x_2 and their estimates $\hat{x}_1$ and $\hat{x}_2$, respectively. The Tracking errors e_1 and e_2 of state x_1 and x_2 are shown in Figures 4, and Figures 6 and 7 show the trajectories of the actual control input u. It can be seen from the figure, compared to removing variable-parameter predefined performance, the

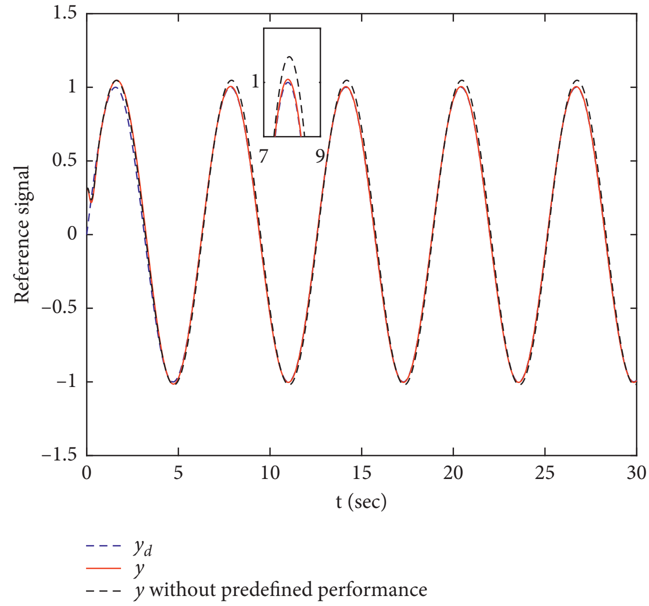

Figure 1: Trajectories of y and y_d.

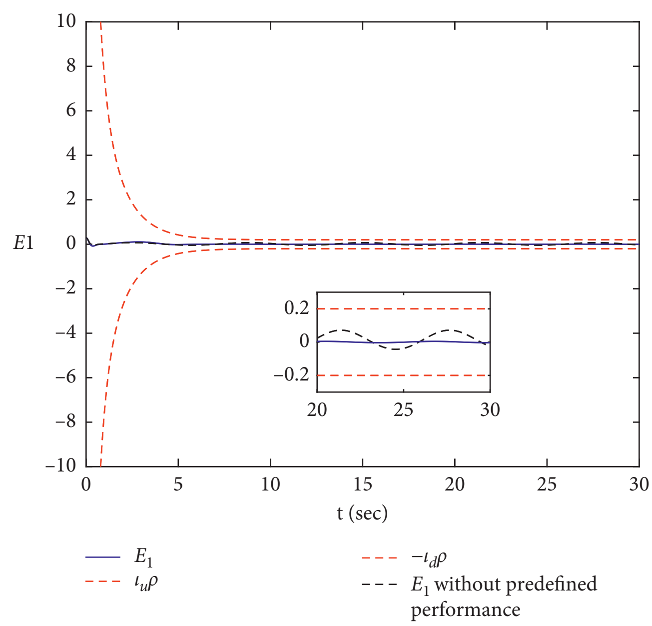

Figure 2: Tracking error E_1 with prescribed performance $\iota_u\rho$ and $-\iota_d\rho$.

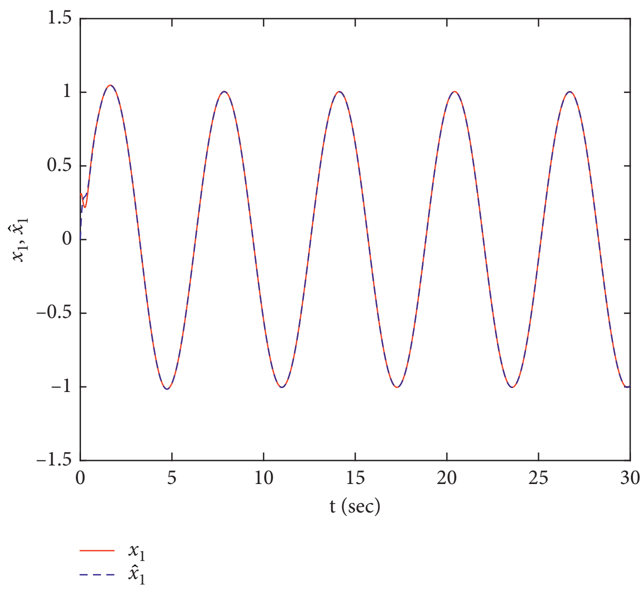

Figure 3: Trajectory of x_1 and $\hat{x}_1$.

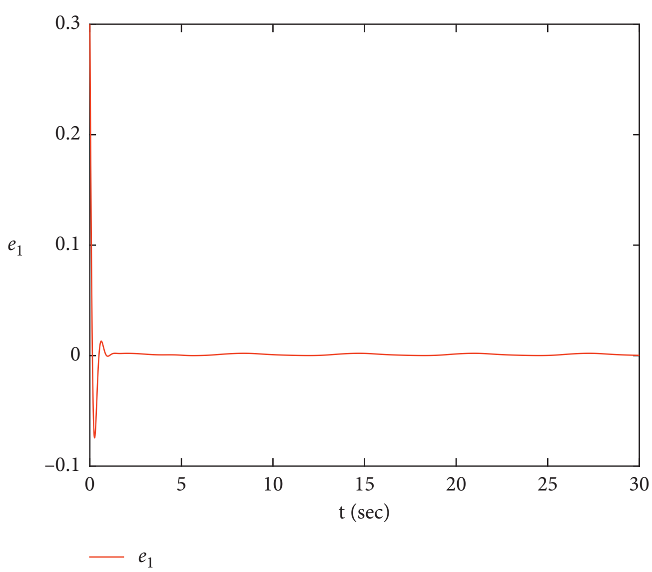

Figure 4: Tracking error e_1.

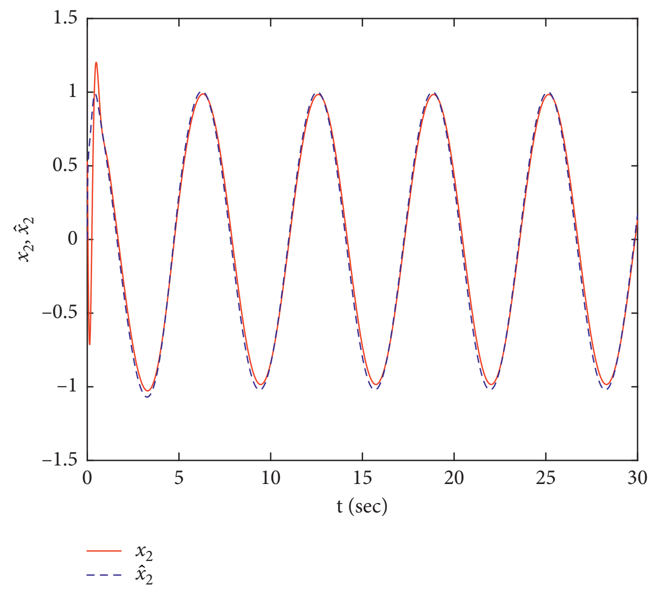

Figure 5: Trajectory of x_2 and $\hat{x}_2$.

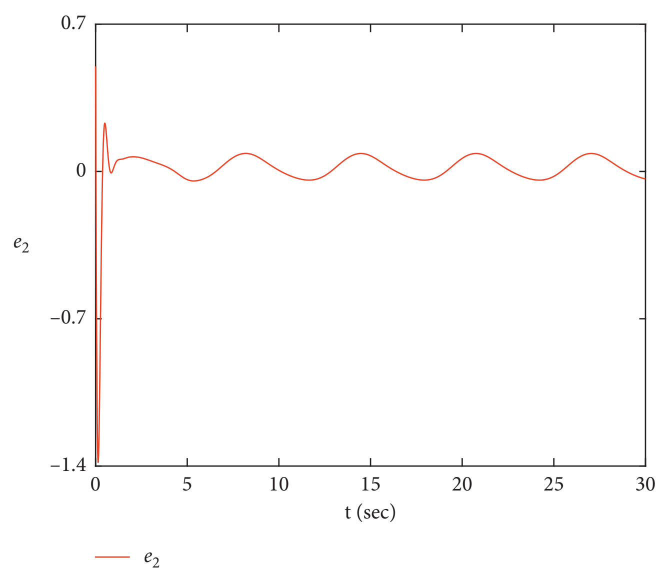

Figure 6: Tracking error e_2.

controller in this paper is obviously better in both the initial oscillation frequency and the tracking effect.

Remark 4. Compared with the existing literature, an adaptive fuzzy predefined performance controller is proposed in this paper, which makes the tracking error convergence in the preset range better. It can be seen from Figure 2 that different from the literatures [34, 37], the novel predefined performance control is a variable-parameter scheme, which relaxes the limitations of known initial error in predefined performance. In addition, from the simulation data, it can be seen that the simultaneous consideration of input delay and pure-feedback system brings great difficulty to controller design. The control method proposed in this paper has excellent control performance, which is shown in Figure 1.

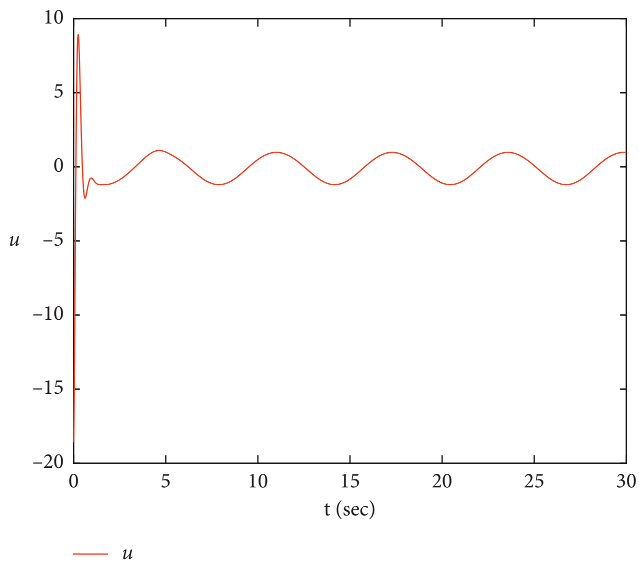

FIGURE 7: The trajectory of actual control input u.

5. Conclusion

In this paper, an observer-based adaptive fuzzy predefined performance controller has been introduced of a class of nonlinear pure-feedback systems with unknown control direction and input delay. A novel predefined performance with variable-parameter scheme has been investigated, which solved the problem of unknown initial error. In order to overcome system complexity caused by input delay and pure-feedback systems, the Pade approximation and mean value theorem has been employed, respectively. In addition, Nussbaum functions have been used to deal with the unknown control direction and a first-order filter has applied to approximate repeated differentiations problem of the virtual controllers. State observer and FLSs have been proposed to estimate the unmeasured states and approximate the unknown nonlinear functions, respectively. Therefore, it has been proved that stability of the entire closed-loop system is SGUUB in limitation of the predefined performance control. The tracking errors have converged within a predefined range, while the observer estimation errors have converged within a small zero region. Finally, the simulation results have verified the effectiveness of the proposed control method. In the future research, an observer-based adaptive fuzzy predefined performance controller will be considered in multiagent systems.

Data Availability

All data in this paper are from Simulink in MATLAB, and all data have been given in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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