Research Article

Three-Prey One-Predator Continuous Time Nonlinear System Model

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In this paper, we propose a new multiple-prey one-predator continuous time nonlinear system model, in which the number of teams of preys is equal to 3; namely, a continuous time three-prey one-predator model is put forward and studied. The fourth-order differential equation is established, in which the prey teams help each other. The equilibrium points and stability are analyzed. When not considering preys help each other, we study the global stability and persistence of the model without help terms. The simulation results of system solutions with help terms corresponding to locally asymptotically stable equilibrium points and without help terms corresponding to globally asymptotically stable equilibrium points are given.

1. Introduction

Nonlinear systems exist in many ways with various research directions, such as chaotic circuit [1–8], neural network [9–15], and image encryption [16–18]. Prey-predator systems also have nonlinear characteristics. In 1965, Holling proposed the functional responses of three different species to simulate predation. These functional responses describe how predators transform the captured prey into their own growth. According to the research, the prey-predator model can be divided into four categories: (i) the first category is the single-prey single-predator model; (ii) the second category is the single-prey multipredator model; (iii) the third category is the multiprey multipredator model; and (iv) the fourth category is the multiprey single-predator model. There are many research studies on the first type of the model. For example, Rosenzweig and MacArthur [19] proposed the Rosenzweig–MacArthur model (R-M model) or the predator-prey model with Holling type II functional response and gave graphical presentation and stability conditions of predator-prey interactions. Hsu and Huang [20] studied global stability of the R-M model. In [21], the conditions of internal equilibrium were given and the stability of internal equilibrium was analyzed. It also discussed certain critical situations, some of which cannot occur in the usual model.

Wang and Jing [22] studied the global stability and the existence of limit cycles of a predator-prey system. The existence of at least two limit cycles was proved by qualitative analysis and Poincaré–Bendixson theory. Zhang and Hou [23] investigated a nonautonomous ratio-dependent predator-prey model with exploited terms. By using the coincidence degree theory, it was proved that there are at least four positive periodic solutions. For the second category, Llibre and Xiao [24] studied the competitive exclusion of two predators and one prey. In the absence of predators, the growth rate of the prey was logistic or linear. The boundedness of the solution was proved by studying the stability of the equilibrium point of the differential equation at infinity. Through the theoretical analysis of the equations, the necessary and sufficient conditions for the existence of the exclusion principle were obtained, and the global dynamic behaviors of the three species in the first octaves were given. Wang et al. [25] studied a class of n -dimensional consumer-resource systems, in which a group of consumers compete for the same resource, and each consumer is mutually beneficial with the resource. Wang and Wu [26] considered the global dynamics of an n -dimensional Lotka–Volterra system in which $(n-1)$ predator species compete for a single prey species. By using the theory of dynamic systems, the global dynamic behaviors of n species in the first octant were

given, and sufficient and necessary conditions for the existence of the competitive exclusion principle were proved, thus extending Volterra's principle and the work of Llibre and Xiao [24]. For the third category, Elettrey and El-Metwally [27] gave models in which two teams of predators interact with two teams of preys. The teams in each group (predators or preys) help each other. Three different versions of the multiteam predator-prey model were proposed. In addition, the equilibrium solutions, the conditions of their local asymptotic stability, and the global stability of the solution of one of the models were studied. For the fourth category model, Elettrey [28] proposed a two-prey one-predator model, in which the prey teams help each other. Its local stability was studied. In the absence of predators, there was no help between the prey teams. The global stability and persistence of the model without help were studied. However, for the continuous time multiple-prey one-predator model, so far, only the above two-prey one-predator model has been proposed. No continuous time multiple-prey one-predator model in which the number of teams of preys is greater than 2 has been proposed. But in practice, in many cases, the number of teams of preys is greater than 2. Therefore, in this paper, we propose a new multiple-prey one-predator model, in which the number of teams of preys is equal to 3; namely, a continuous time three-prey one-predator model is put forward and studied. The fourth-order differential equation is established. The equilibrium points and stability are analyzed.

2. The Model

In this section, we propose a system consisting of three teams of preys with densities x , y , and z , respectively, interacting with one team of predator with densities w . The assumptions of this model are as follows:

- (1) In the absence of any predation, each team of preys grows logistically, that is, $ax(1-x)$, $by(1-y)$, and $cz(1-z)$.
- (2) The effect of the predation is to reduce the prey growth rate by a term proportional to the prey and predator populations, that is, the $-xw$, $-yw$, and $-zw$ terms.
- (3) The teams of preys help each other against the predator, that is, a $xyzw$ term exists.
- (4) In the absence of any prey for sustenance, the predator's death rate results in inverse decay, that is, the term $-dw^2$.
- (5) The prey's contribution to the predator growth rate is exw , fyw , and gzw , which is proportional to the available prey as well as the size of the predator population.

Using the above assumptions, the following model is proposed:

$$\begin{cases} \frac{dx}{dt} = ax(1-x) - xw + xyzw, \\ \frac{dy}{dt} = by(1-y) - yw + xyzw, \\ \frac{dz}{dt} = cz(1-z) - zw + xyzw, \\ \frac{dw}{dt} = -dw^2 + exw + fyw + gzw, \end{cases} \quad (1)$$

where the coefficients a , b , c , d , e , f , and g are positive constants and the initial values $x(0)$, $y(0)$, $z(0)$, and $w(0)$ are all greater than zero. It is clear that the three teams of preys help each other, e.g., in foraging and in early warning against predation. Note that this help occurs only in the presence of predator. This is presented by the term $xyzw$ in the prey equations.

3. The Analysis of the Model

First, let us find the equilibrium point of system (1). In order to find the equilibrium points of equation (1), equation (1) can be set to zero as follows:

$$\begin{cases} ax(1-x) - xw + xyzw = 0, \\ by(1-y) - yw + xyzw = 0, \\ cz(1-z) - zw + xyzw = 0, \\ -dw^2 + exw + fyw + gzw = 0. \end{cases} \quad (2)$$

Solving equation (2), we can obtain the equilibrium point $E(x, y, z, w)$ as follows: $E_0(0, 0, 0, 0)$, $E_1(1, 0, 0, 0)$, $E_2(0, 1, 0, 0)$, $E_3(0, 0, 1, 0)$, $E_4(0, 1, 1, 0)$, $E_5(1, 0, 1, 0)$, $E_6(1, 1, 0, 0)$, $E_7(1, 1, 1, 0)$, $E_8(0, 0, (dc/(dc+g)), (gc/(dc+g)))$, $E_9(0, (db/(db+f)), 0, (fb/(db+f)))$, $E_{10}((da/(da+e)), 0, 0, (ea/(da+e)))$, $E_{11}(0, (bcd-cg+bg/bcd+cf+bg), (bcd+cf-bf/bcd+cf+bg), (bcf+bcg/bcd+cf+bg))$, $E_{12}((ac-d-cg+ag/acd+ce+ag), 0, (acd+ce-ae/acd+ce+ag), (ace+acg/acd+ce+ag))$, $E_{13}((abd-bf+af/abd+be+af), (abd+be-ae/abd+be+af), 0, (abe+abf/abd+be+af))$, and $E_{14}(1, 1, 1, (e+f+g/d))$.

Proposition 1. *Local stability analysis shows that system (1) has eight unstable equilibrium solutions E_0 , E_1 , E_2 , E_3 , E_4 , E_5 , E_6 , and E_7 .*

Proof. The Jacobian matrix of system (1) is given by the following:

$$J = \begin{bmatrix} a(1-2x) - w(1-yz) & xzw & xyw & -x(1-yz) \\ yzw & b(1-2y) - w(1-xz) & xyw & -y(1-xz) \\ yzw & xzw & c(1-2z) - w(1-xy) & -z(1-xy) \\ ew & fw & gw & -2dw + ex + fy + gz \end{bmatrix}. \quad (3)$$

The characteristic equation is as follows:

$$|J - \lambda E| = \begin{vmatrix} a(1-2x) - w(1-yz) - \lambda & xzw & xyw & -x(1-yz) \\ yzw & b(1-2y) - w(1-xz) - \lambda & xyw & -y(1-xz) \\ yzw & xzw & c(1-2z) - w(1-xy) - \lambda & -z(1-xy) \\ ew & fw & gw & -2dw + ex + fy + gz - \lambda \end{vmatrix} = 0. \quad (4)$$

By substituting the point $E_0(0, 0, 0, 0)$ in equation (4), we get $\lambda = 0, a, b$, and c , which has three positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_1(1, 0, 0, 0)$ in equation (4), we get $\lambda = -a, b, c$, and e , which has three positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_2(0, 1, 0, 0)$ in equation (4), we get $\lambda = a, -b, c$, and e , which has three positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_3(0, 0, 1, 0)$ in equation (4), we get $\lambda = a, b, -c$, and e , which has three positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_4(0, 1, 1, 0)$ in equation (4), we get $\lambda = a, -b, -c$, and $f+g$, which has two positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_5(1, 0, 1, 0)$ in equation (4), we get $\lambda = -a, b, -c$, and $e+g$, which has two positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_6(1, 1, 0, 0)$ in equation (4), we get $\lambda = -a, -b, c$, and $e+f$, which has two positive eigenvalues. So, it is an unstable equilibrium point.

Similarly, by substituting the point $E_7(1, 1, 1, 0)$ in equation (4), we get $\lambda = -a, -b, -c$, and $e+f+g$, which has one positive eigenvalue. So, it is an unstable equilibrium point.

So, all of them are unstable equilibrium points.

By substituting the point $E_8(0, 0, (dc/(dc+g)), (gc/(dc+g)))$ in equation (4), we get $\lambda = a - (gc/(dc+g)), b - (gc/(dc+g)), -c$, and $-(dgc/(dc+g))$.

The equilibrium solution $E_8(0, 0, (dc/(dc+g)), (gc/(dc+g)))$ is locally asymptotically stable under the condition: $\begin{cases} a < (gc/(dc+g)) \\ b < (gc/(dc+g)) \end{cases}$.

Numerical simulations agree with these results. Let $a = 0.5, b = 0.5, c = 2, d = 1, e = 1, f = 2$, and $g = 2$. We get the stable solution $(0, 0, 0.5, 1)$ as shown in Figure 1.

Similarly, from equilibrium point $E_9(0, (db/(db+f)), 0, (fb/(db+f)))$, we can obtain the eigenvalue as follows: $\lambda = a - (fb/(db+f)), c - (fb/(db+f)), -b$, and $-(dfb/(db+f))$.

When the following conditions are met and the real part of all eigenvalues is less than 0, it is locally asymptotically stable: $\begin{cases} a < (fb/(db+f)) \\ c < (fb/(db+f)) \end{cases}$.

The above conclusion is proved by numerical simulation, with $a = 0.5, b = 2, c = 0.5, d = 1, e = 1, f = 2$, and $g = 2$. The equilibrium point $(0, 0.5, 0, 1)$ is obtained, and the simulation results are shown in Figure 2.

Similarly, from equilibrium point $E_{10}((da/(da+e)), 0, 0, (ea/(da+e)))$, we can obtain the eigenvalue as follows: $\lambda = -a, b - (ea/(da+e)), c - (ea/(da+e)), -(dea/(da+e))$.

When the following conditions are met, it is locally asymptotically stable: $\begin{cases} b < ea/(da+e) \\ c < ea/(da+e) \end{cases}$.

The above conclusion is proved by numerical simulation, with $a = 2, b = 0.5, c = 0.5, d = 1, e = 2, f = 2$, and $g = 2$. The equilibrium point $(0.5, 0, 0, 1)$ is obtained, and the simulation results are shown in Figure 3.

By substituting the point $E_{11}(0, (bcd - cg + bg/bcd + cf + bg), (bcd + cf - bf/bcd + cf + bg), (bcf + bcf/bcd + cf + bg))$ in matrix (3), we can obtain the Jacobian matrix:

$$J = \begin{vmatrix} a - w(1-yz) & 0 & 0 & 0 \\ yzw & b(1-2y) - w & 0 & -y \\ yzw & 0 & c(1-2z) - w & -z \\ ew & fw & gw & -2dw + fy + gz \end{vmatrix} \quad (5)$$

$$= [a - w(1-yz)] \begin{vmatrix} b(1-2y) - w & 0 & -y \\ 0 & c(1-2z) - w & -z \\ fw & gw & -dw \end{vmatrix}.$$

FIGURE 1: The values $a = 0.5$, $b = 0.5$, $c = 2$, $d = 1$, $e = 1$, $f = 2$, and $g = 2$ are used. So, we get the stable solution ($x = 0$, $y = 0$, $z = 0.5$, and $w = 1$).

FIGURE 2: The values $a = 0.5$, $b = 2$, $c = 0.5$, $d = 1$, $e = 1$, $f = 2$, and $g = 2$ are used. So, we get the stable solution ($x = 0$, $y = 0.5$, $z = 0$, and $w = 1$).

FIGURE 3: The values $a = 2$, $b = 0.5$, $c = 0.5$, $d = 1$, $e = 2$, $f = 2$, and $g = 2$ are used. So, we get the stable solution ($x = 0.5$, $y = 0$, $z = 0$, and $w = 1$).

In addition, the Jacobian characteristic determinant is as follows:

$$|J - \lambda E| = [a - w(1 - yz) - \lambda] \begin{vmatrix} b(1 - 2y) - w - \lambda & 0 & -y \\ 0 & c(1 - 2z) - w - \lambda & -z \\ fw & gw & -dw - \lambda \end{vmatrix} \quad (6)$$

$$= -(A - \lambda) [\lambda^3 + (dw - B - C)\lambda^2 + (CB + gwz + fwy - dwB - dwC)\lambda + dwBC - gwzB + fwyC],$$

where $A = a - w(1 - yz)$, $B = b(1 - 2y) - w$, and $C = c(1 - 2z) - w$.

Using the Routh-Hurwitz condition, the following condition is necessary and sufficient that all roots of the

characteristic equation of system (1) at the equilibrium point have a negative real part:

$$\begin{cases} A < 0, \\ dw - B - C > 0, \\ CB + gwz + fwy - dwB - dwC > 0, \\ dwBC - gwzB + fwyC > 0, \\ (dw - B - C)(CB + gwz + fwy - dwB - dwC) > dwBC - gwzB + fwyC. \end{cases} \quad (7)$$

The above conclusion is proved by numerical simulation, with $a = 0.5$, $b = 2$, $c = 3$, $d = 1$, $e = 2$, $f = 2$, and $g = 2$. The equilibrium point $(0, 0.25, 0.5, 1.5)$ is obtained, and the simulation results are shown in Figure 4.

Similarly, by substituting the point $E_{12}(((acd - cg + ag)/(acd + ce + ag)), 0, ((acd + ce - ae)/(acd + ce + ag)), ((ace + acg)/(acd + ce + ag)))$ in matrix (3), we can obtain the Jacobian matrix:

$$J = \begin{bmatrix} a(1 - 2x) - w & xzw & 0 & -x \\ 0 & b - w(1 - xz) & 0 & 0 \\ 0 & xzw & c(1 - 2z) - w & -z \\ ew & fw & gw & -dw \end{bmatrix} = [b - w(1 - xz)] \begin{bmatrix} a(1 - 2x) - w & 0 & -x \\ 0 & c(1 - 2z) - w & -z \\ ew & gw & -dw \end{bmatrix}. \quad (8)$$

In addition, the Jacobian characteristic determinant is as follows:

$$|J - \lambda E| = [b - w(1 - xz) - \lambda] \begin{vmatrix} a(1 - 2x) - w - \lambda & 0 & -x \\ 0 & c(1 - 2z) - w - \lambda & -z \\ ew & gw & -dw - \lambda \end{vmatrix}$$

$$= -(B_1 - \lambda) [\lambda^3 + (dw - A_1 - C_1)\lambda^2 + (A_1C_1 + zgw + ewx - dwA_1 - dwC_1)\lambda + dwA_1C_1 - zgwA_1 + ewxC], \quad (9)$$

where $\begin{cases} A_1 = a(1 - 2x) - w \\ B_1 = b - w(1 - xz) \\ C_1 = c(1 - 2z) - w \end{cases}$.

Similarly, using the Routh-Hurwitz condition, the following condition is necessary and sufficient that all roots

FIGURE 4: The values $a = 0.5$, $b = 2$, $c = 3$; $d = 1$, $e = 2$, $f = 2$, and $g = 2$ are used. So, we get the stable solution ($x = 0$, $y = 0.25$, $z = 0.5$, and $w = 1.5$).

of the characteristic equation of system (1) at the equilibrium point have a negative real part:

$$\begin{cases} B_1 < 0, \\ dw - A_1 - C_1 > 0, \\ A_1 C_1 + zgw + ewx - dw A_1 - dw C_1 > 0, \\ dw A_1 C_1 - zgw A_1 + ewx C_1 > 0, \\ (dw - A_1 - C_1)(A_1 C_1 + zgw + ewx - dw A_1 - dw C_1) > dw A_1 C_1 - zgw A_1 + ewx C_1. \end{cases} \quad (10)$$

The above conclusion is proved by numerical simulation, with $a = 2$, $b = 0.5$, $c = 3$, $d = 1$, $e = 2$, $f = 2$, and $g = 2$. The equilibrium point (0.25, 0, 0.5, 1.5) is obtained, and the simulation results are shown in Figure 5.

By substituting the point $E_{13}(((ab d - b f + a f)/(ab d + be + a f)), ((ab d + be - a e)/(ab d + be + a f)), 0, ((abe + ab f)/(ab d + be + a f)))$ in matrix (3), we can obtain the Jacobian matrix:

$$J = \begin{bmatrix} a(1-2x) - w & 0 & xyw & -x \\ 0 & b(1-2y) - w & xyw & -y \\ 0 & 0 & c - w(1-xy) & 0 \\ ew & fw & gw & -dw \end{bmatrix} = [c - w(1-xy)] \begin{bmatrix} a(1-2x) - w & 0 & -x \\ 0 & b(1-2y) - w & -y \\ ew & fw & -dw \end{bmatrix}. \quad (11)$$

In addition, the Jacobian characteristic determinant is as follows:

$$\begin{aligned} |J - \lambda E| &= [c - w(1-xy) - \lambda] \begin{vmatrix} a(1-2x) - w - \lambda & 0 & -x \\ 0 & b(1-2y) - w - \lambda & -y \\ ew & fw & -dw - \lambda \end{vmatrix} \\ &= -(C_2 - \lambda) [\lambda^3 + (dw - A_2 - B_2)\lambda^2 + (A_2 B_2 + ewx + fwy - dw B_2 - dw A_2)\lambda + dw A_2 B_2 - ewx B_2 + fwy A_2] = 0, \end{aligned} \quad (12)$$

FIGURE 5: The values $a = 2, b = 0.5, c = 3, d = 1, e = 2, f = 2$, and $g = 2$ are used. So, we get the stable solution ($x = 0.25, y = 0, z = 0.5$, and $w = 1.5$).

where $A_2 = a(1 - 2x) - w, B_2 = b(1 - 2y) - w$, and $C_2 = c - w(1 - xy)$

Using the Routh-Hurwitz condition, the following condition is necessary and sufficient that all roots of the

characteristic equation of system (1) at the equilibrium point have a negative real part:

$$\begin{cases} C_2 < 0, \\ dw - A_2 - B_2 > 0, \\ A_2 B_2 + ewx + fwy - dw B_2 - dw A_2 > 0, \\ dw A_2 B_2 - ewx B_2 + fwy A_2 > 0, \\ (dw - B_2 - A_2)(A_2 B_2 + ewx + fwy - dw B_2 - dw A_2) > dw A_2 B_2 - ewx B_2 + fwy A_2. \end{cases} \quad (13)$$

The above conclusion was proved by numerical simulation, with $a = 2, b = 3, c = 0.5, d = 1, e = 2, f = 2$, and $g = 2$. The equilibrium point $(0.25, 0.5, 0, 1.5)$ is obtained, and the simulation results are shown in Figure 6.

By substituting the point $E_{14}(1, 1, 1, (e + f + g)/d)$ in matrix (3), we can obtain the Jacobian matrix and the Jacobian characteristic determinant as follows:

$$\begin{aligned} |J - \lambda E| &= \begin{vmatrix} -a - \lambda & w & w & 0 \\ w & -b - \lambda & w & 0 \\ w & w & -c - \lambda & 0 \\ ew & fw & gw & -e - f - g - \lambda \end{vmatrix} \\ &= (-e - f - g - \lambda) \begin{vmatrix} -a - \lambda & w & w \\ w & -b - \lambda & w \\ w & w & -c - \lambda \end{vmatrix} \\ &= -(-e - f - g - \lambda) [\lambda^3 + (a + b + c)\lambda^2 + (ab + ac + bc - 3w^2)\lambda + abc - (a + b + c)w^2 - 2w^3] = 0. \end{aligned} \quad (14)$$

FIGURE 6: The values $a=2$, $b=3$, $c=0.5$, $d=1$, $e=2$, $f=2$, and $g=2$ are used. So, we get the stable solution ($x=0.25$, $y=0.5$, $z=0$, and $w=1.5$).

It is known from above equation that $\lambda_1 = -e-f-g < 0$. Using the Routh–Hurwitz condition, the following condition is necessary and sufficient that all roots of the characteristic equation of system (1) at the equilibrium point have a negative real part:

$$\begin{cases} ab + ac + bc - 3w^2 > 0, \\ abc - (a + b + c)w^2 - 2w^3 > 0, \\ abc - (a + b + c)w^2 - 2w^3 < (a + b + c)(ab + ac + bc - 3w^2). \end{cases} \quad (15)$$

When above conditions are met, $E_{14}(1, 1, 1, (e + f + g)/d)$ is locally asymptotically stable.

The above conclusion is proved by numerical simulation, with $a=4$, $b=4$, $c=4$, $d=3$, $e=2$, $f=2$, and $g=2$. The equilibrium point $(1, 1, 1, 2)$ is obtained, and the simulation results are shown in Figure 7.

Now, we consider the above system but without team interaction help terms:

$$\begin{cases} \frac{dx}{dt} = ax(1-x) - xw, \\ \frac{dy}{dt} = by(1-y) - yw, \\ \frac{dz}{dt} = cz(1-z) - zw, \\ \frac{dw}{dt} = -dw^2 + exw + fyw + gzw. \end{cases} \quad (16)$$

It is easy to get its equilibrium point as follows: $x = ((bag - bgc - cfb + afc + dbca)/(dbca + bag + bec + afc))$, $y = ((dbca + bag + bec - agc - aec)/(dbca + bag + bec + afc))$, $z = ((dbca - afb - bea + afc + bec)/(dbca + bag + bec + afc))$, and $w = (cab(f + e + g))/(dbca + bag + bec + afc)$.

FIGURE 7: The values $a=4$, $b=4$, $c=4$, $d=3$, $e=2$, $f=2$, and $g=2$ are used. So, we get the stable solution ($x=1$, $y=1$, $z=1$, and $w=2$).

To prove the global stability of the above equilibrium point (the internal solution) of system (16), we use the following propositions. $\square$

Proposition 2 (see [28]). The system $du/dt = ru(1-u)$ has two equilibrium solutions, $u=0$ and $u=1$, where the constant $r > 0$. The nonzero equilibrium solution $u=1$ is globally stable for all solutions u with the initial condition $u(0) > 0$.

Proposition 3. When a, b, c, d, e, f , and g are all positive, the internal solutions of (16) are globally asymptotically stable for all solutions with the initial conditions $x(0) > 0$, $y(0) > 0$, $z(0) > 0$, and $w(0) > 0$.

Proof. From the first equation in system (16), it is known that $(dx/dt) \leq ax(1-x)$. According to Proposition 2, we can obtain that $x \leq U_1^x$, where $U_1^x = 1$.

From the second equation in system (16), it is known that $(dy/dt) \leq by(1-y)$. According to Proposition 2, we can obtain that $y \leq U_1^y$, where $U_1^y = 1$.

From the third equation in system (16), it is known that $(dz/dt) \leq cz(1-z)$. According to Proposition 2, we can obtain that $z \leq U_1^z$, where $U_1^z = 1$. Therefore, from the fourth equation in system (16), it is known that $(dw/dt) \leq -dw^2 + (eU_1^x + fU_1^y + gU_1^z)w$ and $dw/dt \leq dw((eU_1^x + fU_1^y + gU_1^z)/d - w)$.

According to Proposition 2, we can obtain that $w \leq U_1^w$, where $U_1^w = eU_1^x + fU_1^y + gU_1^z/d$.

On the other hand,

$$\begin{aligned} \frac{dx}{dt} &\geq ax(1-x) - xU_1^w, \\ \frac{dx}{dt} &\geq ax\left(\left(1 - \frac{U_1^w}{a}\right) - x\right). \end{aligned} \quad (17)$$

Using Proposition 2, we can obtain that $x \geq L_1^x$, where $L_1^x = 1 - (U_1^w/a)$.

By the same way, $y \geq L_1^y$, where $L_1^y = 1 - (U_1^w/b)$, and $z \geq L_1^z$, where $L_1^z = 1 - (U_1^w/c)$.

Similarly, $(dw/dt) \geq -dw^2 + (eL_1^x + fL_1^y + gL_1^z)w$ and $(dw/dt) \geq dw((eL_1^x + fL_1^y + gL_1^z/d) - w)$.

Using Proposition 2, we can obtain that $w \geq L_1^w$, where $L_1^w = (eL_1^x + fL_1^y + gL_1^z/d)$.

Continuing the above process, one gets a sequence of upper limits U_i^x, U_i^y, U_i^z , and U_i^w and lower limits L_i^x, L_i^y, L_i^z , and L_i^w , where $i = 2, 3$, related by the relations:

$$\begin{aligned} U_i^x &= 1 - \frac{L_i^w}{a}, \\ U_i^y &= 1 - \frac{L_i^w}{b}, \\ U_i^z &= 1 - \frac{L_i^w}{c}, \\ U_i^w &= \frac{eU_i^x + fU_i^y + gU_i^z}{d}, \\ L_i^x &= 1 - \frac{U_i^w}{a}, \\ L_i^y &= 1 - \frac{U_i^w}{b}, \\ L_i^z &= 1 - \frac{U_i^w}{c}, \\ L_i^w &= \frac{eL_i^x + fL_i^y + gL_i^z}{d}, \quad i = 2, 3, \dots, n, \end{aligned} \quad (18)$$

whose solutions are

$$\begin{cases} U_i^x = L_i^x = (bag - bgc - cfb + afc + dbca)/(dbca + bag + bec + afc) \\ U_i^y = L_i^y = (dbca + bag + bec - agc - aec)/(dbca + bag + bec + afc) \\ U_i^z = L_i^z = (dbca - afb - bea + afc + bec)/(dbca + bag + bec + afc) \\ U_i^w = L_i^w = (cab(f + e + g))/dbca + bag + bec + afc \end{cases}$$

This completes the proof.

The conclusion is proved by numerical simulation. When arbitrarily taking positive number as $a = 4, b = 6, c = 8, d = 2, e = 3, f = 2$, and $g = 2$ and $x(0) = 0.2, y(0) = 0.1, z(0) = 0.3$, and $w(0) = 1.4$, we can obtain internal solution in equation (16) as $(0.475, 0.65, 0.7375, 2.1)$, and the simulation results are shown in Figure 8, which shows that the simulation result agrees with theory result of internal solution (equilibrium point). When arbitrarily taking positive number as $a = 2, b = 4, c = 5, d = 5, e = 2, f = 2$, and $g = 2$ and $x(0) = 1.5, y(0) = 0.1, z(0) = 1.3$, and $w(0) = 1.4$, we can obtain internal solution in equation (16) as $(0.5652, 0.7826, 0.8261, 0.8696)$, and the simulation results are shown in Figure 9, which shows that the simulation result agrees with theory result of internal solution (equilibrium point). $\square$

Proposition 4. System (16) is persistent.

FIGURE 8: Positive parameters $a = 4, b = 6, c = 8, d = 2, e = 3$, and $f = 2$ corresponding to internal solution in equation (16) as $(0.475, 0.65, 0.7375, 2.1)$.

FIGURE 9: Positive parameters $a = 2, b = 4, c = 5, d = 5, e = 2, f = 2$, and $g = 2$ corresponding to internal solution in equation (16) as $(0.5652, 0.7826, 0.8261, 0.8696)$.

Proof. According to Proposition 3, we have proved that for all $t > 0$, all solutions with positive initial conditions satisfy

$$\begin{cases} 0 < L_i^x \leq x \leq U_i^x \\ L_i^y \leq y \leq U_i^y \\ L_i^z \leq z \leq U_i^z \\ L_i^w \leq w \leq U_i^w \end{cases}$$

Thus, system (16) is persistent. $\square$

4. Comparison with Reference [28]

Reference [28] studied the two-prey one-predator model, while this paper studies the three-prey single-predator model. Compared with reference [28], our model in this paper has the following characteristics: (i) the number of differential equations for the system increases by one; (ii) the number of equilibrium points increases by half, from 8 in reference [28] to 15 in this paper; and (iii) the equilibrium point converges faster, and the convergence time of numerical simulation is about 10 s in reference [28] and 2 s in this paper.

5. Conclusions

In this paper, a three-prey single-predator continuous time nonlinear system is studied and a model of fourth-order nonlinear differential equation is proposed. The equilibrium point, the characteristic root, and the stability of the system solution of the differential equation are analyzed. When considering preys help each other, there are 15 equilibria in the system in which 8 equilibria are unstable and other 7 equilibria are locally asymptotically stable. When not considering preys help each other, all equilibria are globally asymptotically stable.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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