Research Article

On Behavioral Response of Microstructural Slip on the Development of Magnetohydrodynamic Micropolar Boundary Layer Flow

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In this paper, the concept of microstructural slip is introduced in the flow of micropolar fluids pertinent to model various physical situations. The flow is modeled by a set of PDEs which are transformed to a nonlinear system of ODEs by employing boundary layer transformations. The system of governing equations is implemented using MATLAB bvp4c function along with the initial-boundary conditions. The code is validated by comparing the computed results in the limiting case with the available literature. Influence of microstructural slip on the skin friction coefficient and Nusselt number along with hydrodynamic and thermal boundary layer profiles is studied and discussed. It is found that, in the presence of microstructural slip, the microrotational velocity boundary layer thickness decreases up to a maximum of 37.5% in its value, in comparison to the case where there is no microstructural slip effect. The results predict that, in the presence of first-order translational slip, the microrotations have shown counterrotational phenomena in comparison to the case where there is no translational slip effect. Moreover, second-order translational slip results in declining the microrotational velocity and associated layer thickness.

1. Introduction

It is worth mentioning that the present slip model contains a true description of slip velocity at the wall in the content of micropolar flow. Translational slip effects have been studied in the literature in a number of physical situations pertinent to different modeling aspects. For instance, reduction in friction, energy conversation, and mimicking biological water channels [1]. The literature survey reveals significant interest in analyzing the slip effects in various geometries for several fluids [2, 3]. Micropolar fluids [4] are a class of polar fluids [5] with microstructure [6]. These fluids are pertinent to model various physical situations [7] and are important to study for many industrial applications [8], for instance,

paper and fiberglass production, extrusion of plastic sheets in the aerodynamical study, polymer processes, and extraction of oil. A detailed review, on the theory and applications of micropolar fluids, was given by Crane [9]. He worked initially on steady, boundary layer flow of an incompressible viscous fluid over a linearly stretching sheet. Heat and mass transfer with suction and blowing was analyzed by Gupta and Gupta [10].

A familiar article of all the aforementioned studies replicated the no-slip boundary condition. However, there may be natural situations where the no-slip boundary condition may not be relevant. The slip flow boundary condition was first introduced by C. L. M. H. Navier more than a century ago. In microsystems, slip

flows show a most important task, for instance, hard disk drive, micropumps, nozzles, and microvalves. He and Cai [11] observed the combined impact of temperature jump and velocity slip on a boundary layer flow towards a flat plate. The influence of velocity slip and temperature jump on a boundary layer flow of MHD pseudoplastic power law fluid over a moving permeable surface was considered by Xinnui et al. [12]. Unsteady boundary layer flow towards a stretching permeable surface was presented by Hosseini et al. [13]. They analyzed that, with rise in thermal and jump parameters, the skin friction coefficient and heat transfer rates diminish. Farhan et al. [14] proposed a single-step implicit block method which is quite suitable for solving nonlinear stiff ordinary differential equations. The proposed algorithm is zero stable and convergent. They investigated the flow numerically and concluded the results for temperature jump and velocity slip. Daniel et al. [15] discussed MHD boundary layer flow of a nanofluid towards a porous sheet for convective boundary condition along with slip effect. Mebarek-Oudina et al. [16] discussed a numerical approach finite volume method via computer code with Fortran programming to improve the phenomena of heat transfer by investigating MHD natural convection in porous cylindrical annulus filled with magnetized nanomaterial. The top and bottom walls are thermally insulated, outer wall at lesser temperature. Mostafa and Shima [17] gave the analysis of flow and heat transfer characteristics of magnetohydrodynamics mixed convection flow of a micropolar fluid past a stretching sheet with slip velocity at the surface and heat generation (absorption). Mahmood et al. [18] used the spectral homotopy analysis method (SHAM) for the solution of heat transfer phenomena in magnetohydrodynamic (MHD) Jeffery–Hamel flow. The two-dimensional viscous incompressible flow is taken into account for both convergent and divergent channels. In their study, a strong agreement of results is shown in comparison with shooting and the differential transform method (DTM). Mukhopadhyay [19] discussed the velocity slip and thermal slip over MHD boundary layer flow and heat transfer towards a porous exponential stretching sheet in presence of a magnetic field. Ali et al. [20] used a modified implicit finite difference scheme for the solution of two-dimensional fractional subdiffusion equation. They carried out the stability and convergence of the proposed scheme and found that the scheme is unconditionally stable, and approximate solution converges to the exact solution. Sohail et al. [21] explored theoretically and numerically heat and mass transfer with irreversibility for the flow of couple stressed fluid passing through a nonlinear stretched surface. A useful discussion is focused for variable thermophysical characteristics in their proposed model. Ali et al. [22–24] explored a new numerical approach, implicit difference scheme, for the variable-order fractional Riemann–Liouville Integral formula for the fractional subdiffusion equation. The one-dimensional time-fractional wave-diffusion equation is also resolved by the abovementioned scheme. They

also discovered a new forth-order implicit difference scheme and used it to solve the two-dimensional time-fractional modified subdiffusion equation. The effectiveness, unconditional stability, convergence, and consistency of the proposed scheme are analyzed by comparing the results using the von Neumann method. Abdelsalam and Sohail [25] described theoretically the heat and mass transfer phenomena of a three-dimensional viscous fluid flow over a nonlinear stretched surface. Optimal homotopic procedure is used to resolve nonlinear system of ordinary differential equations. Several references in this field can be found in [26–28]. The results are compared with the published work, and excellent agreement is found.

The literature discussed above only uses the theory of classical continuum to describe the slip velocity. Although the nonclassical continuum (i.e., micropolar theory) is used by many researchers in the literature, they defined the slip velocity with only the translational degree of freedom (i.e., the translational velocity). Here, for the first time in the literature, the effect rotational degree of freedom has been introduced simply through the kinematic relation in the description of velocity slip based on the fact that the shear stress takes the contribution also from the rotational degree of freedom of the fluid particles. The definition of velocity slip is modified, thereby incorporating the complete description of the micropolar continuum. Moreover, the effects of rotational slip velocity on some physical parameters are shown and discussed. These results are compatible with the actual physical assumptions in the theory which reveals the importance of the research study.

The rest of the paper is organized as follows: in Section 2, microstructural slip (translational and rotational slip) flow conditions are shown along with the flow governing equations. In Section 3, solution methodology is given. In Section 4, the results are shown and discussion is presented. Finally, conclusions are drawn in Section 5.

2. Mathematical Formulation

In this section, we describe the microstructural slip flow boundary conditions along with the flow governing equations. The flow equations are defined as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{(\mu + \kappa)}{\rho} \frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho} \frac{\partial N}{\partial y} - \frac{\sigma B_0^2}{\rho} u, \quad (2)$$

$$u \frac{\partial N}{\partial x} + v \frac{\partial N}{\partial y} = \frac{\Omega}{\rho j} \frac{\partial^2 N}{\partial y^2} - \frac{\kappa}{\rho j} \left(2N + \frac{\partial u}{\partial y} \right), \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}. \quad (4)$$

The boundary conditions are

$$\begin{aligned}
u &= u_w + U_{\text{slip}}, \\
v &= 0, \\
N &= -n \frac{\partial u}{\partial y}, \\
T &= T_w, \text{ at} \\
y &= 0, \\
u &\longrightarrow 0, \\
N &\longrightarrow 0, \\
T &\longrightarrow T_\infty, \text{ as} \\
y &\longrightarrow \infty,
\end{aligned} \tag{5}$$

where U_{slip} is the slip velocity at the surface, the modified form of micropolar kinematics is given as

$$\begin{aligned}
U_{\text{slip}} &= \frac{2}{3} \left(\frac{3 - \alpha l^2}{\alpha} - \frac{3}{2} \frac{1 - l^2}{\kappa_n} \right) \lambda \frac{\partial u}{\partial y} - \frac{1}{4} \left[l^4 + \frac{2}{\kappa_n^2} (1 - l^2) \right] \lambda^2 \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} - r_0 N \right), \\
U_{\text{slip}} &= A \frac{\partial u}{\partial y} + B \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} - r_0 N \right).
\end{aligned} \tag{6}$$

Let us define a similarity variable by

$$\xi = \sqrt{\frac{a}{\nu}} y. \tag{7}$$

The dimensionless variables f , θ , and g are defined as

$$\begin{aligned}
f(\xi) &= \frac{\psi}{\sqrt{a\nu x}}, \\
g(\xi) &= \frac{N}{ax\sqrt{a/\nu}}, \\
\theta(\xi) &= \frac{T - T_\infty}{T_w - T_\infty}.
\end{aligned} \tag{8}$$

The equation of continuity is satisfied if we choose stream function $\psi(x, y)$ such that

$$\begin{aligned}
u &= \frac{\partial \psi}{\partial x}, \\
v &= -\frac{\partial \psi}{\partial y}.
\end{aligned} \tag{9}$$

Using dimensionless variables and similarity transformation, equations (1)–(4) are reduced into the ODEs as follows:

$$(1 + \beta) f''' = f'^2 - f f'' - \beta g' + M f', \tag{10}$$

$$\left(1 + \frac{\beta}{2} \right) g'' = \beta (2g + f'') - f g' + f' g, \tag{11}$$

$$\left(\frac{3R + 4}{3R} \right) \theta'' = -\text{Pr} f \theta', \tag{12}$$

with boundary conditions,

$$\begin{aligned}
f(0) &= 0, \\
f'(0) &= 1 + \gamma f''(0) + \delta f'''(0) + r_0 g'(0), \\
g(0) &= -n f''(0), \\
\theta(0) &= 1, \text{ at} \\
\xi &= 0, \\
f'(\infty) &\longrightarrow 0, \\
g(\infty) &\longrightarrow 0, \\
\theta(\infty) &\longrightarrow 0, \text{ as} \\
\xi &\longrightarrow \infty,
\end{aligned} \tag{13}$$

where the governing parameters are defined by

TABLE 1: Comparison of skin friction coefficient $-f''(0)$ with slip factor γ , when $M = \delta = r_0 = 0.0$.

γ	Present result	Wubshet ²⁵	Sahoo ²¹
0.0	1.0000	1.000000	1.001154
0.1	0.8719	0.872082	0.871447
0.2	0.7762	0.776377	0.774933
0.3	0.7014	0.701548	0.699738
0.5	0.5911	0.591196	0.589195
1.0	0.4301	0.430160	0.428450
2.0	0.2840	0.283980	0.282893
3.0	0.2140	0.214055	0.213314
5.0	0.1448	0.144841	0.144430
10	0.0812	0.081243	0.081091
20	0.0438	0.043790	0.043748

$$\begin{aligned}
Pr &= \frac{\nu}{A}, \\
M &= \frac{\sigma B_0^2}{\rho a}, \\
\beta &= \frac{\kappa}{\mu}.
\end{aligned} \tag{14}$$

Skin friction coefficient and Nusselt number are defined as

$$\begin{aligned}
C_f &= \frac{\tau_w}{\rho u_w^2}, \\
Nu_x &= \frac{x q_w}{k(T_w - T_\infty)},
\end{aligned} \tag{15}$$

where the wall shear stress τ_w and wall heat flux q_w are given by

$$\begin{aligned}
\tau_w &= \left[(\mu + \kappa) \frac{\partial u}{\partial y} + \kappa N \right]_{y=0}, \\
q_w &= -\kappa \left(\frac{\partial T}{\partial y} \right)_{y=0}.
\end{aligned} \tag{16}$$

By using equation (16) in equation (15), one can arrive at

$$\begin{aligned}
C_f \sqrt{Re_x} &= -(1 + \beta(1 - n)) f''(0), \\
\frac{Nu_x}{\sqrt{Re_x}} &= -\theta'(0),
\end{aligned} \tag{17}$$

where $Re_x = ax^2/\nu$ and Nu_x are local Reynolds and local Nusselt numbers, respectively.

3. Solution Methodology and Code Validation

To solve numerically the coupled set of ordinary differential equations (ODEs) in equations (10)–(12) subjected to the boundary conditions in equation (13), we use bvp4c function in MATLAB. For this purpose, the function is modified according to equations (10)–(12) and the boundary conditions in equation (13) are implemented. To validate the implemented code, the physical parameters, i.e., the skin friction coefficient and the Nusselt numbers, are calculated

for different values of parameters. These results are compared with the results obtained by Wubshet²⁵ in the case of translational slip velocity, and a good agreement is found. In this respect, Tables 1–4 are presented as follows.

In Table 1, slip factor γ results in declining the skin friction coefficient which is compatible with the other research studies [29, 30].

In Table 2, heat transfer coefficient is comparatively analyzed numerically.

In Table 3, skin friction coefficient is numerically studied and comparison of results is shown. It depicts that skin friction increases with magnetic number, whereas opposite trend is observed for material parameter. When second-order slip parameter is reduced, the skin friction behaves alike.

In Table 4, the local Nusselt number is numerically examined and the results are compared with Wubshet²⁸. It is seen that local Nusselt number decreases with magnetic number, whereas opposite trend is experienced for material parameter. When second-order slip parameter is lessened, the Nusselt number also shows a behavior like slip parameter.

4. Results and Discussion

In this section, the effect of microstructural slip on the development of microrotation velocity, macroscopic velocity, and temperature boundary layer profiles is shown through tables and graphs and discussion is presented in detail. Moreover, the influence of microstructural slip on the skin friction coefficient and Nusselt number is analyzed. In Table 5, these physical parameters are calculated and shown with varying values of the rotational and translational velocity slip factors. It is found that, with the increase in rotational and translational slip parameters, the values of the skin friction coefficient decrease. Moreover, the values of Nusselt number increase with the increase in the translational and rotational slip parameters. This finding is in accordance with the observation of Wubshet²⁵, where only the effect of translational slip parameter is shown.

In Figure 1, the development of microrotational velocity boundary layer is depicted with varying microstructural slip for three different cases. In the first case, the perimeter $n(0 \leq n \leq 1)$ is associated with the concentration factor; the rotation of the microelements near the stretching sheet is

TABLE 2: Numerical comparison of values of local Nusselt number $-\theta'(0)$ for different values of Pr when $M = \beta = 0.0$ and $n = 0.5$

Pr	Ishak ²⁴	Present result $r_0 = 0, 1$	Wubshet ²⁵
0.72	0.4631	0.4623	0.4636
1.0	0.5820	0.5809	0.5822
3.0	1.1652	1.1634	1.1652
10.0	2.3080	2.3047	2.3080
100.0	7.7657	7.7552	7.7657

TABLE 3: Numerically computed values of skin friction coefficient $-f''(0)$ when $Pr = 1$ and $n = 0.5$ for different values of M, β , and δ .

M	β	δ	$-f''(0)$ Wubshet ²⁵			$-f''(0)$ Present results		
			$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.1	0.1	-1	0.3220	0.2399	0.1914	0.3219	0.2398	0.1913
0.2	0.1	-1	0.3262	0.2434	0.1943	0.3262	0.2433	0.1942
0.3	0.1	-1	0.3293	0.2459	0.1964	0.3293	0.2459	0.1964
0.4	0.1	-1	0.3315	0.2478	0.1979	0.3315	0.2478	0.1979
0.2	1	-1	0.3173	0.2371	0.1896	0.3172	0.2370	0.1895
0.2	2	-1	0.3068	0.2301	0.1846	0.3066	0.2300	0.1845
0.2	3	-1	0.2971	0.2239	0.1802	0.2967	0.2236	0.1800
0.2	4	-1	0.2884	0.2183	0.1763	0.2878	0.2179	0.1759
0.2	2	-1	0.3068	0.2301	0.1846	0.3066	0.2300	0.1845
0.2	2	-2	0.2588	0.2039	0.1683	0.2586	0.2037	0.1681
0.2	2	-3	0.2262	0.1841	0.1551	0.2259	0.1839	0.1549

chosen to be zero, thereby implying the strong concentration [31]. In the second and third cases, $n = 0.5$ and $n = 1$ are chosen, respectively, thereby implying the weak concentration [32]. It is observed that, in the case of strong concentration, where $n = 0$, the thickness of microrotational velocity boundary layer increases with increasing the value of microstructural slip. However, in the case of weak concentration of the rotation of microelements near the stretching surface, the thickness of microrotational velocity boundary layer decreases with increasing the value of microstructural slip.

In Figure 2, the development of microstructural boundary layer profiles near the stretching surface is studied in the presence and absence of magnetic field effect. It is observed that microstructural slip affects the microrotational boundary layer profiles significantly in the presence of magnetic field effect in comparison to the absence of magnetic field effect. Moreover, the microrotational boundary layer profiles decrease in both the cases with increase in microstructural slip.

In Figures 3 and 4, microstructural slip effects on the development of microrotational velocity boundary layer profiles are predicted. Here, an interesting feature of microstructural slip is observed which is that the thickness of microrotational boundary layer profiles is bounded to variation of microstructural slip. We observe that, in the presence of microstructural slip, the microrotational velocity boundary layer thickness decreases up to a maximum of 37.5% in its value, in comparison to the case where there is no microstructural slip effect. Moreover, in a flow situation, microstructural slip effects are bounded to increasing microstructural slip.

TABLE 4: Computed values of local Nusselt number $-\theta'(0)$ when $Pr = 1$ and $n = 0.5$ for different values of M, β , and δ .

M	β	δ	$-\theta'(0)$ Wubshet ²⁵			$-\theta'(0)$ Present results		
			$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.1	0.1	-1	0.3816	0.3432	0.3167	0.3773	0.3370	0.3085
0.2	0.1	-1	0.3603	0.3214	0.2947	0.3540	0.3122	0.2825
0.3	0.1	-1	0.3411	0.3024	0.2761	0.3327	0.2900	0.2599
0.4	0.1	-1	0.3239	0.2859	0.2605	0.3130	0.2702	0.2404
0.2	1	-1	0.4066	0.3623	0.3315	0.4038	0.3579	0.3252
0.2	2	-1	0.4431	0.3954	0.3620	0.4418	0.3934	0.3590
0.2	3	-1	0.4703	0.4206	0.3856	0.4698	0.4199	0.3844
0.2	4	-1	0.4915	0.4407	0.4046	0.4915	0.4408	0.4046
0.2	2	-1	0.4431	0.3954	0.3620	0.4418	0.3934	0.3590
0.2	2	-2	0.4143	0.3767	0.3487	0.4127	0.3742	0.3451
0.2	2	-3	0.3927	0.3616	0.3375	0.3906	0.3538	0.3333

TABLE 5: Computed values of skin friction $-f''(0)$ and local Nusselt number $-\theta'(0)$ for different values of r_0 when $Pr = 0.72$, $n = 0.5$, $M = 0.0$, $\beta = 0.1$, and $\delta = -1$.

r_0	$-f''(0)$			$-\theta'(0)$		
	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$	$\gamma = 1$	$\gamma = 2$	$\gamma = 3$
0.0	0.3162	0.2351	0.1873	0.3211	0.2912	0.2703
0.2	0.3161	0.2350	0.1872	0.3217	0.2917	0.2707
0.5	0.3160	0.2349	0.1871	0.3224	0.2922	0.2717
1.0	0.3158	0.2347	0.1870	0.3230	0.2928	0.2727
1.5	0.3157	0.2346	0.1869	0.3237	0.2934	0.2730
2.0	0.3156	0.2345	0.1868	0.3239	0.2935	0.2731

FIGURE 1: Impact of n and r_0 on $g(\xi)$ when $M = 10, \gamma = \delta = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 2: Impact of M and r_0 on $g(\xi)$ when $n = 0.5, \gamma = \delta = r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 3: Impact of r_0 on $g(\xi)$ when $n = 0.5, \gamma = \delta = 1, r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 4: Impact of r_0 on $g(\xi)$ when $M = 5, n = 0.5, \gamma = \delta = 1, r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 5: Impact of γ and r_0 on $g(\xi)$ when $M = 5, n = 0.5, \delta = 1, r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 6: Impact of δ and r_0 on $g(\xi)$ when $M = 5, n = 0.5, \gamma = 1, r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 7: Impact of γ and r_0 on $f'(\xi)$ when $M = 5, n = 0.5, \delta = 1, r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

FIGURE 8: Impact of $\delta = \gamma$ and r_0 on $f'(\xi)$ when $M = 5, n = 0.5, r_1 = R = 0$, and $\text{Pr} = \beta = 5$.

In Figure 5, the combined effect of second-order translational slip and microstructural slip on the development of microrotational velocity boundary layer is shown. It is observed that the microrotational velocity boundary layer profile in case of combined effect of microstructural slip and second-order translational slip is always lower than the case where there is only microstructural slip and no second-order translational slip. Moreover, it is observed that the variation in the thickness of microrotational velocity boundary layer in case of second-order translational slip and microstructural slip is larger than the case where only microstructural slip is present, thereby implying that the second-order

translational slip do affect the microrotational boundary layer profile. By increasing the second-order translational slip parameter, the thickness of microrotational velocity boundary layer profile decreases.

In Figure 6, the combined effect of first-order translational slip and microstructural slip can be seen, where it is observed that, in the presence of first-order translational slip, the microrotations have shown a counterrotational phenomenon in comparison to the case where there is only microstructural slip effect and no translational slip effect. The variation of the thickness of boundary layer profiles is always larger in case of combined effect of translational and

FIGURE 9: Impact of R and r_0 on $\theta(\xi)$ when $M = \text{Pr} = \beta = 5$, $n = 0.5$, and $\delta = 1$.

FIGURE 10: Impact of Pr and r_0 on $\theta(\xi)$ when $M = \beta = 5$, $n = 0.5$, $\delta = r_1 = 0$, and $\gamma = 1$.

microrotational slip in comparison to the case where is no translational slip, as can be seen in Figures 5 and 6. However, in the absence of first-order translational slip, the counterrotational phenomenon near the stretching surface is not observed.

Figure 7 is plotted to notice the impact of fluid velocity in the absence and presence of microstructural slip. It is depicted that velocity profile decreases for mounting values of γ and increases for mounting values of slip parameter r_0 .

Figure 8 shows the combined influence of $f'(\eta)$ on growing values of slip parameter and fixed values of γ and δ . It is analyzed that the velocity profile increases. Figure 9 is plotted to notice the behavior of temperature profile for varying values of radiation parameter in the presence and

absence of microstructural slip. It is observed that fluid temperature decreases for growing values of radiation parameter in both the cases. Figure 10 shows the impact of Prandtl number on fluid temperature in the absence and presence of microstructural slip, and it is observed that fluid temperature decreases for higher values of Prandtl number in both the cases. In Figure 11, the impact of magnetic parameter M on temperature profile is observed in the absence and presence of microstructural slip. The fluid temperature increases for an increasing value of magnetic parameter. Figure 12 indicates the impact of material parameter β in the absence and presence of microstructural slip. The decline in fluid temperature is noticed.

FIGURE 11: Impact of M and r_0 on $\theta(\xi)$ when $Pr = 3, \beta = R = 5, n = 0.5$, and $\delta = r_1 = 0$.

FIGURE 12: Impact of β and r_0 on $\theta(\xi)$ when $M = R = 5, Pr = 3, n = 0.5, \delta = r_1 = 0$, and $\gamma = 1$.

5. Conclusion

The aim of this paper is to investigate the effects of microstructural slip on the development of hydrodynamic and thermal boundary layers. Moreover, its effect on the skin friction coefficient and Nusselt number of a micropolar flow governed by a set of PDEs along with initial-boundary conditions is studied. The resulting equations are solved numerically using the bvp4c scheme. The main findings of this study can be summarized as follows:

- (i) Microrotational boundary layer thickness decreases in both the cases with increase in microstructural slip
- (ii) In the presence of microstructural slip, the microrotational velocity boundary layer thickness decreases up to a maximum of 37.5% in its value, in

comparison to the case where there is no microstructural slip effect

- (iii) Increasing the second-order translational slip parameter, the thickness of microrotational velocity boundary layer profile decreases
- (iv) In the presence of first-order translational slip, the microrotations have shown a counterrotational phenomenon in comparison to the case where there is only microstructural slip effect and no translational slip effect
- (v) In the absence of first-order translational slip, the counterrotational phenomenon near the stretching surface is not observed
- (vi) Increasing the values of microstructural slip parameter and magnetic parameter reduces the fluid velocity

- (vii) Increasing the values of Prandtl number and radiation parameter decreases the fluid temperature in the absence as well as in the presence of microstructural slip
- (viii) Decrease in temperature profile thickness is noticed for growing values of microstructural slip parameter

Abbreviations

A :	Stretching constant
A and B :	Slip constants
B_0 :	Magnetic field strength
K :	Thermal conductivity of fluid
C_f :	Skin friction coefficient
k :	Coefficient of vortex viscosity
f :	Dimensionless stream function
g :	Dimensionless microrotation function
j :	Microinertia density
Kn :	Knudsen number
Ψ :	Stream function
M :	Magnetic parameter
n :	Microrotation parameter
Nux :	Local Nusselt number
τ_w :	Wall shear stress
Pr :	Prandtl number
Q_w :	Surface heat flux
Re_x :	Local Reynolds number
α :	Momentum accommodation coefficient
γ :	First-order slip parameter
δ :	Second-order slip parameter
η :	Dimensionless similarity variable
N :	Microrotation component normal to xy plane
C_p :	Specific heat
M :	Coefficient of dynamic viscosity
ν :	Coefficient kinematic viscosity
σ :	Electrical conductivity
λ :	The molecular mean free path
Λ :	Thermal diffusivity
β :	Material parameter
ρ :	Fluid density
Ω :	Spin-gradient viscosity
∞ :	Condition at the free stream
w :	Condition at the surface.

Data Availability

Data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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