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Research Article

Finite-Time Tracking Control for Nonstrict-Feedback State-Delayed Nonlinear Systems with Full-State Constraints and Unmodeled Dynamics

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The problem of finite-time tracking control is discussed for a class of uncertain nonstrict-feedback time-varying state delay nonlinear systems with full-state constraints and unmodeled dynamics. Different from traditional finite-control methods, a C^1 smooth finite-time adaptive control framework is introduced by employing a smooth switch between the fractional and cubic form state feedback, so that the desired fast finite-time control performance can be guaranteed. By constructing appropriate Lyapunov-Krasovskii functionals, the uncertain terms produced by time-varying state delays are compensated for and unmodeled dynamics is coped with by introducing a dynamical signal. In order to avoid the inherent problem of "complexity of explosion" in the backstepping-design process, the DSC technology with a novel nonlinear filter is introduced to simplify the structure of the controller. Furthermore, the results show that all the internal error signals are driven to converge into small regions in a finite time, and the full-state constraints are not violated. Simulation results verify the effectiveness of the proposed method.

1. Introduction

During the past few decades, great achievements have been proposed for uncertain nonlinear systems based on adaptive control technique, especially for pure-feedback systems (e.g., see [1–5]) and strict-feedback systems (e.g., see [6–9]) with the lower-triangular structure. Lately, the authors in [10] introduced a more general nonlinear system named nonstrict-feedback nonlinear systems. By employing the variable separation method, the tracking control problem has been well solved. Since then, many control techniques for nonstrict-feedback systems and extensions to other fields were achieved (e.g., see [11–17]).

It is known to all that many practical systems encounter the effect of the constraints, such as the temperature of chemical reactor and physical stoppages. Thus, the research about the systems with state constraints is very meaningful and necessary on account of the existence of state constraints which may undermine the stability of the system. In order to tackle the problem of state constraints, some effective control techniques (e.g., model predictive control (MPC) [18, 19], reference governors (RGs) [20], one-to-one nonlinear mapping (NM) [21–23], and barrier Lyapunov functions (BLFs) [24–28]) have been presented. Due to the fact that MPC and RGs require strong online computing capability to guarantee constraints, this requirement restricts their applications in engineering design. Therefore, one-to-one NM and the BLFs-based methods become the main methods to deal with the constrained nonlinear systems. There exist many significant results which focus on lower-triangular structure nonlinear systems with different constraints (e.g., input constraints [3], output constraints [24], partial-state constraints [25], and full-state constraints [21–23, 26, 27]). In addition, the rate of convergence is also an essential consideration for most practical systems. The works mentioned above only obtain asymptotic or exponential stability with infinite time, which cannot meet the requirement of finite-time control in most practical control systems. As a consequence, a considerable number of meaningful researches (e.g., see [28–33]) have been proposed on finite-time control for nonlinear systems. However, most of the works are to present C^0 finite-time controller by using a backstepping technique together with a nonsmooth fractional feedback design method. In order to achieve a faster convergence rate, the authors in [34] originally proposed a C^1 smooth finite-time adaptive NN controller by using a smooth switch between the fractional and cubic form state feedback. Moreover, there are other significant results presented in [35–41], such that two globally stable adaptive controllers were proposed in [35, 36]. To obtain the tracking accuracy, a practical adaptive fuzzy tracking controller for a class of perturbed nonlinear systems with backlash nonlinearity has been designed in [37]. An adaptive fuzzy output-feedback tracking control technique for switched stochastic pure-feedback nonlinear systems has been presented in [38]. The authors in [39] proposed an observed-based adaptive finite-time tracking control technique for a class of nonstrict-feedback nonlinear systems with input saturation. An adaptive finite-time output-feedback controller for switched pure-feedback nonlinear systems with average dwell time has been given in [40]. A decentralized event-triggered controller for interconnected systems with unknown disturbances has been proposed in [41].

Furthermore, due to the fact that unmodeled dynamics can severely degrade the closed-loop system performance, dealing with the effects of unmodeled dynamics is essential for practical nonlinear control systems. Therefore, several results were proposed by employing backstepping or DSC in [4, 21–23, 42–47]. Generally, unmodeled dynamics was disposed by introducing a dynamic signal in [4, 21–23, 42–46] or a Lyapunov function description in [47].

In addition, time delays frequently occur in some practical engineering systems. As stated in [48], their existence can deteriorate the transient performance and even can destroy the stability of the control systems. Thus, the research on nonlinear time-delay systems has become one of the hot topics and some meaningful results have been achieved during the past decades [49–53]. For uncertain nonlinear time-delay systems, the effective controller was developed originally in [50] by combining the backstepping technique with Lyapunov-Krasovskii functionals. Soon afterward, this method was extended to nonlinear strict-feedback time-delay system with unknown control gain functions [51] and uncertain multi-input/multi-output nonlinear systems with time delays [52]. Later, some improved control schemes based on [50] were proposed (e.g., see [35, 53, 54]).

Although many significant research results on adaptive neural network control for uncertain nonstrict-feedback systems have been obtained in [11–17], their considered systems did not include unmodeled dynamics or full-state constraints. In [21–28], the effective controllers have been designed for the lower-triangular structure nonlinear systems with state constraints and unmodeled dynamics, but their considered systems did not include state delay and their control methods may be invalid to nonstrict-feedback systems on account of subsystem function which contains the whole state variables. Furthermore, the above-mentioned control methods only obtain asymptotic or exponential stability with infinite time. To the best knowledge of the authors, finite-time tracking control for a class of uncertain nonstrict-feedback time-varying state-delayed nonlinear systems with full-state constraints and unmodeled dynamics has not been fully discussed in the literature, which is still open and remains unsolved. In this paper, we are committed to solving the problem mentioned above. The main contributions of the paper are summarized as follows:

(i) In contrast to the existing results reported in [21–28, 47] where the control methods have been proposed for nonlinear strict-feedback or pure-feedback systems with state or output constraints and unmodeled dynamics, a generalization of the results is proposed for a class of nonstrict-feedback state delay systems with state constraints and unmodeled dynamics of which the subsystem function contains the whole state variables. To the best of authors' knowledge, it is the first time to develop an adaptive DSC method for uncertain nonstrict-feedback state delay systems with state constraints and unmodeled dynamics.

(ii) Different from the finite-control methods in [31–33], a C^1 smooth finite-time adaptive control framework is introduced by employing a smooth switch between the fractional and cubic form state feedback reported in [34], so that the desired fast finite-time control performance can be guaranteed. Moreover, unmodeled dynamics is coped with by introducing a dynamical signal and the uncertain terms produced by time-varying state delays are compensated for by constructing appropriate Lyapunov-Krasovskii functionals. The results show that all the error signals are driven to converge into small regions in a finite time, and the full-state constraints are never violated.

The remainder of this paper is organized as follows. In Section 2, the problem formulation and preliminaries are presented. Adaptive DSC design and stability analysis are given in Section 3. Simulation results verify the effectiveness of the proposed control approach in Section 4, followed by Section 5, which concludes this paper.

Notation. In this paper, R denotes a set of real numbers, R^+ denotes a set of nonnegative real numbers, $R^{m\times n}$ denotes a set of $m \times n$ real matrices, R^n denotes a set of n-dimensional real vectors, $\sup(\cdot)$ denotes the least upper bound, $\|\cdot\|$ denotes 2-norm of a vector or matrix, $|\cdot|$ denotes an absolute value of a real number $\cdot$, $\exp(\cdot)$ denotes an exponential function of $\cdot$, and $\log(\cdot)$ denotes the natural logarithm of $\cdot$.

2. Problem Formulation and Preliminaries

2.1. Problem Statement. Consider a class of uncertain nonstrict-feedback state-delayed nonlinear systems with unmodeled dynamics for $i = 1, 2, \ldots, n-1$ in the following form:

$$\begin{cases} \dot{\xi} = q(\xi, x, t), \\ \dot{x}_i = f_i(x) + g_i(\bar{x}_i)x_{i+1} + \delta_i(\xi, x, t) + d_i(\bar{x}_i(t - T_i(t))), \\ \dot{x}_n = f_n(x) + g_n(\bar{x}_n)u + \delta_n(\xi, x, t) + d_n(\bar{x}_n(t - T_n(t))), \\ y = x_1, \end{cases} \quad (1)$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector, $\xi \in R^{n_0}$ is the unmodeled dynamics, and $u, y, T_i(t)$ denote the system input, the system output, and the unknown time-varying delays, respectively. $f_i(x), g_i(\bar{x}_i)$, and $d_i(\bar{x}_i(t - T_i(t)))$ are the unknown smooth functions. Let $\bar{x}_i = [x_1, x_2, \ldots, x_i]^T$ and $\delta_i(\xi, x, t)$ be the unknown uncertain disturbances. All the states x_i are required to remain in the sets $\Omega_{x_i} = \{x_i: |x_i| < k_{c_i}\}$, where k_{c_i} are positive constants.

Remark 1. System (1) is called a nonstrict-feedback form in which the system function $f_i(\cdot)$ and its bounding function contain all the state variables [10]. Apparently, strict-feedback and pure-feedback structures are the special cases of system (1). The methods proposed in [21–28, 31–33, 47] cannot be directly applied to system (1) on account of its nonstrict-feedback structure.

The control objective of this paper is to construct an adaptive NN controller $u(t)$ to make sure that the output y follows the desired trajectory y_r in a finite time, while every state $x_i \in \Omega_{x_i}$ is never violated.

2.2. RBFNN Approximation. In this paper, for $i = 1, \ldots, n$, the unknown smooth nonlinear functions $\bar{F}_i(Z_i)$: $R^m \longrightarrow R$ will be approximated on a compact set $\Omega_i \subset R^m$ by the following RBFNN:

$$\bar{F}_i(Z_i) = W_i^T S_i(Z_i) + \varepsilon_i(Z_i), \quad (2)$$

where Z_i, W_i, l denote input vectors, weight vectors, and NN node number, respectively. $\varepsilon_i(Z_i)$ are the NN inherent approximation errors which are bounded over the compact sets; that is, $\varepsilon_i(Z_i) \le \varepsilon_i$, where ϵ_i are unknown constants and $S_i(Z_i) = [s_1(Z_i), \ldots, s_l(Z_i)]^T$: $\Omega_i \longrightarrow R^l$ are known smooth vector functions with $s_q(Z_i)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_q(Z_i) = \exp\left[\frac{-(Z_i - \mu_q)^T(Z_i - \mu_q)}{\eta_q^2}\right], \quad q = 1, \ldots, l, \quad (3)$$

where $\mu_q = [\mu_{q1}, \ldots, \mu_{qm}]^T$ is the center vector and η_q is the spreads of the Gaussian function. The optimal weight vector W_i is defined as

$$W_i = \arg\min_{\widehat{W}_i \in R^l}\left\{\sup_{Z_i \in \Omega_i}\left|F(Z_i) - \widehat{W}_i^T S(Z)\right|\right\}, \quad (4)$$

where $\widehat{W}_i$ is the estimate of W_i.

2.3. Key Definition and Lemmas

Definition 1 (see [21]). The unmodeled dynamics ξ is said to be exponentially input-state-practically stable (exp-ISpS), that is, for system $\dot{\xi} = q(\xi, x, t)$, if there exist functions $\bar{\alpha}_1, \bar{\alpha}_2$ of class K_∞ and a Lyapunov function $V(\xi)$, such that

$$\bar{\alpha}_1(\|\xi\|) \le V(\xi) \le \bar{\alpha}_2(\|\xi\|), \quad (5)$$

and there exist two constants $c > 0, d \ge 0$ and a class K_∞ function γ, such that

$$\frac{\partial V(\xi)}{\partial \xi} q(\xi, x, t) \le -cV(\xi) + \gamma(|x_1|) + d, \quad \forall t \ge 0, \quad (6)$$

where c and d are known positive constants and $\gamma(\cdot)$ is a known function of class K_∞.

Lemma 1 (see [21]). *If V is an exp-ISpS Lyapunov function for a system $\dot{\xi} = q(\xi, x, t)$, that is, (5) and (6) hold, then, for any constant $\bar{c} \in (0, c)$, any initial instant $t_0 > 0$, any initial condition $\xi_0 = \xi(t_0), r_0 > 0$, and any continuous function $\bar{\gamma}$, such that $\bar{\gamma}(|x_1|) \ge \gamma(|x_1|)$, there exist a finite $T_0 = \max\{0, \log[(V(\xi_0)/r_0)/(c - \bar{c})]\} \ge 0$, a nonnegative function $D(t_0, t)$ defined for all $t \ge t_0$, and a signal described by*

$$\dot{r} = -\bar{c}r + \bar{\gamma}(\|x_1\|) + d, \ r(t_0) = r_0, \quad (7)$$

such that $D(t_0, t) = 0$ for $t \ge t_0 + T_0$ and $V(\xi) \le r(t) + D(t_0, t)$ with $D(t_0, t) = \max\{0, e^{-c(t-t_0)}V(z_0) - e^{-\bar{c}(t-t_0)}r_0)\}$.

Lemma 2 (see [11]). *Let $S(Z)$ be the basis function vector of an RBFNN and Z be the input vector, where $S(Z) = [s_1(Z), \ldots, s_l(Z)]^T$ and $Z = [z_1, \ldots z_n]^T$. For any positive integer $m \le n$, let $Z_m = [z_1, \ldots, z_m]^T$, and the following inequality holds:*

$$\|S(Z)\|^2 \le \|S(Z_m)\|^2. \quad (8)$$

Lemma 3 (see [55]). *For any real numbers $\zeta_1 > 0, \zeta_2 > 0$ and $0 < h < 1$, an extended Lyapunov condition of finite-time stability can be given in the form of fast terminal sliding mode as $\dot{V}(x) + \zeta_1 V(x) + \zeta_2 V^h(x) \le 0$; then, $V(x)$ is in fast finite-time convergent with a finite settling time $T^* \le (1/\zeta_1(1-h))\log((\zeta_1 V^{1-h}(x_0) + \zeta_2)/\zeta_2)$.*

Lemma 4 (see [56]). *For $x, y \in R$, if $0 < h = h_2/h_1 < 1$, where $h_1, h_2 > 0$ are odd integers, then $xy^h \le -\varsigma_1 x^{1+h} + \varsigma_2(x+y)^{1+h}$, where $\varsigma_1 = (1/(1+h))(2^{h-1} - 2^{(h-1)(h+1)})$ and $\varsigma_2 = (1/(1+h))(1 + (2h/(1+h)) + (2^{-(h-1)^2(h+1)}/(1+h))2^{h-1})$.*

Lemma 5 (see [34]). *Consider the dynamic system*

$$\dot{\phi}(t) = -l_1\phi(t) - l_2\phi^h(t) + \varrho(t), \quad (9)$$

where $\phi(t) \in R, 0 < h = (h_2/h_1) < 1$ *(h_1 and h_2 are positive odd integers), l_1 and l_2 are positive constants, and $\varrho(t)$ is a positive function. Then, for any given bounded initial condition $\phi(0) \geq 0$, one has that $\phi(t) \geq 0$, $\forall t \geq 0$.*

Lemma 6 (see [57]). *For $x_i \in R, i = 1, 2, \ldots, n$, and $0 < h \leq 1$, then $(\sum_{i=1}^n |x_i|)^h \leq \sum_{i=1}^n |x_i|^h \leq n^{1-h}(\sum_{i=1}^n |x_i|)^h$.*

To obtain the control objective, the following assumptions are needed.

Assumption 1. The unmodeled dynamics ξ is exp-ISpS.

Assumption 2. There exist unknown nonnegative continuous functions φ_{i1} and nondecreasing continuous functions φ_{i2} such that

$$\left|\delta_i(\xi, x, t)\right| \leq \varphi_{i1}\left(\left\|\bar{x}_i\right\|\right) + \varphi_{i2}(\|\xi\|), \quad \forall(\xi, x, t) \in R^{n_0} \times R^n \times R^+, \quad (10)$$

where $\varphi_{i2}(0) = 0, \quad i = 1, \ldots, n.$

Remark 2. From Definition 1 and Assumption 1, we have $\|\xi\| \leq \bar{\alpha}_1^{-1}(V(\xi))$. According to Lemma 1, there exists a positive constant D_0 such that $\|\xi\| \leq \bar{\alpha}_1^{-1}(r + D_0)$, $\forall t \geq 0$. This inequality will be used to cope with the uncertain terms in the following controller design.

Assumption 3. The sign of $g_i(\bar{x}_i)$ is known, and there exist some unknown positive constants a_i and b_i such that $0 < b_i \leq |g_i(\bar{x}_i)| \leq a_i$. Without loss of generality, this paper assumes that $g_i(\bar{x}_i) > 0$.

Assumption 4. The reference trajectory $y_r(t)$ and its derivatives about time $\dot{y}_r$ and $\ddot{y}_r$ are in a bounded region Ω_d, and there exists a known constant A_0, such that $|y_r| \leq A_0 < k_{c1}$.

Assumption 5. The unknown continuous functions $d_i(\bar{x}_i(t - T_i(t)))$ satisfy the following inequality:

$$d_i\left(\bar{x}_i(t - T_i(t))\right) \leq \sum_{j=1}^{i} \rho_{ij}\left(x_j\left(t - T_j(t)\right)\right), \quad (11)$$

and the time-varying state delays $T_i(t)$ satisfy the inequalities $0 \leq T_i(t) \leq T_{\max}$ and $\dot{T}_i(t) \leq \bar{T}_{\max} < 1$, where $\rho_{ij}(x_j(t - T_j(t)))$ are unknown positive smooth functions and $T_{\max}$ and $\bar{T}_{\max}$ are unknown constants.

3. Adaptive DSC Design and Stability Analysis

3.1. Adaptive DSC Design. Similar to traditional backstepping, the backstepping-design procedure with n steps is developed to construct the adaptive neural controller in this part.

By using the backstepping technique, the proposed adaptive DSC scheme contains n steps as follows.

Step 1. Define the first surface error $z_1 = x_1 - y_r$; the time derivative of z_1 is defined as

$$\dot{z}_1 = f_1(x) + g_1\left(\bar{x}_1\right)x_2 + \delta_1(\xi, x, t) + d_1\left(\bar{x}_1(t - T_1(t))\right) - \dot{y}_r. \quad (12)$$

The virtual control law α_1 and the update law for $\dot{\hat{\varpi}}_1$ are designed as

$$\alpha_1 = -c_1 z_1 - \mu_1 \frac{z_1}{k_{b1}^2 - z_1^2} - \frac{z_1 \hat{\varpi}_1 S_1^T(\Xi_1) S_1(\Xi_1)}{2l_1\left(k_{b1}^2 - z_1^2\right)} - \kappa_1 \beta_1(z_1), \quad (13)$$

$$\dot{\hat{\varpi}}_1 = \rho_1\left(-\sigma_{11}\hat{\varpi}_1 - \sigma_{12}\hat{\varpi}_1^h + \frac{z_1^2 S_1^T(\Xi_1) S_1(\Xi_1)}{2l_1\left(k_{b1}^2 - z_1^2\right)^2}\right), \quad (14)$$

where $c_1, \mu_1, k_{b1}, \kappa_1, \rho_1, \sigma_{11}, \sigma_{12}, l_1$ are positive design parameters, $\hat{\varpi}_1$ is an estimate of ϖ_1, $\tilde{\varpi}_1 = \varpi_1 - b_1\hat{\varpi}_1$, $\varpi_1 = \|W_1\|^2$, b_1 is defined in Assumption 3. $\beta_1(z_1)$ is defined as

$$\beta_1(z_1) = \begin{cases} z_1^h\left(k_{b1}^2 - z_1^2\right)^{(1-h/2)}, & \text{if } |z_1| \geq \tau_1, \\ \iota_{11} z_1 + \iota_{12} z_1^3, & \text{if } |z_1| < \tau_1, \end{cases} \quad (15)$$

where $0 < h = (h_1/h_2) < 1, h_1$ and h_2 are the positive odd integers, $\iota_{11} = \tau_1^{h-1}(k_{b1}^2 - \tau_1^2)^{(1-(h/2))} - \iota_{12}\tau_1^2, \iota_{12} = (1/2\tau_1^3)(h-1)\tau_1^h[(k_{b1}^2 - \tau_1^2)^{(1-(h/2))} + \tau_1^2((k_{b1}^2 - \tau_1^2)^{-(1+(h/2))}]$, and $\tau_1 < k_{b1}$ is a small positive constant.

Consider the BLF candidate V_{z_1} as

$$V_{z_1} = \frac{1}{2}\log\frac{k_{b1}^2}{k_{b1}^2 - z_1^2} + \frac{1}{2b_1\rho_1}\tilde{\varpi}_1^2. \quad (16)$$

Obviously, V_{z_1} is positive definite and continuously differentiable. Based on Assumptions 2 and 5 and Young's inequality, we obtain the time derivative of V_{z_1} as follows:

$$\begin{aligned}
\dot{V}_{z_1} &= \frac{z_1}{k_{b1}^2 - z_1^2}\left[f_1(x) + g_1(\bar{x}_1)x_2 + \delta_1(\xi, x, t) + d_1(\bar{x}_1(t - T_1(t))) - \dot{y}_r\right] - \frac{1}{\rho_1}\tilde{\varpi}_1\dot{\hat{\varpi}}_1, \\
&\leq \frac{z_1}{k_{b1}^2 - z_1^2}\left[f_1(x) + g_1(\bar{x}_1)x_2 - \dot{y}_r\right] + \frac{|z_1|}{k_{b1}^2 - z_1^2}\left[\left[\varphi_{11}(\|\bar{x}_1\|) + \varphi_{12}\left(\bar{\alpha}_1^{-1}(r + D_0)\right)\right] + \rho_{11}(x_1(t - T_1(t)))\right] - \frac{1}{\rho_1}\tilde{\varpi}_1\dot{\hat{\varpi}}_1, \\
&\leq \frac{z_1}{k_{b1}^2 - z_1^2}\left[f_1(x) + g_1(\bar{x}_1)x_2 - \dot{y}_r\right] + \frac{z_1^2}{(k_{b1}^2 - z_1^2)^2}\left[\varphi_{11}(\|\bar{x}_1\|) + \varphi_{12}\left(\bar{\alpha}_1^{-1}(r + D_0)\right)\right]^2 + \frac{1}{4}, \\
&\quad + \frac{z_1^2}{2(k_{b1}^2 - z_1^2)^2} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) - \frac{1}{\rho_1}\tilde{\varpi}_1\dot{\hat{\varpi}}_1, \\
&\leq \frac{z_1}{k_{b1}^2 - z_1^2}\left[f_1(x) + g_1(\bar{x}_1)x_2 + \frac{z_1}{k_{b1}^2 - z_1^2}\left[\varphi_{11}(\|\bar{x}_1\|) + \varphi_{12}\left(\bar{\alpha}_1^{-1}(r + D_0)\right)\right]^2 + \frac{z_1}{2(k_{b1}^2 - z_1^2)} - \dot{y}_r\right], \\
&\quad + \frac{1}{4} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) - \frac{1}{\rho_1}\tilde{\varpi}_1\dot{\hat{\varpi}}_1, \\
&\leq \frac{z_1}{k_{b1}^2 - z_1^2}\left[\bar{F}_1(Z_1) + g_1(\bar{x}_1)x_2\right] + \frac{1}{4} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) - \frac{1}{\rho_1}\tilde{\varpi}_1\dot{\hat{\varpi}}_1, \\
&= \frac{z_1}{k_{b1}^2 - z_1^2}\left[\bar{F}_1(Z_1) + g_1(\bar{x}_1)(z_2 + y_2 + \alpha_1)\right] + \frac{1}{4} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) - \frac{1}{\rho_1}\tilde{\varpi}_1\dot{\hat{\varpi}}_1,
\end{aligned} \tag{17}$$

where

$$\begin{aligned}
\bar{F}_1(Z_1) = f_1(x) &+ \frac{z_1}{k_{b1}^2 - z_1^2}\left[\varphi_{11}(\|\bar{x}_1\|) + \varphi_{12}\left(\bar{\alpha}_1^{-1}(r + D_0)\right)\right]^2 \\
&+ \frac{z_1}{2(k_{b1}^2 - z_1^2) - \dot{y}_r}.
\end{aligned} \tag{18}$$

Note that $\bar{F}_1(Z_1)$ is an unknown continuous function and RBFNN can be used to approximate it. Hence, from (2), the following equation holds:

$$\bar{F}_1(Z_1) = W_1^T S_1(Z_1) + \varepsilon_1(Z_1), \tag{19}$$

where $W_1^T S_1(Z_1)$ is an NN, $|\varepsilon_1(Z_1)| \leq \varepsilon_1, Z_1 = [\bar{x}_n, z_1, r, \dot{y}_r]^T$, and $\varepsilon_1 > 0$ is any given.

By using Young's inequality and Lemma 2, one has

$$\begin{aligned}
\frac{z_1}{k_{b1}^2 - z_1^2}\bar{F}_1(Z_1) &= \frac{z_1}{k_{b1}^2 - z_1^2}\left[W_1^T S_1(Z_1) + \varepsilon_1(Z_1)\right], \\
&\leq \frac{|z_1|}{k_{b1}^2 - z_1^2}\|W_1\|\|S_1(Z_1)\| + \frac{\varepsilon_1(Z_1)z_1}{k_{b1}^2 - z_1^2}, \\
&\leq \frac{|z_1|}{k_{b1}^2 - z_1^2}\|W_1\|\|S_1(\Xi_1)\| + \frac{\varepsilon_1(Z_1)z_1}{k_{b1}^2 - z_1^2}, \\
&\leq \frac{1}{2l_1}\frac{z_1^2}{(k_{b1}^2 - z_1^2)^2}\|W_1\|^2\|S_1(\Xi_1)\|^2 + \frac{l_1}{2} \\
&\quad + \frac{\varepsilon_1 z_1}{k_{b1}^2 - z_1^2}, \\
&= \frac{1}{2l_1}\frac{z_1^2}{(k_{b1}^2 - z_1^2)^2}\varpi_1\|S_1(\Xi_1)\|^2 + \frac{l_1}{2} + \frac{\varepsilon_1 z_1}{k_{b1}^2 - z_1^2},
\end{aligned} \tag{20}$$

where $\Xi_1 = [x_1, z_1, r, \dot{y}_r]^T$.

Substituting (13), (14), and (20) into (17), we can obtain

$$
\begin{aligned}
\dot{V}_{z_1} \le & \frac{g_1(\bar{x}_1)z_1z_2}{k_{b1}^2 - z_1^2} + \frac{g_1(\bar{x}_1)z_1y_2}{k_{b1}^2 - z_1^2} - \frac{c_1g_1(\bar{x}_1)z_1^2}{k_{b1}^2 - z_1^2} - \frac{\mu_1g_1(\bar{x}_1)z_1^2}{\left(k_{b1}^2 - z_1^2\right)^2} - \frac{g_1(\bar{x}_1)z_1^2\widehat{\varpi}_1\|S_1(\Xi_1)\|^2}{2l_1\left(k_{b1}^2 - z_1^2\right)^2} + \frac{l_1}{2}, \\
& - \frac{\kappa_1g_1(\bar{x}_1)z_1\beta_1(z_1)}{k_{b1}^2 - z_1^2} + \frac{z_1^2\varpi_1\|S_1(\Xi_1)\|^2}{2l_1\left(k_{b1}^2 - z_1^2\right)^2} + \frac{1}{4} + \frac{\varepsilon_1z_1}{k_{b1}^2 - z_1^2} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))), \\
& - \frac{1}{\rho_1}\widetilde{\varpi}_1\left[\rho_1\left(-\sigma_{11}\widehat{\varpi}_1 - \sigma_{12}\widehat{\varpi}_1^h + \frac{z_1^2S_1^T(\Xi_1)S_1(\Xi_1)}{2l_1\left(k_{b1}^2 - z_1^2\right)^2}\right)\right], \\
\le & \frac{g_1(\bar{x}_1)z_1z_2}{k_{b1}^2 - z_1^2} + \frac{g_1(\bar{x}_1)z_1y_2}{k_{b1}^2 - z_1^2} - \frac{b_1c_1z_1^2}{k_{b1}^2 - z_1^2} - \frac{\mu_1b_1z_1^2}{\left(k_{b1}^2 - z_1^2\right)^2} - \frac{z_1^2\varpi_1\|S_1(\Xi_1)\|^2 - z_1^2\widetilde{\varpi}_1\|S_1(\Xi_1)\|^2}{2l_1\left(k_{b1}^2 - z_1^2\right)^2}, \\
& + \frac{l_1}{2} - \frac{\kappa_1g_1(\bar{x}_1)z_1\beta_1(z_1)}{k_{b1}^2 - z_1^2} + \frac{z_1^2\varpi_1\|S_1(\Xi_1)\|^2}{2l_1\left(k_{b1}^2 - z_1^2\right)^2} + \frac{1}{4} + \frac{\varepsilon_1z_1}{k_{b1}^2 - z_1^2} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))), \\
& + \sigma_{11}\widetilde{\varpi}_1\widehat{\varpi}_1 + \sigma_{12}\widetilde{\varpi}_1\widehat{\varpi}_1^h - \frac{z_1^2\widetilde{\varpi}_1S_1^T(\Xi_1)S_1(\Xi_1)}{2l_1\left(k_{b1}^2 - z_1^2\right)^2}, \\
\le & \frac{g_1(\bar{x}_1)z_1z_2}{k_{b1}^2 - z_1^2} + \frac{g_1(\bar{x}_1)z_1y_2}{k_{b1}^2 - z_1^2} - \frac{b_1c_1z_1^2}{k_{b1}^2 - z_1^2} - \frac{\mu_1b_1z_1^2}{\left(k_{b1}^2 - z_1^2\right)^2} + \frac{l_1}{2} - \frac{\kappa_1g_1(\bar{x}_1)z_1\beta_1(z_1)}{k_{b1}^2 - z_1^2} + \frac{1}{4}, \\
& + \frac{\varepsilon_1z_1}{k_{b1}^2 - z_1^2} + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) + \sigma_{11}\widetilde{\varpi}_1\widehat{\varpi}_1 + \sigma_{12}\widetilde{\varpi}_1\widehat{\varpi}_1^h.
\end{aligned} \tag{21}
$$

By utilizing Young's inequality, the following inequalities can be obtained:

$$
\begin{aligned}
& \frac{g_1(\bar{x}_1)z_1y_2}{k_{b1}^2 - z_1^2} - \frac{\mu_1b_1z_1^2}{2\left(k_{b1}^2 - z_1^2\right)^2} \le \frac{g_1^2(\bar{x}_1)y_2^2}{2\mu_1b_1} \le \frac{a_1^2y_2^2}{2\mu_1b_1}, \\
& \frac{\varepsilon_1z_1}{k_{b1}^2 - z_1^2} - \frac{\mu_1b_1z_1^2}{2\left(k_{b1}^2 - z_1^2\right)^2} \le \frac{\varepsilon_1^2}{2\mu_1b_1}.
\end{aligned} \tag{22}
$$

Therefore, we have

$$
\begin{aligned}
\dot{V}_{z_1} \le & \frac{g_1(\bar{x}_1)z_1z_2}{k_{b1}^2 - z_1^2} + \frac{a_1^2y_2^2}{2\mu_1b_1} + \frac{\varepsilon_1^2}{2\mu_1b_1} - \frac{b_1c_1z_1^2}{k_{b1}^2 - z_1^2} + \frac{l_1}{2} \\
& - \frac{\kappa_1g_1(\bar{x}_1)z_1\beta_1(z_1)}{k_{b1}^2 - z_1^2} + \frac{1}{4}, \\
& + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) + \sigma_{11}\widetilde{\varpi}_1\widehat{\varpi}_1 + \sigma_{12}\widetilde{\varpi}_1\widehat{\varpi}_1^h.
\end{aligned} \tag{23}
$$

According to the inequality $2b_1\widetilde{\varpi}_1\widehat{\varpi}_1 \le \varpi_1^2 - \widetilde{\varpi}_1^2$ and Lemma 4, one as

$$
\begin{aligned}
\dot{V}_{z_1} \le & \frac{g_1(\bar{x}_1)z_1z_2}{k_{b1}^2 - z_1^2} + \frac{a_1^2y_2^2}{2\mu_1b_1} + \frac{\varepsilon_1^2}{2\mu_1b_1} - \frac{b_1c_1z_1^2}{k_{b1}^2 - z_1^2} + \frac{l_1}{2} \\
& - \frac{\kappa_1g_1(\bar{x}_1)z_1\beta_1(z_1)}{k_{b1}^2 - z_1^2} + \frac{1}{4}, \\
& + \frac{1}{2}\rho_{11}^2(x_1(t - T_1(t))) + \frac{\sigma_{11}}{2b_1}\varpi_1^2 - \frac{\sigma_{11}}{2b_1}\widetilde{\varpi}_1^2 \\
& - \frac{\sigma_{12}\zeta_1}{b_1^h}\widetilde{\varpi}_1^{1+h} + \frac{\sigma_{12}\zeta_2}{b_1^h}\varpi_1^{1+h},
\end{aligned} \tag{24}
$$

where ζ_1 and ζ_2 are defined in Lemma 4.

To deal with the time delay in equation (24), define the Lyapunov-Krasovskii functional as follows:

$$
V_{U_1} = \frac{e^{-\gamma(t - T_{\max})}}{2\left(1 - \bar{T}_{\max}\right)} \int_{t - T_1(t)}^{t} e^{\gamma s}\rho_{11}^2(x_1(s))\mathrm{d}s, \tag{25}
$$

where $\gamma > 0$ is a positive constant. Using Assumption 5, we obtain that the derivative of V_{U_1} is

$$\dot{V}_{U_1} = \frac{e^{-\gamma(t-T_{\max})}}{2(1-\bar{T}_{\max})}\left[e^{\gamma t}\rho_{11}^2(x_1(t)) - e^{\gamma(t-T_1(t))}\rho_{11}^2 \cdot (x_1(t-T_1(t)))(1-\dot{T}_1(t))\right] - \gamma V_{U_1},$$
$$\leq \frac{e^{\gamma T_{\max}}}{2(1-\bar{T}_{\max})}\rho_{11}^2(x_1(t)) - \frac{1}{2}\rho_{11}^2(x_1(t-T_1(t))) - \gamma V_{U_1}. \tag{26}$$

From equations (24) and (26), we have

$$\begin{aligned}\dot{V}_{z_1} + \dot{V}_{U_1} \leq &\frac{g_1(\bar{x}_1)z_1z_2}{k_{b1}^2 - z_1^2} + \frac{a_1^2 y_2^2}{2\mu_1 b_1} + \frac{\varepsilon_1^2}{2\mu_1 b_1} - \frac{b_1c_1z_1^2}{k_{b1}^2 - z_1^2} + \frac{l_1}{2}\\ &- \frac{\kappa_1 g_1(\bar{x}_1)z_1\beta_1(z_1)}{k_{b1}^2 - z_1^2} + \frac{1}{4},\\ &+ \frac{\sigma_{11}}{2b_1}\varpi_1^2 - \frac{\sigma_{11}}{2b_1}\tilde{\varpi}_1^2 - \frac{\sigma_{12}\zeta_1}{b_1^h}\tilde{\varpi}_1^{1+h} + \frac{\sigma_{12}\zeta_2}{b_1^h}\varpi_1^{1+h}\\ &+ \Phi_1 - \gamma V_{U_1},\end{aligned} \tag{27}$$

where $\Phi_1 = (e^{\gamma T_{\max}}/2(1-\bar{T}_{\max}))\rho_{11}^2(x_1(t))$.

To move on, introduce the coordinate transformation

$$\begin{aligned} z_i &= x_i - w_i,\\ y_i &= w_i - \alpha_{i-1},\end{aligned} \tag{28}$$

where z_i, α_{i-1}, and y_i denote the tracking error, the virtual control input, and the boundary layer error for $i = 2, 3, \ldots, n$, respectively. w_i is the output of the following first-order filter:

$$\dot{w}_i = -\tau_{i1}y_i - \tau_{i2}y_i^h, \tag{29}$$

where τ_{i1} and τ_{i2} are the positive design constants and h is defined in (15).

Remark 3. From (29), it can be seen that the proposed filter involves both the linear and fractional terms. In particular, when $\tau_{i1} = 0$ or $\tau_{i2} = 0$, filter (29) degrades into the fractional filter used in [58] and the linear filter as widely used in the literature [21–23], respectively. It is the key to ensure the fast finite-time stability of the closed-loop system, which will be detailed in the following analysis.

Remark 4. As mentioned in [34], by designing ι_{11} and ι_{12} properly, both the virtual control input α_1 and its derivative $\dot{\alpha}_1$ are ensured to be inherently continuous in the set Ω_{x_i}. It means that the virtual control input α_1 defined in (13) is C^1 continuous in the set Ω_{x_i}. From (13), it is not hard to see that α_1 and its derivative $\dot{\alpha}_1$ are the functions of the variables $z_1, \hat{\varpi}_1, \dot{y}_r$ and $z_1, z_2, \hat{\varpi}_1, y_2, \dot{y}_r, \ddot{y}_r$, respectively. Combining the continuity of $\dot{\alpha}_1$ and (28) and (29), it can be seen that there exists a continuous function $\lambda_2(z_1, z_2, \dot{\hat{\varpi}}_1, y_2, \dot{y}_r, \ddot{y}_r)$ which satisfies

$$\dot{y}_2 \leq -\tau_{21}y_2 - \tau_{22}y_2^h + \lambda_2\left(z_1, z_2, \dot{\hat{\varpi}}_1, y_2, \dot{y}_r, \ddot{y}_r\right). \tag{30}$$

Step 2. $(i = 2, 3, \ldots, n-1)$ Define the i^{th} surface error $z_i = x_i - w_i$; the time derivative of z_i is defined as

$$\begin{aligned}\dot{z}_i &= f_i(x) + g_i(\bar{x}_i)x_{i+1} + \delta_i(\xi, x, t) + d_i(\bar{x}_i(t-T_i(t))) - \dot{w}_i,\\ &= f_i(x) + g_i(\bar{x}_i)(z_{i+1} + y_{i+1} + \alpha_i) + \delta_i(\xi, x, t)\\ &\quad + d_i(\bar{x}_i(t-T_i(t))) - \dot{w}_i.\end{aligned} \tag{31}$$

The virtual control law α_i and the update law $\dot{\hat{\varpi}}_i$ are designed as

$$\alpha_i = -c_iz_i - \mu_i\frac{z_i}{k_{bi}^2 - z_i^2} - \frac{z_i\hat{\varpi}_iS_i^T(\Xi_i)S_i(\Xi_i)}{2l_i(k_{bi}^2 - z_i^2)} - \kappa_i\beta_i(z_i), \tag{32}$$

$$\dot{\hat{\varpi}}_i = \rho_i\left(-\sigma_{i1}\hat{\varpi}_i - \sigma_{i2}\hat{\varpi}_i^h + \frac{z_i^2S_i^T(\Xi_i)S_i(\Xi_i)}{2l_i(k_{bi}^2 - z_i^2)^2}\right), \tag{33}$$

where $c_i, \mu_i, k_{bi}, \kappa_i, \rho_i, \sigma_{i1}, \sigma_{i2}, l_i$ are positive design parameters, $\hat{\varpi}_i$ is an estimate of ϖ_i, $\tilde{\varpi}_i = \varpi_i - b_i\hat{\varpi}_i$, $\varpi_i = \|W_i\|^2$, $\beta_i(z_i)$ is defined as

$$\beta_i(z_i) = \begin{cases} z_i^h(k_{bi}^2 - z_i^2)^{(1-h/2)}, & \text{if } |z_i| \geq \tau_i,\\ \iota_{i1}z_i + \iota_{i2}z_i^3, & \text{if } |z_i| < \tau_i,\end{cases} \tag{34}$$

where h is defined in (15), $\iota_{i1} = \tau_i^{h-1}(k_{bi}^2 - \tau_i^2)^{1-h/2} - \iota_{i2}\tau_i^2$, $\iota_{i2} = (1/2\tau_i^3)(h-1)\tau_i^h[(k_{bi}^2 - \tau_i^2)^{(1-h/2)} + \tau_i^2((k_{bi}^2 - \tau_i^2)^{-(1+h/2)})]$, and $\tau_i < k_{bi}$ is a small positive constant.

Consider the BLF candidate V_{z_i} as

$$V_{z_i} = \frac{1}{2}\log\frac{k_{bi}^2}{k_{bi}^2 - z_i^2} + \frac{1}{2b_i\rho_i}\tilde{\varpi}_i^2, \tag{35}$$

where V_{z_i} is also positive definite and continuously differentiable in the set $|z_i| < k_{bi}$. Similar to (17), the time derivative of V_{z_i} is

$$\begin{aligned}\dot{V}_{z_i} \leq &\frac{z_i}{k_{bi}^2 - z_i^2}\left[\bar{F}_i(Z_i) + g_i(\bar{x}_i)(z_{i+1} + y_{i+1} + \alpha_i)\right]\\ &+ \frac{1}{4} - \frac{1}{\rho_i}\tilde{\varpi}_i\dot{\hat{\varpi}}_i,\\ &+ \frac{1}{2}\sum_{j=1}^{i}\rho_{ij}^2(x_j(t-T_j(t))) - \frac{g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i}{k_{b(i-1)}^2 - z_{i-1}^2},\end{aligned} \tag{36}$$

where

$$\begin{aligned}\bar{F}_i(Z_i) = &f_i(x) + \frac{z_i}{k_{bi}^2 - z_i^2}\left[\varphi_{i1}(\|\bar{x}_i\|) + \varphi_{i2}\left(\bar{\alpha}_i^{-1}(r + D_0)\right)\right]^2\\ &- \dot{w}_i + \frac{g_{i-1}(\bar{x}_{i-1})(k_{bi}^2 - z_i^2)z_{i-1}}{k_{b(i-1)}^2 - z_{i-1}^2} + \frac{iz_i}{2(k_{bi}^2 - z_i^2)}.\end{aligned} \tag{37}$$

Note that $\bar{F}_i(Z_i)$ is an unknown continuous function and RBFNN can be used to approximate it. Hence, from (2), the following equation holds:

$$\bar{F}_i(Z_i) = W_i^T S_i(Z_i) + \varepsilon_i(Z_i), \tag{38}$$

where $W_i^T S_i(Z_i)$ is an NN, $|\varepsilon_i(Z_i)| \le \varepsilon_i$, $Z_i = [\bar{x}_n, z_{i-1}, z_i, r, w_{i-1}, \dot{w}_i]^T$, and $\varepsilon_i > 0$ is any given.

By using Young's inequality and Lemma 2, one has

$$\begin{aligned} \frac{z_i}{k_{bi}^2 - z_i^2}\bar{F}_i(Z_i) &= \frac{z_i}{k_{bi}^2 - z_i^2}\left[W_i^T S_i(Z_i) + \varepsilon_i(Z_i)\right], \\ &\le \frac{|z_i|}{k_{bi}^2 - z_i^2}\|W_i\|\|S_i(Z_i)\| + \frac{\varepsilon_i(Z_i) z_i}{k_{bi}^2 - z_i^2}, \\ &\le \frac{|z_i|}{k_{bi}^2 - z_i^2}\|W_i\|\|S_i(\Xi_i)\| + \frac{\varepsilon_i(Z_i) z_i}{k_{bi}^2 - z_i^2}, \\ &\le \frac{1}{2l_i}\frac{z_i^2}{\left(k_{bi}^2 - z_i^2\right)^2}\|W_i\|^2\|S_i(\Xi_i)\|^2 + \frac{l_i}{2} + \frac{\varepsilon_i(Z_i) z_i}{k_{bi}^2 - z_i^2}. \end{aligned} \tag{39}$$

Substituting (32), (33), and (39) into (36), we can obtain

$$\begin{aligned} \dot{V}_{z_i} &\le \frac{g_i(\bar{x}_i) z_i z_{i+1}}{k_{bi}^2 - z_i^2} + \frac{g_i(\bar{x}_i) z_i y_{i+1}}{k_{bi}^2 - z_i^2} - \frac{c_i g_i(\bar{x}_i) z_i^2}{k_{bi}^2 - z_i^2} - \frac{\mu_i g_i(\bar{x}_i) z_i^2}{\left(k_{bi}^2 - z_i^2\right)^2} - \frac{g_i(\bar{x}_i) z_i^2 \widehat{\varpi}_i \|S_i(\Xi_i)\|^2}{2l_i\left(k_{bi}^2 - z_i^2\right)^2} + \frac{l_i}{2} + \frac{1}{4}, \\ &\quad - \frac{\kappa_i g_i(\bar{x}_i) z_i \beta_i(z_i)}{k_{bi}^2 - z_i^2} + \frac{z_i^2 \varpi_i \|S_i(\Xi_i)\|^2}{2l_i\left(k_{bi}^2 - z_i^2\right)^2} + \frac{\varepsilon_i(Z_i) z_i}{k_{bi}^2 - z_i^2} + \frac{1}{2}\sum_{j=1}^{i}\rho_{ij}^2\left(x_j\left(t - T_j(t)\right)\right), \\ &\quad - \frac{g_{i-1}(\bar{x}_{i-1}) z_{i-1} z_i}{k_{b(i-1)}^2 - z_{i-1}^2} - \frac{1}{\rho_i}\widetilde{\varpi}_i\left[\rho_i\left(-\sigma_{i1}\widehat{\varpi}_i - \sigma_{i2}\widehat{\varpi}_i^h + \frac{z_i^2 S_i^T(\Xi_i) S_i(\Xi_i)}{2l_i\left(k_{bi}^2 - z_i^2\right)^2}\right)\right], \\ &\le \frac{g_i(\bar{x}_i) z_i z_{i+1}}{k_{bi}^2 - z_i^2} + \frac{g_i(\bar{x}_i) z_i y_{i+1}}{k_{bi}^2 - z_i^2} - \frac{b_i c_i z_i^2}{k_{bi}^2 - z_i^2} - \frac{\mu_i b_i z_i^2}{\left(k_{bi}^2 - z_i^2\right)^2} - \frac{z_i^2 \varpi_i \|S_i(\Xi_i)\|^2 - z_i^2 \widetilde{\varpi}_i \|S_i(\Xi_i)\|^2}{2l_i\left(k_{bi}^2 - z_i^2\right)^2}, \\ &\quad + \frac{l_i}{2} + \frac{1}{4} - \frac{\kappa_i g_i(\bar{x}_i) z_i \beta_i(z_i)}{k_{bi}^2 - z_i^2} + \frac{z_i^2 \varpi_i \|S_i(\Xi_i)\|^2}{2l_i\left(k_{bi}^2 - z_i^2\right)^2} + \frac{\varepsilon_i(Z_i) z_i}{k_{bi}^2 - z_i^2} + \frac{1}{2}\sum_{j=1}^{i}\rho_{ij}^2\left(x_j\left(t - T_j(t)\right)\right), \\ &\quad + \sigma_{i1}\widetilde{\varpi}_i\widehat{\varpi}_i + \sigma_{i2}\widetilde{\varpi}_i\widehat{\varpi}_i^h - \frac{z_i^2 \widetilde{\varpi}_i S_i^T(\Xi_i) S_i(\Xi_i)}{2l_i\left(k_{bi}^2 - z_i^2\right)^2}\Bigg) - \frac{g_{i-1}(\bar{x}_{i-1}) z_{i-1} z_i}{k_{b(i-1)}^2 - z_{i-1}^2}, \\ &\le \frac{g_i(\bar{x}_i) z_i z_{i+1}}{k_{bi}^2 - z_i^2} + \frac{g_i(\bar{x}_i) z_i y_{i+1}}{k_{bi}^2 - z_i^2} - \frac{b_i c_i z_i^2}{k_{bi}^2 - z_i^2} - \frac{\mu_i b_i z_i^2}{\left(k_{bi}^2 - z_i^2\right)^2} + \frac{l_i}{2} + \frac{1}{4} - \frac{\kappa_i g_i(\bar{x}_i) z_i \beta_i(z_i)}{k_{bi}^2 - z_i^2} + \frac{\varepsilon_i(Z_i) z_i}{k_{bi}^2 - z_i^2}, \\ &\quad + \frac{1}{2}\sum_{j=1}^{i}\rho_{ij}^2\left(x_j\left(t - T_j(t)\right)\right) + \sigma_{i1}\widetilde{\varpi}_i\widehat{\varpi}_i + \sigma_{i2}\widetilde{\varpi}_i\widehat{\varpi}_i^h - \frac{g_{i-1}(\bar{x}_{i-1}) z_{i-1} z_i}{k_{b(i-1)}^2 - z_{i-1}^2}. \end{aligned} \tag{40}$$

By utilizing Young's inequality, the following inequalities can be obtained:

$$\frac{g_i(\bar{x}_i)z_i y_{i+1}}{k_{bi}^2 - z_i^2} - \frac{\mu_i b_i z_i^2}{2(k_{bi}^2 - z_i^2)^2} \le \frac{g_i^2(\bar{x}_i) y_{i+1}^2}{2\mu_i b_i} \le \frac{a_i^2 y_{i+1}^2}{2\mu_i b_i},$$
$$\frac{\varepsilon_i(Z_i)z_i}{k_{bi}^2 - z_i^2} - \frac{\mu_i b_i z_i^2}{2(k_{bi}^2 - z_i^2)^2} \le \frac{\varepsilon_i^2(Z_i)}{2\mu_i b_i} \le \frac{\varepsilon_i^2}{2\mu_i b_i}. \quad (41)$$

Therefore, we have

$$\dot{V}_{z_i} \le \frac{g_i(\bar{x}_i)z_i z_{i+1}}{k_{bi}^2 - z_i^2} + \frac{a_i^2 y_{i+1}^2}{2\mu_i b_i} + \frac{\varepsilon_i^2}{2\mu_i b_i} - \frac{b_i c_i z_i^2}{k_{bi}^2 - z_i^2} - \frac{\kappa_i g_i(\bar{x}_i)z_i \beta_i(z_i)}{k_{bi}^2 - z_i^2},$$
$$+ \frac{1}{2}\sum_{j=1}^{i} \rho_{ij}^2\left(x_j\left(t - T_j(t)\right)\right) + \frac{l_i}{2} + \frac{1}{4} + \sigma_{i1}\tilde{\varpi}_i\hat{\varpi}_i + \sigma_{i2}\tilde{\varpi}_i\hat{\varpi}_i^h - \frac{g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i}{k_{b(i-1)}^2 - z_{i-1}^2}. \quad (42)$$

According to the inequality $2b_i\tilde{\varpi}_i\hat{\varpi}_i \le \varpi_i^2 - \tilde{\varpi}_i^2$ and Lemma 4, one has

$$\dot{V}_{z_i} \le \frac{g_i(\bar{x}_i)z_i z_{i+1}}{k_{bi}^2 - z_i^2} + \frac{a_i^2 y_{i+1}^2}{2\mu_i b_i} + \frac{\varepsilon_i^2}{2\mu_i b_i} - \frac{b_i c_i z_i^2}{k_{bi}^2 - z_i^2} - \frac{\kappa_i g_i(\bar{x}_i)z_i \beta(z_i)}{k_{bi}^2 - z_i^2},$$
$$+ \frac{1}{2}\sum_{j=1}^{i} \rho_{ij}^2\left(x_j\left(t - T_j(t)\right)\right) + \frac{l_i}{2} + \frac{1}{4} + \frac{\sigma_{i1}}{2b_i}\varpi_i^2 - \frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2$$
$$- \frac{\sigma_{i2}\zeta_1}{b_i^h}\tilde{\varpi}_i^{1+h} + \frac{\sigma_{i2}\zeta_2}{b_i^h}\varpi_i^{1+h} - \frac{g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i}{k_{b(i-1)}^2 - z_{i-1}^2}, \quad (43)$$

where ζ_1 and ζ_2 are defined in Lemma 4.

To handle the time delay, define the Lyapunov-Krasovskii functional as follows:

$$V_{U_i} = \frac{e^{-\gamma(t - T_{\max})}}{2(1 - \bar{T}_{\max})}\sum_{j=1}^{i}\int_{t - T_j(t)}^{t} e^{\gamma s}\rho_{ij}^2\left(x_j(s)\right)\mathrm{d}s, \quad (44)$$

where $\gamma > 0$ is a positive constant. By using Assumption 5, we obtain that the derivative of V_{Ui} is

$$\dot{V}_{U_i} = \frac{e^{-\gamma(t - T_{\max})}}{2(1 - \bar{T}_{\max})}\sum_{j=1}^{i}\Big[e^{\gamma t}\rho_{ij}^2\left(x_j(t)\right) - e^{\gamma(t - T_j(t))}\rho_{ij}^2 \cdot \left(x_j\left(t - T_j(t)\right)\right)\left(1 - \dot{T}_j(t)\right)\Big] - \gamma V_{U_i},$$
$$\le \sum_{j=1}^{i}\frac{e^{\gamma T_{\max}}}{2(1 - \bar{T}_{\max})}\rho_{ij}^2\left(x_j(t)\right) - \frac{1}{2}\sum_{j=1}^{i}\rho_{ij}^2\left(x_j\left(t - T_j(t)\right)\right) - \gamma V_{U_i}. \quad (45)$$

From in equations (43) and (45), we have

$$\dot{V}_{z_i} + \dot{V}_{U_i} \le \frac{g_i(\bar{x}_i)z_i z_{i+1}}{k_{bi}^2 - z_i^2} + \frac{a_i^2 y_{i+1}^2}{2\mu_i b_i} + \frac{\varepsilon_i^2}{2\mu_i b_i} - \frac{b_i c_i z_i^2}{k_{bi}^2 - z_i^2} - \frac{\kappa_i g_i(\bar{x}_i)z_i\beta_i(z_i)}{k_{bi}^2 - z_i^2} + \frac{l_i}{2} + \frac{1}{4} + \frac{\sigma_{i1}}{2b_i}\varpi_i^2,$$
$$- \frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2 - \frac{\sigma_{i2}\zeta_1}{b_i^h}\tilde{\varpi}_i^{1+h} + \frac{\sigma_{i2}\zeta_2}{b_i^h}\varpi_i^{1+h} - \frac{g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i}{k_{b(i-1)}^2 - z_{i-1}^2} + \Phi_i - \gamma V_{U_i}, \quad (46)$$

where $\Phi_i = \sum_{j=1}^{i}(e^{\gamma T_{\max}}/(2(1 - \bar{T}_{\max})))\rho_{ij}^2(x_j(t))$.

Similar to the analysis in Remark 4, there exists a continuous function $\lambda_{i+1}(\bar{z}_{i+1}, \bar{\hat{\varpi}}_i, y_2, y_3, \backslash\backslash \ldots, y_{i+1}, \dot{y}_r, \ddot{y}_r)$ which satisfies

$$\dot{y}_{i+1} \le -\tau_{(i+1)1}y_{i+1} - \tau_{(i+1)2}y_{i+1}^h + \lambda_{i+1} \cdot \left(\bar{z}_{i+1}, \bar{\hat{\varpi}}_i, y_2, y_3, \ldots, y_{i+1}, \dot{y}_r, \ddot{y}_r\right). \quad (47)$$

Step 3. Define the n^{th} surface error $z_n = x_n - w_n$; the time derivative of z_n is defined as

$$\dot{z}_n = f_n(x) + g_n(\bar{x}_n)u + \delta_n(\xi, x, t) + d_n\left(\bar{x}_n\left(t - T_n(t)\right)\right) - \dot{w}_n. \quad (48)$$

The actual control law u and the update law $\dot{\hat{\varpi}}_n$ are designed as

$$u = -c_n z_n - \mu_n\frac{z_n}{2(k_{bn}^2 - z_n^2)} - \frac{z_n\hat{\varpi}_n S_n^T(\Xi_n)S_n(\Xi_n)}{2l_n(k_{bn}^2 - z_n^2)} - \kappa_n\beta_n(z_n), \quad (49)$$

$$\dot{\hat{\varpi}}_n = \rho_n\left(-\sigma_{n1}\hat{\varpi}_n - \sigma_{n2}\hat{\varpi}_n^h + \frac{z_n^2 S_n^T(\Xi_n)S_n(\Xi_n)}{2l_n(k_{bn}^2 - z_n^2)^2}\right), \quad (50)$$

where $c_n, \mu_n, k_{bn}, \kappa_n, \rho_n, \sigma_{n1}, \sigma_{n2}, l_n$ are positive design parameters, $\hat{\varpi}_n$ is an estimate of ϖ_n, $\tilde{\varpi}_n = \varpi_n - b_n\hat{\varpi}_n$, $\varpi_n = \|W_n\|^2, \beta_n(z_n)$ is defined as

$$\beta_n(z_n) = \begin{cases} z_n^h\left(k_{bn}^2 - z_n^2\right)^{(1-h/2)}, & \text{if } |z_n| \ge \tau_n, \\ \iota_{n1}z_n + \iota_{n2}z_n^3, & \text{if } |z_n| < \tau_n, \end{cases} \quad (51)$$

where h is defined in (15), $\iota_{n1}=\tau_n^{h-1}(k_{bn}^2-\tau_n^2)^{1-h/2}-\iota_{n2}\tau_n^2$, $\iota_{n2}=(1/2\tau_n^3)(h-1)\tau_n^h[(k_{bn}^2-\tau_n^2)^{1-h/2}+\tau_n^2((k_{bn}^2-\tau_n^2)^{-(1+h/2)})]$, and $\tau_n<k_{bn}$ is a small positive constant.

Consider the BLF candidate V_{z_n} as

$$V_{z_n}=\frac{1}{2}\log\frac{k_{bn}^2}{k_{bn}^2-z_n^2}+\frac{1}{2b_n\rho_n}\tilde{\varpi}_n^2. \quad (52)$$

Similar to (17) and (36), we can obtain the time derivative of V_{z_n} as follows:

$$\dot{V}_{z_n}\leq\frac{z_n}{k_{bn}^2-z_n^2}\left[\bar{F}_n(Z_n)+g_n(\bar{x}_n)u\right]+\frac{1}{4}+\frac{1}{2}\sum_{j=1}^n\rho_{nj}^2 \cdot\left(x_j(t-T_j(t))\right)-\frac{1}{\rho_n}\tilde{\varpi}_n\dot{\hat{\varpi}}_n-\frac{g_{n-1}(\bar{x}_{n-1})z_{n-1}z_n}{k_{b(n-1)}^2-z_{n-1}^2}, \quad (53)$$

where

$$\bar{F}_n(Z_n)=f_n(x)+\frac{z_n}{k_{bn}^2-z_n^2}\left[\varphi_{n1}(\|\bar{x}_n\|)+\varphi_{n2}\left(\bar{\alpha}_n^{-1}(r+D_0)\right)\right]^2-\dot{w}_n, +\frac{g_{n-1}(\bar{x}_{n-1})(k_{bn}^2-z_n^2)z_{n-1}}{k_{b(n-1)}^2-z_{n-1}^2}+\frac{nz_n}{2(k_{bn}^2-z_n^2)}. \quad (54)$$

Note that $\bar{F}_n(Z_n)$ is an unknown continuous function and RBFNN can be used to approximate it. Hence, from (2), the following equation holds:

$$\bar{F}_n(Z_n)=W_n^TS_n(Z_n)+\varepsilon_n(Z_n), \quad (55)$$

where $W_n^TS_n(Z_n)$ is an NN, $|\varepsilon_n(Z_n)|\leq\varepsilon_n$, $Z_n=[\bar{x}_n,z_{n-1},z_n,r,w_{n-1},\dot{w}_n]^T$, and $\varepsilon_n>0$ is any given.

By using Young's inequality and Lemma 2, one has

$$\begin{aligned}\frac{z_n}{k_{bn}^2-z_n^2}\bar{F}_n(Z_n)&=\frac{z_n}{k_{bn}^2-z_n^2}\left[W_n^TS_n(Z_n)+\varepsilon_n(Z_n)\right],\\&\leq\frac{|z_n|}{k_{bn}^2-z_n^2}\|W_n\|\|S_n(Z_n)\|+\frac{\varepsilon_n(Z_n)z_n}{k_{bn}^2-z_n^2},\\&\leq\frac{|z_n|}{k_{bn}^2-z_n^2}\|W_n\|\|S_n(\Xi_n)\|+\frac{\varepsilon_n(Z_n)z_n}{k_{bn}^2-z_n^2},\\&\leq\frac{1}{2l_n}\frac{z_n^2}{(k_{bn}^2-z_n^2)^2}\|W_n\|^2\|S_n(\Xi_n)\|^2+\frac{l_n}{2}\\&\quad+\frac{\varepsilon_n(Z_n)z_n}{k_{bn}^2-z_n^2},\\&=\frac{1}{2l_n}\frac{z_n^2}{(k_{bn}^2-z_n^2)^2}\varpi_n\|S_n(\Xi_n)\|^2\\&\quad+\frac{l_n}{2}+\frac{\varepsilon_n(Z_n)z_n}{k_{bn}^2-z_n^2},\end{aligned} \quad (56)$$

where $\Xi_n=Z_n=[\bar{x}_n,z_{n-1},z_n,r,w_{n-1},\dot{w}_n]^T$.

Substituting (49), (50), and (56) into (53), we can obtain

$$\begin{aligned}\dot{V}_{z_n}\leq&-\frac{b_nc_nz_n^2}{k_{bn}^2-z_n^2}-\frac{\mu_nb_nz_n^2}{2(k_{bn}^2-z_n^2)^2}-\frac{\kappa_ng_n(\bar{x}_n)z_n\beta_n(z_n)}{k_{bn}^2-z_n^2}\\&+\frac{l_n}{2}+\frac{1}{4}+\frac{\varepsilon_n(Z_n)z_n}{k_{bn}^2-z_n^2},\\&+\frac{1}{2}\sum_{j=1}^n\rho_{nj}^2\left(x_j(t-T_j(t))\right)+\sigma_{n1}\tilde{\varpi}_n\hat{\varpi}_n+\sigma_{n2}\tilde{\varpi}_n\hat{\varpi}_n^h\\&-\frac{g_{n-1}(\bar{x}_{n-1})z_{n-1}z_n}{k_{b(n-1)}^2-z_{n-1}^2}.\end{aligned} \quad (57)$$

By utilizing Young's inequality, the following inequality can be obtained:

$$\frac{\varepsilon_n(Z_n)z_n}{k_{bn}^2-z_n^2}-\frac{\mu_nb_nz_n^2}{2(k_{bn}^2-z_n^2)^2}\leq\frac{\varepsilon_n^2(Z_n)}{2\mu_nb_n}\leq\frac{\varepsilon_n^2}{2\mu_nb_n}. \quad (58)$$

Therefore, we have

$$\begin{aligned}\dot{V}_{z_n}\leq&-\frac{b_nc_nz_n^2}{k_{bn}^2-z_n^2}+\frac{\varepsilon_n^2}{2\mu_nb_n}-\frac{\kappa_ng_n(\bar{x}_n)z_n\beta_n(z_n)}{k_{bn}^2-z_n^2}\\&+\frac{l_n}{2}+\frac{1}{4}+\frac{1}{2}\sum_{j=1}^n\rho_{nj}^2\left(x_j(t-T_j(t))\right),\\&+\sigma_{n1}\tilde{\varpi}_n\hat{\varpi}_n+\sigma_{n2}\tilde{\varpi}_n\hat{\varpi}_n^h-\frac{g_{n-1}(\bar{x}_{n-1})z_{n-1}z_n}{k_{b(n-1)}^2-z_{n-1}^2}.\end{aligned} \quad (59)$$

According to the inequality $2b_n\tilde{\varpi}_n\hat{\varpi}_n\leq\varpi_n^2-\tilde{\varpi}_n^2$ and Lemma 4, one has

$$\begin{aligned}\dot{V}_{z_n}\leq&-\frac{b_nc_nz_n^2}{k_{bn}^2-z_n^2}+\frac{\varepsilon_n^2}{2\mu_nb_n}-\frac{\kappa_ng_n(\bar{x}_n)z_n\beta_n(z_n)}{k_{bn}^2-z_n^2}\\&+\frac{l_n}{2}+\frac{1}{4}+\frac{1}{2}\sum_{j=1}^n\rho_{nj}^2\left(x_j(t-T_j(t))\right),\\&+\frac{\sigma_{n1}}{2b_n}\varpi_n^2-\frac{\sigma_{n1}}{2b_n}\tilde{\varpi}_n^2-\frac{\sigma_{n2}\zeta_1}{b_n^h}\tilde{\varpi}_n^{1+h}+\frac{\sigma_{n2}\zeta_2}{b_n^h}\varpi_n^{1+h}\\&-\frac{g_{n-1}(\bar{x}_{n-1})z_{n-1}z_n}{k_{b(n-1)}^2-z_{n-1}^2},\end{aligned} \quad (60)$$

where ζ_1 and ζ_2 are defined in Lemma 4.

To handle the time delay, define the Lyapunov-Krasovskii functional as follows:

$$V_{U_n}=\frac{e^{-\gamma(t-T_{\max})}}{2(1-\bar{T}_{\max})}\sum_{j=1}^n\int_{t-T_j(t)}^te^{\gamma s}\rho_{nj}^2(x_j(s))\mathrm{d}s, \quad (61)$$

where $\gamma > 0$ is a positive constant. By using Assumption 5, we obtain that the derivative of V_{Un} is

$$\dot{V}_{U_n} = \frac{e^{-\gamma(t-T_{\max})}}{2(1-\bar{T}_{\max})}\sum_{j=1}^{n}\left[e^{\gamma t}\rho_{nj}^2(x_j(t)) - e^{\gamma(t-T_j(t))}\rho_{nj}^2 \cdot (x_j(t-T_j(t)))(1-\dot{T}_j(t))\right] - \gamma V_{U_n},$$
$$\leq \sum_{j=1}^{n}\frac{e^{\gamma T_{\max}}}{2(1-\bar{T}_{\max})}\rho_{nj}^2(x_j(t)) - \frac{1}{2}\sum_{j=1}^{n}\rho_{nj}^2(x_j(t-T_j(t))) - \gamma V_{U_n}. \quad (62)$$

From equations (60) and (62), we have

$$\dot{V}_{z_n} + \dot{V}_{U_n} \leq -\frac{b_n c_n z_n^2}{k_{bn}^2 - z_n^2} + \frac{\varepsilon_n^2}{2\mu_n b_n} - \frac{\kappa_n g_n(\bar{x}_n) z_n \beta_n(z_n)}{k_{bn}^2 - z_n^2} + \frac{l_n}{2} + \frac{1}{4} + \frac{\sigma_{n1}}{2b_n}\varpi_n^2 - \frac{\sigma_{n1}}{2b_n}\tilde{\varpi}_n^2, - \frac{\sigma_{n2}\zeta_1}{b_n^h}\tilde{\varpi}_n^{1+h} + \frac{\sigma_{n2}\zeta_2}{b_n^h}\varpi_n^{1+h} - \frac{g_{n-1}(\bar{x}_{n-1}) z_{n-1} z_n}{k_{b(n-1)}^2 - z_{n-1}^2} + \Phi_n - \gamma V_{U_n}, \quad (63)$$

where $\Phi_n = \sum_{j=1}^{n}(e^{\gamma T_{\max}}/(2(1-\bar{T}_{\max})))\rho_{nj}^2(x_j(t))$.

3.2. Stability Analysis. In this subsection, we present the stability analysis of the resulting closed-loop system. The main results are presented by the following theorem.

Theorem 1 *Consider the nonlinear system (1) with Assumptions 1–5. Let the actual control input and the NN adaptive law be designed as (49) and (50), respectively. If the initial conditions satisfy $V(0) \leq \Delta, |z_i(0)| \leq k_{bi}$, in which $\Delta > k_{bi}$ is any positive constant for $i = 1, 2, \ldots, n$ and k_{bi} are properly chosen, such that $k_{c1} > k_{b1} + A_0$ and $k_{ci} > \bar{w}_i + k_{bi}$ with $\bar{w}_i = \sup\{w_i\}$ for $i = 2, 3, \ldots, n$, one has that all internal signals $z_i, \tilde{\varpi}_i$ and y_{i+1} in the closed-loop system are semi-globally uniformly ultimately bounded and the tracking error will converge into the arbitrarily small regions in a finite time. Meanwhile, each state x_i will remain in the set Ω_{x_i}; that is, the full-state constraints are never violated.*

Proof. Construct the overall Lyapunov function candidate

$$V = \sum_{i=1}^{n} V_{z_i} + \sum_{i=1}^{n} V_{U_i} + \sum_{i=1}^{n-1} V_{y_i}, \quad (64)$$

where $V_{y_i} = (a_i^2/2b_i)y_{i+1}^2$ and V_{z_i}, V_{U_i} are defined in (35) and (44), respectively.

From (28), (29), and (47), the derivative of $\sum_{i=1}^{n-1} V_{y_i}$ is

$$\sum_{i=1}^{n-1}\dot{V}_{y_i} \leq -\sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\left(\tau_{(i+1)1} y_{i+1}^2 + \tau_{(i+1)2} y_{i+1}^{1+h}\right) + \sum_{i=1}^{n-1}\frac{a_i^2}{b_i} y_{i+1}\lambda_{i+1}\left(\bar{z}_{i+1}, \bar{\hat{\varpi}}_i, y_2, y_3, \ldots, y_{i+1}, \dot{y}_r, \ddot{y}_r\right). \quad (65)$$

Define a compact set as $\Omega_n = \{(\bar{z}_n, \bar{\hat{\varpi}}_n, y_2, y_3, \ldots, y_n) : V \leq \Delta\}$ with Δ being a positive constant. If $V \leq \Delta$, together with Assumption 4 and (65), it can be obtained that there exists a positive constant Λ_{i+1} $(i = 1, 2, \ldots, n-1)$, such that $\lambda_{i+1}(\cdot) \leq \Lambda_{i+1}$ on the compact set $\Omega_n \times \Omega_d$. Then, applying Young's inequality to (65) yields

$$\sum_{i=1}^{n-1}\dot{V}_{y_i} \leq -\sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\left(\tau_{(i+1)1} - \frac{1}{2\chi_{i+1}}\right) y_{i+1}^2 - \sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\tau_{(i+1)2} y_{i+1}^{1+h} + \sum_{i=1}^{n-1}\frac{a_i^2}{2b_i}\Lambda_{i+1}^2\chi_{i+1}, \quad (66)$$

where χ_{i+1} are positive constants.

According to the above analysis, we can obtain the derivative of the overall Lyapunov function candidate V as

$$\dot{V} \leq -\sum_{i=1}^{n}\frac{b_i c_i z_i^2}{k_{bi}^2 - z_i^2} - \sum_{i=1}^{n}\frac{\kappa_i g_i(\bar{x}_i) z_i \beta_i(z_i)}{k_{bi}^2 - z_i^2} - \sum_{i=1}^{n}\frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2 - \sum_{i=1}^{n}\frac{\sigma_{i2}\zeta_1}{b_i^h}\tilde{\varpi}_i^{1+h} - \sum_{i=1}^{n-1}\hat{\tau}_{(i+1)1} y_{i+1}^2, - \sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\tau_{(i+1)2} y_{i+1}^{1+h} - \gamma\sum_{i=1}^{n} V_{U_i} + d_0, \quad (67)$$

where

$$\hat{\tau}_{(i+1)1} = \frac{a_i^2}{b_i}\left(\tau_{(i+1)1} - \frac{1}{2\mu_i} - \frac{1}{2\chi_{i+1}}\right),$$
$$d_0 = \sum_{i=1}^{n}\left(\frac{\varepsilon_i^2}{2\mu_i b_i} + \frac{l_i}{2} + \frac{1}{4} + \frac{\sigma_{i1}}{2b_i}\varpi_i^2 + \frac{\sigma_{i2}\zeta_2}{b_i^h}\varpi_i^{1+h} + \Phi_i\right) + \sum_{i=1}^{n-1}\left(\frac{a_i^2}{2b_i}\Lambda_{i+1}^2\chi_{i+1}\right). \quad (68)$$

Here, we choose $\tau_{(i+1)1} > (1/2\mu_i) + (1/2\chi_{i+1})$, such that $\hat{\tau}_{(i+1)1} > 0$.

From the definition of $\beta_i(z_i)$ $(i = 1, 2, \ldots, n)$ in (15), (34), and (51), the following two cases should be considered.

Case 1: When $|z_i| < \tau_i, i = 1, 2, \ldots, n$, substituting $\beta_i(z_i) = \iota_{i1} z_i + \iota_{i2} z_i^3$ into (67) gives

$$\dot{V} \le -\sum_{i=1}^{n}\left(c_i+\kappa_i\iota_{i1}\right)\frac{b_i z_i^2}{k_{bi}^2-z_i^2}-\sum_{i=1}^{n}\frac{\kappa_i b_i \iota_{i2} z_i^4}{k_{bi}^2-z_i^2}-\sum_{i=1}^{n}\frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2 -\sum_{i=1}^{n}\frac{\sigma_{i2}\zeta_1}{b_i^h}\tilde{\varpi}_i^{1+h}, -\sum_{i=1}^{n-1}\hat{\tau}_{(i+1)1}y_{i+1}^2-\sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\tau_{(i+1)2}y_{i+1}^{1+h}-\gamma\sum_{i=1}^{n}V_{Ui}+d_0, \le -\sum_{i=1}^{n}\left(c_i+\kappa_i\iota_{i1}\right)\frac{b_i z_i^2}{k_{bi}^2-z_i^2}-\sum_{i=1}^{n}\frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2 -\sum_{i=1}^{n-1}\hat{\tau}_{(i+1)1}y_{i+1}^2-\gamma\sum_{i=1}^{n}V_{U_i}+d_0. \tag{69}$$

Noting (69), we can have

$$\dot{V} \le -\upsilon V + d_0, \tag{70}$$

with $\upsilon = \min\{2b_1(c_1+\kappa_1\iota_{11}),\ldots,2b_n(c_n+\kappa_n\iota_{n1}),\rho_1\sigma_{11},\ldots,\rho_n\sigma_{n1},\gamma,(2b_1\hat{\tau}_{(2,1)}/a_1^2),\ldots,(2b_{n-1}\hat{\tau}_{(n,1)}/a_{n-1}^2)\}$, which further implies that all the internal signals are uniformly ultimately bounded.

Case 2: When $\|z_i\| \ge \tau_i, i=1,2,\ldots,n$, substituting $\beta_i(z_i)=z_i^h(k_{bi}^2-z_i^2)^{(1-h/2)}$ into (67) gives

$$\dot{V} \le -\sum_{i=1}^{n}\frac{b_i c_i z_i^2}{k_{bi}^2-z_i^2}-\sum_{i=1}^{n}\frac{\kappa_i g_i(\bar{x}_i)z_i^{1+h}}{\left(k_{bi}^2-z_i^2\right)^{(1+h/2)}}-\sum_{i=1}^{n}\frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2-\sum_{i=1}^{n}\frac{\sigma_{i2}\zeta_1}{b_i^h}\tilde{\varpi}_i^{1+h}-\sum_{i=1}^{n-1}\hat{\tau}_{(i+1)1}y_{i+1}^2, -\sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\tau_{(i+1)2}y_{i+1}^{1+h}-\frac{\gamma}{2}\sum_{i=1}^{n}V_{Ui}-\frac{\gamma}{2}\left[\sum_{i=1}^{n}V_{U_i}\right]^{(1+h/2)}+\frac{\gamma}{2}\frac{1-h}{2}\left(\frac{1+h}{2}\right)^{(1+h/1-h)}+d_0, \le -\sum_{i=1}^{n}\frac{b_i c_i z_i^2}{k_{bi}^2-z_i^2}-\sum_{i=1}^{n}\frac{\kappa_i g_i(\bar{x}_i)z_i^{1+h}}{\left(k_{bi}^2-z_i^2\right)^{(1+h/2)}}-\sum_{i=1}^{n}\frac{\sigma_{i1}}{2b_i}\tilde{\varpi}_i^2-\sum_{i=1}^{n}\frac{\sigma_{i2}\zeta_1}{b_i^h}\tilde{\varpi}_i^{1+h}-\sum_{i=1}^{n-1}\hat{\tau}_{(i+1)1}y_{i+1}^2, -\sum_{i=1}^{n-1}\frac{a_i^2}{b_i}\tau_{(i+1)2}y_{i+1}^{1+h}-\frac{\gamma}{2}\sum_{i=1}^{n}V_{Ui}-\frac{\gamma}{2n^{(1-h/2)}}\left[\sum_{i=1}^{n}V_{U_i}\right]^{(1+h/2)}+\frac{\gamma}{2}\frac{1-h}{2}\left(\frac{1+h}{2}\right)^{(1+h/1-h)}+d_0, \le -\upsilon_1 V-\upsilon_2 V^{(1+h/2)}+d_1, \tag{71}$$

where

$$\upsilon_1 = \min\left\{2b_1c_1,\ldots,2b_nc_n,\sigma_{11}\rho_1,\ldots,\sigma_{n1}\rho_n,\frac{\gamma}{2},\frac{2b_1\hat{\tau}_{21}}{a_1^2},\ldots,\frac{2b_{n-1}\hat{\tau}_{n1}}{a_{n-1}^2}\right\}, \upsilon_2 = \min\left\{\begin{array}{c} b_1c_12^{(1+h/2)},\ldots,b_nc_n2^{(1+h/2)},\frac{\sigma_{12}\zeta_1}{b_1^h}\left(\frac{2b_1\rho_1}{\sigma_{11}}\right)^{(1+h/2)},\ldots,\frac{\sigma_{n2}\zeta_1}{b_n^h}\left(\frac{2b_n\rho_n}{\sigma_{n1}}\right)^{(1+h/2)}, \\ \frac{\gamma}{2n^{(1-h/2)}},\frac{a_1^2\tau_{22}}{b_1}\left(\frac{2b_1}{a_1^2}\right)^{(1+h/2)},\ldots,\frac{a_{n-1}^2\tau_{n2}}{b_{n-1}}\left(\frac{2b_{n-1}}{a_{n-1}^2}\right)^{(1+h/2)}\end{array}\right\}, d_1 = d_0+\frac{\gamma}{2}\frac{1-h}{2}\left(\frac{1+h}{2}\right)^{(1+h/1-h)}. \tag{72}$$

By virtue of [[59], Th.5.2], there always exists a finite-time t^*, such that $V \geq (2d_1/v_2)^{(1+h/2)}$ for all $t \in [0, t^*]$. Thus, for all $t \in [0, t^*]$, one has $\dot{V} \leq -v_1 V - (v_2/2)V^{(1+h/2)}$, and it then comes from Lemma 3 that the fast finite-time stability of the closed-loop system can be ensured with a finite settling time $T^* \leq (2/(v_1(1-h)))\log((2v_1 V^{(1-h/2)}(0) + v_2)/v_2)$. Furthermore, it is readily seen that $t^* \leq T^*$. Therefore, $\forall t > T^*, V \leq (2d_1/v_2)^{(2/1+h)}$. Then, the internal error signals z_i, $\tilde{\varpi}_i$, and y_{i+1} will converge into the following compact sets:

$$\begin{aligned} |z_i| &\leq k_{bi}\left(1 - e^{-2(2d_1/v_2)^{(2/1+h)}}\right)^{(1/2)}, \quad i = 1, \ldots, n, \\ |\tilde{\varpi}_i| &\leq (2\rho_i b_i)^{(1/2)}\left(\frac{2d_1}{v_2}\right)^{(1/1+h)}, \quad i = 1, \ldots, n, \\ |y_{i+1}| &\leq \left(\frac{2b_i}{a_i^2}\right)^{(1/2)}\left(\frac{2d_1}{v_2}\right)^{(1/1+h)}, \quad i = 1, \ldots, n-1, \end{aligned} \tag{73}$$

in a finite-time T^* with $T^* \leq (2/(v_1(1-h)))\log((2v_1 V^{(1-h/2)}(0) + v_2)/v_2)$. It is readily seen that the regions (73) can be made as small as possible by adjusting $(2d/v_2)$ with proper control parameters.

Then, we will prove that the full-state constraints are never violated. According to [[60], Lemma 1], we can conclude from (70) and (71) that $|z_i| \leq k_{bi}, i = 1, \ldots, n$, for all $t \geq 0$. Noting that $|y_r| \leq A_0$ from Assumption 4 and $z_1 = x_1 - y_r$, we have that $|x_1| \leq k_{b1} + A_0$. To get $x_2 \leq k_{c2}$, we need to show the boundedness of w_2. From (73), one has that y_2 is bounded and $b_1\hat{\varpi}_1 = \varpi_1 - \tilde{\varpi}_1$ is also bounded. With the proper choices of ι_1 and ι_2, α_1 is a continuous function of $\hat{\varpi}_1, x_1$, and $\dot{y}_r$. Then, there exists an upper bound $\bar{w}_2$, such that $w_2 = |y_2 + \alpha_1| \leq \bar{w}_2$. From $z_2 = x_2 - w_2$ and $z_2 < k_{b2}$, we get that $|x_2| \leq |z_2| + |w_2| \leq k_{c2}$. Similarly and iteratively, we have that $\alpha_{i-1}$and y_i for $i = 3, \ldots, n$ are bounded, which together with $z_i < k_{bi}$ ensures that $|x_i| \leq k_{ci}, i = 3, \ldots, n$. Therefore, each state $x_i, i = 3, \ldots, n$ will remain in the set Ω_{x_i}. The proof is completed. □

4. Simulation Results

Example 1. Consider the following nonlinear system:

$$\begin{cases} \dot{\xi} = -\xi + 0.5x_1^2 \sin(x_1 t), \\ \dot{x}_1 = x_2 e^{-0.5x_1} + \left(1 + x_1^2\right)x_2 + \delta_1(\xi, x_1, x_2, t) + 2x_1^2(t - T_1(t)), \\ \dot{x}_2 = x_1 x_2^2 + 2.5u(t) + \delta_2(\xi, x_1, x_2, t) + 0.2x_2^2(t - T_2(t)), \\ y = x_1, \end{cases} \tag{74}$$

where $\delta_1(\xi, x_1, x_2, t) = 0.2\xi x_1 \sin(x_2 t), \delta_2(\xi, x_1, x_2, t) = 0.1\xi \cos(0.5x_2 t), T_1(t) = 0.2(4 + \sin t), T_2 = 4 + 0.5\sin t$, and the dynamic signal $\dot{r} = -r + 2.5x_1^4 + 0.625$. The desired tracking trajectory $y_r = \sin(0.5t)$. u is the control input. The design parameters of the controller are taken as $k_{b1} = 0.4, k_{b2} = 2, c_1 = 10, c_2 = 15, \tau_1 = \tau_2 = 0.01, \sigma_{11} = \sigma_{12} = 0.01, \rho_1 = \rho_2 = 50, l_1 = l_2 = 1$, $T = 0.001, \mu_1 = \mu_2 = 3, \kappa_1 = \kappa_2 = 0.5, h = 0.6$. There are 68 nodes with the center placed on $[-2,2] \times [-2,2] \times [-2,2] \times [-2,2]$ and the width of Gaussian functions is $\eta_1 = 1$ in the first RBF vector. There are 85 nodes with the center placed on $[-2,2] \times [-2,2] \times [-2,2] \times [-2,2] \times [-2,2]$ and the width of Gaussian functions is $\eta_2 = 1$ in the second RBF vector. With the initial conditions, $x_1(0) = 0.2, x_2(0) = 0.1, w_1(0) = 0.1, \hat{\varpi}_1 = 2, \hat{\varpi}_2 = 0.5, r(0) = 0.1$. Simulation results are shown in Figures 1–6. The profiles of the system output y and the desired signal y_r are shown in Figure 1, which indicates that the output y follows the specified desire trajectory y_r. From Figure 2, we know that all state constraints are not violated.

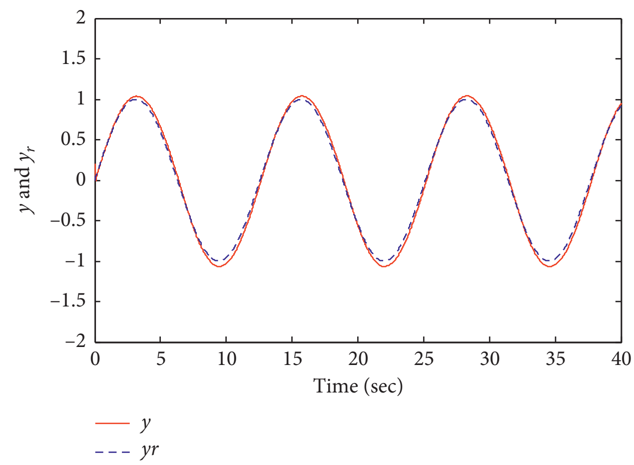

Figure 1: Output y and desired trajectory y_r.

Example 2. A Spring-Mass-Damper system is provided in this part. The system model is as follows:

$$\begin{cases} \dot{p} = V, \\ M\dot{V} = -KP - CV + F, \end{cases} \tag{75}$$

where P, V *and* F are the position, the velocity, and the force applied to the object, respectively. Let $x_1 = P, x_2 = V, u = F$. Assuming that the controlled system (75) gives unmodeled dynamics and time delay, let $\delta_1(\xi, x_1, x_2, t) = 0.2\xi x_1 \sin(x_2 t), \delta_2(\xi, x_1, x_2, t) = 0.1\xi \cos(0.5x_2 t), T_1(t) = 0.2(4 + \sin t)$, $T_2 = 4 + 0.5\sin t$, and the dynamic signal $\dot{r} = -r + 2.5x_1^4 + 0.625$. Then, system (75) can be rewritten as

$$\begin{cases} \dot{\xi} = -\xi + 0.5x_1^2 \sin(x_1 t), \\ \dot{x}_1 = x_2 + \delta_1(\xi, x_1, x_2, t) + 2x_1^2(t - T_1(t)), \\ \dot{x}_2 = -\dfrac{K}{M}x_1 - \dfrac{C}{M}x_2 + \dfrac{1}{M}u(t) + \delta_2(\xi, x_1, x_2, t) + 0.2x_2^2(t - T_2(t)), \\ y = x_1, \end{cases} \tag{76}$$

The desired tracking trajectory $y_r = \sin(0.5t) + 0.5\sin(t)$. The design parameters of the controller are taken as $k_{b1} = 0.4, k_{b2} = 2, c_1 = 10, c_2 = 15, \tau_1 = \tau_2 = 0.01, \sigma_{11} = \sigma_{12} = 0.01, \rho_1 = \rho_2 = 50, l_1 = l_2 = 1, T = 0.001, \mu_1 = \mu_2 = 3, \kappa_1 = \kappa_2 = 0.5, \quad h = 0.6$.

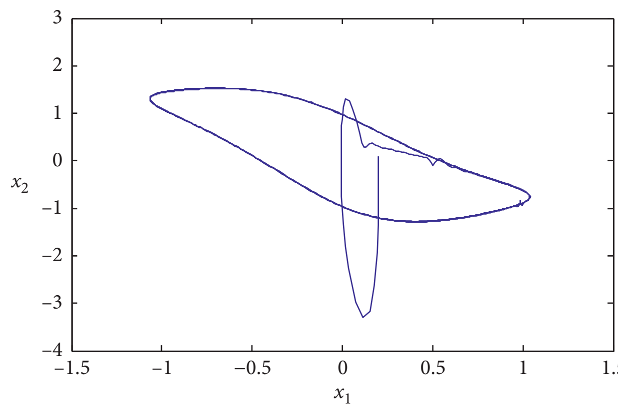

Figure 2: Phase portrait of states x_1 and x_2.

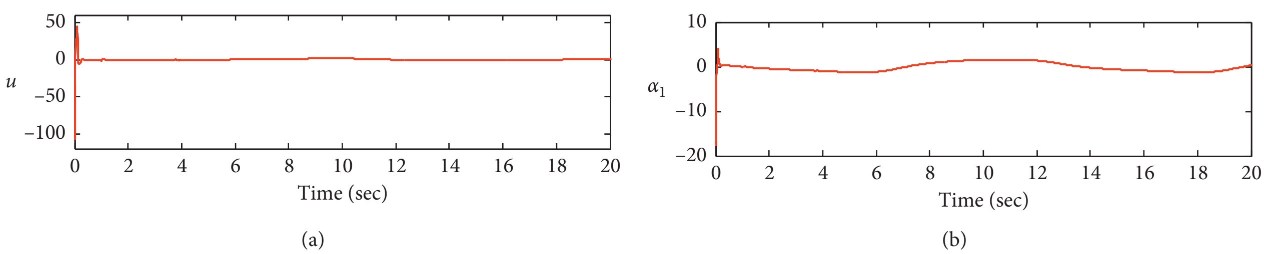

Figure 3: Profiles of control inputs u and α_1.

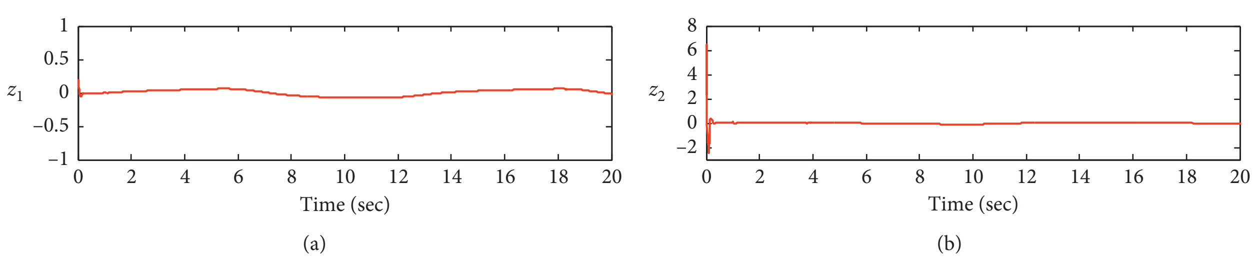

Figure 4: Profiles of the tracking errors z_1 and z_2.

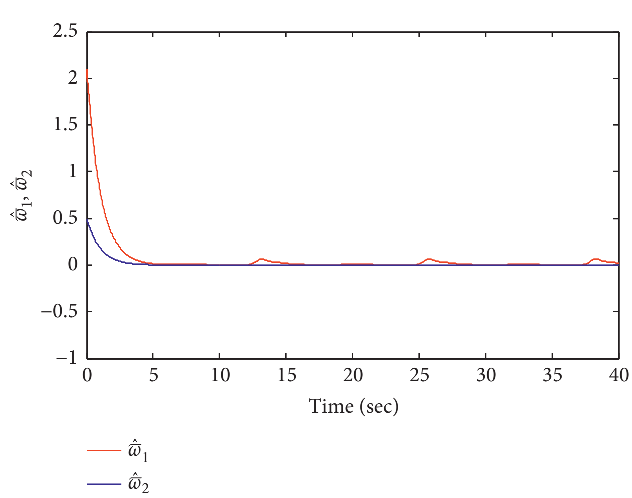

Figure 5: Estimated parameters $\widehat{\varpi}_1$ and $\widehat{\varpi}_2$.

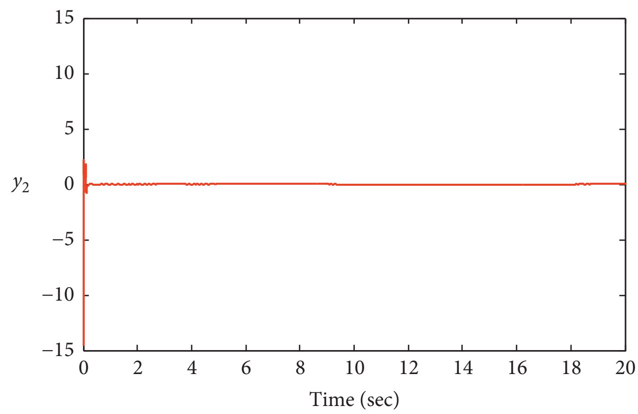

FIGURE 6: Profiles of the boundary layer error y_2.

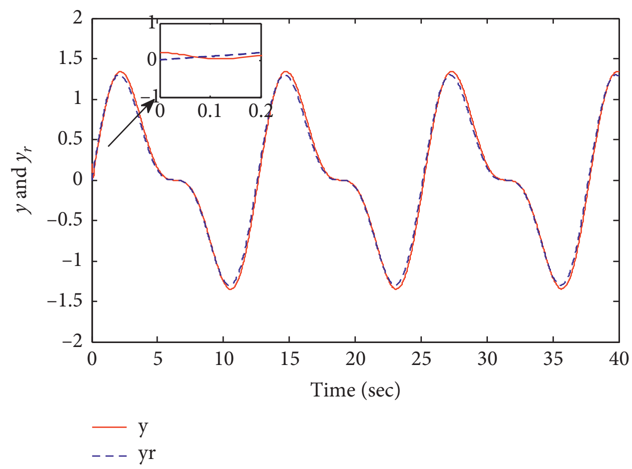

FIGURE 7: Output y and desired trajectory y_r.

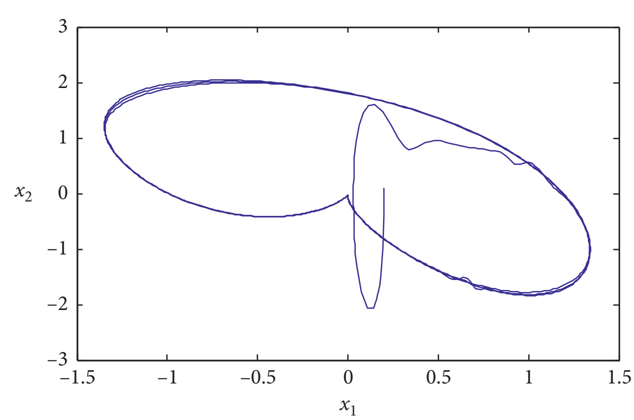

FIGURE 8: Phase portrait of states x_1 and x_2.

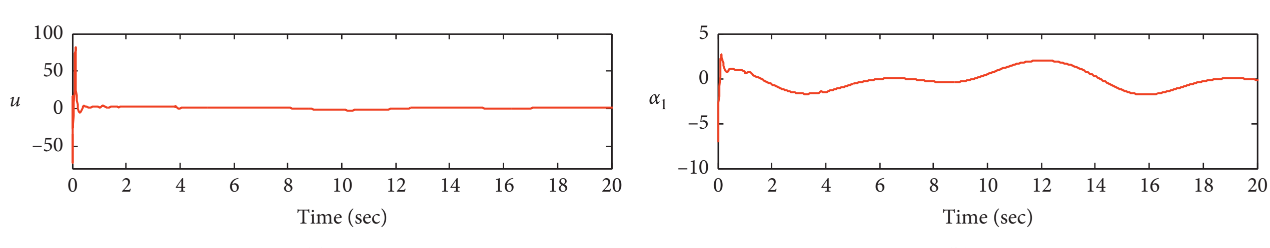

FIGURE 9: Profiles of control inputs u and α_1.

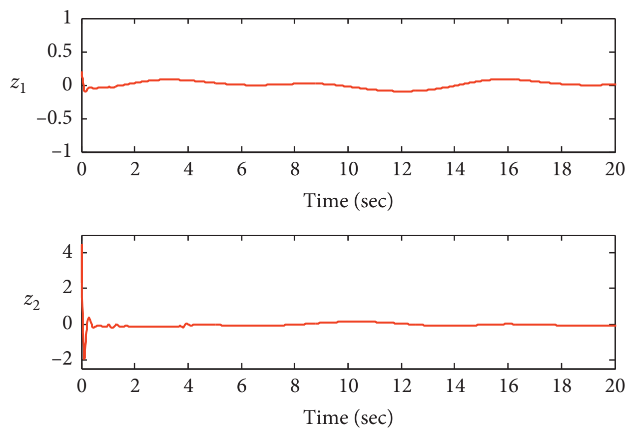

Figure 10: Profiles of the tracking errors z_1 and z_2.

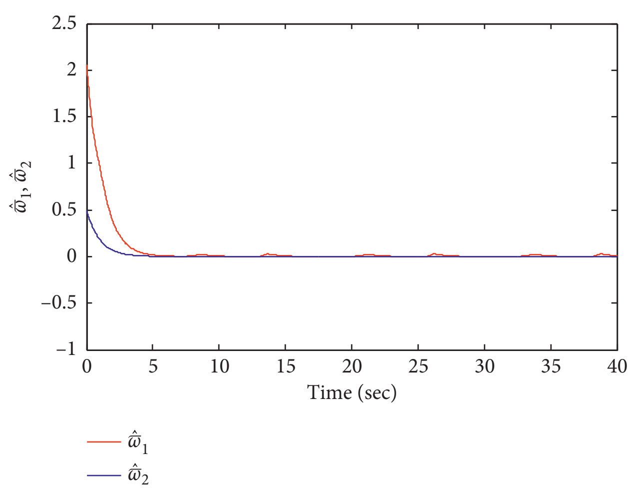

Figure 11: Estimated parameters $\widehat{\omega}_1$ and $\widehat{\omega}_2$.

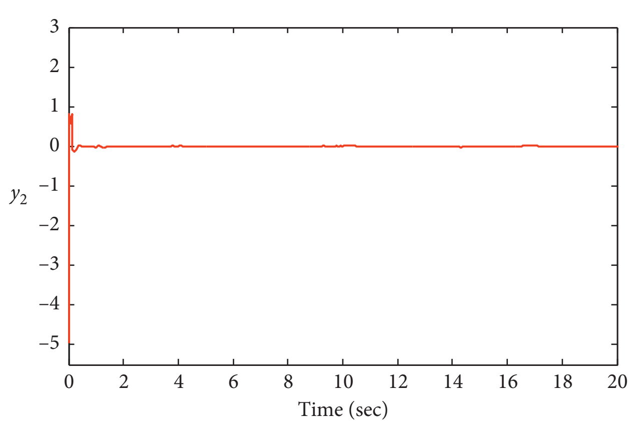

Figure 12: Profiles of the boundary layer error y_2.

There are 68 nodes with the center placed on $[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]$ and the width of Gaussian functions is $\eta_1 = 1$ in the first RBF vector. There are 85 nodes with the center placed on $[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]\times[-2,2]$ and the width of Gaussian functions is $\eta_2 = 1$ in the second RBF vector. With the initial conditions, $x_1(0) = 0.2, x_2(0) = 0.1, w_1(0) = 0.1, \widehat{\omega}_1 = 2, \widehat{\omega}_2 = 0.5, r(0) = 0.1$. Simulation results are shown in Figures 7–12.

5. Conclusions

The problem of finite-time tracking control for a class of uncertain nonstrict-feedback state-delayed nonlinear systems with full-state constraints and unmodeled dynamics has been proposed in this paper. Unmodeled dynamics is dealt with by introducing a dynamical signal and the uncertain terms produced by time-varying state delays are compensated for by constructing appropriate Lyapunov-Krasovskii functionals. By utilizing a smooth switch between the fractional and cubic form state feedback, novel C^1 smooth finite-time NN control laws have been provided for nonlinear systems with full-state constraints. Based on a modified DSC method and adaptive NN control, together with the BLFs, the fast finite-time control performance of the closed-loop nonlinear systems can be ensured, while the full-state constraints are never violated. Theoretical proofs and experimental simulation show that all the internal signals in the closed-loop system are uniformly bounded, and the tracking error signals can converge into compact sets in a finite time with sufficient accuracy, respectively. To extend this control scheme to solve the finite-time tracking control problem for some more complicated systems, such as MIMO nonlinear systems, switched nonlinear systems are also the direction of our future efforts.

Data Availability

This paper is a theoretical study and no data were used to support this study.

Conflicts of Interest

The authors declare that they do not have any financial or nonfinancial conflicts of interest.

Acknowledgments

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