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Research Article

Proportional PDC Design-Based Robust Stabilization and Tracking Control Strategies for Uncertain and Disturbed T-S Model

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This paper presents a proportional parallel distributed compensation (PPDC) design to the robust stabilization and tracking control of the nonlinear dynamic system, which is described by the uncertain and perturbed Takagi–Sugeno (T-S) fuzzy model. The proposed PPDC control design can greatly reduce the number of adjustable parameters involved in the original PDC and separate them from the feedback gain. Furthermore, the process of finding the common quadratic Lyapunov matrix is simplified. Then, the global asymptotic stability with decay rate and disturbance attenuation of the closed-loop T-S model affected by uncertainties and external disturbances are discussed using the H_∞ synthesis and linear matrix inequality (LMI) tools. Finally, to illustrate the merit of our purpose, numerical simulation studies of stabilizing and tracking an inverted pendulum system are presented.

1. Introduction

During the last decade, the fuzzy logic control has attracted rapidly growing attention from both the academic and industrial communities [1–3]. Specially, the Takagi–Sugeno (T-S) fuzzy model has been extensively used to investigate nonlinear control systems [4, 5]. This model is described by a set of fuzzy If-then rules with fuzzy sets in the antecedents and linear dynamics models in the consequent [6–8]. The overall model of the complex system is achieved by fuzzy interpolating these linear models through nonlinear fuzzy membership functions. Moreover, the stability study of this class of systems has been usually based on the use of the Lyapunov direct method. The obtained stability conditions are in general given in terms of linear matrix inequalities (LMI), which can efficiently be solved by convex programming techniques [9–11]. The overall controller of the T-S fuzzy model used the parallel distributed compensation (PDC) approach which used multiple linear state feedback controllers corresponding to the local models via fuzzy rules [12, 13].

Throughout this work, the PPDC control scheme is employed to design the fuzzy controllers from the T-S fuzzy model. On the one hand, most of the results obtained have focused on the stabilization and the tracking problems by using the normal PDC approach and very few of them are concerned with the stability problem of satisfying the decay rate and the disturbance attenuation problem [14–17]. On the other hand, for r rules, the total number of unknown parameters is reduced to $r + mn$, compared with the normal PDC rmn where r, m, and n are the numbers of the rules, the inputs, and the state variables, respectively [18].

In addition, uncertainties, unknown parameters, and external disturbances are frequently a source of instability and encountered in various complex systems [19–21]. In particular, the design of the robust fuzzy control of the T-S fuzzy model has received considerable interest in the literature [3, 22–24]. Based on the PDC approach, some researchers treated the stability analysis, stabilization, and tracking control problems of this class of systems [25–27].

In this study, H_∞ state feedback synthesis and reference model tracking control schemes are expanded to include nonlinear systems described as uncertain and disturbed the T-S fuzzy model by using the PPDC approach. However, we consider both the stability problem of satisfying the decay rate and the disturbance attenuation. Hence, we obtain sufficient conditions, expressed in LMI terms, for the existence of robust fuzzy controllers. The main contributions of this paper are summarized as follows:

(i) Compared with the existing results obtained with the normal PDC approach [9, 13, 22, 23, 28–31], our control design is carried out based on the T-S fuzzy model via the PPDC scheme, which can significantly reduce the number of parameters in PDC.

(ii) In the proposed PPDC control design, the proportional coefficients are first assigned. Then, the common quadratic Lyapunov matrix and the feedback gains are directly obtained from the LMI constraints that consider not only stability but also other control performances such as speed of response, attenuation of the disturbances effect, and structured uncertainties. As a result, the solution of the LMI is more flexible and the controller design is more feasible.

This paper is organized as follows: Section 2 introduces the uncertain T-S fuzzy model and the proportional PDC-based robust fuzzy H_∞ stabilization design scheme. In Section 3, sufficient stability conditions are developed to ensure the stability of the augmented system with a reference model tracking. A simulation example is considered in Section 4 to illustrate the merit of the designed H_∞ controllers.

2. Proportional PDC-Based Robust Fuzzy H_∞ Stabilization Design Scheme

We consider a nonlinear affine system:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where f and g are nonlinear functions of the state $x \in \mathfrak{R}^n$ and $u \in \mathfrak{R}^m$ is the control input. Then, its fuzzy dynamic model can be described by fuzzy If-then rules which represented local linear input-output relations [7, 8, 32, 33]. The ith rule is of the following form.

2.1. Plant Rule i

If x_1 is $M_{i1}(x_1)$ and . . . and x_n is $M_{in}(x_n)$,

then $\dot{x} = \overline{A}_i x + \overline{B}_{1i}u + \overline{B}_{2i}w$ and $z = C_i x + D_{1i}u + D_{2i}w$, (2)

for $i = 1, 2, \ldots r$, where $w \in \mathfrak{R}^q$ is the disturbance input, $z \in \mathfrak{R}^p$ is the controlled output, M_{ij} is the fuzzy set, and r is the number of If-then rules. $\overline{A}_i \in \mathfrak{R}^{n\times n}$, $\overline{B}_{1i} \in \mathfrak{R}^{n\times m}$, $\overline{B}_{2i} \in \mathfrak{R}^{n\times q}$, $C_i \in \mathfrak{R}^{p\times n}$, $D_{1i} \in \mathfrak{R}^{p\times m}$, and $D_{2i} \in \mathfrak{R}^{p\times q}$ are real matrices verifying

$$\begin{cases} \overline{A}_i = A_i + \Delta A_i(t), \\ \overline{B}_{1i} = B_{1i} + \Delta B_{1i}(t), \\ \overline{B}_{2i} = B_{2i} + \Delta B_{2i}(t), \end{cases} \tag{3}$$

in which $\Delta A_i(t)$, $\Delta B_{1i}(t)$, and $\Delta B_{2i}(t)$ represent the time-varying parameter uncertainties defined as follows:

$$\left[\Delta A_i(t) \;\; \Delta B_{1i}(t) \;\; \Delta B_{2i}(t)\right] = H_i F(t)\left[E_{1i} \;\; E_{2i} \;\; E_{3i}\right], \tag{4}$$

where H_i, E_{1i}, E_{2i}, and E_{3i} are known constant real matrices of appropriate dimensions. $F(t)$ is a matrix function, which is bounded by: $F^T(t)F(t) \le I$ where I is the matrix identity of an appropriate dimension.

The resulting state $\dot{x}$ and the final output z of the T-S fuzzy model are inferred by using the center of gravity method for defuzzification as follows:

$$\begin{cases} \dot{x} = \sum_{i=1}^{r} h_i(x)\left(\overline{A}_i x + \overline{B}_{1i}u + \overline{B}_{2i}w\right), \\ z = \sum_{i=1}^{r} h_i(x)\left(C_i x + D_{1i}u + D_{2i}w\right), \end{cases} \tag{5}$$

where $h_i = (\mu_i(x)/\sum_{i=1}^{r}\mu_i(x))$ and $\mu_i = \prod_{j=1}^{n} M_{ij}(x_j)$, for $i = 1, 2, \ldots, r$.

It should be noted that

$$\begin{cases} \sum_{i=1}^{r} h_i(x) = 1, \\ h_i(x) \ge 0. \end{cases} \tag{6}$$

For the fuzzy controller design, we supposed that all the states are measurable and the studied system is locally controllable. Then, we applied a proportional PDC-based compensator for each local fuzzy model (2) as follows.

2.2. Control Rule i

If x_1 is $M_{i1}(x_1)$ and . . . and x_n is $M_{in}(x_n)$, then $u = -k_i K x$, (7)

where $K \in \mathfrak{R}^{m\times n}$ is the local state feedback matrix and k_i, for $i = 1, 2, \ldots, r$, represent the proportional adjustable coefficients which differ with different control rules.

Then, the overall fuzzy controller u is defined by

$$u = -\sum_{i=1}^{r} h_i(x) k_i K x. \tag{8}$$

Substituting controller (8) into the T-S fuzzy system given by (5), the final closed-loop model is described by

$$\begin{cases} \dot{x} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x)h_j(x)\left(\overline{G}_{ij}x + \overline{B}_{2i}w\right), \\ z = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x)h_j(x)\left(F_{ij}x + D_{2i}w\right), \end{cases} \tag{9}$$

where $\overline{G}_{ij} = G_{ij} + \Delta G_{ij}(t)$, $G_{ij} = A_i - k_j B_{1i} K$, $\Delta G_{ij}(t) = \Delta A_i(t) - k_j \Delta B_{1i}(t) K$, and $F_{ij} = C_i - k_j D_{1i} K$.

Consequently, we presented in Theorem 1 sufficient stability conditions that guarantee the stability of the considered T-S model (9) and achieve a prescribed level of disturbance attenuation γ for all admissible uncertainties such that

$$\left\|T_{zw}\right\|_\infty = \sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2} < \gamma, \tag{10}$$

where $\|z\|_2 = \int_0^\infty z^T z \mathrm{d}t$.

Theorem 1. *The equilibrium of the closed-loop fuzzy model (9) described by using the proportional PDC controller (8) is globally asymptotically stable with decay rate α and satisfying the performance objective (10) if there exists a common positive definite matrix X, a matrix R, and positive constants $\tau_1 \sim \tau_3$ verifying the LMI formulation:*

$$\begin{aligned} &\underset{X,R,\gamma}{\text{maximize}} \quad \alpha \\ &\text{subject to:} \left\{ \begin{matrix} \lambda_{11} & * & * & * & * \\ B_{2i}^T & -\gamma^2 I + \tau_1 E_{3i}^T E_{3i} & * & * & * \\ C_i X - k_j D_{1i} R & D_{2i} & -I & * & * \\ E_{1i} X & 0 & 0 & -\tau_2^{-1} I & * \\ k_j E_{2i} R & 0 & 0 & 0 & -\tau_3^{-1} I \end{matrix} \right\} < 0, \end{aligned} \tag{11}$$

where $\lambda_{11} = A_i X - k_j B_{1i} R + (*) + 2\alpha X + (\tau_1^{-1} + \tau_2^{-1} + \tau_3^{-1}) H_i H_i^T$.

Furthermore, the common state feedback matrix K, shown in (8), is given by

$$K = RP. \tag{12}$$

Proof 1. In order to prove Theorem 1, we used the quadratic Lyapunov function given by

$$V(x) = x^T P x, \tag{13}$$

and verifying the control performance

$$\dot{V}(x) \leq -2\alpha V(x), \tag{14}$$

where $P = P^T > 0$ and $\alpha > 0$ is the largest Lyapunov exponent. □

Additionally, the stability of the closed-loop T-S fuzzy model is satisfied under the H_∞ performance, given in (10), with the attenuation index γ if

$$\dot{V}(x) + 2\alpha V(x) + z^T z - \gamma^2 w^T w < 0. \tag{15}$$

According to equations (9), (13), and (14), the development of the above matrix inequality leads to

$$\sum_{i=1}^{r} \sum_{j=1}^{r} h_i(x) h_j(x) x_w^T \varpi_{ij} x_w < 0, \tag{16}$$

where $x_w = \begin{pmatrix} x \\ w \end{pmatrix}$ and $\varpi_{ij} = \begin{pmatrix} P\overline{G}_{ij} + (*) + 2\alpha P + F_{ij}^T F_{ij} & * \\ \overline{B}_{2i}^T P + D_{2i}^T F_{ij} & D_{2i}^T D_{2i} - \gamma^2 I \end{pmatrix}$.

As all $h_i(x) \in [0,1]$, condition (16) gives $\varpi_{ij} < 0$. Then, by denoting $X = P^{-1}$, pre- and postmultiplying to it by the positive definite matrix $diag(X, I)$, respectively, and using the Schur complement, see Appendix A, we obtain

$$\begin{pmatrix} \overline{G}_{ij} X + (*) + 2\alpha X & * & * \\ \overline{B}_{2i}^T & -\gamma^2 I & * \\ F_{ij} X & D_{2i} & -I \end{pmatrix} < 0. \tag{17}$$

It is clear that the matrix inequality (17) contains certain Ψ_{ij} and uncertain $\Delta\Psi_{ij}$ parts. Thereby, it can be transformed into the following form:

$$\Psi_{ij} + \Delta\Psi_{ij}(t) < 0, \tag{18}$$

with $\Psi_{ij} = \begin{pmatrix} G_{ij} X + (*) + 2\alpha X & * & * \\ B_{2i}^T & -\gamma^2 I & * \\ F_{ij} X & D_{2i} & -I \end{pmatrix}$ and $\Delta\Psi_{ij}(t) = \begin{pmatrix} \Delta G_{ij}(t) X + (*) & * & * \\ \Delta B_{2i}^T(t) & 0 & * \\ 0 & 0 & 0 \end{pmatrix}$.

Using the uncertainties defined in (3), $\Delta\Psi_{ij}$ becomes

$$\Delta\Psi_{ij}(t) = \begin{pmatrix} H_i F(t) \left(E_{1i} - k_j E_{2i} K\right) + (*) & * & * \\ E_{3i}^T F^T(t) H_i^T & 0 & * \\ 0 & 0 & 0 \end{pmatrix}. \tag{19}$$

It is worth pointing out that $\Delta\Psi_{ij}$ contained antidiagonal terms. However, to transform them into diagonal terms, we used the appropriate lemma, as presented in Appendix B. By denoting $R = K X$, it follows that

$$\Delta\Psi_{ij}(t) \leq \begin{pmatrix} \eta_{11} & * & * \\ 0 & \tau_1 E_{3i}^T E_{3i} & * \\ 0 & 0 & 0 \end{pmatrix}, \tag{20}$$

where $\eta_{11} = (\tau_1^{-1} + \tau_2^{-1} + \tau_3^{-1}) H_i H_i^T + \tau_2 X E_{1i}^T E_{1i} X + \tau_3 k_j^2 R^T E_{2i}^T E_{2i} R$.

Furthermore, according to Ψ_{ij} and (20), the matrix inequality (18) can be transformed as

$$\begin{pmatrix} G_{ij}X + (*) + 2\alpha X + \eta_{11} & * & * \\ B_{2i}^T & -\gamma^2 I + \tau_1 E_{3i}^T E_{3i} & * \\ F_{ij}X & D_{2i} & -I \end{pmatrix} < 0. \tag{21}$$

After some manipulations using the Schur complement, we complete the proof and we get an optimization problem involving LMI, as illustrated in (11).

3. Design of Robust Fuzzy H_∞ Tracking Control

In this section, we treated the robust fuzzy H_∞ tracking problem for the considered global model (5) to the following reference model [11, 13, 30]:

$$\dot{x}_e = A_e x_e + B_e e, \tag{22}$$

where $A_e \in \mathfrak{R}^{n\times n}$ is an asymptotically stable matrix, $B_e \in \mathfrak{R}^n$ is an input matrix, $x_e \in \mathfrak{R}^n$ is the state of the reference model, and $e \in \mathfrak{R}$ is a bounded reference input. We applied then the following local control law.

3.1. Control Rule i

If x_1 is $M_{i1}(x_1)$ and ... and x_n is $M_{in}(x_n)$, then $u = -l_i L\varepsilon$, (23)

where $\varepsilon = x_e - x$ is the tracking error. Then, the final fuzzy controller u is defined as

$$u = -\sum_{i=1}^{r} h_i(x) l_i L\varepsilon. \tag{24}$$

Substituting controller (24) into the state x of the T-S model (5), we obtained the following augmented model:

$$\dot{x}_a = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x) h_j(x)\left(\overline{\Upsilon}_{ij} x_a + \overline{\Phi}_i \varphi\right), \tag{25}$$

where $x_a = \begin{pmatrix} \varepsilon \\ x_e \end{pmatrix}$, $\varphi = \begin{pmatrix} w \\ e \end{pmatrix}$, $\overline{\Upsilon}_{ij} = \Upsilon_{ij} + \Delta\Upsilon_{ij}(t)$, $\overline{\Phi}_i = \Phi_i + \Delta\Phi_i(t)$, $\Upsilon_{ij} = \begin{pmatrix} A_i + l_j B_{1i} L & A_e - A_i \\ 0 & A_e \end{pmatrix}$, $\Delta\Upsilon_{ij}(t) = \begin{pmatrix} \Delta A_i(t) + l_j \Delta B_{1i}(t) L & -\Delta A_i(t) \\ 0 & A_e \end{pmatrix}$, $\Phi_i = \begin{pmatrix} -B_{2i} & B_e \\ 0 & B_e \end{pmatrix}$, and $\Delta\Phi_i(t) = \begin{pmatrix} -\Delta B_{2i}(t) & 0 \\ 0 & 0 \end{pmatrix}$.

Consequently, we presented in Theorem 2 sufficient stability conditions in terms of LMI that guarantee the stability of the augmented T-S model (25) and ensure a good H_∞ tracking performance as

$$\left\| T_{x_a\varphi} \right\|_\infty = \sup_{\|\varphi\|_2 \neq 0} \frac{\|x_a\|_2}{\|\varphi\|_2} < \gamma. \tag{26}$$

Theorem 2. *The equilibrium of the augmented closed-loop fuzzy model (25) is globally asymptotically stable, and the H_∞ tracking control performance, shown in (26), is guaranteed if there exist symmetric positive definite matrices X_1 and P_2, a matrix V, and positive constants γ and $\mu_1 \sim \mu_4$ verifying the LMI formulation:*

$$\begin{aligned} &\underset{X_1, P_2, V, \gamma}{\text{maximize}} \quad \alpha \\ &\text{subject to:} \quad \begin{pmatrix} \theta_{11} & * & * & * & * & * & * \\ A_e^T - A_i^T & \theta_{22} & * & * & * & * & * \\ -B_{2i}^T & 0 & -\gamma^2 I + \mu_2 E_{3i}^T E_{3i} & * & * & * & * \\ B_e^T & B_e^T P_2 & 0 & -\gamma^2 I & * & * & * \\ X_1 & 0 & 0 & 0 & -Q^{-1} & * & * \\ E_{1i} X_1 & 0 & 0 & 0 & 0 & -\mu_3^{-1} I & * \\ l_j E_{2i} V & 0 & 0 & 0 & 0 & 0 & -\mu_4^{-1} I \end{pmatrix} < 0, \end{aligned} \tag{27}$$

with

(i) $\theta_{11} = A_i X_1 + l_j B_{1i} V + (*) + 2\alpha X_1 + (\mu_1^{-1} + \mu_2^{-1} + \mu_3^{-1} + \mu_4^{-1}) H_i H_i^T$,

(ii) $\theta_{22} = P_2 A_e + (*) + 2\alpha P_2 + \mu_1 E_{1i}^T E_{1i}$.

Furthermore, the common feedback matrix L, shown in (24), is given by

$$L = V P_1. \tag{28}$$

Proof 2. In order to prove Theorem 2, we considered the following candidate Lyapunov function:

$$V(x_a) = x_a^T P x_a, \tag{29}$$

and verifying the control performance

$$\dot{V}(x_a) \leq -2\alpha V(x_a), \tag{30}$$

where $P = P^T > 0$ and $\alpha > 0$. □

Then, the corresponding closed-loop model (25) is globally asymptotically stable and the decay rate is at least α if

$$\dot{V}(x_a) + 2\alpha\dot{V}(x_a) + x_a^T\overline{Q}x_a - \gamma^2\varphi^T\varphi < 0, \tag{31}$$

where $\overline{Q} = \begin{pmatrix} Q & 0 \\ 0 & 0 \end{pmatrix}$. The control problem is to minimize

$$\int_0^\infty \varepsilon^T Q\varepsilon \le \gamma^2 \int_0^\infty \varphi^T\varphi. \tag{32}$$

Using (25), (29), and (30), the condition above becomes

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(x)h_j(x)x_a^T\Omega_{ij}x_a < 0, \tag{33}$$

where $\Omega_{ij} = \begin{pmatrix} P\overline{\Upsilon}_{ij} + (*) + 2\alpha P + \overline{Q} & * \\ \overline{\Phi}_i^T P & -\gamma^2 I \end{pmatrix}$.

As all $h_i(x) \in [0,1]$, the matrix inequality (33) gives $\Omega_{ij} < 0$. Then, by assuming that $P = \mathrm{diag}(P_1 P_2)$, we obtain

$$\begin{pmatrix} \Lambda_{11} + \Delta\Lambda_{11}(t) & * & * & * \\ \left(A_e^T - \overline{A}_i^T\right)P_1 & P_2A_e + (*) + 2\alpha P_2 & * & * \\ -\overline{B}_{2i}^T P_2 & 0 & -\gamma^2 I & * \\ B_e^T P_2 & B_e^T P_2 & 0 & -\gamma^2 I \end{pmatrix} < 0, \tag{34}$$

with

(i) $\Lambda_{11} = P_1(A_i + l_j B_{1i}L) + (*) + 2\alpha P_1 + Q$,

(ii) $\Delta\Lambda_{11}(t) = P_1(\Delta A_i(t) + l_j\Delta B_{1i}(t)L) + (*)$.

Thereafter, the inequality matrix (34) can be transformed into the following form:

$$\Sigma_{ij} + \Delta\Sigma_{ij}(t) < 0, \tag{35}$$

with

(i) $\Sigma_{ij} = \begin{pmatrix} \Lambda_{11} & * & * & * \\ (A_e^T - \overline{A}_i^T)P_1 & P_2A_e + (*) + 2\alpha P_2 & * & * \\ -B_{2i}^T P_2 & 0 & -\gamma^2 I & * \\ B_e^T P_2 & B_e^T P_2 & 0 & -\gamma^2 I \end{pmatrix}$,

(ii) $\Delta\Sigma_{ij}(t) = \begin{pmatrix} \Delta\Lambda_{11}(t) & * & * & * \\ -\Delta A_i^T(t)P_1 & 0 & * & * \\ -\Delta B_{2i}^T(t)P_1 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{pmatrix}$.

Using the uncertainties defined in (3), the matrix $\Delta\Sigma_{ij}(t)$ can be rewritten as

$$\Delta\Sigma_{ij}(t) = \begin{pmatrix} P_1H_iF(t)\left(E_{1i} + l_jE_{2i}L\right) + (*) & * & * & * \\ -E_{1i}^T F^T(t)H_i^T P_1 & 0 & * & * \\ -E_{3i}^T F^T(t)H_i^T P_1 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{36}$$

By using the lemma, as presented in Appendix B, $\Delta\Sigma_{ij}$ is increased as follows:

$$\Delta\Sigma_{ij}(t) \le \begin{pmatrix} \xi_{11} & * & * & * \\ 0 & \mu_1 E_{1i}^T E_{1i} & * & * \\ 0 & 0 & \mu_2 E_{3i}^T E_{3i} & * \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{37}$$

where $\xi_{11} = (\mu_1^{-1} + \mu_2^{-1} + \mu_3^{-1} + \mu_4^{-1})P_1H_iH_i^T P_1 + \mu_3 E_{1i}^T E_{1i} + \mu_4 l_j^2 L^T E_{2i}^T E_{2i} L$.

Therefore, according to Σ_{ij} and (37), the matrix inequality (35) can be transformed as

$$\begin{pmatrix} \Lambda_{11} + \xi_{11} & * & * & * \\ \left(A_e^T - \overline{A}_i^T\right)P_1 & P_2A_e + (*) + 2\alpha P_2 + \mu_1 E_{1i}^T E_{1i} & * & * \\ -B_{2i}^T P_2 & 0 & -\gamma^2 I + \mu_2 E_{3i}^T E_{3i} & * \\ B_e^T P_2 & B_e^T P_2 & 0 & -\gamma^2 I \end{pmatrix} < 0. \tag{38}$$

By denoting $X_1 = P_1^{-1}$ and $V = LX_1$ and pre- and postmultiplying to (38) by the positive definite matrix $\mathrm{diag}(X_1, I, I, I)$, respectively, we obtain

$$\begin{pmatrix} \upsilon_{11} & * & * & * \\ \left(A_e^T - \overline{A}_i^T\right)P_1 & P_2A_e + (*) + 2\alpha P_2 + \mu_1 E_{1i}^T E_{1i} & * & * \\ -B_{2i}^T P_2 & 0 & -\gamma^2 I + \mu_2 E_{3i}^T E_{3i} & * \\ B_e^T P_2 & B_e^T P_2 & 0 & -\gamma^2 I \end{pmatrix} < 0, \tag{39}$$

where $\upsilon_{11} = A_i X_1 + l_j B_{1i} V + (*) + 2\alpha X_1 + X_1 Q X_1 + (\mu_1^{-1} + \mu_2^{-1} + \mu_3^{-1} + \mu_4^{-1}) H_i H_i^T + \mu_3 X_1 E_{1i}^T E_{1i} X_1 + \mu_4 l_j^2 V^T E_{2i}^T E_{2i} V$.

Using then the Schur complement, we complete the proof and we get an optimization problem involving LMI, as illustrated in (27).

Remark. It should be mentioned that the above results on the design problem for T-S fuzzy models are based on an implicit assumption that the controller will be implemented exactly. However, uncertainties or inaccuracies do occur in the implementation of a designed filter or controller. In recent years, there are considerable studies on the robust nonfragile H_∞ filtering problem of T-S fuzzy models [20, 21, 24].

4. Numerical Simulation

In this section, the proposed control strategies are verified for an inverted pendulum system. Its mathematical model is described by the following nonlinear equations [11, 13, 34]:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = \dfrac{(m+M)(-dx_2 + mgl\sin(x_1)) - (m^2l^2/2)x_2^2\sin(2x_1)}{(m+M)(J+ml^2) - m^2l^2\cos^2(x_1)} - \dfrac{ml\cos(x_1)}{(m+M)(J+ml^2) - m^2l^2\cos^2(x_1)}u, \end{cases} \tag{40}$$

where $x_1 = \theta(\text{rad})$ is the angle from the vertical position and $x_2 = \dot{\theta}(\text{rad/s})$ is the angular velocity, $g = 9.81\,\text{m/s}^2$ is the gravity constant, $M = 2.4\,\text{kg}$ is the mass of the cart, $m = 0.23\,\text{kg}$ is the mass of the pendulum, $l = 0.36\,\text{m}$ is the pole length, $J = 0.099\,\text{kg m}^2$ is the moment of inertia of the pole, and $d = 0.005\,\text{Nms/rad}$ is the pendulum damping coefficient.

For simplicity, we suppose that $m^2l^2\cos^2(x_1) \ll (m+M)(J+ml^2)$ and we use two rules:

Plant rule 1: if x_1 is about 0, then $\dot{x} = \overline{A}_1 x + \overline{B}_{11} u + \overline{B}_{21} w$

Plant rule 2: if x_1 is about $\pm(\pi/3)$, then $\dot{x} = \overline{A}_2 x + \overline{B}_{12} u + \overline{B}_{22} w$

with

$$A_1 = \begin{pmatrix} 0 & 1 \\ \dfrac{mgl}{J+ml^2} & \dfrac{-d}{J+ml^2} \end{pmatrix},$$

$$B_{11} = \begin{pmatrix} 0 \\ -\dfrac{ml}{(m+M)(J+ml^2)} \end{pmatrix},$$

$$B_{21} = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 1 \\ \dfrac{mgl\sin(\pi/3)}{J+ml^2} & \dfrac{-d}{J+ml^2} \end{pmatrix},$$

$$B_{12} = \begin{pmatrix} 0 \\ \dfrac{ml\cos(\pi/3)}{(m+M)(J+ml^2)} \end{pmatrix},$$

$$B_{22} = \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

$$\begin{aligned} \Delta A_1(t) &= H_1 F(t) E_{11}, \\ \Delta A_2(t) &= H_2 F(t) E_{12}, \\ F(t) &= \sin(t), \\ H_1 &= \begin{pmatrix} 0 \\ -\dfrac{m^2l^2}{(m+M)(J+ml^2)} \end{pmatrix}, \\ H_2 &= \begin{pmatrix} 0 \\ -\dfrac{m^2l^2\sin(2(\pi/3))}{2(m+M)(J+ml^2)} \end{pmatrix}, \\ E_{11} &= E_{12} = \begin{pmatrix} a & 0 \end{pmatrix}, \\ \Delta B_{11} &= \Delta B_{12} = \Delta B_{21} = \Delta B_{22} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \\ a &= \max(x_2^2) = 0.05. \end{aligned} \tag{41}$$

Using Theorem 1 with $\gamma = 0.5$, $\tau_1 = 2.9$, $\tau_2 = 1.7$, $\tau_3 = 0.9$, $k_1 = 0.62$, and $k_2 = 0.85$, we obtain $\alpha = 1.4106$, $P = \begin{bmatrix} 4.1656 & 0.7511 \\ 0.7511 & 0.1546 \end{bmatrix}$, $R = \begin{bmatrix} -151.3583 & 137.6572 \end{bmatrix}$, and $K = \begin{bmatrix} -527.0948 & -92.4086 \end{bmatrix}$.

The responses of the state variables $x_1(t) = \theta(t)(\text{rad})$ and $x_2(t) = \dot{\theta}(t)(\text{rad/s})$ are shown in Figures 1 and 2, respectively. The evolution of the force $F(t)(N)$ is depicted in Figure 3, while the disturbance input signal is given by $w(t) = \begin{cases} -0.05 & 5 \le t \le 7 \\ 0 & \text{otherwise} \end{cases}$.

It is shown from the simulation results that the designed controller $u = -\sum_{i=1}^{2} h_i(x) k_i K x$ is efficient and guarantees the rapid global stability of the closed-loop fuzzy model (9).

In the following, we present a comparative study between the obtained results related to designing robust fuzzy

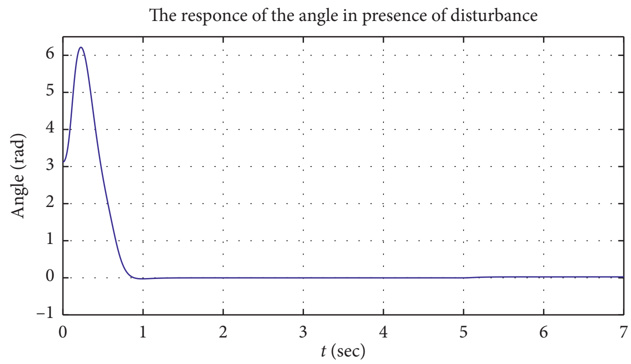

Figure 1: Response of the state variable $x_1(t) = \theta(t)$.

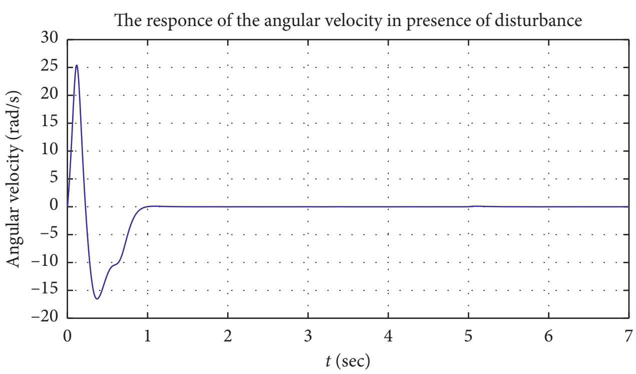

Figure 2: Response of the state variable $x_2(t) = \dot{\theta}(t)$.

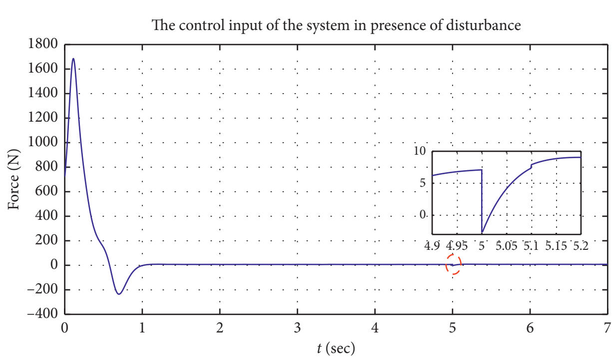

Figure 3: Evolution of the control signal $F(t)$.

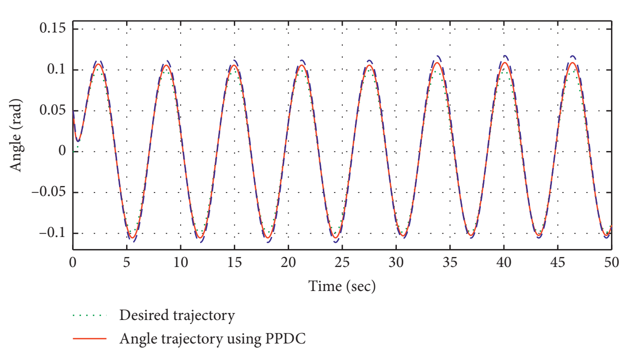

Figure 4: Angle tracking trajectories using PDC and PPDC approaches.

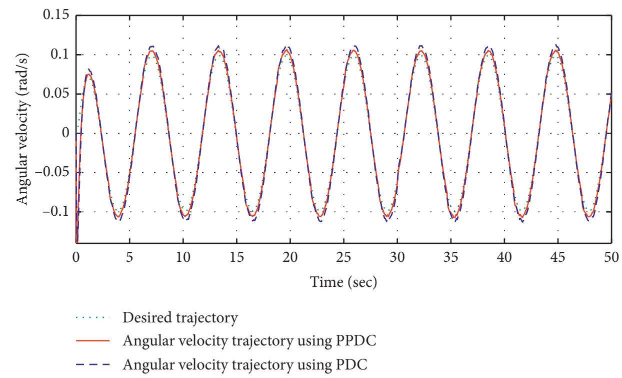

Figure 5: Angular velocity trajectories using PDC and PPDC approaches.

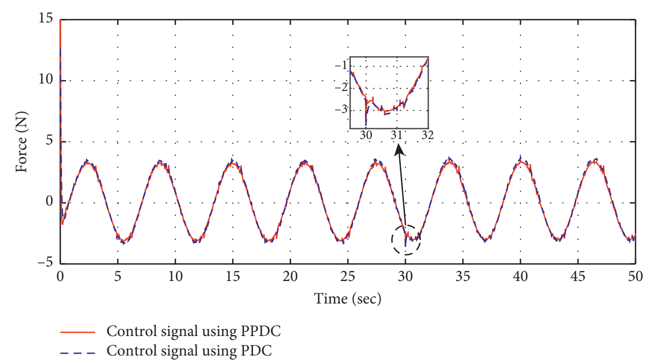

Figure 6: Control signals using PDC and PPDC approaches.

tracking controllers using the proposed PPDC approach and the original PDC as developed in [13].

By using Theorem 2 with $\gamma = 0.5$, $\mu_1 = 2.9$, $\mu_2 = 1.7$, $\mu_3 = 0.9$, $\mu_4 = 0.5$, $l_1 = 0.47$, $l_2 = 0.83$, $Q = \begin{pmatrix} 0.35 & 0 \\ 0 & 0.03 \end{pmatrix}$, $A_e = \begin{pmatrix} 0 & 1 \\ -(15/4) & -4 \end{pmatrix}$, and $B_e = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, we obtain $\alpha = 1.2984$, $P_1 = \begin{bmatrix} 5.0572 & 1.1914 \\ 1.1914 & 0.3065 \end{bmatrix}$, $P_2 = \begin{bmatrix} 4.7786 & -0.1363 \\ -0.1363 & 4.4632 \end{bmatrix}$, $R = \begin{bmatrix} 157.6185 & -69.5819 \end{bmatrix}$, and $L = \begin{bmatrix} 714.2054 & 166.4582 \end{bmatrix}$.

The designed robust fuzzy controller $u = -\sum_{i=1}^{2} h_i(x) l_i L\varepsilon$ quadratically stabilizes the uncertain and perturbed augmented model (25) and ensures a good H_∞ tracking performance, as presented in (26).

For comparison, by solving the LMI constraints in [13], we obtain $L_1 = \begin{bmatrix} 264.5209 & 65.0914 \end{bmatrix}$, $L_2 = \begin{bmatrix} 264.5209 & 65.0914 \end{bmatrix}$, $P_1 = \begin{bmatrix} 3.7228 & 0.9739 \\ 0.9739 & 0.2871 \end{bmatrix}$, and $P_2 = \begin{bmatrix} 4.6364 & -0.9691 \\ -0.9691 & 3.1186 \end{bmatrix}$.

The responses of the state variables $x_1(t) = \theta(t)$ (rad) and $x_2(t) = \dot{\theta}(t)$ (rad/s), the control signal $F(t)(N)$, and the quadratic error tracking are depicted in Figures 4–7, respectively, with as initial conditions $x_1(0) = 0.05$ and $x_2(0) = 0$. The disturbance input signal is a random number chosen from an uniform distribution on the interval $[0, 0.1]$ and given by $w(t) = \begin{cases} -0.05 & 30 \le t \le 50 \\ 0 & \text{otherwise} \end{cases}$.

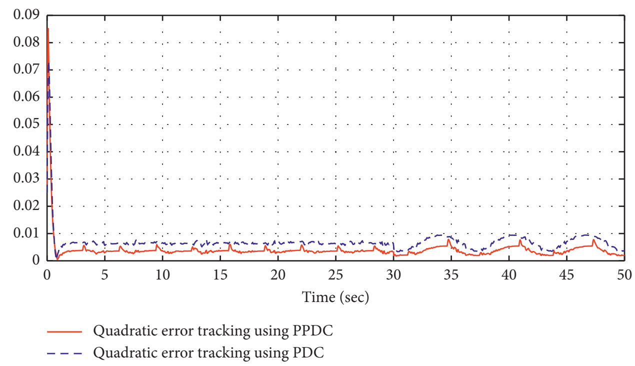

FIGURE 7: Quadratic error tracking using PDC and PPDC approaches.

It is observed that in contrast with the normal PDC approach, the PPDC one can rapidly achieve desired responses despite the presence of uncertainties and disturbances. Moreover, magnitudes of the quadratic error tracking using the PDC design are larger than those of PPDC.

5. Conclusion

This paper has presented the robust state feedback synthesis and model reference tracking control for the nonlinear dynamic system described as the T-S fuzzy model affected by uncertainties and external disturbances using the proportional PDC design. Its interest is to reduce the number of adjustable parameters in the normal PDC one. Based on the quadratic Lyapunov function with a decay rate, sufficient stability conditions have been obtained using the H_∞ criterion and LMI tools that guarantee the stability of the closed-loop T-S model and ensure a good robust performance. Moreover, the solution of finding the common positive definite matrix was simplified and proportional coefficient design was separated from the feedback matrix parameters. Finally, an inverted pendulum system was considered to show the effectiveness of the designed H_∞ fuzzy controllers.

Appendix

A. Schur Complement

Given the matrix inequality $\begin{pmatrix} M & * \\ L & Q \end{pmatrix} < 0$ where M and Q are invertible symmetrical matrices, it is equal to each of the following inequalities:

(i) $Q < 0, M - L^T Q^{-1} L < 0$,

(ii) $Q < 0, M - L^T Q^{-1} L < 0$.

B. Lemma

For matrices A and B with appropriate dimensions and a positive real constant τ, the following matrix inequality holds: $A^T B + AB^T \leq \tau A^T A + \tau^{-1} B^T B$.

Data Availability

No data were used to support this study.

Additional Points

Notations. (i) A symmetric matrix $\begin{pmatrix} M & * \\ N & T \end{pmatrix} < 0$ stands for $\begin{pmatrix} M & N^T \\ N & T \end{pmatrix} < 0$, (ii) $M + (*) < 0$ stands for $M + M^T < 0$, and (iii) $h_i(x) = h_i(x(t))$.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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