Research Article

Some Nonlinear Fractional PDEs Involving β -Derivative by Using Rational $\exp(-\Omega(\eta))$ -Expansion Method

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In this article, some new nonlinear fractional partial differential equations (PDEs) (the space-time fractional order Boussinesq equation; the space-time $(2+1)$ -dimensional breaking soliton equations; and the space-time fractional order SRLW equation) have been considered, in which the treatment of these equations in the diverse applications are described. Also, the fractional derivatives in the sense of β -derivative are defined. Some fractional PDEs will convert to consider ordinary differential equations (ODEs) with the help of transformation β -derivative. These equations are analyzed utilizing an integration scheme, namely, the rational $\exp(-\Omega(\eta))$ -expansion method. Different kinds of traveling wave solutions such as solitary, topological, dark soliton, periodic, kink, and rational are obtained as a by product of this scheme. Finally, the existence of the solutions for the constraint conditions is also shown. The outcome indicates that some fractional PDEs are used as a growing finding in the engineering sciences, mathematical physics, and so on.

1. Introduction

Traveling wave and soliton solutions are one of the most interesting and fascinating areas of research in different fields of engineering and physical sciences. This type of solutions, basically structured by solving nonlinear partial differential equations (NLPDEs), arises in physical phenomena such as optic, acoustic wave, and mechanic sciences. Consequently, it is imperative to address the dynamics of these soliton solutions from a mathematical aspect. This will lead to a deeper understanding of the engineering perspective of these solutions [1–10]. Some of the authors investigated novel models of partial differential equations containing the Fokas–Lenells equation [11], the General Satisfaction Index [12], the multiasset option pricing problem [13], the affine jump-diffusion model [14], and the $(1+1)$ -dimensional coupled nonlinear Schrödinger equation [15]. An important analysis in signal analysis including the transfer function, Bode diagram, Nyquist plot, and Nichols plot was obtained based on the Laplace transform by Atangana and Akgül [16]. As a result, in the past few years,

several effective, rising, and realistic methods have been initiated and dilated to extract closed form solutions to the NLPDEs; for example, the semi-inverse principle method is one of the powerful methods many scholars studied including the fractional nonlinear model of the low-pass electrical transmission lines [17] and the higher-order nonlinear Schrödinger equation with the non-Kerr nonlinear term [18]. Xie et al. worked on nonlinear optical soliton containing a variable-coefficient AB system in the geophysical fluids or nonlinear optics [19], studied the turbulence for a coupled nonlinear Schrödinger system to find the rogue waves solutions [20], and described the propagation of ultrashort optical pulses in a birefringent fiber in the coupled Hirota equations [21]. Rizvi et al. investigated the optical solitons for the Biswas–Milovic equation by the new extended auxiliary equation method [22] and the modulated compressional dispersive alfvén and heisenberg ferromagnetic spin chains [23]. Some authors studied the conformable fractional derivative on some of the conformable time-fractional variant bussinesq equations [24], the space-time fractional simplified MCH and SRLW

equations [25], and the fractional and classical GEW-Burgers equations [26] by utilizing the analytical methods.

In this paper, we will study the different kinds of traveling wave solutions in generalized nonlinear Schrödinger equation modeling which describe the model of few-cycle pulse propagation in metamaterials with parabolic law of nonlinearity from a purely mathematical viewpoint. Therefore, the importance of this paper will be to extract exact traveling wave solution for the nonlinear model. This model is described by the decoupled nonlinear Schrödinger equation (NLSE) with group velocity mismatch, group velocity dispersion, and spatio-temporal dispersion. There are several integration tools available to solve the model. Many nonlinear Schrödinger equations have been examined with regards to soliton theory, where complete integrability was emphasized by various analytical techniques. A few methods are improved $\tan(\phi/2)$ -expansion method, the homotopy analysis method, the variation iteration principle, the exp-

function method, the generalized (G'/G) -expansion approach, the Hirota bilinear method, simplest equation method, etc. [1–46]. The space-time fractional order Boussinesq equation (STFBE) with quadratic-cubic nonlinearity involving the β -derivative [47, 48] is presented as follows:

$$D_t^{2\alpha}\Psi + bD_x^{2\alpha}\Psi + \beta D_x^{2\alpha}(\Psi^2) + \gamma D_x^{4\alpha}\Psi = 0, \quad t > 0, 0 < \alpha \leq 1, \quad (1)$$

where $D_t^{2\alpha}(\cdot) = (d^{2\alpha}/dt^{2\alpha})(\cdot)$ $\Psi(x, t)$ is the vertical deflection, β is the coefficient controlling nonlinearity and the quadratic nonlinearity, and γ is the dispersion parameter depending on the rigidity characteristics of the material and compression [47, 48]. Furthermore, we take the space-time $(2+1)$ -dimensional breaking soliton equations (STFBSEs) [48, 49] in the following:

$$\begin{aligned} D_t^{2\alpha}\Psi + aD_{x^2y}^{3\alpha}\Psi + 4a\Psi(D_x^\alpha\Omega) + 4a\Omega(D_x^\alpha\Psi) &= 0, \quad t > 0, 0 < \alpha \leq 1, \\ D_y^\alpha\Psi - D_x^\alpha\Omega &= 0. \end{aligned} \quad (2)$$

Mohyud-Din and Bibi [48] used the exponential rational function method to get exact solutions of nonlinear STFBSEs. Also, Wen and Zheng [49] implemented the new fractional subequation method to nonlinear STFBSEs.

Finally, a few of authors including Mohyud-Din and Bibi [48], Xu [50], and Alzaidy [51] considered the space-time fractional order SRLW equation (STFSRLWE) as below:

$$D_t^{2\alpha}\Psi + D_x^{2\alpha}\Psi + \Psi D_t^\alpha(D_x^\alpha\Psi) + D_t^\alpha\Psi D_x^\alpha\Psi + D_t^{2\alpha}(D_x^{2\alpha}\Psi) = 0, \quad t > 0, 0 < \alpha \leq 1. \quad (3)$$

The rest of this paper is structured as follows. The initial definitions are given in Section 2. In Section 3, an overview of the integration scheme is given along with the analysis of the model. In Sections 4, 5, and 6, respectively, the STFBE, STFBSEs, and STFSRLWE have been employed to obtain the rational dark, periodic, kink-singular, cupson, and travelling wave solutions with the considered method. In Section 7, discussion of mitigating internet bottleneck of parabolic law

of nonlinearity has been offered. In Section 8, the conclusion has been given.

2. Initial Definitions

Definition 1 (definition of β -derivative). Suppose $\chi: [0; 1) \rightarrow \mathbf{R}$, then the β derivative of χ of order α is defined as

$$D_t^\alpha(\chi)(t) = \lim_{\epsilon \rightarrow 0} \frac{\chi[t + \epsilon(t + (1/\Gamma(\alpha)))^{1-\alpha}] - \chi(t)}{\epsilon}, \quad \alpha \in (0, 1], t > 0. \quad (4)$$

The properties and new theorems will used as follows.

Theorem 1. Suppose $\alpha \in (0, 1]$; let χ and ψ be α -differentiable at a point t ; therefore,

- (1) $D_t^\alpha(a\chi(t) + b\psi(t)) = aD_t^\alpha(\chi(t)) + bD_t^\alpha(\psi(t))$, for $a, b \in \mathbf{R}$
- (2) $D_t^\alpha(c) = 0$, for $c \in \mathbf{R}$
- (3) $D_t^\alpha(\chi(t)\psi(t)) = \chi(t)D_t^\alpha(\psi(t)) + \psi(t)D_t^\alpha(\chi(t))$

$$(4) \quad D_t^\alpha(\chi(t)/\psi(t)) = \chi(t)D_t^\alpha(\psi(t)) - \psi(t)D_t^\alpha(\chi(t))/\psi^2(t)$$

$$(5) \quad D_t^\alpha\chi(t) = (t + (1/\Gamma(\alpha)))^{1-\alpha}(d\chi(t)/dt)$$

Theorem 2. Suppose $\chi: [0; 1) \rightarrow \mathbf{R}$ be a function such that χ is differentiable and also α -differentiable. Assume ψ be a differentiable function defined in the range of χ . Therefore, we have

$$D_t^\alpha (\chi \circ \psi)(t) = (t + (1/\Gamma(\alpha)))^{1-\alpha} \psi'(t) \chi'(\psi(t)), \quad (5)$$

where prime denotes the classical derivatives with respect to t .

The proofs of the above β -derivative properties were obviously presented in [52]. Also, improvement of fractional derivative has been made in works of Atangana and Baleanu in Refs. [53, 54].

3. The Rational $\exp(-\Omega(\eta))$ -Expansion Method

This section elucidates a systematic explanation of rational $\exp(-\Omega(\eta))$ -expansion method so that it can be further applied to optical metamaterials with parabolic nonlinearity in order to furnish its exact solutions.

Step 1: the following NLPDE,

$$\mathcal{N}(u, u_x, u_t, u_{xx}, u_{tt}, \dots) = 0, \quad (6)$$

can be transformed into an ordinary differential equation (ODE)

$$\mathcal{Q}(U, U', \nu U', U'', \nu^2 U'', \dots) = 0, \quad (7)$$

by using the suitable transformation $\eta = x + \nu t$, where ν is the free parameter which would be calculated subsequently.

Step 2: assume the solution of the ODE to be of the following form:

$$U(\eta) = \frac{\sum_{j=0}^N A_j F^j(\eta)}{\sum_{j=0}^M B_j F^j(\eta)}, \quad (8)$$

where $F(\eta) = \exp(-\Omega(\eta))$ and A_j ($0 \leq j \leq N$) and B_j ($0 \leq j \leq M$) are constants to be determined, such that $A_N, B_M \neq 0$; $\Omega = \Omega(\eta)$ satisfying the ODE is given below:

$$\Omega' = \mu F^{-1}(\eta) + F(\eta) + \lambda. \quad (9)$$

The special cases formed from the solutions of the ODE given in equation (9) are mentioned below.

Solution 1. If $\mu \neq 0$ and $\lambda^2 - 4\mu > 0$, then we have

$$\Omega(\eta) = \ln \left(-\frac{\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh \left(\frac{\sqrt{\lambda^2 - 4\mu}}{2} (\eta + E) \right) - \frac{\lambda}{2\mu} \right), \quad (10)$$

where E is integral constant.

Solution 2. If $\mu \neq 0$ and $\lambda^2 - 4\mu < 0$, then we have

$$\Omega(\eta) = \ln \left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan \left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2} (\eta + E) \right) - \frac{\lambda}{2\mu} \right). \quad (11)$$

Solution 3. If $\mu = 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu > 0$, then we obtain

$$\Omega(\eta) = -\ln \left(\frac{\lambda}{\exp(\lambda(\eta + E)) - 1} \right). \quad (12)$$

Solution 4. If $\mu \neq 0$, $\lambda \neq 0$, and $\lambda^2 - 4\mu = 0$, then we obtain

$$\Omega(\eta) = \ln \left(-\frac{2\lambda(\eta + E) + 4}{\lambda^2(\eta + E)} \right). \quad (13)$$

Solution 5. If $\mu = 0$, $\lambda = 0$, and $\lambda^2 - 4\mu = 0$, then we obtain $\Omega(\eta) = \ln(\eta + E)$,

$$\Omega(\eta) = \ln(\eta + E), \quad (14)$$

where A_j ($0 \leq j \leq N$) and B_j ($0 \leq j \leq M$); λ and μ are the constants to be explored later. The values N and M are determined by equalizing the maximum order nonlinear term and the maximum order partial derivative term appearing in (7). If N and M are the rational, then the appropriate transformations can be applied to conquer these hurdles.

Step 3. Put (8) into equation (7), as well as the values of N and M determined in previous step, into (8). Gathering coefficients of all the powers of $F(\eta)$ and then equating every coefficient with zero, we derive a set of overdetermined nonlinear algebraic equations for $A_0, B_0, A_1, B_1, \dots, A_N, B_M, \lambda$, and μ .

Here, it is important to note that E is the integration constant. We have the following relations as

$$\Psi(\eta) \approx \delta F^{N-M}, \quad (15)$$

$$\begin{aligned} \Psi'(\eta) &\approx \delta F^{N-M-1} F' = -\delta(\mu F^{N-M-1} + \lambda F^{N-M} + F^{N-M+1}) \\ &\approx -\delta F^{N-M+1}, \end{aligned} \quad (16)$$

$$(\Psi''(\eta)) \approx \delta F^{N-M+2}, \quad (17)$$

$$(\Psi(\eta))^2 \approx \delta^4 F^{2N-2M}, \quad (18)$$

where $\delta = (A_N/B_M)$. Balancing Ψ'' with Ψ^2 yields

$$F^{N-M+2} \approx \Psi''(\xi) = (\Psi(\eta))^2 \approx F^{2N-2M}. \quad (19)$$

We can determine values of N and M as follows:

$$N - M + 2 = 2N - 2M \implies N = M + 2. \quad (20)$$

4. The STFBE

Consider the nonlinear space-time fractional order Bousinesq equation by the following variable change as

$$\eta = k \left(x + \frac{1}{\Gamma(\alpha)} \right)^\alpha - \omega \left(t + \frac{1}{\Gamma(\alpha)} \right)^\alpha. \quad (21)$$

Then, equation (1) is transformed to

$$\omega^2 \Psi'' + bk^2 \Psi'' + \beta k^2 (\Psi^2)'' + \gamma k^4 \Psi''' = 0, \quad (22)$$

in which $\Psi' = d\Psi/d\eta$. By integrating equation (22) twice with respect to η reads as

$$(\omega^2 + bk^2) \Psi + \beta k^2 \Psi^2 + \gamma k^4 \Psi'' = 0. \quad (23)$$

With the help of relations (15)–(18), we balance the term of nonlinear and derivative of higher order in equation (23) and we attain $N = M + 2$. We investigated two following cases as follows.

4.1. Case I: $N = 2, M = 0$. The IEFM allows us to recruit the substitutions:

$$\Psi(\eta) = \frac{A_0 + A_1 F(\eta) + A_2 F^2(\eta)}{B_0}. \quad (24)$$

Plugging (24) along with (9) into equation (23) and equating all the coefficients of powers of $F(\eta)$ to be zero, one gains a system of algebraic equations. Solving the considered system by the help of Maple the following results will be obtained.

Set I:

$$k = k,$$

$$\omega = \omega,$$

$$A_0 = A_0,$$

$$B_0 = B_0,$$

$$A_1 = 2 \frac{\sqrt{-9b\gamma k^2 B_0^2 - 6\beta\gamma k^2 A_0 B_0 - 9\gamma\omega^2 B_0^2}}{\beta}, \quad (25)$$

$$A_2 = -\frac{6\gamma k^2 B_0}{\beta},$$

$$\lambda = -\frac{1}{3} \frac{\sqrt{-9b\gamma k^2 B_0^2 - 6\beta\gamma k^2 A_0 B_0 - 9\gamma\omega^2 B_0^2}}{\gamma k^2 B_0},$$

$$\mu = -\frac{1}{6} \frac{\beta A_0}{\gamma k^2 B_0},$$

$$\lambda^2 - 4\mu = -\frac{bk^2 + \omega^2}{\gamma k^4},$$

where A_0 and B_0 are arbitrary values. Imposing the solution set (25) into (24) can be concluded in the following cases.

Subcase IA:

With the help of (10), the dark soliton solution to STFBE is deduced as

$$\Psi_1(x, t) = \frac{A_0}{B_0} - \frac{(2/B_0) \sqrt{-9b\gamma k^2 B_0^2 - 6\beta\gamma k^2 A_0 B_0 - 9\gamma\omega^2 B_0^2}/\beta}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)} - \frac{6\gamma k^2}{\beta \left[\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)\right]^2}, \quad (26)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be noted that this soliton exist for

$$\gamma k^4 (bk^2 + \omega^2) < 0. \quad (27)$$

As seen in Figure 1, we can observe dark behavior in the $-20 < x < 20$ range. Different interpretations and studies can be made for this situation. In Figure 1(c), for three cases $\alpha = 0.5$, $\alpha = 0.75$, and $\alpha = 0.99$, the fractional model of equation has been plotted and compared.

Subcase IB:

With the help of (11), the periodic wave solution to STFBE is concluded as

$$\Psi_2(x, t) = \frac{A_0}{B_0} + \frac{(2/B_0) \sqrt{-9b\gamma k^2 B_0^2 - 6\beta\gamma k^2 A_0 B_0 - 9\gamma\omega^2 B_0^2}/\beta}{\left(\sqrt{-\lambda^2 + 4\mu/2\mu}\right) \tan\left(\left(\sqrt{-\lambda^2 + 4\mu/2}\right)\eta\right) - (\lambda/2\mu)} - \frac{6\gamma k^2}{\beta \left[\left(\sqrt{-\lambda^2 + 4\mu/2\mu}\right) \tan\left(\left(\sqrt{-\lambda^2 + 4\mu/2}\right)\eta\right) - (\lambda/2\mu)\right]^2}, \quad (28)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be understood that this soliton is available for

$$\gamma k^4 (bk^2 + \omega^2) > 0. \quad (29)$$

As seen in Figure 2, we can observe periodic behavior in the $-20 < x < 20$ range. Different interpretations and studies can be made for this situation. In Figure 2(c), for three cases $\alpha = 0.5$, $\alpha = 0.75$, and $\alpha = 0.99$ at the space $0 < x < 10$ the fractional model of equation have been plotted and compared.

Subcase IC:

FIGURE 1: The plot of (26) $\Psi_1(x, t)$ at $A_0 = 1, B_0 = 2, a = 3, b = 2, k = 1.5, \omega = 2, \beta = 3, \gamma = -1$, and $\alpha = 1$ for (a) 3D plot and (b) and (c) 2D plot (red $\alpha = 0.5$, blue $\alpha = 0.75$, and green $\alpha = 0.99$).

FIGURE 2: The plot of (28) $\Psi_2(x, t)$ at $A_0 = 1, B_0 = 2, a = 1, b = 2, k = 1.5, \omega = 2, \beta = 3, \gamma = -1$, and $\alpha = 1$ for (a) 3D plot and (b) and (c) 2D plot (red $\alpha = 0.5$, blue $\alpha = 0.75$, and green $\alpha = 0.99$).

With the help of (12), the kink-singular wave solution to STFBE is concluded as

$$\Psi_3(x, t) = \frac{2\lambda\sqrt{-9b\gamma k^2 - 9\gamma\omega^2}}{\beta(\exp(\lambda\eta) - 1)} - \frac{6\gamma\lambda^2 k^2}{\beta[\exp(\lambda\eta) - 1]^2}, \quad (30)$$

$$\lambda = -\frac{\sqrt{-9b\gamma k^2 - 9\gamma\omega^2}}{3\gamma k^2},$$

in which $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be noted that this soliton exist for

$$\gamma(bk^2 + \omega^2) < 0. \quad (31)$$

Subcase ID:

With the help of (13), the rational cupson wave solution to STFBE is obtained as

$$\Psi_4(x, t) = \frac{A_0}{B_0} - \frac{2\lambda^2\eta\sqrt{-6\beta\gamma k^2 A_0 B_0}}{B_0\beta(2\lambda\eta + 4)} - \frac{6\gamma k^2(2\lambda\eta + 4)^2}{\beta\lambda^4\eta^2}, \quad (32)$$

in which $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. Also, it should be noted that, for getting the soliton solution in the rational analysis of the function Ψ , we have the relation as follows:

$$b = -\frac{\omega^2}{k^2}. \quad (33)$$

Set II:

$$\begin{aligned}
k &= k, \\
\omega &= \frac{1}{6} \frac{\sqrt{-3\gamma(12b\gamma k^2 B_0^2 - 24\beta\gamma k^2 A_0 B_0 - \beta^2 A_1^2)}}{\gamma B_0}, \\
A_0 &= A_0, \\
B_0 &= B_0, \\
A_1 &= A_1, \\
A_2 &= -\frac{6\gamma k^2 B_0}{\beta}, \\
\lambda &= -\frac{\beta A_1}{6\gamma k^2 B_0}, \\
\mu &= -\frac{\beta(36\gamma k^2 A_0 B_0 + \beta A_1^2)}{72\gamma^2 k^4 B_0^2}, \\
\lambda^2 - 4\mu &= \frac{\beta(24\gamma k^2 A_0 B_0 + \beta A_1^2)}{12\gamma^2 k^4 B_0^2},
\end{aligned} \tag{34}$$

where A_0, B_0 , and A_1 are arbitrary constants. Imposing the solution set (34) into (24), one can conclude the following cases.

Subcase IIA:

With the help of (10), the hyperbolic function solution to the STFBE is deduced as

$$\begin{aligned}
\Psi_5(x, t) &= \frac{A_0}{B_0} - \frac{A_1/B_0}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)} \\
&\quad - \frac{6\gamma k^2/\beta}{\left[\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)\right]^2},
\end{aligned} \tag{35}$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + (1/6) (\sqrt{-3\gamma(12b\gamma k^2 B_0^2 - 24\beta\gamma k^2 A_0 B_0 - \beta^2 A_1^2)}/\gamma B_0)(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be noted that this soliton is available for

$$\beta(24\gamma k^2 A_0 B_0 + \beta A_1^2) > 0. \tag{36}$$

Subcase IIB:

With the help of (11), the periodic wave solution to the STFBE is obtained as

$$\begin{aligned}
\Psi_6(x, t) &= \frac{A_0}{B_0} + \frac{A_1/B_0}{\left(\sqrt{-\lambda^2 + 4\mu/2\mu}\right)\tan\left(\left(\sqrt{-\lambda^2 + 4\mu/2}\right)\eta\right) - (\lambda/2\mu)} \\
&\quad - \frac{6\gamma k^2/\beta}{\left[\left(\sqrt{-\lambda^2 + 4\mu/2\mu}\right)\tan\left(\left(\sqrt{-\lambda^2 + 4\mu/2}\right)\eta\right) - (\lambda/2\mu)\right]^2},
\end{aligned} \tag{37}$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + (1/6) (\sqrt{-3\gamma(12b\gamma k^2 B_0^2 - 24\beta\gamma k^2 A_0 B_0 - \beta^2 A_1^2)}/\gamma B_0)(t + (1/\Gamma(\alpha)))^\alpha + E$. Moreover, indeed need to be assured to warrant the rational analysis of the function Ψ as

$$\beta(24\gamma k^2 A_0 B_0 + \beta A_1^2) < 0. \tag{38}$$

Subcase IIC:

With the help of (12), the kink-singular solution to the STFBE is obtained as

$$\Psi_7(x, t) = \frac{A_1\lambda}{B_0(\exp(\lambda\eta) - 1)} - \frac{6\gamma\lambda^2 k^2}{\beta[\exp(\lambda\eta) - 1]^2}, \quad \lambda = -\frac{\beta A_1}{6\gamma k^2 B_0}, \tag{39}$$

in which $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega = (\sqrt{-3\gamma(12b\gamma k^2 B_0^2 - \beta^2 A_1^2)}/6\gamma B_0)(t + (1/\Gamma(\alpha)))^\alpha + E$ and B_0 is optional arbitrary. One can note that these solitons are available for

$$\gamma(12b\gamma k^2 B_0^2 - \beta^2 A_1^2) < 0. \tag{40}$$

Subcase IID:

With the help of (13), the rational cupson solution to the STFBE is gained as

$$\Psi_8(x, t) = \frac{A_0}{B_0} - \frac{\sqrt{-(24\gamma k^2 A_0 B_0/\beta)\lambda^2\eta}}{B_0(2\lambda\eta + 4)} - \frac{6\gamma k^2 \lambda^4 \eta^2}{\beta(2\lambda\eta + 4)^2}, \tag{41}$$

in which $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \sqrt{-bk^2}(t + (1/\Gamma(\alpha)))^\alpha + E$. Also, indeed need to be assured to warrant the rational analysis of the function Ψ as

$$\gamma A_0 B_0 \beta < 0. \tag{42}$$

4.2. Case II: $N = 3, M = 1$. The IEFM allows us to recruit the substitutions:

$$\Psi(\eta) = \frac{A_0 + A_1 F(\eta) + A_2 F^2(\eta) + A_3 F^3(\eta)}{B_0 + B_1 F(\eta)}. \tag{43}$$

Plugging (43) along with (9) into equation (1) and equating all the coefficients of powers of $F^k(\eta)$, $k = 0, 1, \dots, 7$ to be zero, one gains a system of

algebraic equations. Solving the considered system with the help of Maple the following results will be obtained.

Set I:

$$\begin{aligned}
 k &= k, \\
 \omega &= \omega, \\
 A_0 &= -\frac{B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)}{2\beta k^2}, \\
 B_0 &= B_0, \\
 A_1 &= -\frac{3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1}{2\beta k^2}, \\
 B_1 &= B_1, \\
 A_2 &= -\frac{6\gamma k^2 (\lambda B_1 + B_0)}{\beta}, \\
 A_3 &= -\frac{6\gamma k^2 B_1}{\beta}, \\
 \lambda &= \lambda, \\
 \mu &= \frac{\gamma k^4 \lambda^2 - bk^2 - \omega^2}{4\gamma k^4}, \\
 \lambda^2 - 4\mu &= \frac{bk^2 + \omega^2}{\gamma k^4},
 \end{aligned} \tag{44}$$

$$\begin{aligned}
 \Psi_1(x, t) &= \frac{-\left(B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2\right) - \left(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1/2\beta k^2\right)\Delta(\eta) - (6\gamma k^2 (\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\
 \Delta(\eta) &= -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},
 \end{aligned} \tag{45}$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be noted that this soliton is available for

$$\gamma(bk^2 + \omega^2) > 0. \tag{46}$$

where A_0 and B_0 are arbitrary constants. Imposing the solution set (44) into (24), one can conclude the following cases.

Subcase IA:

With the help of (10), the exact solution to the STFBE is deduced as

Subcase IB:

With the help of (11), the periodic wave solution to the STFBE is concluded as

$$\begin{aligned}
 \Psi_2(x, t) &= \frac{-\left(B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2\right) - \left(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1/2\beta k^2\right)\Delta(\eta) - (6\gamma k^2 (\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\
 \Delta(\eta) &= -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},
 \end{aligned} \tag{47}$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. Also, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\gamma(bk^2 + \omega^2) < 0. \tag{48}$$

Subcase IC:

With the help of (12), the rational kink-singular solution to the STFBE is obtained as

$$\Psi_3(x, t) = \frac{-(B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2) - (3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1/2\beta k^2)\Delta(\eta) - (6\gamma k^2(\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = \frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},$$
(49)

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha) + E$. It should be noted that this soliton is available for

$$\lambda = \frac{\sqrt{\gamma(bk^2 + \omega^2)}}{\gamma k^2},$$
(50)

$$\gamma(bk^2 + \omega^2) > 0.$$

Subcase ID:

With the help of (13), the exact rational cupson solution to the STFBE is gained as

$$\Psi_4(x, t) = \frac{-(B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2) - (3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1/2\beta k^2)\Delta(\eta) - (6\gamma k^2(\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = \frac{\lambda^2(\eta + E)}{2\lambda(\eta + E) + 4},$$
(51)

in which $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha)$. It should be noted that these solitons exist for

$$b = -\frac{\omega^2}{k^2}.$$
(52)

$$A_3 = -\frac{6\gamma k^2 B_1}{\beta}, \lambda = \lambda, \mu = \frac{\gamma k^4 \lambda^2 + bk^2 + \omega^2}{4\gamma k^4}, \lambda^2 - 4\mu = -\frac{bk^2 + \omega^2}{\gamma k^4},$$
(53)

Set II:

$$k = k,$$

$$\omega = \omega,$$

$$A_0 = -\frac{3B_0(3\gamma k^4 \lambda^2 + bk^2 + \omega^2)}{2\beta k^2},$$

$$B_0 = B_0,$$

$$A_1 = -\frac{3(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 + bk^2 B_1 + \omega^2 B_1)}{2\beta k^2},$$

$$A_2 = \frac{6\gamma k^2(\lambda B_1 + B_0)}{\beta},$$

where A_0, B_0 , and B_1 are arbitrary constants. Imposing the solution set (53) into (24), one can obtain the following cases.

Subcase IIA:

With the help of (10), the exact dark soliton solution to the STFBE is deduced as

$$\Psi_5(x, t) = \frac{-(3B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2) - (3(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1)/2\beta k^2)\Delta(\eta) - (6\gamma k^2(\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = \frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},$$
(54)

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. Also, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\gamma(bk^2 + \omega^2) < 0. \quad (55)$$

$$\Psi_6(x, t) = \frac{-(3B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2) - (3(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1)/2\beta k^2)\Delta(\eta) - (6\gamma k^2(\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\eta\right) + (\lambda/2\mu)},$$

(56)

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\gamma(bk^2 + \omega^2) > 0. \quad (57)$$

$$\Psi_7(x, t) = \frac{-(3B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2) - (3(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1)/2\beta k^2)\Delta(\eta) - (6\gamma k^2(\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = \frac{\exp((\sqrt{-\gamma(bk^2 + \omega^2)}/\gamma k^2)(\eta + E)) - 1}{\sqrt{-\gamma(bk^2 + \omega^2)}/\gamma k^2},$$

(58)

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha$. It should be noted that this soliton is available for

$$\lambda = \frac{\sqrt{-\gamma(bk^2 + \omega^2)}}{\gamma k^2}, \quad (59)$$

$$\gamma(bk^2 + \omega^2) < 0.$$

$$\Psi_8(x, t) = \frac{-(3B_0(3\gamma k^4 \lambda^2 - bk^2 - \omega^2)/2\beta k^2) - (3(3\gamma k^4 \lambda^2 B_1 + 12\gamma k^4 \lambda B_0 - bk^2 B_1 - \omega^2 B_1)/2\beta k^2)\Delta(\eta) - (6\gamma k^2(\lambda B_1 + B_0)/\beta)\Delta(\eta)^2 - (6\gamma k^2 B_1/\beta)\Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = \frac{\lambda^2(\eta + E)}{2\lambda(\eta + E) + 4},$$

(60)

in which $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha$. Also, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$b = -\frac{\omega^2}{k^2}. \quad (61)$$

Subcase IB:

With the help of (11), the periodic wave solution to the STFBE is concluded as

Subcase IIC:

With the help of (12), the kink-singular solution to the STFBE is obtained as

Subcase IID:

With the help of (13), the exact rational solution to the STFBE is resulted as

5. The STFBSEs

Consider the nonlinear space-time (2 + 1)-dimensional breaking soliton equations by the following variable change as

$$\eta = k\left(x + \frac{1}{\Gamma(\alpha)}\right)^\alpha + \omega\left(y + \frac{1}{\Gamma(\alpha)}\right)^\alpha - c\left(t + \frac{1}{\Gamma(\alpha)}\right)^\alpha. \quad (62)$$

Then, equation (2) is transformed to

$$-c\Psi' + ak^2\omega''' + 4ak\Psi\Omega' + 4ak\Psi'\Omega = 0, \quad (63)$$

$$\omega\Psi' - k\Omega' = 0, \quad (64)$$

in which $\Psi' = d\Psi/d\eta$ and $\Omega' = d\Omega/d\eta$. By integrating equation (64) once with respect to η , we obtain

$$\omega\Psi - k\Omega = 0. \quad (65)$$

Inserting equation (65) into first equation of system (63) and integrating the obtained equation once with respect to η , we obtain

$$-c\Psi + ak^2\omega\Psi' + 4a\omega\Psi^2 = 0. \quad (66)$$

With the help of relations (15)–(18), we balance the term of nonlinear and derivative of higher order in equation (64), and we attain $N = M + 2$. We investigate the following two cases.

5.1. Case I: $N = 2, M = 0$. The IEFM allows us to recruit the substitutions:

$$\Psi(\eta) = \frac{A_0 + A_1F(\eta) + A_2F^2(\eta)}{B_0}. \quad (67)$$

Plugging (67) along with (9) into equation (2) and equating all the coefficients of powers of $F(\eta)$ to be zero, one gains a system of algebraic equations. Solving the considered system with the help of Maple the following results will be obtained.

Set I:

$$\begin{aligned} \lambda &= 0, \\ \mu &= \mu, \\ k &= k, \\ c &= 2\sqrt{14}a\omega k^2\mu, \\ \omega &= \omega, \\ A_0 &= \left(-1 + \frac{1}{4}\sqrt{14}\right)B_0\mu k^2, \\ B_0 &= B_0, \\ A_1 &= 0, \\ A_2 &= -\frac{1}{4}k^2B_0, \end{aligned} \quad (68)$$

where B_0 is arbitrary constant. Imposing the solution set (68) into (24), one can conclude the following cases.

Subcase IA:

With the help of (10), the singular solution to the STFBSEs is deduced as

$$\begin{aligned} \Psi_1(x, y, t) &= \left(-1 + \frac{1}{4}\sqrt{14}\right)\mu k^2 + \frac{1}{4}k^2\mu \coth^2(\sqrt{-\mu}\eta + E), \\ \Omega_1(x, y, t) &= \left(-1 + \frac{1}{4}\sqrt{14}\right)\mu\omega k + \frac{1}{4}\omega k\mu \coth^2(\sqrt{-\mu}\eta + E), \end{aligned} \quad (69)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(y + (1/\Gamma(\alpha)))^\alpha - 2\sqrt{14}a\omega k^2\mu(t + (1/\Gamma(\alpha)))^\alpha$. Also, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\mu < 0. \quad (70)$$

As seen in Figure 3, we can observe dark behavior in the $-10 < x < 20$ range. Different interpretations and studies can be made for this situation. In Figure 3(b), for one case $\alpha = 1$, the fractional model of the equation has been plotted and compared for the two-dimensional plot.

Subcase IB:

With the help of (11), the periodic solution to the STFBSEs is resulted as

$$\begin{aligned} \Psi_2(x, y, t) &= \left(-1 + \frac{1}{4}\sqrt{14}\right)\mu k^2 - \frac{1}{4}k^2\mu \cot^2(\sqrt{\mu}\eta + E), \\ \Omega_2(x, y, t) &= \left(-1 + \frac{1}{4}\sqrt{14}\right)\mu\omega k - \frac{1}{4}\omega k\mu \cot^2(\sqrt{\mu}\eta + E), \end{aligned} \quad (71)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(y + (1/\Gamma(\alpha)))^\alpha - 2\sqrt{14}a\omega k^2\mu(t + (1/\Gamma(\alpha)))^\alpha$. It should be noted that this soliton exists for

$$\mu > 0. \quad (72)$$

As seen in Figure 4, we can observe periodic behavior in the $-10 < x < 10$ range. Different interpretations and studies can be made for this situation. In Figure 4(c), for three cases $\alpha = 0.5$, $\alpha = 0.75$, and $\alpha = 0.99$, the fractional model of the equation has been plotted and compared.

Set II:

$$\begin{aligned} \lambda &= 0, \\ \mu &= \frac{8}{3} \frac{A_1^2}{k^4 B_0^2}, \\ k &= k, \\ c &= \frac{16}{3} \frac{i a \omega A_1^2}{k^2 B_0^2}, \\ A_0 &= \frac{2}{3} \frac{(-1 + i) A_1^2}{B_0 k^2}, \\ B_0 &= B_0, \\ A_1 &= A_1, \\ A_2 &= -\frac{1}{4} k^2 B_0, \end{aligned} \quad (73)$$

FIGURE 3: The plot of (69) $\Psi_1(x, y, t)$ at $A_0 = 1, B_0 = 2, a = 3, b = 2, k = 1.5, \omega = 2, \beta = 3, \gamma = -1, y = 2$, and $\alpha = 1$ for (a) 3D plot and (b) 2D plot.

FIGURE 4: The plot of (69) $\Psi_2(x, y, t)$ at $A_0 = 1, B_0 = 2, a = 3, b = 2, k = 1.5, \omega = 2, \beta = 3, \gamma = -1, y = 2$, and $\alpha = 1$ for (a) 3D plot and (b) and (c) 2D plot (red $\alpha = 0.5$, blue $\alpha = 0.75$, and green $\alpha = 0.99$).

where B_0 is arbitrary constant. Imposing the solution set (73) into (24), one can conclude the following cases.

Subcase IA:

With the help of (10), the exact solution to the model is deduced as

$$\Psi_3(x, y, t) = \frac{2}{3} \frac{(-1+i)A_1^2}{B_0^2 k^2} + \frac{A_1}{B_0} \sqrt{-\mu} \coth(\sqrt{-\mu} \eta + E) + \frac{1}{4} k^2 \mu \coth^2(\sqrt{-\mu} \eta + E), \quad (74)$$

$$\Omega_3(x, y, t) = \frac{2}{3} \frac{(-1+i)A_1^2}{B_0^2 \omega k} + \frac{A_1 \omega}{B_0 k} \sqrt{-\mu} \coth(\sqrt{-\mu} \eta + E) + \frac{1}{4} k^2 \mu \coth^2(\sqrt{-\mu} \eta + E),$$

$$\mu < 0. \quad (75)$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(y + (1/\Gamma(\alpha))^\alpha - (16/3)(i\omega A_1^2/k^2 B_0^2)(t + (1/\Gamma(\alpha))^\alpha))^\alpha$. It should be noted that this soliton exist for

Subcase IB:

With the help of (11), the exact solution to the model is concluded as

$$\begin{aligned}\Psi_4(x, y, t) &= \frac{2}{3} \frac{(-1+i)A_1^2}{B_0^2 k^2} + \frac{A_1}{B_0} \sqrt{\mu} \cot(\sqrt{\mu} \eta + E) - \frac{1}{4} k^2 \mu \cot^2(\sqrt{\mu} \eta + E), \\ \Omega_4(x, y, t) &= \frac{2}{3} \frac{(-1+i)A_1^2}{B_0^2 \omega k} + \frac{A_1 \omega}{B_0 k} \sqrt{\mu} \cot(\sqrt{\mu} \eta + E) - \frac{1}{4} \omega k \mu \cot^2(\sqrt{\mu} \eta + E),\end{aligned}\quad (76)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(y + (1/\Gamma(\alpha)))^\alpha - (16/3)(i a \omega A_1^2 / k^2 B_0^2)(t + (1/\Gamma(\alpha)))^\alpha$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\mu > 0. \quad (77)$$

5.2. Case II: $N = 3, M = 1$. The IEFM allows us to recruit the substitutions

$$v(\eta) = \frac{A_0 + A_1 F(\eta) + A_2 F^2(\eta) + A_3 F^3(\eta)}{B_0 + B_1 F(\eta)}. \quad (78)$$

Plugging (78) along with (9) into equation (3) and equating all the coefficients of powers of $F^k(\eta), k = 0, 1, \dots, 7$ to be zero, one gains a system of algebraic equations. Solving the considered system, with the help of Maple, the following results can be obtained.

Set I:

$$k = k,$$

$$\omega = \omega,$$

$$A_0 = -\frac{3}{2} k^2 \mu B_0,$$

$$B_0 = B_0,$$

$$A_1 = -\frac{3}{2} k^2 (\lambda B_0 + \mu B_1),$$

$$A_2 = -\frac{3}{2} k^2 (\lambda B_1 + B_0),$$

$$A_3 = -\frac{3}{2} k^2 B_1, \quad (79)$$

$$B_1 = B_1,$$

$$\lambda = \lambda,$$

$$\mu = \mu,$$

$$c = a \omega k^2 (\lambda^2 - 4\mu),$$

where A_0, B_0 , and B_1 are arbitrary constants. Imposing the solution set (44) into (78), one can conclude the following cases.

Subcase IA:

With the help of (10), the exact solution to the model is obtained as

$$\begin{aligned}\Psi_5(x, y, t) &= \frac{-(3/2)k^2 \mu B_0 - (3/2)k^2 (\lambda B_0 + \mu B_1) \Delta(\eta) - (3/2)k^2 (\lambda B_1 + B_0) \Delta(\eta)^2 - (3/2)k^2 B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)}, \\ \Omega_5(x, y, t) &= \frac{-(3/2)\omega k \mu B_0 - (3/2)\omega k (\lambda B_0 + \mu B_1) \Delta(\eta) - (3/2)\omega k (\lambda B_1 + B_0) \Delta(\eta)^2 - (3/2)\omega k B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)}, \\ \Delta(\eta) &= \frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},\end{aligned}\quad (80)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(y + (1/\Gamma(\alpha)))^\alpha - a\omega k^2(\lambda^2 - 4\mu)(t + (1/\Gamma(\alpha)))^\alpha + E$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\lambda^2 - 4\mu > 0. \quad (81)$$

Subcase IB:

With the help of (11), the exact solution to the model is concluded as

$$\begin{aligned} \Psi_6(x, y, t) &= \frac{-(3/2)k^2\mu B_0 - (3/2)k^2(\lambda B_0 + \mu B_1)\Delta(\eta) - (3/2)k^2(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)k^2B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\ \Omega_6(x, y, t) &= \frac{-(3/2)\omega k\mu B_0 - (3/2)\omega k(\lambda B_0 + \mu B_1)\Delta(\eta) - (3/2)\omega k(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)\omega k^2B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \end{aligned} \quad (82)$$

$$\Delta(\eta) = -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(y + (1/\Gamma(\alpha)))^\alpha - a\omega k^2(\lambda^2 - 4\mu)(t + (1/\Gamma(\alpha)))^\alpha + E$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

Subcase IC:

With the help of (12), the kink-singular solution to the STFBSEs is obtained as

$$\lambda^2 - 4\mu < 0. \quad (83)$$

$$\Psi_7(x, y, t) = \frac{-(3/2)k^2(\lambda B_0)\Delta(\eta) - (3/2)k^2(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)k^2B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)},$$

$$\Delta(\eta) = \frac{\exp(\lambda(\eta + E)) - 1}{\lambda},$$

(84)

$$\Omega_7(x, y, t) = \frac{-(3/2)\omega k(\lambda B_0)\Delta(\eta) - (3/2)\omega k(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)\omega k^2B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)},$$

$$\Delta(\eta) = \frac{\exp(\lambda(\eta + E)) - 1}{\lambda},$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(y + (1/\Gamma(\alpha)))^\alpha - a\omega k^2\lambda^2(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be noted that this soliton is available for

Set II:

$$\lambda^2 - 4\mu < 0. \quad (85)$$

$$\begin{aligned}
A_0 &= -\frac{1}{4}B_0k^2(\lambda^2 + 2\mu), \\
B_0 &= B_0, \\
A_1 &= -\frac{1}{4}k^2(\lambda^2 B_1 + 6\lambda B_0 + 2\mu B_1), \\
A_2 &= -\frac{3}{2}k^2(\lambda B_1 + B_0), \\
k &= k, \\
\omega &= \omega, \\
A_3 &= -\frac{3}{2}k^2 B_1, \\
B_1 &= B_1, \\
\lambda &= \lambda, \\
\mu &= \mu, \\
c &= -a\omega k^2(\lambda^2 - 4\mu),
\end{aligned} \tag{86}$$

where A_0, B_0 , and B_1 are arbitrary constants. Imposing the solution set (86) into (24), one can conclude the following cases.

Subcase IIA:

With the help of (10), the exact dark solution to the STFBSEs is obtained as

$$\begin{aligned}
\Psi_8(x, y, t) &= \frac{-(1/4)B_0k^2(\lambda^2 + 2\mu) - (1/4)k^2(\lambda^2 B_1 + 6\lambda B_0 + 2\mu B_1)\Delta(\eta) - (3/2)k^2(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)k^2 B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)}, \\
\Omega_8(x, y, t) &= \frac{-(1/4)B_0\omega k^2(\lambda^2 + 2\mu) - (1/4)\omega k^2(\lambda^2 B_1 + 6\lambda B_0 + 2\mu B_1)\Delta(\eta) - (3/2)\omega k^2(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)\omega k^2 B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)}, \\
\Delta(\eta) &= -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},
\end{aligned} \tag{87}$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(y + (1/\Gamma(\alpha))^\alpha + a\omega k^2(\lambda^2 - 4\mu)(t + (1/\Gamma(\alpha))^\alpha + E)$. Also, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\lambda^2 - 4\mu > 0. \tag{88}$$

Subcase IIB:

With the help of (11), the periodic wave solution to the STFBSEs is obtained as

$$\begin{aligned}
\Psi_9(x, y, t) &= \frac{-(1/4)B_0k^2(\lambda^2 + 2\mu) - (1/4)k^2(\lambda^2 B_1 + 6\lambda B_0 + 2\mu B_1)\Delta(\eta) - (3/2)k^2(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)k^2 B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)}, \\
\Delta(\eta) &= -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},
\end{aligned} \tag{89}$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(y + (1/\Gamma(\alpha))^\alpha + a\omega k^2(\lambda^2 - 4\mu)(t + (1/\Gamma(\alpha))^\alpha + E)$ and A_0 and B_0 are optional arbitrarities. It should be noted that these solitons exist for

$$\lambda^2 - 4\mu < 0. \tag{90}$$

Subcase IIC:

With the help of (12), the exact solution to the model is obtained as

$$\begin{aligned}\Psi_{10}(x, y, t) &= \frac{-(1/4)B_0k^2\lambda^2 - (1/4)k^2(\lambda^2B_1 + 6\lambda B_0)\Delta(\eta) - (3/2)k^2(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)k^2B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\ \Delta(\eta) &= \frac{\exp(\lambda(\eta + E)) - 1}{\lambda}, \\ \Omega_{10}(x, y, t) &= \frac{-(1/4)B_0\omega k\lambda^2 - (1/4)k^2(\lambda^2B_1 + 6\lambda B_0)\Delta(\eta) - (3/2)\omega k(\lambda B_1 + B_0)\Delta(\eta)^2 - (3/2)\omega k B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\ \Delta(\eta) &= \frac{\exp(\lambda(\eta + E)) - 1}{\lambda},\end{aligned}\tag{91}$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(y + (1/\Gamma(\alpha))^\alpha + a\omega k^2\lambda^2(t + (1/\Gamma(\alpha))^\alpha + E$

6. The STFSRLWE

Take the nonlinear space-time fractional SRLW equation with the following variable change:

$$\eta = k\left(x + \frac{1}{\Gamma(\alpha)}\right)^\alpha - \omega\left(t + \frac{1}{\Gamma(\alpha)}\right)^\alpha. \tag{92}$$

Then, equation (3) is integrated respect to η and simplified to obtain

$$2(\omega^2 + k^2)\Psi + k\omega\Psi^2 + 2k^2\omega^2\Psi'' = 0, \tag{93}$$

in which $\Psi' = d\Psi/d\eta$. With the help of relations (15)–(18), we balance the term of nonlinear and derivative of higher order in equation (93), and we attain $N = M + 2$. We investigate the two following cases,

6.1. Case I: $N = 2, M = 0$. The IEFM allows us to recruit the substitutions:

$$\Psi(\eta) = \frac{A_0 + A_1F(\eta) + A_2F^2(\eta)}{B_0}. \tag{94}$$

Plugging (94) along with (9) into equation (93) and equating all the coefficients of powers of $F(\eta)$ to be zero, one gains a system of algebraic equations. Solving the considered system with the help of Maple, the following results will be obtained.

Set I:

$$k = k,$$

$$\omega = \omega,$$

$$A_0 = \frac{1}{48} \frac{48k^2B_0^2 + 48\omega^2B_0^2 - A_1^2}{k\omega B_0},$$

$$B_0 = B_0,$$

$$A_1 = A_1, \tag{95}$$

$$A_2 = -12k\omega B_0,$$

$$\lambda = -\frac{1}{12} \frac{A_1}{k\omega B_0},$$

$$\mu = -\frac{144k^2B_0^2 + 144\omega^2B_0^2 - A_1^2}{576k^2\omega^2B_0^2},$$

$$\lambda^2 - 4\mu = \frac{k^2 + \omega^2}{k^2\omega^2},$$

where A_0, B_0 , and B_1 are arbitrary constants. Imposing the solution set (95) into (24), one can conclude the following cases.

Subcase IA:

With the help of (10), the exact dark soliton solution to the STFSRLWE is deducted as

$$\Psi_1(x, t) = \frac{1}{48} \frac{48k^2 B_0^2 + 48\omega^2 B_0^2 - A_1^2}{k\omega B_0^2} - \frac{A_1/B_0}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\eta\right) + (\lambda/2\mu)} - \frac{12k\omega}{\left[\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\eta\right) + (\lambda/2\mu)\right]^2}, \quad (96)$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha + E)$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$k^2 + \omega^2 > 0. \quad (97)$$

As seen in Figure 5, we can observe dark behavior in the $-20 < x < 20$ range. Different interpretations and studies can be made for this situation. In Figure 5(c), for three cases $\alpha = 0.5$, $\alpha = 0.75$, and $\alpha = 0.99$, the fractional model of the equation has been plotted and compared.

Subcase IB:

With the help of (11), the kink-singular solution to the model is concluded as

$$\Psi_2(x, t) = -2 \frac{k^2 + \omega^2}{k\omega} + \frac{12\sqrt{k^2 + \omega^2} B_0}{B_0} \Delta(\eta) - 12k\omega \Delta^2(\eta), \quad \Delta(\eta) = \frac{\lambda}{\exp(\lambda(\eta + E)) - 1}, \quad (98)$$

in which $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha + E)$. Likewise, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\lambda = -\frac{\sqrt{k^2 + \omega^2}}{k\omega}, \quad k\omega \neq 0. \quad (99)$$

As seen in Figure 6, we can observe periodic behavior in the $-20 < x < 20$ range. Different interpretations and studies can be made for this situation. In Figure 6(c), for three cases $\alpha = 0.5$, $\alpha = 0.75$, and $\alpha = 0.99$, the fractional model of the equation has been plotted and compared.

Set II:

$$k = k,$$

$$\omega = \omega,$$

$$A_0 = -\frac{1}{48} \frac{48k^2 B_0^2 + 48\omega^2 B_0^2 + A_1^2}{k\omega B_0},$$

$$B_0 = B_0,$$

$$A_1 = A_1,$$

$$A_2 = -12k\omega B_0,$$

(100)

$$\lambda = -\frac{1}{12} \frac{A_1}{k\omega B_0},$$

$$\mu = \frac{144k^2 B_0^2 + 144\omega^2 B_0^2 + A_1^2}{576k^2 \omega^2 B_0^2},$$

$$\lambda^2 - 4\mu = -\frac{k^2 + \omega^2}{k^2 \omega^2},$$

where A_0, B_0 , and B_1 are arbitrary constants. Imposing the solution set (100) into (24), one can obtain the following cases.

Subcase IIA:

With the help of (10), the exact periodic wave solution to the STFSRLWE is deduced as

$$\Psi_3(x, t) = \frac{1}{48} \frac{48k^2 B_0^2 + 48\omega^2 B_0^2 + A_1^2}{k\omega B_0^2} + \frac{A_1/B_0}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\eta\right) + (\lambda/2\mu)} - \frac{12k\omega}{\left[\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\eta\right) + (\lambda/2\mu)\right]^2}, \quad (101)$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha + E)$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$k^2 + \omega^2 > 0. \quad (102)$$

Subcase IB:

FIGURE 5: The plot of (96) $\Psi_1(x, t)$ at $A_0 = 1, B_0 = 2, a = 3, b = 2, k = 1.5, \omega = 2, \beta = 3, \gamma = -1$, and $\alpha = 1$ for (a) 3D plot and (b) and (c) 2D plot (red $\alpha = 0.5$, blue $\alpha = 0.75$, and green $\alpha = 0.99$).

FIGURE 6: The plot of (98) $\Psi_2(x, t)$ at $A_0 = 1, B_0 = 2, a = 3, b = 2, k = 1.5, \omega = 2, \beta = 3, \gamma = -1$, and $\alpha = 1$ for (a) 3D plot and (b) and (c) 2D plot (red $\alpha = 0.5$, blue $\alpha = 0.75$, and green $\alpha = 0.99$).

With the help of (11), the exact solution to the STFSRLWE is obtained as

$$\Psi_4(x, t) = -2 \frac{k^2 + \omega^2}{k\omega} + \frac{12\sqrt{-k^2 - \omega^2}B_0}{B_0} \Delta(\eta) - 12k\omega\Delta^2(\eta), \quad \Delta(\eta) = \frac{\lambda}{\exp(\lambda(\eta + E)) - 1}, \quad (103)$$

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. Likewise, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\lambda = -\frac{\sqrt{-k^2 - \omega^2}}{k\omega}, \quad k\omega \neq 0. \quad (104)$$

6.2. Case II: $N = 3, M = 1$. The IEFM allows us to recruit the substitutions:

$$\Psi(\eta) = \frac{A_0 + A_1F(\eta) + A_2F^2(\eta) + A_3F^3(\eta)}{B_0 + B_1F(\eta)}. \quad (105)$$

Plugging (105) along with (9) into equation (93) and equating all the coefficients of powers of $F^k(\eta), k = 0, 1, \dots, 7$ to be zero, one gains a system of algebraic equations. Solving the considered system with the help of Maple, the following results will be obtained.

Set I:

$$\begin{aligned}
A_0 &= -\frac{\lambda(3k^2\lambda^2\omega^2B_1^2 + 21k^2\lambda\omega^2B_0B_1 + 24k^2\omega^2B_0^2 + 3k^2B_1^2 - k\omega A_1B_1 + 3\omega^2B_1^2) + 9k^2B_0B_1 - 2k\omega A_1B_0 + 9\omega^2B_0B_1}{\omega k B_1}, \\
k &= k, \\
\omega &= \omega, \\
A_1 &= -\frac{-2(-\lambda^2\omega^2k^2 - k^2 - \omega^2)B_1 + \lambda^2B_1\omega^2k^2 + 12k^2\lambda\omega^2B_0 + (k^2 + \omega^2)B_1}{k\omega}, \\
A_2 &= -12\lambda B_1\omega k - 12B_0\omega k, \\
\lambda &= \lambda, \\
\mu &= -\frac{1}{4} \frac{-\lambda^2\omega^2k^2 - k^2 - \omega^2}{k^2\omega^2}, \\
\lambda^2 - 4\mu &= -\frac{k^2 + \omega^2}{k^2\omega^2}, \\
A_3 &= -12\omega k B_1, \\
B_0 &= B_0, \\
B_1 &= B_1,
\end{aligned} \tag{106}$$

where B_0 and B_1 are arbitrary constants. Imposing the solution set (106) into (24), one can obtain the following cases.

Subcase IIA:

With the help of (11), the exact periodic wave solution to the STFSRLWE is deducted as

$$\begin{aligned}
\Psi_5(x, t) &= \frac{A_0 + A_1\Delta(\eta) - 12k\omega(\lambda B_1 + 12B_0)\Delta(\eta)^2 - 12\omega k B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\
\Delta(\eta) &= \frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\tan\left(\left(\sqrt{\lambda^2 - 4\mu/2}\right)\eta\right) + (\lambda/2\mu)},
\end{aligned} \tag{107}$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha + E)$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$k\omega \neq 0. \tag{108}$$

Subcase IB:

With the help of (12), the kink-singular solution to the STFSRLWE is obtained as

$$\begin{aligned}
\Psi_6(x, t) &= \frac{A_0 + A_1\Delta(\eta) - 12k\omega(\lambda B_1 + 12B_0)\Delta(\eta)^2 - 12\omega k B_1\Delta(\eta)^3}{B_0 + B_1\Delta(\eta)}, \\
\Delta(\eta) &= \frac{\sqrt{-k^2 - \omega^2}}{k\omega \left[\exp\left(\left(\sqrt{-k^2 - \omega^2}/k\omega\right)\eta\right) - 1 \right]},
\end{aligned} \tag{109}$$

where $\eta = k(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha + E)$. It should be noted that this soliton is available for

$$k\omega \neq 0. \quad (110)$$

Subcase IIC:

With the help of (13), the exact rational solution to the STFSRLWE is gained as

$$\Psi_7(x, t) = \frac{A_0 - (3\lambda^2 B_1 \omega^4 + 12\lambda B_0 \omega^4 B_1 / \sqrt{-\omega} \omega) \Delta(\eta) - 12\omega \sqrt{-\omega} (\lambda B_1 + B_0) \Delta(\eta)^2 - 12\omega \sqrt{-\omega} B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$A_0 = \frac{(2k^2 \lambda^3 B_1^2 + 4\lambda^3 B_1^2 \omega^2 + 12\lambda^2 \omega^2 B_0 B_1^2 + 27\lambda^2 B_0 B_1 \omega^2 + 24\lambda \omega^2 B_0^2 B_1 + 24B_0^2 k^2 \lambda + 6B_1 B_0) \sqrt{-\omega}}{B_1}, \quad (111)$$

$$\Delta(\eta) = \frac{\lambda^2 (\eta + E)}{2\lambda (\eta + E) + 4},$$

where $\eta = \sqrt{-\omega}(x + (1/\Gamma(\alpha))^\alpha + \omega(t + (1/\Gamma(\alpha))^\alpha + E)$. Also, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\omega < 0. \quad (112)$$

Set II:

$$A_0 = -\frac{\lambda(3k^2 \lambda^2 \omega^2 B_1^2 + 21k^2 \lambda \omega^2 B_0 B_1 + 24k^2 \omega^2 B_0^2 + 3k^2 B_1^2 - k\omega A_1 B_1 + 3\omega^2 B_1^2) + B_0(9k^2 B_1 - 2k\omega A_1 + 9\omega^2 B_1)}{\omega k B_1},$$

$$k = k,$$

$$\omega = \omega,$$

$$A_1 = -\frac{-2(-\lambda^2 \omega^2 k^2 + k^2 + \omega^2) B_1 + \lambda^2 B_1 \omega^2 k^2 + 12k^2 \lambda \omega^2 B_0 + (k^2 + \omega^2) B_1}{k\omega},$$

$$A_2 = -12\lambda B_1 \omega k - 12B_0 \omega k,$$

$$\lambda = \lambda, \quad (113)$$

$$\mu = -\frac{1}{4} \frac{-\lambda^2 \omega^2 k^2 + k^2 + \omega^2}{k^2 \omega^2},$$

$$\lambda^2 - 4\mu = \frac{k^2 + \omega^2}{k^2 \omega^2},$$

$$A_3 = -12\omega k B_1,$$

$$B_0 = B_0,$$

$$B_1 = B_1,$$

where B_0 and B_1 are arbitrary constants. Imposing the solution set (113) into (24), one can conclude the following cases.

Subcase IIA:

With the help of (11), the exact dark soliton solution to the STFSRLWE is deduced as

$$\Psi_8(x, t) = \frac{A_0 + A_1 \Delta(\eta) - 12k\omega(\lambda B_1 + 12B_0)\Delta(\eta)^2 - 12\omega k B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = -\frac{1}{\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right) \tanh\left(\left(\sqrt{\lambda^2 - 4\mu/2\mu}\right)\eta\right) - (\lambda/2\mu)},$$
(114)

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. It should be noted that this soliton is available for

$$k\omega \neq 0. \quad (115)$$

Subcase IIB:

With the help of (12), the exact kink-singular solution to the STFSRLWE is obtained as

$$\Psi_9(x, t) = \frac{A_0 + A_1 \Delta(\eta) - 12k\omega(\lambda B_1 + 12B_0)\Delta(\eta)^2 - 12\omega k B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$\Delta(\eta) = \frac{\sqrt{k^2 + \omega^2}}{k\omega \left[\exp((\sqrt{k^2 + \omega^2}/k\omega)\eta) - 1 \right]},$$
(116)

where $\eta = k(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha + E$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$k\omega \neq 0. \quad (117)$$

Subcase IIC:

With the help of (13), the exact rational cupson solution to the STFSRLWE is gained as

$$\Psi_{10}(x, t) = \frac{A_0 - (3\lambda^2 B_1 \omega^4 + 12\lambda B_0 \omega^4 B_1 / \sqrt{-\omega} \omega) \Delta(\eta) - 12\omega \sqrt{-\omega} (\lambda B_1 + B_0) \Delta(\eta)^2 - 12\omega \sqrt{-\omega} B_1 \Delta(\eta)^3}{B_0 + B_1 \Delta(\eta)},$$

$$A_0 = \lambda(3\lambda^2 \omega^4 B_1^2 + 21\lambda B_0 \omega^4 B_1 + 24\omega^4 B_0^2 + 6\omega^2 B_1^2 - \sqrt{-\omega\omega} A_1 B_1) + B_0(18\omega^2 B_1 - 2\sqrt{-\omega} \omega A_1),$$
(118)

$$\Delta(\eta) = -\frac{\lambda^2 (\eta + E)}{2\lambda (\eta + E) + 4},$$

where $\eta = \sqrt{-\omega}(x + (1/\Gamma(\alpha)))^\alpha + \omega(t + (1/\Gamma(\alpha)))^\alpha$. Moreover, indeed need to be assured to warrantee the rational analysis of the function Ψ as

$$\omega < 0. \quad (119)$$

Remark. This paper finds many novel hyperbolic, trigonometric, kink, and kink-singular soliton solutions to govern the model. These mathematical properties come from trigonometric and hyperbolic function properties. In this sense, from the mathematical and physical points of views, these results play an important role in explaining wave propagation of considered models. We obtained more and better results compared with the results of [48] as Mohyud-din and Bibi used the exponential rational function method.

7. Discussion and Remark

This paper finds many novel hyperbolic, trigonometric, kink, and kink-singular soliton solutions to govern model. With the help of some calculations, surfaces of results reported have been observed in Figures 1–6. In Figures 1–6, two dimension plots used the spaces $x = -2$, $x = 0$, and $x = 2$, respectively. These figures are depended on the family conditions which are important physically. It has been investigated that all figures plotted have symbolized the space-time fractional order Boussinesq equation, the space-time (2 + 1)-dimensional breaking soliton equations, and the space-time fractional order SRLW equation. These mathematical properties come from trigonometric and hyperbolic function properties. In this sense, from the mathematical and physical points of views, these results play an important role in explaining waves propagation and nonlinear ocean

problems. Hence, when we consider surfaces plotted in this paper have proved such physical meaning of the solutions.

8. Conclusion

In this paper, the traveling wave solutions of different kinds, which are solitary, topological, dark, kink, singular, periodic, and rational solutions to the model, for the some nonlinear fractional partial differential equations (the STFBE; the STFBSEs; and the STFSRLWE) involving the β -derivative have been obtained. The integration mechanism, namely, the rational $\exp(-\Omega(\eta))$ -expansion scheme is used for the considered models. It is quite visible that this integration scheme has its limitation. Thus, this paper provides a lot of encouragement for future research with β -derivative for other models. Afterwards, other extra solution methods will be applied to obtain optical and singular soliton solutions to the nonlinear model. In addition, some fractional PDEs are converted to ordinary differential equations by with the help of transformation β -derivative. Finally, the existence of the solutions for the constraint conditions is also presented. The constructed results may be helpful in explaining the physical meaning of the studied models and other related as a growing finding in the engineering sciences, mathematical physics, and so forth. The results are beneficial to the study of the wave propagation. All calculations in this paper have been made quickly with the aid of the Maple.

Data Availability

The datasets supporting the conclusions of this article are included within the article and its additional file.

Conflicts of Interest

The author declares that she has no conflicts of interest.

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