Structured $H_\infty$ Control of an Electric Power Steering System

Hongbo Zhou (✉), Aiping Pang (✉), Jing Yang (✉) and Zhen He (✉)

1 College of Electrical Engineering, Guizhou University, Guiyang, Guizhou 550025, China
2 School of Astronautics, Harbin Institute of Technology, Harbin 150000, China

Correspondence should be addressed to Aiping Pang; 417524788@qq.com

Received 8 May 2020; Revised 27 May 2020; Accepted 9 June 2020; Published 25 July 2020

Copyright © 2020 Hongbo Zhou et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Electric power steering (EPS) systems are prone to oscillations because of a very small phase angle margin, so a stable controller is required to increase the stability margin. In addition, the EPS system has parameter disturbances in the gain of the torque map under different conditions, which requires a certain degree of robustness in the control design. This paper synthesizes the multidimensional performance requirements considering the stability margin, robustness, and bandwidth of the system to form an $H_\infty$ optimization matrix with multidimensional performance output in using the structured $H_\infty$ control design. The structured $H_\infty$ controller not only retains the characteristics of traditional $H_\infty$ controllers with excellent robust performance and high stability margin but also has a lower order, which can be better applied in practice. Based on the performance requirements of the system and practical implementation, the structured $H_\infty$ controllers with different orders were designed, and the feasibility of the structured controller was confirmed through comparison and theoretical analysis.

1. Introduction

The electric power steering (EPS) system is a steering system supported by motors, which offers drivers lighter steering experience. Comparing with hydraulic power steering (HPS), EPS has many advantages including better fuel efficiency, smaller size, and the feeling of steering easily, in addition, the capability to combine other electric control systems in the car with itself, so most cars are equipped with an EPS system [1]. When the driver turns the steering wheel, the torque sensor detects the steering angle and torque and sends a voltage signal to the electronic control unit. The electronic control unit sends instructions to the motor control unit based on the torque voltage signal, direction of rotation, and speed signal detected by the torque sensor, so that the motor outputs the steering booster torque of the corresponding size and torque.

Although the EPS system has many advantages, designing a suitable controller for EPS is a challenging problem for many reasons. Torque map is the main component of the EPS controller. The torque map is a gain function between the measured torque from the steering wheel and the assist torque provided by the motor. It determines how much steering torque the motor assists. The shape of the torque map determines the driver’s driving feeling [2]. Generally, since the torque required to steer is maximum when parking, the slope of the torque map is steepest at zero speed, and then decreases as the speed increases. When driving at low speed, the high gain of the controller and the nonlinearity of the torque map cause the instability and vibration [3–5]. Due to the dynamic uncertainty (unmodeled dynamic characteristics) and parameter uncertainty of the EPS system, the controller must be robust. Even for the same type of vehicle, the system parameters of each vehicle will be different, so the tuning of the parameters also faces huge challenges [6]. In addition, the steering system is in an extremely sensitive state to interact with the driver’s hand, so a good controller design should eliminate unwanted vibrations.

There are many researches on the EPS system controller and various EPS controllers are proposed to ensure the system stability. In [7], the authors analyze the stability conditions based on the EPS model and use a structured structure compensator to realize the system stability and torque vibration minimization. In [8], the authors use frequency weighted damping compensator to improve the phase margin of the system, improving the stability of the system, but the phase margin is limited. In [9], the authors use an integral sliding mode controller to generate the power
torque so that the system can achieve stability and improve the
damping characteristics of the system. In [10], the authors analyze the stability of a system with approximately
linear torque diagrams and nonlinear torque diagrams,
propose criteria for designing a stable compensator, and give
lead-lag compensators of different orders. The lead-lag compensator with different parameters is applied together
with the torque map for vehicle experiments.

However, the previous control design has some limita-
tions. Firstly, most researches approximate the nonlinear
torque diagram as a simple linear gain without analyzing the
influence of nonlinearity on the stability of the system. In
addition, the main concern of these designs is whether the
control system is stable or not, without considering the
robustness and control performance comprehensively. $H_\infty$
control can consider many aspects of the design require-
ments, such as robust stability, system bandwidth require-
ments, output performance, and so on. The study [11] gives a
$H_\infty$ controller that enhances the close-loop robustness of the
system and improves the steering comfort, but its limitation
is that the order of the $H_\infty$ controller is too high to realize in
practical application. In recent years, Apkarian et al. pro-
posed a new structured $H_\infty$ comprehensive control method
[12, 13]. Compared with the traditional $H_\infty$ control method,
the advantage of structured $H_\infty$ control is that the structure
or order of the controller can be set in advance. In other
words, the controller meets the performance requirements
and simultaneously has a relatively simple structure.

Aiming at the stability and comprehensive performance
of the EPS system, this paper adopts a structured $H_\infty$ control method and gives the controller design and parameter
optimization results under a given torque diagram. Taking a
cylindrical EPS system as an example, we analyze the system
performance under two sources of instability with large gain
at low speed and nonlinearity caused by torque diagram and
design a structured $H_\infty$ controller according to the system
performance requirements. First, considering that the high
order of the traditional $H_\infty$ controller is not conducive to the
actual production, we determined the order and structure of the controller and designed the controller structure of 2nd order to 4th order. Then, we selected ap-
propriate weight functions according to the performance
requirements of the system. Finally, we obtained the optimal
parameters that met the system performance requirements
through simulation calculations and verified the theoretical
design through simulation analysis.

2. EPS System Model

According to the different positions of power supply, the
EPS system can be divided into three types: steering column
type, pinion type, and rack type. In this paper, we will take
the column EPS system (C-EPS) as an example. It is mainly
composed of four parts: steering wheel, column, motor, and
rack. The steering wheel and steering column are connected
by a torque sensor including an elastic torsion bar, and the
motor and rack are respectively connected to the steering
column by a reduction mechanism (in this case, it is a worm
reduction mechanism) and a pinion. The dynamic model is
shown in Figure 1, and the meaning of each variable and
parameter throughout this paper is shown in the figures and
is defined in Table 1.

The equations of motion of each part of the system are
listed as in the following equations:

\[
\begin{align*}
J_1 \ddot{\theta}_1 + C_1 \dot{\theta}_1 + K (\theta_1 - \theta_2) &= \tau_h, \\
J_2 \ddot{\theta}_2 + C_2 \dot{\theta}_2 + K (\theta_2 - \theta_1) &= \tau_{gear} - \tau_{pinion}, \\
J_m \ddot{\theta}_m + C_m \dot{\theta}_m &= \frac{\tau_m - \tau_{gear}}{N}, \\
M_r \ddot{x}_r + C_r \dot{x}_r &= \frac{\tau_{pinion}}{r_p} - F_{load}.
\end{align*}
\]

The gear ratio of rack, pinion, and worm gear are shown in the following equation:

\[
\begin{align*}
r_p \dot{\theta}_2 &= x_r, \\
N \dot{\theta}_m &= \theta_m.
\end{align*}
\]

Equations (2)–(4) can be simplified to a lumped mass
equation as shown in the following equation:

\[
J_2 \ddot{\theta}_2 + C_2 \dot{\theta}_2 + K (\theta_2 - \theta_1) = \tau_a - \tau_p.
\]

where equivalent moment of inertia $J_2$, equivalent damping
coefficient $C_2$, equivalent boost torque $\tau_a$, and equivalent
load torque $\tau_l$ are determined in the following equation:

\[
\begin{align*}
J_2 &= J_c + N^2 J_m + r_p M_r, \\
C_2 &= C_c + N^2 C_m + r_p C_r, \\
\tau_a &= N \tau_m, \\
\tau_l &= r_p F_{load}.
\end{align*}
\]

The system block diagram of the EPS system is shown in
Figure 2, which describes the relationship between the
system’s external input (steering wheel torque and equiv-
alent load torque) and the system state variables (steering
wheel angle and steering column angle).

As shown in Figure 2, $h$ is the EPS controller consisting
of a torque map and a compensation controller. $\tau_s$ is the
measure torque on the torque sensor, and $\tau_{a, ref}$ is the ref-
ence value of the boost torque calculated by the controller,
listed as in the following equations:

\[
\begin{align*}
\tau_s &= K (\theta_1 - \theta_2), \\
\tau_{a, ref} &= h \cdot \tau_s.
\end{align*}
\]

The transfer function from steering wheel torque to
output angle $\theta_1$ is as follows:

\[
G_w(s) = \frac{1}{J_1 s^2 + C_1 s}.
\]

The transfer function from the steering column moment
to output angle $\theta_2$ is as follows:

\[
G_c(s) = \frac{1}{J_2 s^2 + C_2 s}.
\]
The mathematical model of the engine in the system can be expressed as low-pass filter with a cutoff frequency of $\omega_m$ as follows:

$$G_m(s) = \frac{\tau_a}{\tau_{a,\text{ref}} + s} = \frac{\omega_m}{s + \omega_m}. \quad (12)$$

The parameters adopted for the EPS system in literature [7] are shown in Table 2:

### Table 2: Value of the parameters of EPS.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>143.24</td>
</tr>
<tr>
<td>$J_1$ ($\text{kg} \cdot \text{m}^2$)</td>
<td>0.044</td>
</tr>
<tr>
<td>$C_1$ ($\text{Nm} \cdot \text{s} / \text{rad}$)</td>
<td>0.25</td>
</tr>
<tr>
<td>$J_2$ ($\text{kg} \cdot \text{m}^2$)</td>
<td>0.11</td>
</tr>
<tr>
<td>$C_2$ ($\text{Nm} \cdot \text{s} / \text{rad}$)</td>
<td>1.35</td>
</tr>
<tr>
<td>$\omega_m$ (rad/s)</td>
<td>200\pi</td>
</tr>
</tbody>
</table>

### 3. Structured $H_\infty$ Controller Design

In this case, the control design of the EPS system involves multiple control objectives, so the design of the structured $H_\infty$ is adopted. It not only retains the synthesis of traditional $H_\infty$ design but also can be weighted for each performance requirement to form a diagonal matrix with multidimensional performance output for performance synthesis optimization [14, 15].

A complete structured $H_\infty$ control design can be generally divided into three steps: first, the performance requirements of the system should be analyzed according to the control objectives; then, the appropriate weighting functions should be selected according to the specific performance requirements and optimization objectives, and multiple weighting performance requirements should be formed into a diagonal optimization matrix; finally, a structured control with adjustable parameters should be determined in the light of the actual needs and design objectives [16–19]. The structured $H_\infty$ controller satisfying the comprehensive performance requirements is obtained by solving the optimal controller parameters.

#### 3.1. Controller Structure.

The control structure of the system is shown in Figure 3. In the EPS system, the driver inputs the steering angle signal $\theta_1$ from the steering wheel, and the control center gets the steering column measurement torque $\tau_s$ from the torque sensor, and inputs it into the controller to obtain the desired torque boosting torque $\tau_{a,\text{ref}}$. Where the value of $\tau_{a,\text{ref}}^*$ before passing through the stability controller is given by the following equation:

$$\tau_{a,\text{ref}}^* = \left\{ \begin{array}{ll} 0, & 0 \leq \tau_s \leq \tau_{a,0}, \\ K_v \cdot \tau_{s,\text{car}} (\tau_s - \tau_{s,\text{ref}}), & \tau_{a,0} < \tau_s. \end{array} \right. \quad (13)$$

The torque map has a dead zone below $\tau_{a,0}$ to prevent the system from being too sensitive to the driver’s small-angle steering, especially at high speeds. It is the dead zone that causes the nonlinearity of the system. In the parking state, the driver needs a larger assist torque, while at high speed, it needs less assist torque, so $K_v$ decreases as the vehicle speed increases. Compared to the torque map in Figure 3, the torque map applied to a conventional vehicle is a smoother curve, but to simplify the analysis, we use a proportional compensator due to the instability of the system caused by the high gain and nonlinearity of the torque map at low
speeds. We use a structured $H_{\infty}$ controller as a stability controller behind the torque map, which provides the controller with dynamic characteristics while ensuring stability and robustness and suppressing the vibration of the entire EPS system. The structured $H_{\infty}$ compensator is shown in the following equation:

$$C = \prod_{i=1}^{n} C_i(s) = \prod_{i=1}^{n} \frac{s|b_i| + 1}{s|a_i| + 1}, \quad n = 1, 2, \ldots, L,$$

where $a_i, b_i$ are parameters to be optimized and $n$ in subscript indicates the order of the controller.

3.2. Stability Analysis and Weight Function Choice. By the analysis, the performance requirements of the control design are as follows: stability margin, robust stability, and system bandwidth.

3.2.1. Stability Margin. From the system models (1)–(12), the phase margin is only $\gamma = 8.54^\circ$, and the stability margin of the system is too small. In order to improve driving comfort, the first performance requirement of the controller design is stability margin.

$T_1$ is the transfer function from $r$ to $e$ shown in Figure 4 where $r$ is the disturbance signal and $e$ is an error signal. The stability of the system is the distance of the transfer function from the critical stability point. It is also the upper limit of the gain toward the sensitivity function [19, 20]. It required the following:

$$\|W_1(s)T_1(s)\|_{\infty} \leq \gamma_1.$$

In formula (15), $\gamma_1$ is the norm index and $W_1(s)$ is the weighting function. The upper limit of the stability margin is given as 0.8, $W_1(s) = 0.8$.

3.2.2. Bandwidth Requirement. Except for the stability, it is also necessary to consider the appropriate bandwidth of the system. $T_3$ represents the transfer function from $r$ to $\tau_r$. The system bandwidth requirement is as follows:

$$\|W_2(s)T_2(s)\|_{\infty} \leq \gamma_2.$$

To limit the bandwidth of the system, the weighted function $W_2(s)$ is selected as the following high-pass filtering form:

$$W_2(s) = \frac{2s}{s + 1200}.$$

3.2.3. Robust Stability. Considering the uncertainty of the power moment ratio $K_v$ of EPS system due to different external conditions and the nonlinearity of the dead zone, $\tau_a^*$ is expressed as follows:

$$\tau_a^* = \tau \left(\frac{K_v}{2} + K_v \delta\right), \quad |\delta| \leq 1,$$

where $\delta$ is the perturbation parameter.

The control structure diagram of the system according to equation (18) is shown in Figure 4.

According to the principle of minimum gain, the robust stability of the system needs to satisfy the condition of the following equation:

$$\|T_{\infty}\|_{\infty} < 1.$$

Therefore, the second performance requirement of the system is robust stability. $T_3$ is the transfer function from $\omega$ to $z$.

The requirement of the system for robust stability is as follows:

$$\|W_3(s)T_3(s)\|_{\infty} \leq \gamma_3,$$

where the weighting function is $W_3(s) = 1$.

For the $H_{\infty}$ control optimization problem where the order has been fixed and has selected the weighting function, the control performance requirements of the EPS system are comprehensively considered and the minimum value satisfying (21) is obtained by optimizing the adjustable parameters $a_i$ and $b_i$:

$$\|H\|_{\infty} \leq \min\{\gamma_1\},$$

$$H = \text{diag} \left( W_1 T_1, W_2 T_2, W_3 T_3 \right).$$

At this time, the adjustable parameters obtained are the optimal parameters of the system controller.

When the optimal parameters in the structured $H_{\infty}$ controller are obtained, $T_1, T_2$, and $T_3$ in (21) are expressed as the following linear fraction form $P_1, P_2$, and $P_3$ in (22), where the parameters in the structured controller ($C$) are extracted for optimization by the method of linear fractional transformation (LFT) [21–24]:

$$\begin{align*}
T_1 &= F_1(P_1, C), \\
T_2 &= F_1(P_2, C), \\
T_3 &= F_1(P_3, C).
\end{align*}$$

3.3. Controller Design Results. In the parking state which means $h = K_v$, the phase margin without compensator is only $\gamma = 8.54^\circ$, which is shown in Figure 5. We decided to use at
least a second-order compensator as a controller in order to ensure the stability margin (phase margin) is more than 45°.

According to the system performance requirements and stability analysis, we designed the structured compensator of different orders as the controller in turn. As shown in (14), when \( n = 2 \), the controller is a second-order compensator (controller 1); when \( n = 3 \), the controller is a third-order compensator (controller 2); when \( n = 4 \), the controller is a fourth-order compensator (controller 3). The parameters used in structured \( H_{\infty} \) controllers and index norms of each controller are shown as in Table 3.

Under normal circumstances, we assume that the system can achieve good performance when the \( \gamma \) value is less than 1. As the controller order increases, the \( \gamma \) value we get will become smaller and smaller. The value of \( \gamma \) of the second-order controller still exceeds 1, and the value of \( \gamma \) is already less than 1 when the order of controller is increased to the third order. Although the \( \gamma \) value is still decreasing when the order increased to the fourth order, the degree of reduction is not obvious. Therefore, we think it is not necessary to increase the controller order, so we have only designed controllers with 2–4 orders.

### 4. Simulation Analysis

The simulation environment is built using Simulink. It consists of steering machinery, controller, motor, and road disturbances. In the parking state, the road disturbance is shown in the following equation:

\[
\tau_{t,k} = K_{park}(\theta_{t,k} - \theta_{l,k}),
\]

\[
\theta_{t,k} = \begin{cases} 
\theta_{2,k} - \theta_{\text{max}}, & \theta_{2,k} - \theta_{l,k-1} > \theta_{\text{max}}, \\
\theta_{2,k} + \theta_{\text{max}}, & \theta_{2,k} - \theta_{l,k-1} > -\theta_{\text{max}}, \\
\theta_{l,k-1}, & \text{otherwise}.
\end{cases}
\]

In the driving state, the road disturbance is proportional to the steering angle as in the following equation:

\[
\tau_{t,k} = K_{\text{drive}}\theta_{2,k},
\]

In the following, we will apply the compensation controllers of different orders (2nd, 3rd, 4th order) obtained previously to simulate the EPS system. We will compare and analyze the performance of the system in terms of stability margin, bandwidth, and robust stability under the action of three controllers, similar to designing a controller.

#### 4.1. Simulation Analysis of Stability Margin

Under the action of different structured \( H_{\infty} \) controllers, the open-loop Nichols diagram of the system is shown in Figure 6. Under the action of controllers with different structures, the stability margin of the system has been greatly improved. With the increase of the order of controller, we find that the stability margin of the sensitivity function can meet the performance requirements of the system, but the difference between different orders is not obvious.

#### 4.2. Simulation Analysis of Bandwidth

Figure 7 shows the Bode diagram under the action of the controller (1, 2, 3), where the full line is the Bode graph of \( T_2 \), and the dotted line corresponds to the weighted function \( W_2 \). The magnitude of \( T_2 \) decreases rapidly before reaching the turning frequency of \( 1/W_2 \). Under the restriction of the weighting function \( W_2 \), the overall amplitude of the system is all below the amplitude of \( 1/W_2 \), and all of the controllers meet the bandwidth limitation of the control objective.

The corresponding stability margins are shown in Table 4. For the improvement of the phase margin, the effects of controllers 2 and 3 are almost similar, and the phase angle margin of the system is greatly increased compared to the controller 1. There is no obvious difference between the three controllers for increasing the amplitude margin. As the structured controller order and phase angle margin increase, we can see that the sheer frequency of the system is continuously decreasing.

![Nichols chart of EPS without compensator](image)

**Figure 5:** Nichols chart of EPS without compensator.

### Table 3: Controllers design parameter.

<table>
<thead>
<tr>
<th>Controller</th>
<th>( a_1 )</th>
<th>( b_1 )</th>
<th>( a_2 )</th>
<th>( b_2 )</th>
<th>( a_3 )</th>
<th>( b_3 )</th>
<th>( a_4 )</th>
<th>( b_4 )</th>
<th>( c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>537</td>
<td>8</td>
<td>2</td>
<td>40.2</td>
<td>—</td>
<td>2</td>
<td>—</td>
<td>—</td>
<td>1.4340</td>
</tr>
<tr>
<td>2</td>
<td>1039</td>
<td>5.39</td>
<td>403</td>
<td>127</td>
<td>18.3</td>
<td>135</td>
<td>513</td>
<td>14.6</td>
<td>1.0768</td>
</tr>
<tr>
<td>3</td>
<td>1019</td>
<td>2.22</td>
<td>0.5</td>
<td>146</td>
<td>168</td>
<td>513</td>
<td>9621</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The symbol “—” indicates that the value is the default value.
4.3. Simulation Analysis of Robust Stability. In order to verify the robustness of the designed controllers, the perturbation parameter is given to $\delta = 0.86$, $K_v = 30$, and a sine function with an amplitude of $\pi$ rad and a frequency of $\pi$ rad/s is given at the input command of steering angle for simulation. Figure 8 shows the comparison between the sinusoidal input and output measured values $\tau_s$. The dotted line is the sinusoidal input, which is the angle of the steering wheel, and the solid line is the output torque of the steering column. Due to the nonlinearity of the torque graph, we can see that the output steering torque has a significant chattering phenomenon at the dead zone characteristics.

Obviously, all control designs have good robustness and can effectively suppress the oscillations generated by the system. The oscillation amplitude of the system is limited within the $0.7 \text{ N} \cdot \text{m}$ under controller 1 but $0.2 \text{ N} \cdot \text{m}$ under controller 3. By comparison, we can conclude that controller 1 is relatively weak in suppressing chattering, while controllers 2 and 3 perform well in eliminating chattering. And as the controller’s order increases, its ability to suppress chatter and its robust stability also become more outstanding.

Based on the analysis of the simulation results of the stability margin, bandwidth, and robust stability, the three controllers designed with different orders (2nd–4th order) can satisfy the performance requirements. It is undeniable that with the increase of the controller order, we can conclude that the phase margin, the bandwidth, and the robust stability of

![Nichols chart](image)

**Figure 6:** Nichols plots of EPS with (a) controller 1, (b) controller 2, and (c) controller 3.
the system have improved significantly. Controller 2 (third-order) has performed very well in all aspects, and controller 3 (fourth-order) is even better in terms of robust stability. Moreover, controllers 2 and 3 are achievable in practical production applications and have engineering application value while meeting the system performance requirements.
5. Conclusion

The low stability margin of the EPS system and the perturbation of parameters in the torque map will cause control problems such as robustness and bandwidth requirements. Based on the $H_\infty$ control method, this paper selects an appropriate weighting function to limit the stability margin and bandwidth of the system by analyzing the system performance requirements. We present a structured $H_\infty$ controller with a higher stability margin, good robustness, and lower order. The simulation results show that the controller designed in this paper has good robustness, can reach the required stability margin, and can suppress the system oscillation within the range of 0.5 N·m. The design ideas adopted in this paper and the selection of weighting functions can provide some reference for a wide range of control systems.

Data Availability

The data used to support the findings of this study are included within the article. Other data or programs used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Nos. 61790562, 61861007 and 61640014), the Guizhou Provincial Science and Technology Foundation (QianHe [2020]1Y273 and [2020]1Y266),
Industrial Project of Guizhou Province (QianxueweiHe ZDXK[2015][8]), Postgraduate Case Library (KCALK201708), and Important Subject of Guizhou Province (QianxueweiHe ZDXK[2015][8]).

References


