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*Research Article*

# Stochastic Stability Criteria for Neutral Distributed Parameter Systems with Markovian Jump

**Yanbo Li**,[1] **Chao-Yang Chen**,[2,3] **and Chengqun Li**[1]

[1]*School of Information and Statistics, Guangxi University of Finance and Economics, Nanning 530001, China*
[2]*School of Information and Electrical Engineering, Hunan University of Science and Technology, Xiangtan 411201, China*
[3]*Center for Polymer Studies and Department of Physics, Boston University, Boston, MA 02215, USA*

Correspondence should be addressed to Chao-Yang Chen; ouzk@163.com

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This paper deals with the problem of stochastic stability for a class of neutral distributed parameter systems with Markovian jump. In this model, we only need to know the absolute maximum of the state transition probability on the principal diagonal line; other transition rates can be completely unknown. Based on calculating the weak infinitesimal generator and combining Poincare inequality and Green formula, a stochastic stability criterion is given in terms of a set of linear matrix inequalities (LMIs) by the Schur complement lemma. Because of the existence of the neutral term, we need to construct Lyapunov functionals showing more complexity to handle the cross terms involving the Laplace operator. Finally, a numerical example is provided to support the validity of the mathematical results.

## 1. Introduction

Time-delay models are popular in all kinds of fields such as demography, biology, economics, and chemistry. Neutral systems as a special type of time-delay systems are often encountered because these systems have a wider application value than the general time-delay systems in many dynamical systems such as bioengineering systems, dynamic systems of offshore platform, and dynamic economic models. Hence, there are so many investigations about time-delay systems [1–7]. As we all know, the systems inevitably receive the impact of sudden changes in the environment, abrupt failure of components, unexpected changes in system parameters, and so on. These random diversifications usually follow the law of Markov jump. These systems are called Markovian jump systems. Markovian jump systems spur investigators' consuming interest [8, 9].

The stability and performance of stochastic systems are quite different from those of deterministic systems [10–12]. More recently, neutral-type Markovian jump systems have aspirated considerable attention. All kinds of analysis methods have been used to discover the stochastic stability criteria of neutral-type Markovian jump systems such as Lyapunov–Krasovskii functional approach, reciprocally convex combination inequality method, and stochastic analysis theory in [13–16]. For less conservative results, the delay-dependent stability has been discussed in [17–20]. Other control methods have also been extensively and thoroughly studied such as robust delay-dependent $H_\infty$ control [21, 22], nonfragile control [23–25], sliding mode control [26, 27], $H_\infty$ sliding mode control [28], and the references therein.

The transition rates in many references mentioned above have been supposed to be completely known. But it is very hard to acquire the accurate transfer probability, and even if the exact transfer probability can be obtained, the cost is also very huge. So the study of Markovian jump systems with general unknown transition rates [29–50] has appealed to a great many scholars. Stability, stabilization, and robust control of Markovian jump systems with partially unknown transition have been reported in [29–33]. Stability analysis for neutral Markovian jump systems with partially unknown transition probabilities has been proposed in [34, 35]. Kao et al. and Yang et al. [36, 37] have settled the delay-dependent stability for Markovian jump systems with partially unknown transition probabilities and Markovian jump neutral stochastic systems with general unknown transition rates, respectively. Singular Markovian jump systems with general incomplete transition probabilities have been presented in [38–40]. Finite-time stochastic stability and control of Markovian jump systems with general incomplete transition probabilities have been discussed in [41–44]. Stabilization of discrete-time Markovian jump systems with partially unknown transition probabilities was probed in [45].

In parallel, many researchers have extensively studied the time-delay distributed parameter systems [46–54]. Three main approaches of time-delay distributed parameter systems are semigroup theory [47], matrix norm theory [48], and linear matrix inequality theory (LMI) [49]. The semigroup method cannot guarantee the system to be of a fine dynamic character and performance index undergoing two transformations. It is not easy to apply the results to practical problems by matrix norm. The problem of exponential stability and stabilization [50], sliding mode control [51], and feedback control [52] has been proposed in terms of the linear matrix inequality approach. However, less attention has been paid to the distributed parameter systems with Markovian jump, especially neutral distributed parameter systems with Markovian jump which requires a lot of research to be performed.

Based on previous discussions, we are considering the problem of stochastic stability of a class of Markovian jumping neutral distributed parameter systems in this paper. The linear matrix inequality approach together with the Lyapunov functional method is employed to develop stochastic stability criteria for the described systems. The results are given in a group of linear matrix inequalities (LMIs).

## 2. Problem Formulation and Preliminaries

Consider the neutral distributed parameter systems with Markovian jump of the following form:

$$\begin{aligned}\frac{\partial}{\partial t}[W(x,t)-C(r(t))W(x,t-\sigma)] &= D(r(t))\Delta W(x,t)\\ &\quad + A(r(t))W(x,t)\\ &\quad + A_1(r(t))W(x,t-\tau),\end{aligned} \tag{1}$$

where $(x,t)\in\Omega\times R_+$, $\Omega=\{x,\|x\|<l<+\infty\}\subset R^m$ is the bounded domain with smooth boundary $\partial\Omega$, and $\operatorname{mes}\Omega>0$. Also,

$$\nabla W(x,t)=\operatorname{col}(\nabla w_1(x,t),\nabla w_2(x,t),\ldots,\nabla w_n(x,t)), \tag{2}$$

where $\nabla=((\partial/\partial x_1),(\partial/\partial x_2),\ldots,(\partial/\partial x_m))$ is the gradient operator.
$W(x,t)=\operatorname{col}(w_1(x,t),w_2(x,t),\ldots,w_n(x,t))\in R^n$ is the state function, and $\Delta=\sum_{k=1}^{m}\partial^2/\partial x_k^2$ is the Laplace operator on $\Omega$.

The initial value and boundary value conditions satisfy

$$\begin{aligned} &W(x,t)=0,\ (x,t)\in\partial\Omega\times[-\gamma,+\infty),\\ &W(x,t)=\psi(x,t),\ (x,t)\in\Omega\times[-\gamma,0],\\ &\frac{\partial W(x,t)}{\partial n}=0,\ (x,t)\in\partial\Omega\times[-\gamma,+\infty),\end{aligned} \tag{3}$$

where $\gamma=\max\{\sigma,\tau\}$ and $\sigma>0$ and $\tau>0$ are constants. $n$ is the unit outward normal vector of $\partial\Omega$, and $\psi(x,t)$ is the smooth function. $\tau>0$, $\sigma>0$, and $D(r(t))>0$ are constants; $A(r(t))$, $C(r(t))$, and $A_1(r(t))$ are constant matrices.

Let $\{r(t),t\ge 0\}$ be a right-continuous Markov process and take values in a finite set $F=\{1,2,\ldots,N\}$ with transition probability matrix $\Pi=(\pi_{ij})$; the mode transition probabilities are defined as follows:

$$P_r(r(t+h)=j\mid r(h)=i)=\begin{cases}\pi_{ij}h+o(h), & i\neq j,\\ 1+\pi_{ii}h+o(h), & i=j,\end{cases} \tag{4}$$

where $h>0$ and $\lim_{\delta\longrightarrow 0}(o(h)/h)=0$. $\pi_{ij}\ge 0, i\neq j$ donates the transition rate from mode $i$ to mode $j$ in the time interval $h$ and $\pi_{ii}=-\sum_{j=1,j\neq i}^{N}\pi_{ij}$. For each $r(t)=i\in F$, let $A(r(t))=A_i$, $A_1(r(t))=A_{1i}$, $D(r(t))=D_i$, and $C(r(t))=C_i$. Then, we can represent system (1) in the following form:

$$\begin{aligned}\frac{\partial}{\partial t}[W(x,t)-C_iW(x,t-\sigma)] &= D_i\Delta W(x,t)+A_iW(x,t)\\ &\quad + A_{1i}W(x,t-\tau).\end{aligned} \tag{5}$$

**Lemma 1** (see [53] (Green formula)). *Let $\Omega\subset R^n$ be the bounded domain with smooth boundary $\partial\Omega$, $n$ is the unit outward normal vector of $\partial\Omega$, and $G\subset\Omega$ is the smooth subdomain. If $u,v\in C^2(\overline{G})$, then*

$$\int_G u\Delta v\,dx=\int_{\partial\Omega}u\frac{\partial v}{\partial n}ds-\int_\Omega\nabla u\nabla v\,dx, \tag{6}$$

*where $\nabla$ is the Hamilton operator and ds is the area element over the boundary region.*

**Lemma 2** (see [54] (Friedrichs's inequality)). *Let $w\in C_0^1(\Omega)$ and $\Omega$ be included in the closed region $\Omega_1$: $0\le x_i\le l\,(i=1,2,\ldots,n)$. Then,*

$$\int_\Omega w^2(x)dx\le\int_\Omega\sum_{i=1}^{n}\left(\frac{\partial w}{\partial x}\right)^2dx=c\int_\Omega|\nabla w|^2dx, \tag{7}$$

*where $c=l^2/n$.*

**Lemma 3** (see [55]). *Let $V_1, V_2$, and $V_3$ be the real matrices and $V_3=V_3^T>0$; then, for an arbitrary given scalar $\alpha>0$, the following inequality holds:*

$$V_2^TV_1+V_1^TV_2\le\alpha^{-1}V_1^TV_3^{-1}V_1+\alpha V_2^TV_3V_2. \tag{8}$$

## 3. Main Results

**Theorem 1.** *Given matrices $A_i, A_{1i}$, and $C_i$, time-delay constants $\tau > 0$ and $\sigma > 0$ and constant $D_i > 0$, the neutral distributed parameter systems with Markovian jump (5) is stochastically stable. If there exist positive symmetric matrices $P_i$, $Q_i$, $M$, $N$, $R$, and $Z$ and positive scalars $\alpha_i$, such that for any Markovian jump mode $i \in F$, the following linear matrix inequalities (LMIs) hold:*

$$\begin{pmatrix} (\alpha_i - 2D_i)I & D_iC_i \\ D_iC_i^{\mathrm{T}} & -\alpha_i I \end{pmatrix} < 0,$$

$$\Theta = \begin{pmatrix} \Pi_1 & 0 & -A_i^{\mathrm{T}}C_i & 0 & 0 & 0 \\ 0 & -P_i & -A_{1i}^{\mathrm{T}}C_i & 0 & 0 & 0 \\ * & * & -Q_i & 0 & 0 & 0 \\ 0 & 0 & 0 & \Pi_2 & A_{1i}^{\mathrm{T}} & D_iC_i - C_i^{\mathrm{T}}A_i \\ 0 & 0 & 0 & * & -M & -C_i^{\mathrm{T}}A_{1i} \\ 0 & 0 & 0 & * & * & -N \end{pmatrix} < 0, \tag{9}$$

*where* $\beta = \max\{|\pi_{ii}|, i \in F\}$, $\Pi_1 = A_i^{\mathrm{T}} + A_i + P_i + Q_i + \beta\tau R + \beta\sigma Z$, $\Pi_2 = M + N + A_i + A_i^{\mathrm{T}} - 2D_iI$,

$$P_i < R, \tag{10}$$

$$Q_i < Z. \tag{11}$$

*Proof.* For system (5), we construct the following stochastic Lyapunov functional:

$$V_i(t, W(x,t)) = \sum_{n=1}^{9} V_{\mathrm{in}}, \tag{12}$$

where

$$\begin{aligned}
V_{1i} &= \int_\Omega Y^{\mathrm{T}}(x,t)Y(x,t)\mathrm{d}x, \quad \text{where } Y(x,t) = W(x,t) - C_iW(x,t-\sigma), \\
V_{i2} &= \int_\Omega \int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)P_iW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x, \\
V_{i3} &= \int_\Omega \int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)Q_iW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x, \\
V_{i4} &= \int_\Omega \int_{t-\tau}^{t} \left(\nabla W^{\mathrm{T}}(x,\theta)\right)M\left(\nabla W^{\mathrm{T}}(x,\theta)\right)^{\mathrm{T}}\mathrm{d}\theta\,\mathrm{d}x, \\
V_{i5} &= \int_\Omega \int_{t-\sigma}^{t} \left(\nabla W^{\mathrm{T}}(x,\theta)\right)N\left(\nabla W^{\mathrm{T}}(x,\theta)\right)^{\mathrm{T}}\mathrm{d}\theta\,\mathrm{d}x, \\
V_{i6} &= \int_\Omega \left(\nabla Y^{\mathrm{T}}(x,\theta)\right)\left(\nabla Y^{\mathrm{T}}(x,\theta)\right)^{\mathrm{T}}\mathrm{d}x, \\
V_{i7} &= \alpha_i \int_\Omega \int_{t-\sigma}^{t} (\Delta W(x,\theta))^{\mathrm{T}}\Delta W(x,\theta)\mathrm{d}\theta\,\mathrm{d}x, \\
V_{i8} &= \beta_i \int_\Omega \int_{-\tau}^{0} \int_{t+s}^{t} W^{\mathrm{T}}(x,\theta)RW(x,\theta)\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}x, \\
V_{i9} &= \beta_i \int_\Omega \int_{-\sigma}^{0} \int_{t+s}^{t} W^{\mathrm{T}}(x,\theta)ZW(x,\theta)\mathrm{d}\theta\,\mathrm{d}s\,\mathrm{d}x.
\end{aligned} \tag{13}$$

Let $L$ be the weak infinitesimal generator; then, we calculate

$$LV_i(t, W(x,t)) = \sum_{n=1}^{9} LV_{\mathrm{in}}, \tag{14}$$

where

$$\begin{aligned}
LV_{1i} &= 2\int_\Omega Y^{\mathrm{T}}(x,t)\frac{\partial Y(x,t)}{\partial t}\mathrm{d}x \\
&= 2D_i\int_\Omega W^{\mathrm{T}}(x,t)\Delta W(x,t)\mathrm{d}x \\
&\quad + \int_\Omega W^{\mathrm{T}}(x,t)\left(A_i^{\mathrm{T}} + A_i\right)W(x,t)\mathrm{d}x \\
&\quad + 2\int_\Omega W^{\mathrm{T}}(x,t)A_{1i}W(x,t-\tau)\mathrm{d}x \\
&\quad - 2D_i\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x \\
&\quad - 2\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}A_iW(x,t)\mathrm{d}x \\
&\quad - 2\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}A_{1i}W(x,t-\tau)\mathrm{d}x.
\end{aligned} \tag{15}$$

Applying Lemma 1 and Lemma 2,

$$\begin{aligned}
2D_i\int_\Omega W^{\mathrm{T}}(x,t)\Delta W(x,t)\mathrm{d}x &= \sum_{k=1}^{n}\sum_{l=1}^{n}\left[\int_{\partial\Omega} w_k(x,t)\frac{\partial w_l(x,t)}{\partial n}\mathrm{d}s - \int_\Omega \nabla w_k(x,t)\nabla w_l(x,t)\mathrm{d}x\right] \\
&= -\sum_{k=1}^{n}\sum_{l=1}^{n}\int_\Omega \nabla w_k(x,t)\nabla w_l(x,t)\mathrm{d}x \\
&= -2D_i\int_\Omega \nabla W^{\mathrm{T}}(x,t)\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x,
\end{aligned} \tag{16}$$

$$\begin{aligned}
&-2D_i\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x \\
&= -2D_i\int_\Omega \sum_{l=1}^{n}\sum_{m=1}^{n} w_m(x,t-\sigma)c_{ml}\nabla\cdot(\nabla w_l(x,t))\mathrm{d}x \\
&= 2D_i\sum_{l=1}^{n}\sum_{m=1}^{n}\int_\Omega \nabla w_m(x,t-\sigma)c_{ml}\cdot\nabla w_l(x,t)\mathrm{d}x \\
&= 2D_i\sum_{l=1}^{n}\sum_{m=1}^{n}\sum_{k=1}^{m}\int_\Omega \frac{\partial w_m(x,t-\sigma)}{\partial x_k}c_{ml}\frac{\partial w_l(x,t)}{\partial x_k}\mathrm{d}x \\
&= 2D_i\int_\Omega \nabla W^{\mathrm{T}}(x,t-\sigma)C_i\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x.
\end{aligned} \tag{17}$$

Substituting (16) and (17) into (15), we have

$$
\begin{aligned}
LV_{1i} = & -2D_i\int_\Omega \nabla W^{\mathrm{T}}(x,t)\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x \\
& + \int_\Omega W^{\mathrm{T}}(x,t)\left(A_i^{\mathrm{T}}+A_i\right)W(x,t)\mathrm{d}x \\
& + 2\int_\Omega W^{\mathrm{T}}(x,t)A_{1i}W(x,t-\tau)\mathrm{d}x \\
& + 2D_i\int_\Omega \nabla W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x \\
& - 2\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}A_iW(x,t)\mathrm{d}x \\
& - 2\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}A_{1i}W(x,t-\tau)\mathrm{d}x,
\end{aligned}
\tag{18}
$$

$$
\begin{aligned}
LV_{i2}(t,W(x,t)) = & \int_\Omega W^{\mathrm{T}}(x,t)P_iW(x,t)\mathrm{d}x \\
& - \int_\Omega W^{\mathrm{T}}(x,t-\tau)P_iW(x,t-\tau)\mathrm{d}x \\
& + \int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)W \\
& \cdot (x,\theta)\mathrm{d}\theta\,\mathrm{d}x.
\end{aligned}
\tag{19}
$$

Noticing $\pi_{ij}>0\,(i\neq j)$ and combining $P_j<R$ and $\beta=\max\{|\pi_{ii}|, i\in F\}$, we derive the following inequality:

$$
\begin{aligned}
& \int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)\left(\sum_{j=1}^{N}\pi_{ij}P_j\right)W(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
& \leq \int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)\left(\sum_{j=1,i\neq j}^{N}\pi_{ij}P_j\right)W(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
& = -\pi_{ii}\int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)P_jW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
& \leq \beta\int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)P_jW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
& \leq \beta\int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)RW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x.
\end{aligned}
\tag{20}
$$

Then,

$$
\begin{aligned}
LV_{i2}(t,W(x,t)) \leq & \int_\Omega W^{\mathrm{T}}(x,t)P_iW(x,t)\mathrm{d}x \\
& - \int_\Omega W^{\mathrm{T}}(x,t-\tau)P_iW(x,t-\tau)\mathrm{d}x \\
& + \beta\int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)RW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x.
\end{aligned}
\tag{21}
$$

For the same reason, we can also obtain

$$
\begin{aligned}
& \int_\Omega\int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)\left(\sum_{j=1}^{N}\pi_{ij}Q_j\right)W(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
& \leq \beta\int_\Omega\int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)Q_jW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
& \leq \beta\int_\Omega\int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)ZW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x.
\end{aligned}
\tag{22}
$$

So

$$
\begin{aligned}
LV_{i3}(t,W(x,t)) = & \int_\Omega W^{\mathrm{T}}(x,t)Q_iW(x,t)\mathrm{d}x \\
& - \int_\Omega W^{\mathrm{T}}(x,t-\sigma)Q_iW(x,t-\sigma)\mathrm{d}x \\
& + \int_\Omega\int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)\left(\sum_{j=1}^{N}\pi_{ij}Q_j\right) \\
& \cdot W(x,\theta)\mathrm{d}\theta\,\mathrm{d}x \\
\leq & \int_\Omega W^{\mathrm{T}}(x,t)Q_iW(x,t)\mathrm{d}x \\
& - \int_\Omega W^{\mathrm{T}}(x,t-\sigma)Q_iW(x,t-\sigma)\mathrm{d}x \\
& + \beta\int_\Omega\int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)ZW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x,
\end{aligned}
\tag{23}
$$

$$
\begin{aligned}
LV_{i4} = & \int_\Omega \nabla W^{\mathrm{T}}(x,t)M\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x \\
& - \int_\Omega \nabla W^{\mathrm{T}}(x,t-\tau)M\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)^{\mathrm{T}}\mathrm{d}x,
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
LV_{i5} = & \int_\Omega \nabla W^{\mathrm{T}}(x,t)N\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x \\
& - \int_\Omega \nabla W^{\mathrm{T}}(x,t-\sigma)N\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x,
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
LV_{i6} = & 2\int_\Omega\left(\nabla\frac{\partial Y^{\mathrm{T}}(x,t)}{\partial t}\right)^{\mathrm{T}}\left(\nabla Y^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x \\
= & -2\int_\Omega \frac{\partial Y^{\mathrm{T}}(x,t)}{\partial t}\left(\Delta Y^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x \\
= & -2D_i\int_\Omega (\Delta W(x,t))^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x \\
& + 2D_i\int_\Omega (\Delta W(x,t))^{\mathrm{T}}C_i\Delta W(x,t-\sigma)\mathrm{d}x \\
& - 2\int_\Omega W^{\mathrm{T}}(x,t)A_i^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x \\
& - 2\int_\Omega W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x \\
& + 2\int_\Omega W^{\mathrm{T}}(x,t)A_i^{\mathrm{T}}C_i\Delta W(x,t-\sigma)\mathrm{d}x \\
& + 2\int_\Omega W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}C_i\Delta W(x,t-\sigma)\mathrm{d}x.
\end{aligned}
\tag{26}
$$

Through applying Lemma 3, the following inequality holds:

$$\begin{aligned}&2D_i\int_\Omega (\Delta W(x,t))^{\mathrm{T}} C_i \Delta W(x,t-\sigma)\mathrm{d}x\\&\leq \alpha_i^{-1}\int_\Omega (\Delta W(x,t))^{\mathrm{T}} D_i^2 C_i C_i^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x\\&\quad+\alpha_i\int_\Omega (\Delta W(x,t-\sigma))^{\mathrm{T}}(\Delta W(x,t-\sigma))\mathrm{d}x.\end{aligned}\tag{27}$$

Taking advantage of Lemma 1 and Lemma 2, we obtain

$$-2\int_\Omega W^{\mathrm{T}}(x,t)A_i^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x = 2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t)\right)A_i\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x,\tag{28}$$

$$2\int_\Omega W^{\mathrm{T}}(x,t)A_i^{\mathrm{T}}C_i\Delta W(x,t-\sigma)\mathrm{d}x = -2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t)\right)C_i^{\mathrm{T}}A_i\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x,\tag{29}$$

$$-2\int_\Omega W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x = 2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)A_{1i}\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x,\tag{30}$$

$$2\int_\Omega W^{\mathrm{T}}(x,t-\tau)A_{1i}^{\mathrm{T}}C_i\Delta W(x,t-\sigma)\mathrm{d}x = -2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)C_i^{\mathrm{T}}A_{1i}\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x.\tag{31}$$

Combining (26)–(31) together yields

$$\begin{aligned}LV_{i6}\leq&\int_\Omega (\Delta W(x,t))^{\mathrm{T}}\left(\alpha_i^{-1}D_i^2C_iC_i^{\mathrm{T}}-2D_iI\right)\Delta W(x,t)\mathrm{d}x\\&+\alpha_i\int_\Omega (\Delta W(x,t-\sigma))^{\mathrm{T}}(\Delta W(x,t-\sigma))\mathrm{d}x\\&+2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t)\right)A_i\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x\\&+2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)A_{1i}\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x\\&-2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t)\right)C_i^{\mathrm{T}}A_i\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x\\&-2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)C_i^{\mathrm{T}}A_{1i}\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x,\end{aligned}\tag{32}$$

$$\begin{aligned}LV_{i7}=&\alpha_i\int_\Omega (\Delta W(x,t))^{\mathrm{T}}\Delta W(x,t)\mathrm{d}x\\&-\alpha_i\int_\Omega (\Delta W(x,t-\sigma))^{\mathrm{T}}(\Delta W(x,t-\sigma))\mathrm{d}x,\end{aligned}\tag{33}$$

$$\begin{aligned}LV_{i8}=&\beta\tau\int_\Omega W^{\mathrm{T}}(x,t)RW(x,t)\mathrm{d}x\\&-\beta\int_\Omega\int_{t-\tau}^{t} W^{\mathrm{T}}(x,\theta)RW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x,\end{aligned}\tag{34}$$

$$\begin{aligned}LV_{i9}=&\beta\sigma\int_\Omega W^{\mathrm{T}}(x,t)ZW(x,t)\mathrm{d}x\\&-\beta\int_\Omega\int_{t-\sigma}^{t} W^{\mathrm{T}}(x,\theta)ZW(x,\theta)\mathrm{d}\theta\,\mathrm{d}x.\end{aligned}\tag{35}$$

Synthesizing (18), (21), (23)–(25), and (32)–(35), the following inequality holds:

$$\begin{aligned}LV(t,W(x,t))\leq&\int_\Omega (\Delta W(x,t))^{\mathrm{T}}\left(\alpha_iI+\alpha_i^{-1}D_i^2C_iC_i^{\mathrm{T}}\right.\\&\left.-2D_iI\right)\Delta W(x,t)\mathrm{d}x\\&+\int_\Omega W^{\mathrm{T}}(x,t)\Pi_1W(x,t)\mathrm{d}x\\&+\int_\Omega \nabla W^{\mathrm{T}}(x,t)\Pi_2\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x\\&-\int_\Omega W^{\mathrm{T}}(x,t-\tau)P_iW(t-\tau)\mathrm{d}x\\&-\int_\Omega W^{\mathrm{T}}(x,t-\sigma)Q_iW(t-\sigma)\mathrm{d}x\\&-\int_\Omega \nabla W^{\mathrm{T}}(x,t-\tau)M\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)^{\mathrm{T}}\mathrm{d}x\\&-\int_\Omega \nabla W^{\mathrm{T}}(x,t-\sigma)N\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x\\&+2\int_\Omega \nabla W^{\mathrm{T}}(x,t-\sigma)\left(D_iC_i^{\mathrm{T}}-A_i^{\mathrm{T}}C_i\right)\\&\cdot\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x\\&+2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)A_{1i}\left(\nabla W^{\mathrm{T}}(x,t)\right)^{\mathrm{T}}\mathrm{d}x\\&-2\int_\Omega\left(\nabla W^{\mathrm{T}}(x,t-\tau)\right)C_i^{\mathrm{T}}A_{1i}\\&\cdot\left(\nabla W^{\mathrm{T}}(x,t-\sigma)\right)^{\mathrm{T}}\mathrm{d}x\\&-2\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}A_iW(x,t)\mathrm{d}x\\&-2\int_\Omega W^{\mathrm{T}}(x,t-\sigma)C_i^{\mathrm{T}}A_{1i}W(x,t-\tau)\mathrm{d}x.\end{aligned}\tag{36}$$

Select appropriate $\alpha_i > 0$ such that $\alpha_i I + \alpha_i^{-1} D_i^2 C_i C_i^{\mathrm{T}} - 2 D_i I < 0$, then we transform it into (9) by the Schur complement lemma.

Set

$$X(t) = \begin{pmatrix} W(x,t) \\ W(x,t-\tau) \\ W(x,t-\sigma) \\ (\nabla W^{\mathrm{T}}(x,t))^{\mathrm{T}} \\ (\nabla W^{\mathrm{T}}(x,t-\tau))^{\mathrm{T}} \\ (\nabla W^{\mathrm{T}}(x,t-\sigma))^{\mathrm{T}} \end{pmatrix}. \tag{37}$$

Then,

$$LV(x, W(x,t)) \le \int_{\Omega} X^{\mathrm{T}}(t) \Theta X(t) \mathrm{d}x < 0. \tag{38}$$

The proof is concluded. □

*Remark 1.* Because of the simultaneous existence of neutral term and Markov jump, the stochastic stability of neutral distributed parameter systems with jump is given by finding the maximum value of absolute value on the main diagonal of the state transition probability matrix. Compared with the general time-delay system, the derivation process of the system is complex, but the conclusion is simple.

## 4. Examples

Consider a neutral distributed parameter system (5) with two Markovian jump modes and the parameters as follows:

$$\begin{aligned} A_1 &= \begin{pmatrix} -1 & 0 \\ 0 & -0.9 \end{pmatrix}, \\ A_2 &= \begin{pmatrix} -1 & 0 \\ 0 & -0.1 \end{pmatrix}, \\ A_{11} &= \begin{pmatrix} 1 & 0.1 \\ -0.5 & 1 \end{pmatrix}, \\ A_{12} &= \begin{pmatrix} -1 & 2 \\ 0 & -1 \end{pmatrix}, \\ C_1 &= \begin{pmatrix} -0.3 & 0.5 \\ 0 & -0.2 \end{pmatrix}, \\ C_2 &= \begin{pmatrix} -0.2 & 0.3 \\ -0.1 & -1.6 \end{pmatrix}, \\ D_1 &= D_2 = 1, \\ \tau &= 0.1, \\ \sigma &= 0.2. \end{aligned} \tag{39}$$

The transition rate matrix is defined by

$$\Pi = \begin{pmatrix} -1 & 1 \\ 0.5 & -0.5 \end{pmatrix}. \tag{40}$$

Obviously, the absolute maximum of the principle diagonal line is 1. We get feasible solutions by solving linear matrix inequalities (9)–(12) in Theorem 1 and obtain the following parameters:

$$\begin{aligned} M &= \begin{pmatrix} 2.4183 & 0.3114 \\ 0.3114 & 0.4770 \end{pmatrix}, \\ N &= \begin{pmatrix} 0.6491 & 0.0022 \\ 0.0022 & 1.1412 \end{pmatrix}, \\ P_1 &= \begin{pmatrix} 17.3237 & 1.3197 \\ 1.3197 & 21.9271 \end{pmatrix}, \\ P_2 &= \begin{pmatrix} 35.0493 & 15.3697 \\ 15.3697 & 80.9490 \end{pmatrix}, \\ Q_1 &= \begin{pmatrix} 0.0358 & -0.0829 \\ -0.0829 & 0.2264 \end{pmatrix}, \\ Q_2 &= \begin{pmatrix} 0.0844 & -0.1372 \\ -0.1372 & 0.5661 \end{pmatrix}, \\ R &= \begin{pmatrix} 5.2693 & 0.2587 \\ 0.2587 & 3.0645 \end{pmatrix}, \\ Z &= \begin{pmatrix} 3.6731 & 0.2179 \\ 0.2179 & 1.4507 \end{pmatrix}. \end{aligned} \tag{41}$$

Obviously matrices $P_1, P_2, Q_1, Q_2, M, N, R$, and $Z$ are positive. The effectiveness of the method of Theorem 1 is illustrated.

## 5. Conclusion

First, we choose a set of appropriate Lyapunov stochastic functionals; some of them show more complexity in dealing with the neutral cross terms containing the Laplace operator. Then, by taking advantage of boundary conditions, Green formula, and Schur complement lemma, we obtain a sufficient condition for stochastic stability of the studied model in this paper via linear matrix inequalities. At last, a numerical example is given to prove the effectiveness of the proposed method.

## Data Availability

The simulation data used to support the findings of this study are included within the article.

## Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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