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Research Article

Research on the Complexity of Game Model about Recovery Pricing in Reverse Supply Chain considering Fairness Concerns

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A reverse recycling supply chain consisting of two recyclers is established in this paper, which takes into account the fact that the recyclers will consider the issue of fair concern in pricing. The paper discusses the local stability of the Nash equilibrium point in this price game model showing that the fair concern factors will reduce the stable area of the system. The paper also discusses the impacts of the sensitivity of the recovery price and the price cross coefficient on the stable area of the system. Through the method of system simulation and use of some indicators, such as the singular attractor, bifurcation diagram, attraction domain, power spectrum, and maximum Lyapunov exponent, the characteristics of the system at different times will be illustrated.

1. Introduction

With the information technology innovation and the deep integration of global economy in the postindustrial era, product replacement has become more frequent, resulting in more and more waste products, environmental degradation, and resource shortages. The green innovation strategy is a new idea for achieving green development and an inevitable choice for enterprise upgrading [1]. Closed-loop supply chain, a new type of logistics management mode, realizes the recycling and reuse of waste products. At present, closed-loop supply chain management has attracted widespread attention from various scholars [2].

Scholars such as Savaskan et al. [3, 4] have conducted a more comprehensive study of the recycling models. Firstly, they made analysis on three recycling models in a closed-loop supply chain; secondly, they made an analysis on the selection of the optimal recycling channels in the manufacturers' closed-loop supply chains. Hong and Yeh [5] examined the decision-making problems of the closed-loop supply chain when the retailers and the third-party recyclers make their recycling separately and pointed out that the channel recovery rate, manufacturers' profits, and total channel profits for the retailers are not always better than those when the third parties are responsible for recycling. Choi et al. [6] studied the decision-making problem of the closed-loop supply chains under different channel forces and held that the overall performance of the closed-loop supply chains dominated by retailers was the best. On the basis of symmetric and asymmetric information, Wei et al. [7] constructed four decision models for the closed-loop supply chain under the power of two channels, namely, the manufacturer-led and retailer-led.

According to the above literature, decision makers for closed-loop supply chain are completely rational and take profit maximization as the decision objective. However, Kahnema, a behavioral economist, found that when people pay attention to their own interests, they also pay attention to the interests of others around them and show great attention to fairness [8]. A large number of experimental results show that the members in the game are generally willing to give up part of their interests for achievement to reach a fair result because of their fairness concerns. In the case of the ultimatum game, Ruffle [9] made several analysis on the decision in the case of the ultimatum game; thus, if one party thinks the other party's plan is unfair, the former will make a decision to reject the plan. In addition, many experiments, such as trust game experiment, authoritarian game experiment, and public goods game experiment, show that people have the tendency to display fairness concern behaviors. The study conducted by Fehr and Schmidt [10] showed that the disadvantaged decision-makers pay more attention to their own benefits and compare their benefits with those of other decision-makers, in an attempt to obtain more equity through cooperation. Similarly, through other experiments, Loch and Wu [11] also showed that, in general, members in a disadvantaged position would be more inclined to focus on their own benefits and find ways to coordinate their sense of fairness by making comparison with the benefits from the other party. Tversky and Kahneman [12], on the contrary, believed that, in many cases, the existence of equity concerns has been common in organizations. In the process of operation, enterprises also constantly pay attention to the fact of whether or not their profits are "fair" compared with those of other enterprises. Therefore, the fairness concern behavior impacts on the decision-making subject of the supply chain to a certain extent. Haitao Cui et al. [13] proposed the equity concern in the newsboy model, which indicates that when the behavior of members of the supply chain display equity concern behaviors, suppliers can stimulate the coordination of the supply chain by the utilization of the wholesale price which is above the marginal cost. Li et al. [14] made study on the distribution fairness in the reverse supply chain. A simple reverse supply chain, which consisted of one recycler and one remanufacturer, was established and then extended to the situation in which one remanufacturer and two recyclers were being involved; the study made discussion on the impact which was made on the transfer prices and optimal decision-making by the fairness factors.

Without a doubt, the supply chain is a complex economic system, which entails features of human participation, being open and possessing information feedback function, but at the same time, it is not able to realize the changes in the competition process in accordance with the predesigned blueprint and preset orbit. Instead, these changes can only be realized through the interaction and game of the main players in the system. Although many studies have shown that static optimization is a stable state of dynamic evolution and can also be regarded as the fixed point of a dynamic system, the system often fails to achieve static optimization due to the disordered competition caused by different individual interests in the supply chain.

On the basis of the Cournot model, Rand [15] first found that the game results of oligarchs sometimes did not converge to the equilibrium point but presented periodic or chaotic solutions. Subsequently, a large number of scholars have conducted extensive research on the Cournot model, and the construction of the bounded rationality and incomplete information to the Cournot model is one such study, which speaks the complex behavior of decision makers. Based on Puu and Marín [16, 17], research on the production adjustment process of the Cournot model with the impacts of the elastic demand function came, and they came to the conclusion that complex phenomena such as bifurcation and chaos also occurred in the model. In respect of Bischi and Kopel [18, 19], they also introduced the bounded rationality to the Cournot game model. Additionally, further studies were made on the impacts of the output adjustment rate on the system stability by making analysis on the critical curve and attraction domain. Agiza et al. [20] also studied mapping the symmetry of Cournot model. Xin et al. [21, 22] proposed a fractional-order energy resources demand-supply system and proposed a projective synchronization scheme.

With the improvement in the demand function, Ahmed and Hegazi [23, 24] established a duopoly game model at nonlinear cost and extended the duopoly model to a multidimensional model. In effect, Elsadany [25] studied the impacts of delay decision on Cournot model and found that the appropriate employment of delay strategy could make the system become more stable. Other scholars also introduced the chaos theory into the study of the supply chain game. Li and Ma [26, 27] studied the long-term price competition in the multichannel supply chain and found some complex phenomena like bifurcation and chaos. Li et al. [28] also conducted studies on the chaos phenomenon in the closed-loop supply chain and the control of the chaos theory effectively in the system.

This paper ultimately discussed the impacts of equity fairness concerns on the stable domain of the system and made simulation study on the system in the case of the supply chain system for reverse recovery: (1) the manufacturer does not participate in the recovery but gives subsidies to the recyclers; (2) it is assumed that one of the retailers is concerned about equity.

The structure of this paper is as follows: in the first part, the literature is being summarized; in the second part, the price game model of two recyclers would be constructed; in the third part, analysis on the local and global stability of equilibrium points should be given; in the fourth part, the relation between the equilibrium point and the parameters would be studied with the help of the simulation technique and the characteristics of system in chaos would be presented; and in the fifth part, the conclusion of the paper should be given.

2. Problem Description and Model Building

2.1. Model Description. In this article, two oligopoly recyclers in a reverse supply chain market and a competition model will be established for the study of the product recycling. Competition usually occurs through price strategies, and essentially, the competition between the two oligopolies conforms to the Bertrand game model. The structure of supply chain is as shown in Figure 1.

2.2. Symbol Description

a_1 and a_2 represent the amount of used products that consumers are willing to recycle when the price is 0; to some extent, this amount reflects the environmental awareness of consumers and the recycling influence of each recycler.

p_1 and p_2 represent the recycling prices of two recyclers, respectively.

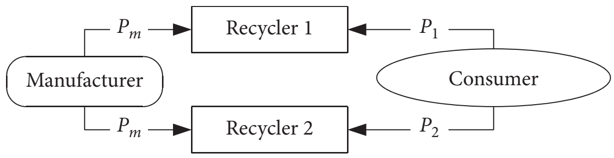

Figure 1: Game model for recycling supply chain.

q_1 and q_2 represent the recycling quantity of two recyclers respectively.

b represents the consumers' sensitivity to recycling prices.

d reflects the price cross coefficient between response channels.

c_1 and c_2 mean the unit cost of the two recyclers, respectively. For simplification of the analysis, we assume $c_1 = c_2 = c$.

p_m represents the subsidy given by the manufacturer to the recycler for the recycled product per unit.

For the purpose of making the model economically meaningful, we assume $a_1, a_2, b, d > 0$.

2.3. Model Construction. We assume that, in reality, two recyclers make recycling of waste products together. According to the concept of the Bertrand game model, the quantity of recycled products is related to the recycling price. When there exists more than one recycling company, the quantity is also related to the recycling prices provided by other recyclers. The model can be expressed as

$$\begin{aligned} q_1 &= a_1 + bp_1 - dp_2, \\ q_2 &= a_2 + bp_2 - dp_1. \end{aligned} \tag{1}$$

The model shows that when the recycler from one channel raises the price, the product recycling volume of the other channel will increase. The profit of the two recyclers can be written as

$$\begin{aligned} \pi_1 &= (p_m - p_1 - c_1)q_1, \\ \pi_2 &= (p_m - p_2 - c_2)q_2. \end{aligned} \tag{2}$$

In making price decisions, recyclers will not only consider their own profits but also the profits of their competitors. Recyclers are unwilling to determine the profit distribution of the supply chain by strength but are more willing to express concerns about fairness by directly comparing the profits with those of other recyclers. By means of dependence on the reference point, this article tries to characterize the retailer's fair concern; that is, one recycler will use the profit of another recycler as the reference point for its own profits with the purpose of showing its perception of fair concern. By introduction of λ as a fair concern coefficient, the utility function of the recycler is as follows:

$$\begin{cases} U_1 = \pi_1 - \lambda(\pi_2 - \pi_1), \\ U_2 = \pi_2. \end{cases} \tag{3}$$

The formula shows that, for recycler 1, when the profit of recycler 2 is greater than that of recycler 1, the utility of recycler 1 will decrease. λ, a fair concern coefficient, reflects the sensitivity of recycler 1 to the profit gap between competitors and themselves. Recycler 2 uses profit maximization as its decision criterion. When $\lambda = 0$, it could be stated that recycler 1 was fair and neutral.

From the formula, we can get the marginal utility of the recycler as

$$\begin{cases} \dfrac{\partial U_1}{\partial p_1} = dp_2 - bp_1 - a_1 - \lambda(a_1 + bp_1 - dp_2 + b(c + p_1 - p_m) + d(c + p_2 - p_m)) + b(p_m - c - p_1), \\ \\ \dfrac{\partial U_2}{\partial p_2} = dp_1 - bp_2 - a_2 + b(p_m - c - p_2). \end{cases} \tag{4}$$

From the formula (4), we can get

$$\begin{cases} p_1^* = -\dfrac{2a_1b(1+\lambda) + 2b^2(c - p_m - \lambda p_m + c\lambda) + a_2d + bd(c - p_m - 2\lambda p_m + 2c\lambda)}{4b^2(1+\lambda) - d^2}, \\ \\ p_2^* = \dfrac{2a_2b(1+\lambda) + 2b^2(c - p_m - \lambda p_m + c\lambda)}{4b^2(1+\lambda) - d^2}. \end{cases} \tag{5}$$

In reality, corporate decisions could be limited by objective conditions like the individual ability of the decision maker, which shows that it is impossible for the decision maker to obtain all the information in the market. Here, we make assumptions that the recycler is to be boundedly rational; price decision can be adjusted within a reasonable range in the next cycle. Recyclers would make prediction and determination on the price of the next period based on the profit margin. In other words, if the marginal profit is positive in the period t, the recycler will raise its price in the period $t+1$. Conversely, recyclers will lower their prices. So, we can build a corresponding dynamic model:

$$\begin{cases} p_1(t+1) = p_1(t) + \alpha_1 p_1(t)\big(dp_2(t) - bp_1(t) - a_1 - \lambda\big(a_1 + bp_1(t) - dp_2(t) + b\big(c + p_1(t) - p_m\big) \\ +d\big(c + p_2(t) - p_m\big)\big) + b\big(p_m - c - p_1(t)\big)\big), \\ p_2(t+1) = p_2(t) + \alpha_2 p_2(t)\big(dp_1(t) - bp_2(t) - a_2 + b\big(p_m - c - p_2(t)\big)\big). \end{cases} \quad (6)$$

3. Stability Analysis of the Equilibrium Points

3.1. Market Equilibrium. According to the definition of fixed point, $p_i(t+1) = p_i(t)$, $(i = 1, 2)$, we can get the equilibrium point of the system as

$$\begin{aligned} E_1 &= (0, 0), \\ E_2 &= \left(0, \frac{a_2 + bc - bp_m}{-2b}\right), \\ E_3 &= \left(\frac{a_1 + bc - bp_m + \lambda(a_1 + bc - bp_m + cd - dp_m)}{-2b(\lambda + 1)}, 0\right), \\ E^* &= \left(-\frac{2a_1 b(1+\lambda) + 2b^2(c - p_m - \lambda p_m + c\lambda) + a_2 d + bd(c - p_m - 2\lambda p_m + 2c\lambda)}{4b^2(1+\lambda) - d^2}, \frac{2a_2 b(1+\lambda) + 2b^2(c - p_m - \lambda p_m + c\lambda)}{4b^2(1+\lambda) - d^2}\right). \end{aligned} \quad (7)$$

Since the pricing cannot be negative in reality, in order to ensure that the equilibrium point has economic meaning, the value range of the parameters should meet $E_1, E_2, E_3, E^* \geq 0$. Obviously, E_1, E_2, and E_3 are the boundary equilibrium solution, and E^* is the only NASH equilibrium solution.

3.2. Local Stability Analysis of Equilibrium Points. For the purpose of making analysis on the local stability of the equilibrium point, we make calculation for the Jacobian matrix of the system:

$$\begin{pmatrix} 1 + \alpha_1 f_1 & dp_1\alpha_1 \\ dp_2\alpha_2 & 1 + \alpha_2 f_2 \end{pmatrix}. \quad (8)$$

In this matrix,

$$\begin{aligned} f_1 &= dp_2 - 2bp_1 - a_1 - \lambda(a_1 + 2bp_1 - dp_2 \\ &\quad + b(c + 2p_1 - p_m) + d(c + p_2 - p_m)) + b(p_m - c - 2p_1), \\ f_2 &= dp_1 - 2bp_2 - a_2 + b(p_m - c - 2p_2). \end{aligned} \quad (9)$$

The stability of the equilibrium point is determined by the properties of the eigenvalue corresponding to the equilibrium point in the Jacobian matrix. When the equilibrium points E_1, E_2, E_3, and E^* are substituted into the matrix, we can get the following theorem.

Theorem 1. *The equilibrium point E_1 is a stable equilibrium point.*

Proof. Substitute E_1 into the following matrix:

$$\begin{pmatrix} 1 + \alpha_1(-a_1 - \lambda(a_1 + b(c - p_m) + d(c - p_m)) + b(p_m - c)) & 0 \\ 0 & 1 + \alpha_2(-a_2 + b(p_m - c)) \end{pmatrix}. \quad (10)$$

By calculation, we get to know that the two characteristic roots of the corresponding characteristic equation for the matrix are

$$r_1 = 1 + \alpha_1 (-a_1 - \lambda(a_1 + b(c - p_m) + d(c - p_m)) + b(p_m - c)),$$
$$r_2 = 1 + \alpha_2 (-a_2 + b(p_m - c)). \quad (11)$$

Since the value of each parameter satisfies the condition that the four equilibrium points could be positive, we can get $|r_{1,2}| > 1$, which shows that the eigenvalues of the characteristic equation are usually greater than 1 when E_1 has been in correspondence with Jacobian matrix. According to the stability judgment condition of equilibrium point, E_1 is an unstable equilibrium point. □

Theorem 2. *The equilibrium points E_2 and E_3 are unstable saddle points.*

Proof. Substitute the equilibrium point E_2 into the matrix, the two characteristic roots of the corresponding characteristic equation could be calculated as

$$r_1 = 1 - \alpha_1\left(2a_1 b(1+\lambda) + 2b^2(c - p_m - \lambda p_m + c\lambda) + a_2 d + bd(c - p_m - 2\lambda p_m + 2c\lambda)\right) > 1,$$
$$r_2 = 1 + \alpha_2 (a_2 + bc - bp_m) < 1. \quad (12)$$

According to the judgment condition of stability for equilibrium point, equilibrium point E_2 is an unstable saddle point. In the same way, E_3 is also an unstable saddle point. □

Theorem 3. *The local stability of the Nash equilibrium point E^* is related to the speed of price adjustment α_1 and α_2.*

Proof. We will plug E^* in and get

$$J(E^*) = \begin{pmatrix} 1 + \alpha_1 h_1 & dp_1\alpha_1 \\ dp_2\alpha_2 & 1 + \alpha_2 h_2 \end{pmatrix}, \quad (13)$$

in which

$$h_1 = dp_2^* - 2bp_1^* - a_1 - \lambda(a_1 + 2bp_1^* - dp_2^* + b(c + 2p_1^* - p_m) + d(c + p_2^* - p_m)) + b(p_m - c - 2p_1^*),$$
$$h_2 = dp_1^* - 2bp_2^* - a_2 + b(p_m - c - 2p_2^*). \quad (14)$$

In order to determine the stable region of the Nash equilibrium point E^* regarding the speed of price adjustment α_1 and α_2, firstly we should obtain the characteristic equation $\lambda'^2 + A\lambda\prime + B = 0$ corresponding to its Jacobian matrix, among that

$$A = 2 + h_1\alpha_1 + h_2\alpha_2,$$
$$B = 1 + h_1\alpha_1 + h_2\alpha_2 + \left(h_1 h_2 - d^2 p_1^* p_2^*\right)\alpha_1\alpha_2. \quad (15)$$

According to Jury's argument for determining stability, which is based on the Nash equilibrium of a discrete system, the local stability E^* is determined by the formula

$$\begin{cases} 1 - A + B > 0, \\ 1 + A + B > 0, \\ 1 - B > 0. \end{cases} \quad (16)$$

Substitute the value of the parameters A and B to get

$$\begin{cases} 2 - (h_1 h_2 - d^2 p_1^* p_2^*)\alpha_1\alpha_2 > 0, \\ 4 + 2h_1\alpha_1 + 2h_2\alpha_2 + (h_1 h_2 - d^2 p_1^* p_2^*)\alpha_1\alpha_2 > 0, \\ 1 + h_1\alpha_1 + h_2\alpha_2 > 0, \end{cases} \quad (17)$$

In the formula, after determining the values of the other parameters α_1 and α_2, the local stability of E^* is obtained if and only if the parameters α_1 and α_2 satisfy the formula. To satisfy all the values of this inequality formula, α_1 and α_2 means the stability domain of the Nash equilibrium point E^* related to parameters α_1 and α_2. If the value of (α_1, α_2) is in the stable region, $(p_1(t), p_2(t))$ will keep stable at the point E^* after a long game. If the value of (α_1, α_2) is not in the stable region, after a series of games, the system will gradually lose stability and the market price will become difficult for prediction. This shows that when recyclers continue to speed up the price adjustment in order to obtain greater own profits, market competition will become disordered. □

4. Numerical Simulation

For better understanding of the model, visual demonstration will be made for the long-term competition of the system by the means of numerical simulation. Taking the actual competition of recyclers into consideration, we make it possible as follows:

$$a_1 = a_2 = 1,$$
$$b = 1,$$
$$d = 0.3,$$
$$c = 1,$$
$$p_m = 5,$$
$$\lambda = 0.3. \quad (18)$$

At this point, Nash equilibrium point is like $E^* = (1.843, 1.777)$.

4.1. Relationship between the Stability of the Equilibrium Points and the Parameters. As is shown in Figure 2, for (α_1, α_2), the local stable area of the Nash equilibrium point is the light blue part in the figure, which indicates that if and only if the value of the price adjustment speed (α_1, α_2) is within this stable range, the price $(p_1(t), p_2(t))$ will eventually get stable at $(1.843, 1.777)$ after the long-term competition.

For the study of the impact of fairness concerns on the equilibrium point and stability of the system, we take the values of λ 0, 0.5, and 1, respectively, which represent different degrees of fair concern. As shown in Figure 3, we can

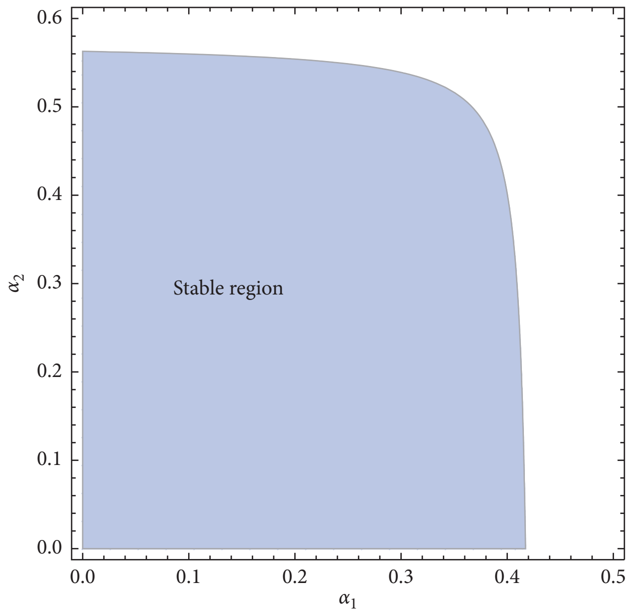

Figure 2: Local stability region of Nash equilibrium about (α_1, α_2).

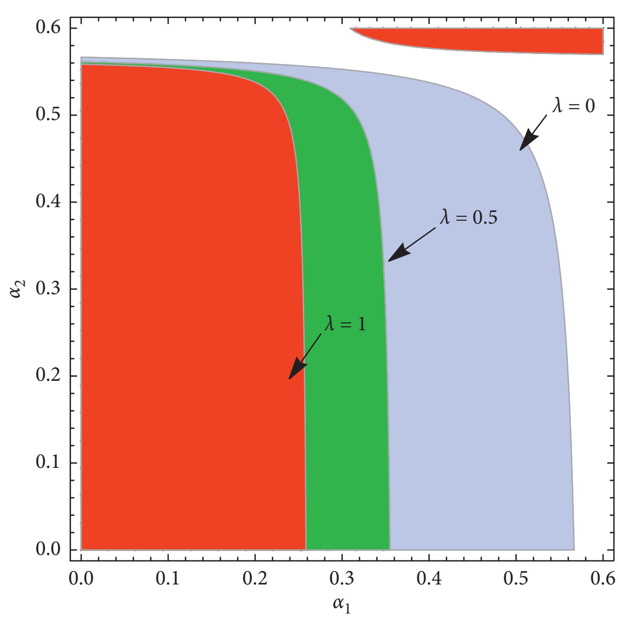

Figure 3: When λ is different, the local stability region of the Nash equilibrium points at (α_1, α_2).

also obtain the stable domains of the system when λ takes different values. The corresponding Nash equilibrium points are shown in Table 1.

In Figure 3, the red, green, and light blue scopes represent the corresponding values, respectively, when $\lambda = 0, 0.5$, and 1. From Figure 3, we can know that when the value of λ increases, that is to say, recycler 1 being more concerned about the sense of fairness, the stable region of the system will get smaller. This shows that recycler 1 will adopt a fierce competition strategy for obtaining fair utility. As a result, this more intense pricing strategy will make it more difficult for the market to maintain its stability.

Table 1: Nash equilibrium.

λ	$\lambda = 0$	$\lambda = 0.5$	$\lambda = 1$
Nash equilibrium	(1.765, 1.765)	(1.878, 1.782)	(1.934, 1.79)

When the recyclers continue to speed up the price adjustment, the market will become unstable, and the system will become bifurcated or even chaotic. Figures 4 and 5 show the bifurcation diagrams indicating price changes of recycler 1 and recycler 2 with changes in price adjustment speed, respectively. From Figure 4, we can learn that when the price adjustment speed of recycler 1 is relatively low, with limited times of game, the price will get stable at the Nash equilibrium point $(1.843, 1.777)$.

When the price adjustment speed gets increased and the doubling period makes bifurcation for the first time and two equilibrium solutions appeared in the system, then followed by four times period, eight times period, and so on, the system finally entered into the chaotic state. Figure 5 shows that the system will show a similar change along with the change in the price adjustment speed for the recycler 2.

By making a comparison between Figures 4 and 5, we can easily find another phenomenon: although the recyclers could gain the preferential advantage to some degrees in price competition through continuous speeding up of price adjustment, when the system enters into chaos, the party, which continues to speed up the price adjustment, would experience a huge price fluctuation, while at the same time, another party who employs the "follow strategy" will experience a smaller price fluctuation.

The type of system bifurcation, the periodic behavior of the solution, and the path to chaos are analyzed by means of the parameter 2D bifurcation diagram. First, we use the price input adjustment coefficient as the bifurcation parameter. Figure 6 shows a two-dimensional bifurcation diagram of the system, among which blue scope represents the system's stable domain, that is, the 1-period solution; red scope represents the 2-period solution, green scope for the 3-period solution, pink scope for the 4-period solution, light blue scope for the 5-period solution, purple scope for the 6-period solution domain, yellow scope for the 7-period solution domain, brown scope for the 8-period solution domain, dark purple scope for the 9-period solution, and dark green scope for the 10-period solution; gray scope represents the chaotic region of the system, and white scope indicates that the system variables have overflowed and no meaning exists. From Figure 6, we could see that the faster the price adjustment speed becomes (that is, the more frequent the price adjustment), the more unstable the entire system will be, and the market is more prone to enter into chaos. From Figure 6, we may also see that the system can enter into chaos in two ways: firstly, the system will lead to chaos through a period-doubling bifurcation channel which is composed of those red, pink, purple, and brown scopes, called flip bifurcation; secondly, the system leads to chaos through the odd cycle which is represented by the green and light blue scopes. Finally, those intermittent odd cycle points can be found from Figure 6.

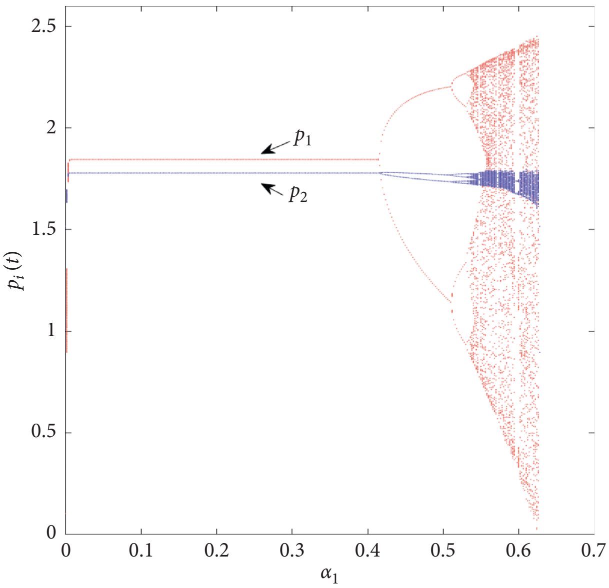

FIGURE 4: Price bifurcation diagram of the system with variations in α_1.

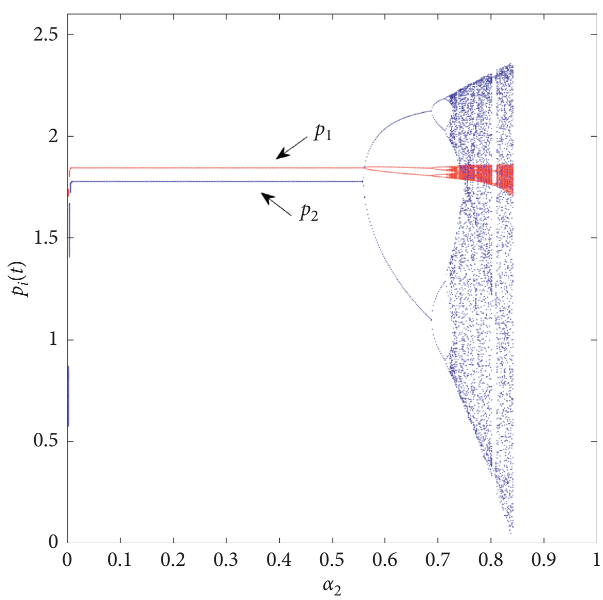

FIGURE 5: Price bifurcation diagram of the system with variations in α_2.

Figure 7 shows a two-dimensional bifurcation diagram which reflects consumers' sensitivity to recycling prices and the speed α_1 of recycling price adjustment. It can be concluded from Figure 7 that when b becomes larger, that is, the more sensitive the consumer is to the recycling price, the narrower the blue area will become in the figure, which indicates that the stability region of the system is decreasing. The result shows that if companies can reduce the consumer's sensitivity to prices by means of advertising or improvement of consumers' environmental awareness, leading to more consumers' awareness of the importance of recycling products, they can effectively reduce consumers'

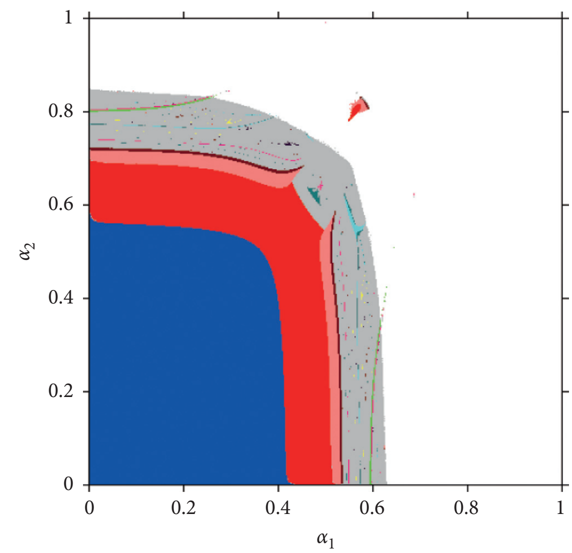

FIGURE 6: Two-dimensional bifurcation of the system with changes in (α_1, α_2).

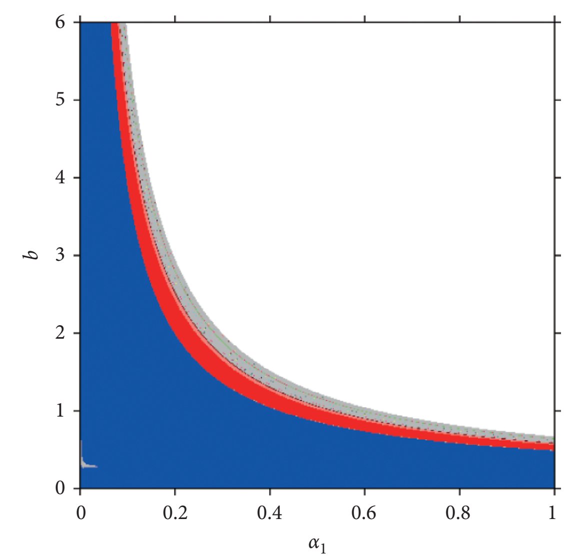

FIGURE 7: Impact of consumers' sensitivity to recycling prices on system's stability.

perception of products, increase the speed of price adjustment for themselves, gain more competitive advantages, and surely create more space.

Figure 8 shows a two-dimensional bifurcation diagram of the cross-elasticity of prices between channels and the speed of adjustment of recycling prices α_1. It can be learned from Figure 8 that the larger the price cross-coefficient d becomes between channels, the smaller the system's stable region will get. This also shows that when consumers are more sensitive to price factors, for recyclers, the strategic space, which is employed to increase competitive advantage through price adjustment, will become smaller. At the same time, it also shows that if the manufacturer could make effective reduction of the competition between two recyclers

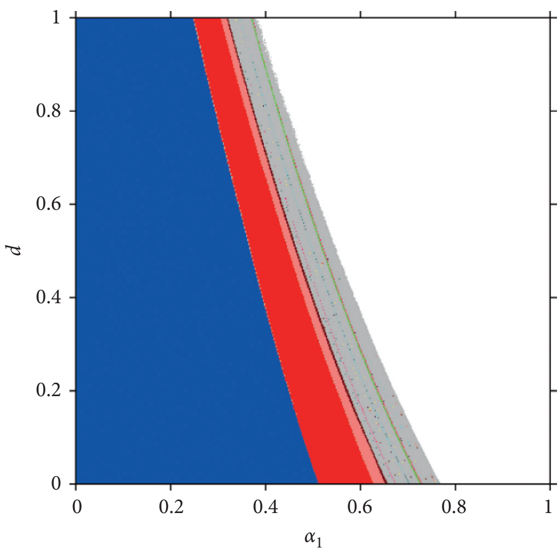

FIGURE 8: Effects of price cross coefficient on system's stability.

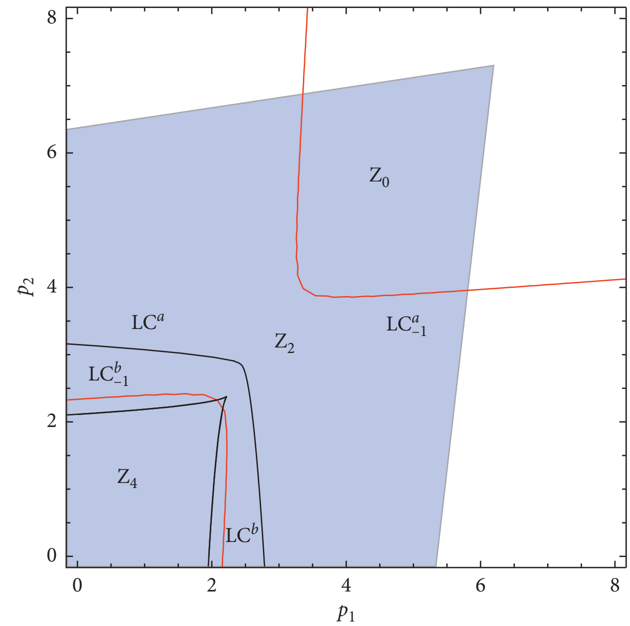

FIGURE 9: Attraction domain for equilibrium point when $\alpha_1 = 0.1$ and $\alpha_2 = 0.1$.

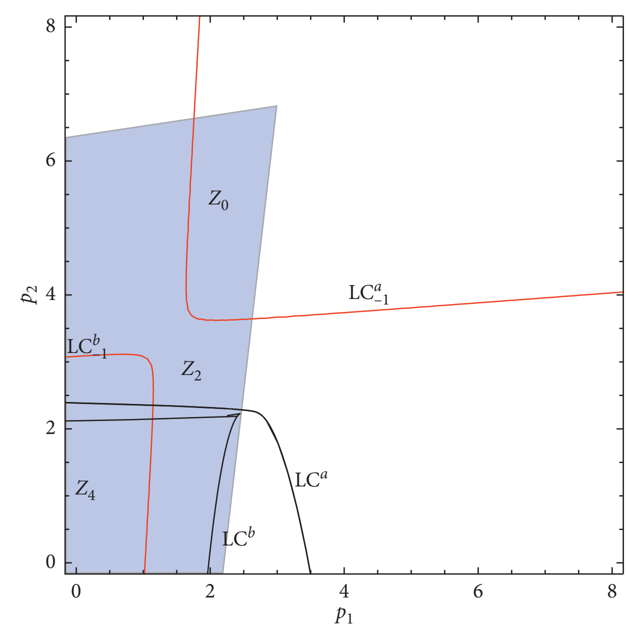

FIGURE 10: Attraction domain for equilibrium point when $\alpha_1 = 0.55$ and $\alpha_2 = 0.1$.

by means of reasonable setting of the recycling sites for two competing recyclers, he could reduce the price cross-coefficient between channels with the result that the market will become more stable. All these factors, which include consumer's sensitivity to the recycling price, the cross-elasticity of the price between channels and the retailer's recycling price adjustment speed α_2 are similar to those in Figures 7 and 8, so they will not be mentioned here again.

In fact, the initial value of the price is not necessarily close to the equilibrium point of the market. Therefore, it is necessary to make analysis on the global stability of the system (6). Figure 9 shows the attractive domain when the equilibrium points are $\alpha_1 = 0.1$ and $\alpha_2 = 0.1$. The LC curve is a trajectory of points that are mapped once and have 2 or more images. The set of these images is defined as LC_{-1}. The LC curve divides the plane into different regions Z_0, Z_2, and Z_4 by the number of images [29], and the LC_{-1} set belongs to the set of points whose determinant Jacobian value is 0. So, we can get

$$LC_{-1} \subseteq J_0 = \left\{ (p_m, p_r) \subset R^2 \,|\, \det J(p_m, p_r) = 0 \right\}. \quad (19)$$

System (19) defines the mapping M so that we can get $LC = M(LC_{-1})$. At the same time, since the price should be nonnegative in reality, that is, $p_m, p_r > 0$ we define a feasible region:

$$R_1 = \left\{ \left((p_m, p_r) \subset R^2 \,|\, p_m > 0, p_r > 0 \right) \right\}. \quad (20)$$

Figure 9 shows the attractive domain of the Nash equilibrium point at that time when $\alpha_1 = 0.1$ and $\alpha_2 = 0.1$. In Figure 9, the gray area represents a feasible attractor area that satisfies the publicity (20). By making a comparison between Figures 9 and 10, we could find that the attraction domain will change from simple connection to multiple connection with the increased adjustment speed for recycler 1, and the feasible area in the direction of p_1 will also be significantly reduced which also leads to significant reduction of the entire feasible area.

4.2. Characteristics of the System in Chaos. Figure 11 shows the maximum Lyapunov index in correspondence with Figure 4 as the price adjustment coefficient α_1 increases. The maximum Lyapunov index can characterize the degree of separation between two points starting at the same time and running over time. When the system is in a stable state, the maximum Lyapunov index of the system is less than zero; when the system is in the chaotic state, the maximum Lyapunov index of the system is greater than zero. From

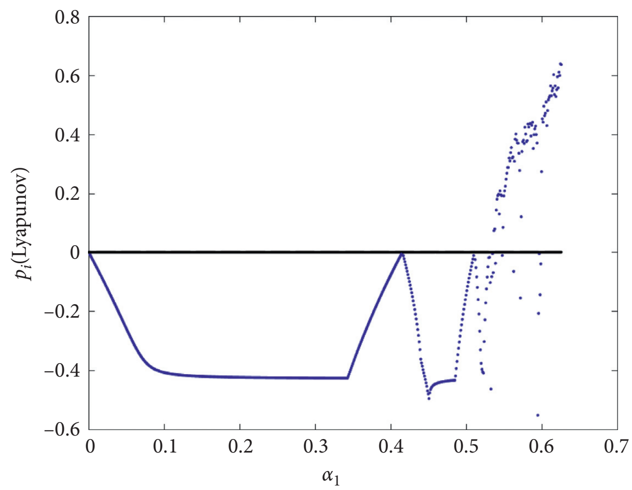

FIGURE 11: The system's largest Lyapunov index.

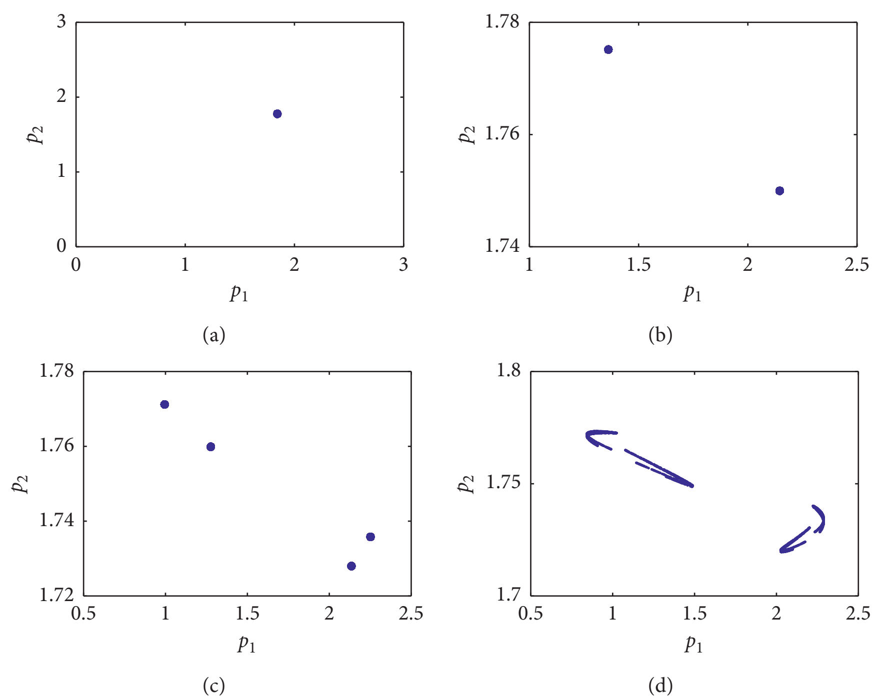

FIGURE 12: Formation process for attractor in system: (a) $\alpha_1 = 0.1, \alpha_2 = 0.1$; (b) $\alpha_1 = 0.46, \alpha_2 = 0.1$; (c) $\alpha_1 = 0.52, \alpha_2 = 0.1$; (d) $\alpha_1 = 0.54, \alpha_2 = 0.1$.

Figure 11, we can clearly see that when the maximum Lyapunov exponent is equal to 0 for the first time, the system enters into a double period bifurcation, and when the maximum Lyapunov exponent is greater than 0, it indicates that the system has entered into a chaotic state.

When the system is in chaos, another characteristic is that the system has singular attractors. The strange attractor is the result of the overall stability and local instability of the system, and it has self-similarity and fractal structure. Figure 12 shows the formation process of the singular attractor in this model at 0.1, 0.46, 0.52, and 0.54 and $\alpha_2 = 0.1$, and the system experienced a stable period, a double period, a quadruple period, and then entered into the chaotic state. Figure 13 corresponds to the rules of price changes in different periods of the system. Figure 13(a) shows the price changes over time when the system is in a stable state. After a limited number of games, the price of the system will stabilize at the Nash equilibrium point. Figures 13(b) and 13(c) show the price changes in the system in the two-cycle and four-cycle cycles, respectively. Figure 13(d) shows the price change over time when the system is in the chaotic state. It is clearly illustrated that compared with price in the stable state, the pricing decision becomes uncertain, disordered, and unpredictable when the system is in the chaotic state.

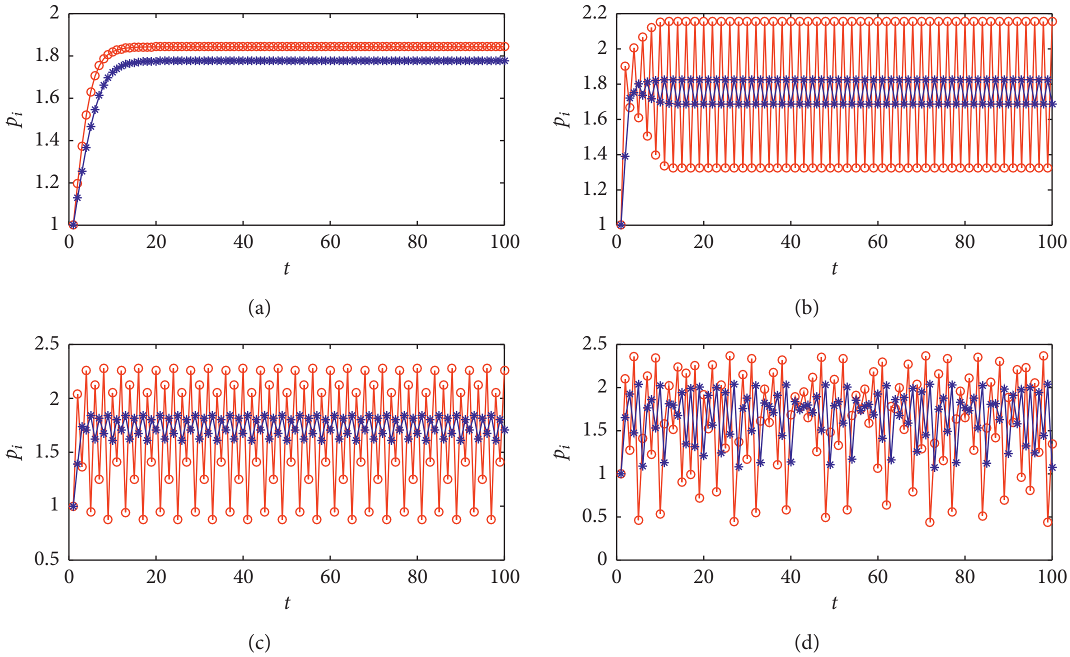

FIGURE 13: Price power spectrum of the system at different times.

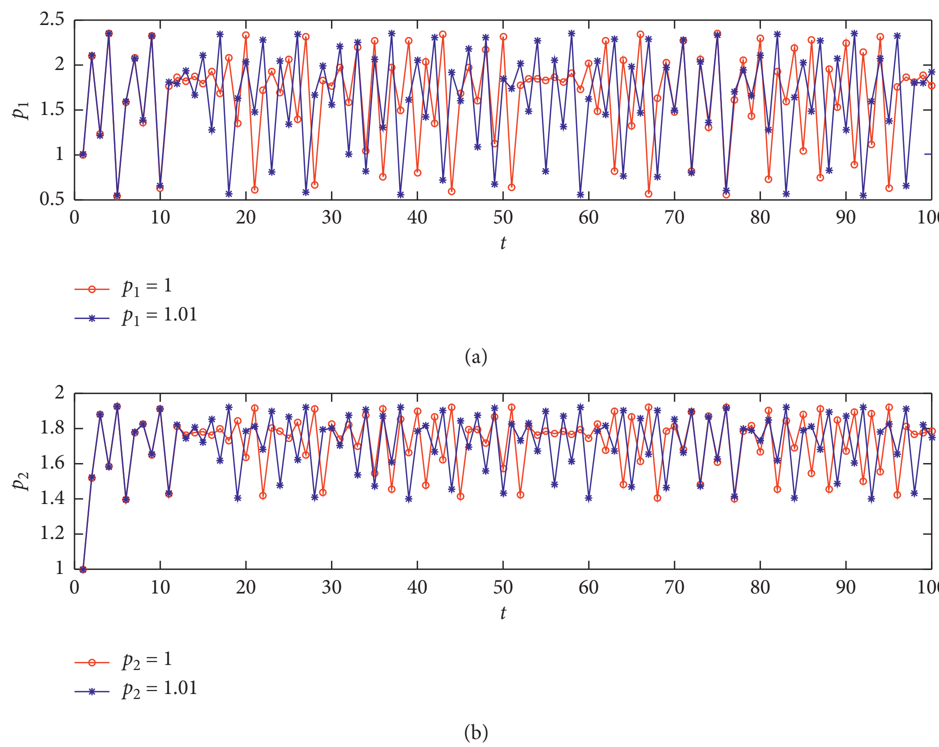

FIGURE 14: Sensitivity of the system to initial values: (a) $\alpha_1 = 0.56, \alpha_2 = 0.4$; (b) $\alpha_1 = 0.56, \alpha_2 = 0.4$.

Sensitive initial value is another important characteristic when the system is in chaos; that is, the evolution result of the system has extremely sensitive dependence on the initial value, which is what we often call the butterfly effect. Figure 14 shows the evolution of the recovery price over time when the initial pricing of recycler 1 and recycler 2 is 1 and 1.01 and when the system is in a chaotic state ($\alpha_1 = 0.56, \alpha_2 = 0.4$). We get to know that even the initial value has only a slight difference. However, over time, the price competition has undergone a long-term evolution process, and its process has become very different.

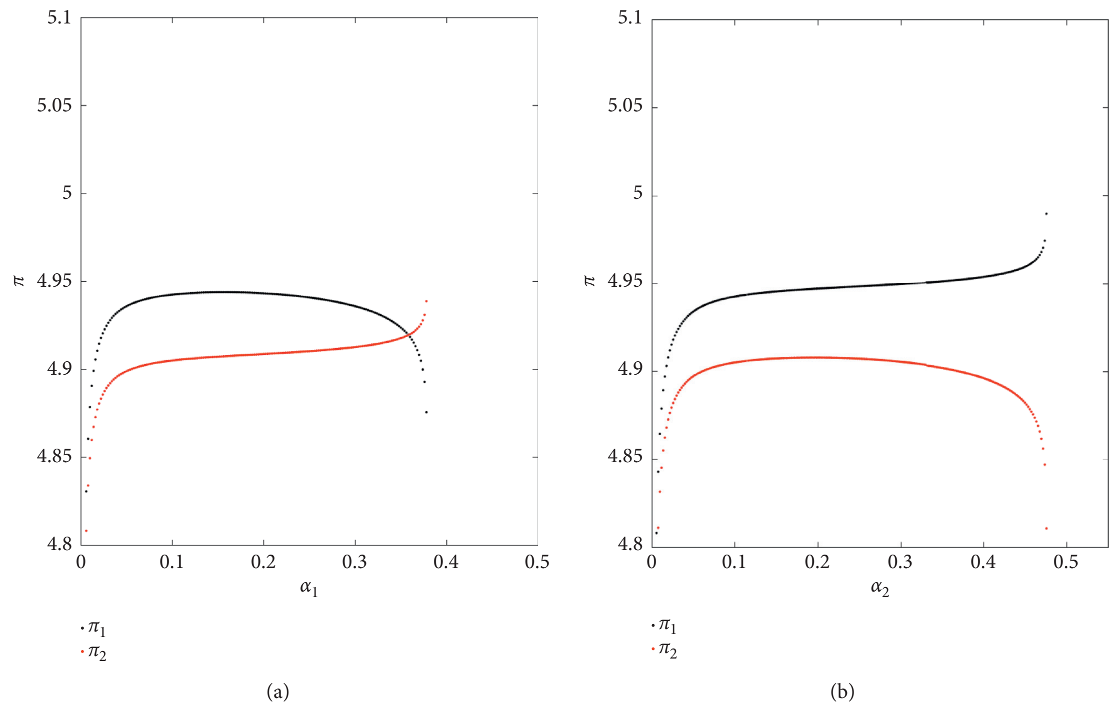

FIGURE 15: Changes in recyclers' profits with that of the price adjustment speed.

4.3. Impact of Price Adjustment Speed on Recyclers' Profits. Figure 15 shows the changes in the profits of two recyclers with the speed of price adjustment. From Figure 15(a), we find that as the price adjustment speed of recycle 1 continues to accelerate, the profit of recycler 1 starts to decline. It is when the system is chaotic, it declines rapidly, but at the same time, the profit of recycler 2 is rising. Comparing to the enlightenment given in Figure 4, this shows that when recycler 1 speeds up the price adjustment to obtain a greater competitive advantage, exaggerated price fluctuations have also affected their own profits. Figure 15(b) and Figure 5 say that when recycler 2 speeds up the price adjustment, its profit also decreases.

5. Conclusion

This article establishes a reverse supply chain consisting of two recyclers. The two recyclers make competition through price strategies. We assume that one of the retailers is of fair concern, which makes the competition for recycling of products more intense. Through analysis on the equilibrium point stability, we find three unstable bounded equilibrium points and a Nash equilibrium point with local stability. Then, the simulation study of the system is performed. So, the following conclusions are made:

(1) With the increase in the price adjustment speed for recyclers, some complex phenomena like bifurcation and chaos will appear in the system during the long-term process of competition. In this paper, the characteristics of the system in different periods are simulated numerically through means of the bifurcation diagram, the maximum Lyapunov index, and the power spectrum diagram of price changes.

(2) The fairness concerns of recyclers have a significant impact on the stability of the system. It is found that when the fair concern coefficient of the recycler becomes larger, the recyclers will care more about the sense of fairness, and the result of it is that recyclers may adopt a more aggressive price competition strategy, which may also make the system become more likely to lose its stability. By making analysis on the stability of the system, we find that amounts of the stability area for the system has decreased significantly.

(3) Speeding up the price adjustment is a common business strategy for companies to gain competitive advantage. However, in a reverse supply chain where there is a fair concern, speeding up the price adjustment will not only cause complex phenomena such as chaos in the system but also actively accelerate the recovery of price adjustment speed. Not only does the price fluctuate greatly during chaos but profits also significantly decrease after complex behaviors such as bifurcation and chaos occurred in the system. At the same time, the relative profit of recyclers who have not actively adjusted the price adjustment rate has increased. This conclusion is different from many previous studies, which indicates that although it is easier to actively adjust prices to obtain a competitive advantage, we must also strive to maintain a competitive balance in the

market. Once the market loses its stability, continual speeding up of the price adjustment will only have negative effects on its own profits but positive effects on the profits of the opponent.

In our research, the impact of fairness factors on the complexity of the system has been taken into consideration, but many other behavioral factors that could have some influence on the retailer's decision-making still account for a large proportion. At the same time, fractional order equations, being an important form of demand function, also means an important research direction for the study which is mainly about the operators' behavior of reverse supply chain in the future.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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