Hindawi
Complexity
Volume 2020, Article ID 9768361, 5 pages
https://doi.org/10.1155/2020/9768361

Research Article

DOA Estimation without Source Number for Cyber-Physical Interactions

Jianzhong Li[1], **Xiaobo Gu**[1], **Ruidian Zhan**[2], **Xiaoming Xiong**[1], and **Yuan Liu**[1]

[1] *School of Automation, Guangdong University of Technology, Guangzhou 510000, China*
[2] *Chipeye Microelectronics Foshan Ltd., Foshan, China*

Correspondence should be addressed to Yuan Liu; eeliuyuan@gdut.edu.cn

Received 23 May 2020; Revised 7 June 2020; Accepted 18 June 2020; Published 8 July 2020

Guest Editor: Hang Su

In this paper, a direction of arrival (DOA) estimator is proposed to improve the cyber-physical interactions, which is based on the second-order statistics without a priori knowledge of the source number. The impact of noise will firstly be eliminated. Then the relationship between the processed covariance matrix and the steering matrix is studied. By applying the elementary column transformation, an oblique projector will be designed without the source number. At last, a rooting method will be adopted to estimate the DOAs with the constructed projector. Simulation results show that the proposed method performs as well as other methods, which requires that the source number must be known.

1. Introduction

Nowadays, the requirement for different objects to communicate with each other is rapidly rising in many fields of the practical life. However, the network combining all the objects is very complicated, and the communications among different nodes faces many problems [1]. On one hand, energy diffusion of the transmitted signal would lower down the quality of communication. The energy collected by the target receiver is very poor in this situation. On the other hand, the currently adopted omnidirectional antennas also increase the risk of being attacked during the communication. Therefore, it is very important to omit the signal in the desired direction, and the technique of finding direction of arrival (DOA) can help to improve the performance of cyber-physical interactions.

DOA estimation has been an important research topic for array signal processing [2–4]. This research topic is widely applied in many fields such as sonar and electronic surveillance [5], where the signals are often nonstationary [6]. The wavefront of a far-field source signal can be assumed to be plane when it impinges on the receiver array. Each source can be localized with its corresponding DOA.

Plenty of researchers over the world have been making efforts to contribute to the research of DOA estimation, and there are already many achievements, such as the multiple signal classification (MUSIC) in [7], estimating signal parameters via rotational invariance techniques (ESPRIT) in [8], root-MUSIC in [9], and oblique projection operator method (LOFNS) in [10]. However, all these existing methods require a priori knowledge of the source number to guarantee their successful application.

In this paper, we propose a method to localize far-field sources without any priori knowledge of the source number. Firstly, the impact of noise is eliminated by taking advantage of the property of nonstationary signal. Then the relationship between the steering matrix and covariance matrix is studied. The elementary column transformation is applied to get rid of the dependency of the source number and an orthogonal matrix is designed based on this relationship. At last, a rooting method is applied for the estimation of DOA, reducing the computational complexity.

The rest of this paper is organized as follows. Section 2 presents the signal model and some common assumptions. In Section 3, the proposed method is described in detail. The complexity analysis is also given to illustrate the improvement of the proposed method. In Section 4, several simulations are provided. At last, the conclusion of the whole paper is made in Section 5.

In this paper, T represents the transpose operation, H the conjugate transpose, and * the complex conjugate. A bold capital letter symbolizes a matrix, and a bold letter in lowercase stands for a vector, such as $\mathbf{A}$ and $\mathbf{a}$, respectively.

2. Signal Model

As shown in Figure 1, K narrow-band far-field source signals are received with a uniform linear array (ULA). $M+1$ sensors are distributed in the array with intersensor spacing being d. The output of the mth $(m \in [0, M])$ sensor can be expressed as

$$y_m(t) = \sum_{k=1}^{K} s_k(t) e^{j\omega_{mk}} + n_m(t), \quad t = 1, 2, \ldots, T,$$
$$\omega_k = -\frac{2\pi d}{\lambda} \sin\theta_k, \tag{1}$$

where θ_k is the DOA of the kth source, λ is the wavelength of the signal satisfying $\lambda \geq 2d$, and $n_m(t)$ is the corresponding Gaussian white noise with the variance σ^2. The noises are assumed to be independent from each other and from all the source signals. Written in matrix form, the received signal can be expressed as

$$\mathbf{y}(t) = \mathbf{A}(\theta)\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where

$$\begin{aligned}
\mathbf{y}(t) &= \left[y_0(t), y_1(t), \ldots, y_M(t)\right]^T, \\
\mathbf{s}(t) &= \left[s_1(t), s_2(t), \ldots, s_K(t)\right]^T, \\
\mathbf{A}(\theta) &= \left[\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_K)\right], \\
\mathbf{a}(\theta_k) &= \left[e^{j[(0)\omega_k]}, e^{j[(1)\omega_k]}, \ldots, e^{j(M)\omega_k}\right]^T, \\
\mathbf{n}(t) &= \left[n_0(t), n_1(t), \ldots, n_M(t)\right]^T.
\end{aligned} \tag{3}$$

Without loss of generality, we make the following assumptions, which are the same as those in [10–17]:

(1) The kurtosis of the source signal is nonzero

(2) All the DOAs are different from each other

(3) The Gaussian noise $n_m(t)$ is independent of the source signals, and the K source signals are independent of each other

3. Proposed Algorithm

Generally in order to estimate DOAs with an oblique projection, the source number should be known to divide the covariance matrix, like LOFNS in [10]. Indeed, after a deep analysis into the basic principle of the oblique projection based methods, we observed that the division with the source number is mainly to ensure that the desired matrix is full column rank, such that the inverse operation can work properly while constructing the oblique projection. Therefore, we propose a rooting method to localize sources with nonstationary signal that can get rid of the dependence of the source number.

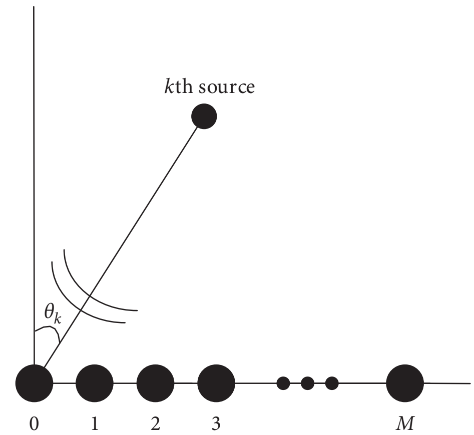

Figure 1: Far-field source localization with ULA.

When the signal is nonstationary and the noise is stationary, two covariance matrices can be obtained with two different group of snapshots as follows:

$$\begin{aligned}
\mathbf{R}_1 &= \frac{2}{T} \sum_{t=1}^{T/2} \left[\mathbf{A}(\theta)\mathbf{y}(t)\mathbf{y}^H(t)A^H(\theta) + \mathbf{n}(t)\mathbf{n}^H(t)\right] \\
&= \mathbf{A}(\theta)\mathbf{R}_{s1}\mathbf{A}^H(\theta) + \sigma^2\mathbf{I}, \\
\mathbf{R}_2 &= \frac{2}{T} \sum_{t=(T/2)+1}^{T} \left[\mathbf{A}(\theta)\mathbf{y}(t)\mathbf{y}^H(t)\mathbf{A}^H(\theta) + \mathbf{n}(t)\mathbf{n}^H(t)\right] \\
&= \mathbf{A}(\theta)\mathbf{R}_{s2}\mathbf{A}^H(\theta) + \sigma^2\mathbf{I},
\end{aligned} \tag{4}$$

where $\mathbf{R}_1$ is estimated with the first $T/2$ snapshots and $\mathbf{R}_2$ with the last $T/2$ ones. The impact of the noise can be eliminated by constructing another matrix [18]:

$$\mathbf{R} = \mathbf{R}_1 - \mathbf{R}_2 = \mathbf{A}(\theta)\mathbf{R}_s\mathbf{A}^H(\theta), \tag{5}$$

where $\mathbf{R}_s = \mathbf{R}_{s1} - \mathbf{R}_{s2} \neq \mathbf{0}$.

From (5), it can be learned that all the columns of $\mathbf{R}$ are the linear combination of the whole steering matrix. The relationship between $\mathbf{R}$ and $\mathbf{A}(\theta)$ can still be maintained if we apply the elementary column transformation to $\mathbf{R}$:

$$\frac{r_{p(M+1-p)}}{r_{(p+q)(M+1-p)}}\mathbf{r}_{p+q} - \mathbf{r}_p \longrightarrow \mathbf{r}_{p+q}, \tag{6}$$

where $p \in [1, M]$, $q \in [1, M+1-p]$, $\mathbf{r}_p$ is the pth column of $\mathbf{R}$, and r_{pq} is the pqth element. For every iteration, we fix the parameter p, and make q traverse all the legal values. The procedure will be repeated before we get a matrix which is upper triangular-like. All the nonzero columns we obtain after the transformation operation can form a maximal linearly independent subsystem of $\mathbf{R}$, and can form a full column-rank matrix $\mathbf{U}_s$. Construct an oblique projection as

$$\mathbf{U}_n = \mathbf{I}_{M+1} - \mathbf{U}_s\left(\mathbf{U}_s^H\mathbf{U}_s\right)^{-1}\mathbf{U}_s^H, \tag{7}$$

where $\mathbf{I}_{M+1}$ is an identity matrix of $(M+1) \times (M+1)$. It can be easily calculated that

$$\mathbf{U}_s^H \mathbf{U}_n = \mathbf{0}_{M+1}. \tag{8}$$

Due to the fact that $\mathbf{U}_s$ and $\mathbf{A}(\theta)$ span the same column space, (8) equals

$$\mathbf{A}^H(\theta)\mathbf{U}_n = \mathbf{0}_{M+1}. \tag{9}$$

Based on this orthogonality, a spectrum can be used for DOA estimation. In order to avoid the spectrum search which is computationally expensive, we propose the application of a rooting method. A polynomial is designed as follows:

$$f(z) = z^{2M+1}\mathbf{a}^T\left(z^{-1}\right)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(z), \tag{10}$$

where $z = e^{j\omega}$. The roots closest to the unit circle are the desired ones for the estimation of ω:

$$\widehat{\omega}_k = \angle\left(z_k\right), \tag{11}$$

where $\angle(z)$ is to take the angle of z. The DOAs of the sources are estimated through

$$\widehat{\theta}_k = -\frac{\lambda\omega_k}{2\pi d}. \tag{12}$$

4. Simulation

In this section, the performance of the proposed method will be studied, which is examined with the root mean square error (RMSE). The definition of RMSE is given by

$$\text{RMSE} = \sqrt{\frac{\sum_{n=1}^{N}\left|\widehat{\theta}_n - \theta_{\text{true}}\right|^2}{N}}, \tag{13}$$

where $\widehat{\theta}_n$ represents the DOA estimate of the nth simulation, θ_{true} is the real DOA and N means the total number of Monte Carlo simulations. The performance of the proposed method will be compared with other methods such as root-MUSIC in [9] and LOFNS in [10]. Two sources are considered in the simulations, whose DOAs are 6° and 23°, respectively. The inner space between sensors of the array $d = \lambda/2$. For the proposed method, the source number K can be unknown. For the other methods, K must be an exact priori knowledge. Specifically, the Cramer-Rao lower bound (CRLB) is also illustrated to make a better comparison.

The first simulation is designed to examine the relationship between the RMSEs and signal-noise radio (SNR), which is defined as

$$\text{SNR} = 10\log_{10}\frac{\sum_{k=1}^{K} P_{s_k}}{\sigma^2}, \tag{14}$$

with P_{s_k} being the signal power. Assume that there are 8 sensors in the array (i.e. $M = 7$) and 400 snapshots are received with the array. The SNR varies form 0 dB to 30 dB. As shown in Figures 2 and 3, by eliminating the effect of noise, both the proposed method and LOFNS outperform root-MUSIC, even though LOFNS does not perform well when the SNR is low. The proposed method shows a robust output for all the SNR. As for the estimates of different

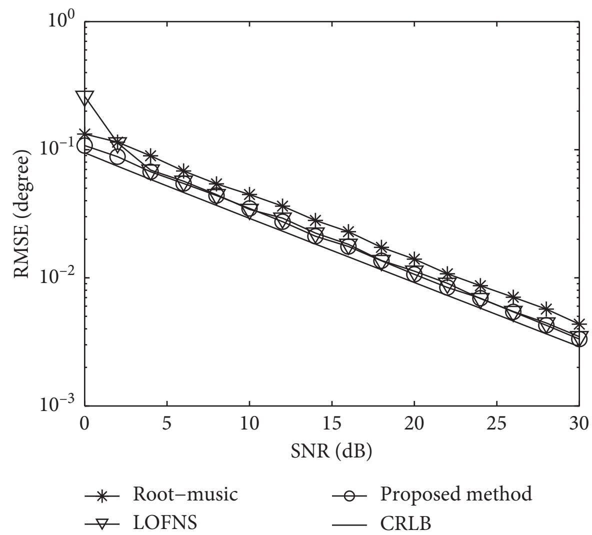

Figure 2: RMSEs versus SNR: 1st source.

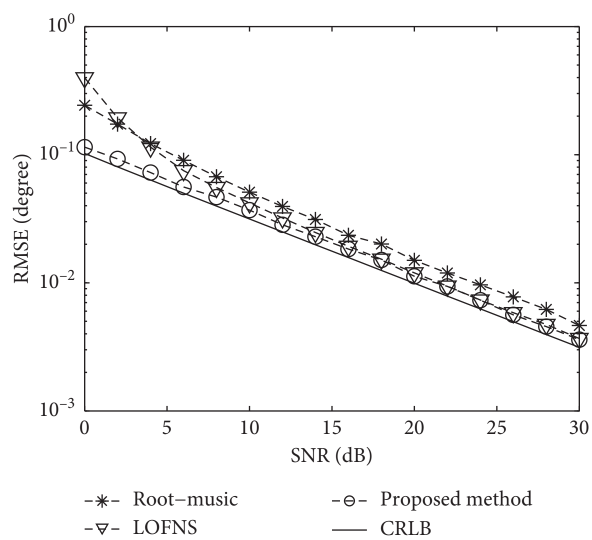

Figure 3: RMSEs versus SNR: 2nd source.

sources, we can see that the RMSEs with the same method are almost the same. The different directions of sources do not affect the estimation accuracy.

The second simulation studies the RMSEs in terms of the number of snapshots. Set the SNR at 15 dB, and the number of snapshots changes from 10 to 10000. The array is the same as that in the first simulation. The corresponding results are displayed in Figures 4 and 5. Similar to those RMSEs in Figures 2 and 3, the proposed method provides the most robust performance while root-MUSIC performs the poorest.

The computational efficiency will be studied in the third simulation. 1000 simulations are run in a PC, whose CPU is Intel (R) Core (TM) I7 8700 3.2 GHz and RAM is 8 GB, to get the total proceeding time for different methods. The results with 400 snapshots are shown in Table 1. We can see that the results of proceeding time of the three methods are almost

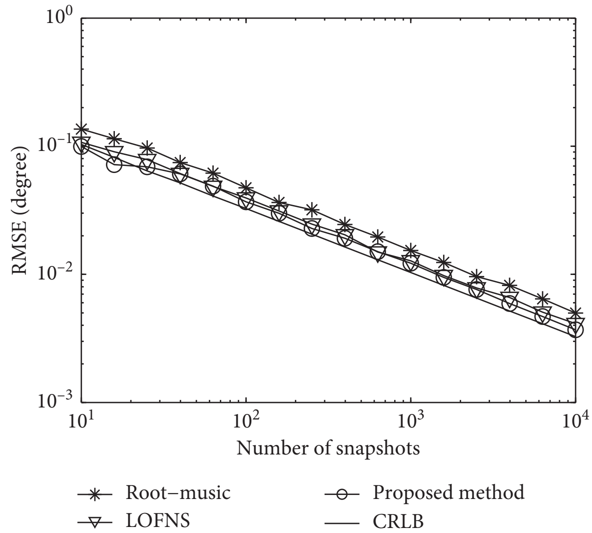

Figure 4: RMSEs versus number of snapshots.

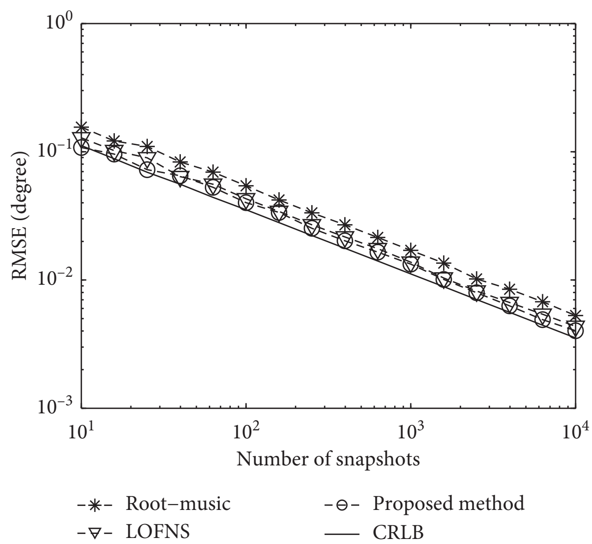

Figure 5: RMSEs versus number of snapshots.

Table 1: Total proceeding time with Matlab for 1000 times (in second).

Methods	Root-music	LOFNS	Proposed method
$M = 5$	0.3645	0.3545	0.4575
$M = 7$	0.5551	0.6222	0.6636
$M = 9$	0.6923	0.6929	0.7018
$M = 11$	0.7757	0.8646	0.7518
$M = 13$	1.1122	1.1589	1.1240
$M = 15$	1.7455	1.7939	1.7756

the same. As for the average proceeding time for one simulation, the difference is negligible.

5. Conclusion

In order to get rid of the dependency on the source number, a rooting method based on the oblique projection is proposed in this paper. By taking advantage of the property of nonstationary signal, the effect of noise can be eliminated. The elementary column transformation is applied and a matrix orthogonal with the steering matrix is constructed without the number of sources. At last, a rooting method is applied with the constructed orthogonal matrix. Simulation results verify the effectiveness of the proposed method, which can perform better than other existing methods even with the known exact source number.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the Key-Area Research and Development Program of Guangdong Province (2019B010145001, 2019B010142001, and 2019B010141002) and Science and Technology Program of Guangdong (2019A141401005).

References

[1] V. Behzadan, "Cyber-physical attacks on uas networks-challenges and open research problems," 2017, http://arxiv.org/abs/1702.01251.

[2] F.-G. Yan, S. Liu, J. Wang, M. Jin, and Y. Shen, "Fast doa estimation using co-prime array," *Electronics Letters*, vol. 54, no. 7, pp. 409-410, 2018.

[3] H. Krim and M. Viberg, "Two decades of array signal processing research: the parametric approach," *IEEE Signal Processing Magazine*, vol. 13, no. 4, pp. 67–94, 1996.

[4] Z. Zheng, J. Sun, W.-Q. Wang, and Y. Haifen, "Classification and localization of mixed near-field and far-field sources using mixed-order statistics," *Signal Processing*, vol. 143, pp. 134–139, 2018.

[5] A. Liu, Q. Yang, X. Zhang, and W. Deng, "Modified root music for co-prime linear arrays," *Electronics Letters*, vol. 54, no. 15, pp. 949–951, 2018.

[6] S. Chen, X. Dong, G. Xing, Z. Peng, W. Zhang, and G. Meng, "Separation of overlapped non-stationary signals by ridge path regrouping and intrinsic chirp component decomposition," *IEEE Sensors Journal*, vol. 17, no. 18, pp. 5994–6005, 2017.

[7] R. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Transactions on Antennas and Propagation*, vol. 34, no. 3, pp. 276–280, 1986.

[8] H. Chen, W. Zhu, and M. Swamy, "Real-valued esprit for two-dimensional doa estimation of noncircular signals for acoustic vector sensor array," in *Proceedings of the 2015 IEEE International Symposium on Circuits and Systems (ISCAS)*, May 2015.

[9] B. Friedlander, "The root-MUSIC algorithm for direction finding with interpolated arrays," *Signal Processing*, vol. 30, no. 1, pp. 15–29, 1993.

[10] W. Zuo, J. Xin, N. Zheng, and A. Sano, "Subspace-based localization of far-field and near-field signals without

eigendecomposition," *IEEE Transactions on Signal Processing*, vol. 66, no. 17, pp. 4461–4476, 2018.

[11] A. Ahmed, Y. D. Zhang, and B. Himed, "Effective nested array design for fourth-order cumulant-based DOA estimation," in *Proceedings of the Radar Conference*, pp. 0998–1002, Seattle, WA, USA, May 2017.

[12] Y. Hu, Y. Liu, and X. Wang, "DOA estimation of coherent signals on coprime arrays exploiting fourth-order cumulants," *Sensors*, vol. 17, no. 4, 2017.

[13] Z. Zheng, M. Fu, W.-Q. Wang, and H. C. So, "Mixed far-field and near-field source localization based on subarray cross-cumulant," *Signal Processing*, vol. 150, pp. 51–56, 2018.

[14] R. Challa and S. Shamsunder, "High-order subspace-based algorithms for passive localization of near-field sources," in *Proceedings of the Conference Record of the Twenty-Ninth Asilomar Conference on Signals, Systems and Computers*, November 1995.

[15] J. M. Mendel, "Tutorial on higher-order statistics (spectra) in signal processing and system theory: theoretical results and some applications," *Proceedings of the IEEE*, vol. 79, no. 3, pp. 278–305, 1991.

[16] K. Wang, L. Wang, J. R. Shang, and X. X. Qu, "Mixed near-field and far-field source localization based on uniform linear array partition," *IEEE Sensors Journal*, vol. 16, no. 22, pp. 8083–8090, 2016.

[17] Z. Zheng, M. Fu, D. Jiang, W.-Q. Wang, and S. Zhang, "Localization of mixed far-field and near-field sources via cumulant matrix reconstruction," *IEEE Sensors Journal*, vol. 18, no. 18, pp. 7671–7680, 2018.

[18] A. Paulraj and T. Kailath, "Eigenstructure methods for direction of arrival estimation in the presence of unknown noise fields," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 34, no. 1, pp. 13–20, 1986.