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Research Article

Nonlinear Cointegration and Asymmetric Adjustment in Purchasing Power Parity of the USA, Germany, and Pakistan

Kashif Ali,[1] Hafsa Hina,[2] Muhammad Ijaz ⓘ,[3] and Mahmoud El-Morshedy ⓘ[4,5]

[1]Department of Statistics, University of Peshawar, Peshawar, Pakistan
[2]Department of Econometrics and Statistics, PIDE, Islamabad, Pakistan
[3]Department of Mathematics and Statistics, The University of Haripur, Haripur, Pakistan
[4]Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam Bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia
[5]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

Correspondence should be addressed to Muhammad Ijaz; ijaz.statistics@gmail.com

The current study explores nonlinear cointegration as well as asymmetric adjustment to investigate the long-run purchasing power parity in three major trading partners of Pakistan. The ESTAR and LSTAR models were used to investigate the behavior of the nominal exchange rates. The findings declared that series follows the nonlinear exchange rate. The asymmetric behavior of the exchange rate allows the threshold cointegration model to be implemented. In the case of Pakistan-China, the result suggests that long-run PPP holds. As a result, trading will be more profitable if the exchange rate is varied in relation to major trading partners rather than just the US dollar.

1. Introduction

Globalization and financial liberalization have significantly increased the role of international finance and trade, whereas small open economies like Pakistan can only creep the benefits of globalization and financial liberalization if they are maintaining a stable exchange rate. The exchange rate volatility has a direct impact on the profitability of multinational corporations and financial institutions. A stable exchange rate may help in reducing the operational risk of enterprise and financial institutions, by guiding the evaluation of the performance of investments, financing, and hedging decision [1]. Instability in the exchange rate is emerged due to poor economic fundamentals, internal and external shocks to the economy, and changes in existing policy structures, which lead to change in the behavior of the exchange rate series from linear to nonlinear.

The purchasing power parity (PPP) can explain the long-run equilibrium of the exchange rate. PPP serves as a standard for calculating exchange rates and determining whether real exchange shocks diminish over time. The concept of PPP is closely related to the behavior of exchange rates, which often help to detect whether implemented policies are responsible for disequilibrium in an open economy or not. The PPP theory of exchange rate which is originally advanced by Cassel [2] states that, in the floating exchange rates system, where the trade is free and transport, speculative expectations costs and capital flows are absent; the nominal exchange rate is the ratio of that two countries price level.

Aminifard [4] studied the trade relations and long-run purchasing power parity (PPP) between the selected East Asian Countries and Iran, Korea, and the World Economy; Zhao [5] worked on the nonlinear cointegration; Enders [6] and Patel [7] used the cointegration test by Engle and Granger [8], but they fail to accept the validity of long-run PPP. Ahmad and Khan [9] studied the long-run PPP for Pakistan and for some Asian countries and verified the failure of the PPP theory. Basher and Mohsin [10] also test the relative form of PPP using panel cointegration and reject

the PPP hypothesis. The problem at the core of this methodology was that when the data-generating process is nonlinear, then standard unit root tests, as well as cointegration tests, have insufficient power to detect stationarity. Another problem with traditional cointegration tests is that it assumes symmetric adjustment, but it is quite possible that an exchange of country is asymmetric. For more detailed study, we refer to the studies by Abdul Qayyum et al. [11]; Abuaf [12]; Basher and Razzaque [13]; Chumrusphonlert and Enders [14]; Enders [6]; Enders [15]; Franses [16]; Hansen and Seo [17]; Husain [18]; Hylleberg et al. [19]; Liu [20]; Miron and Beaulieu [21]; Sollis [22]; and Tong [23].

Nonlinearity in the exchange rate allows for a more general form of PPP where the adjustments of foreign as well as domestic prices need not be proportional and symmetric to exchange rate. In recent times, the Engle–Granger linear cointegration has been extended into a threshold framework by many researchers such as Johansen's [24] research allows for threshold short-run dynamics in error-correction models. Balke and Fomby [25] introduce a threshold cointegration model that accommodates for threshold adjustments towards a long-run (LR) equilibrium while Enders and Falk [15] employ a similar model to investigate the purchasing power parity (PPP).

As we discussed above that many researchers test the long-run PPP by the method of Engle–Granger [8] and Johansen and Juselius [26], some of the results in favor of PPP and some of them reject the PPP. After the development of the threshold cointegration technique by Balke and Fomby [25] and Hansen and Soe [27], many researchers such as Chumrusphonlert and Enders [14]; Enders and Siklos [28]; and Henson and Seo [27] had tested the long-run PPP by applying the nonlinear cointegration method. Phiri [29] defined the nonlinear effects in purchasing power parity, Ang et al. [30]; discussed deviations in PPP and exchange rate, Arize et al. [31] discussed the nonlinear ARDl approach and PPP for more than 80 countries, Tipoy et al. [32] worked out the exchange rate misalignment and economic growth using nonlinear panel cointegration, Bahmani-Oskooee et al. [33] explored the asymmetric cointegration approach for the China and its trading partners, Iqbal et al. [34] studied the asymmetric nonlinear cointegration for Pakistan and its other trading partners. Thus, the existence of the long-run PPP is supported by most of the research studies.

In practice, the linear models are preferring to use when the dependent variables are linear in nature. In the current study, the dependent variables, i.e., the exchange rates of Pak-China, Pak-USA, and Pak-Germany were tested by using the STAR model and found nonlinear. Thus, it is allowed us to use the nonlinear cointegration procedure.

The main objective of the research is to evaluate the nominal exchange rate adjustment via PPP theory for the top three trading partners of Pakistan (USA, Germany, and China), instead of assuming linear cointegration as done by many researchers in the case of Pakistan. This study contributes to the existing literature of Pakistan, in the case of Pakistan; no one incorporated the threshold cointegration in

the purchasing power parity hypothesis. In particular, the linear behavior of the nominal exchange rate will examine through exponential smooth threshold autoregressive (ESTAR) and logistic smooth threshold autoregressive (LSTAR) models. Furthermore, the speed adjustment of the exchange rate toward the long-run equilibrium is also being measured through the threshold error-correction model.

1.1. Novelty and Significance of the Study.

1.1. Novelty and Significance of the Study. In existing literature, many researchers have treated the exchange rate as a linear model. This study shows that the exchange rate follows a nonlinear model in the case of Pakistan. For this purpose, we have considered the nonlinear cointegration technique so that to delineate the long-run relationship of the exchange rate of Pakistan with that of the toped three trader countries, i.e., USA, China, and Germany. Moreover, the main aim of the study is to analyze the theory of PPP by considering the exchange rate of the top three trading partners of Pakistan.

2. Materials and Methodology

2.1. Data and Variable Description. This dataset consists of monthly observations covering the period from 1982 to 2013. The nominal exchange rate of Pak rupee is measured against per unit of US dollar, China yuan, and German euro and denoted by EX_{P-U}, EX_{P-C} and EX_{P-G}. p is the consumer price index of Pakistan and p_U^*, p_C^*, and p_G^* are the CPI for USA, China, and Germany. The dataset is taken from the IMF website.

2.2. Econometric Modeling for PPP. The advocates of the law of one price claim that PPP holds internationally for the same bundle of goods through the adjustment of the exchange rate. The relative form of PPP is defined by Cassel [2] with the following mathematical form:

$$\Delta e_t = \beta_0 + \beta_1 \left(\Delta p_t - \Delta p_t^* \right) + \varepsilon_t, \tag{1}$$

where "Δ" is the first difference of the series; p, p^*, and ε_t stand for the natural log of the domestic price, foreign price, and nominal exchange rate, respectively, while ε_t is the noise.

2.3. Unit Root Test of Beaulieu and Miron. This technique is used to check whether the data under consideration is stationary or nonstationary. The unit root test by Beaulieu and Miron [21] is usually used to detect the nonseasonal and seasonal unit root in monthly data. Beaulieu and Miron [21] proposed an auxiliary regression model for the identification of unit root given as follows:

$$y_{13T} = \alpha + \beta t + \sum_{i=1}^{11} \gamma_i s_i + \sum_{i=1}^{12} \pi_i y_{it-1} + \varepsilon_t, \tag{2}$$

where y_t is the series of interest; the deterministic part is consisting of any combination of constant (α), a linear trend (t), and seasonal dummies (s_i). ε_t is the white noise error term. Table 1 illustrates all possible hypotheses for the monthly nonseasonal and seasonal unit roots.

TABLE 1: Null and alternative hypotheses for monthly unit root test.

Test of unit root at different frequencies	Null and alternative hypotheses
Zero frequency unit root	H_0^A, $\pi_1 = 0$ vs H_1^A, $\pi_1 < 0$
Biannual frequency unit root	H_0^B, $\pi_2 = 0$ vs H_1^B, $\pi_2 < 0$
Seasonal frequency unit root	H_0^j, $\pi_i = \pi_{i+1} = 0$ vs H_1^j, at least one of them is not equal to zero where $j = C, D, E, F, G$ and $i = 3, 4, 5, \ldots., 12$

The existence of seasonal unit roots can be tested by t-statistic in each frequency with the null hypotheses $H_0 = \pi_i = 0$, where $i = 2, 3, \ldots, 12$, or by means of F-statistic associated with joint hypotheses $H_0 = \pi_i = \pi_{i+1} = 0$, where $i = 3, 5, 7, 9, 11$.

2.4. Testing Linearity of Variables.

The STAR model is used to check whether the behavior of series is linear or nonlinear. Initially, the STAR model was proposed by Chan and Tong [35]; Terasvirta [36] and others. STAR model has further two subclasses, i.e., logistic STAR (LSTAR) model, where the function of weights has logistic function and the exponential STAR (ESTAR) model, in which the function of weights has the form of an exponential function.

The LSTAR model is defined by

$$y_t = \partial_0 + \partial_1 y_{t-1} + \cdots + \partial_p y_{t-p} + K\left[\beta_0 + \beta_1 y_{t-1} + \cdots + \beta_p y_{t-p}\right] + \varepsilon_t, \tag{3}$$

where

$$K = \left[1 + \exp\left(-\gamma\left(y_{t-d} - c\right)\right)\right]^{-1}. \tag{4}$$

In (4), the parameter γ is called the smoothness parameter, which is responsible for the smoothness of G, while G is bounded as $0 \le K \le 1$. c is the threshold parameter. y_{t-d} is called the transition variable (d is the delay parameter).

The ESTAR is the same as (3), but the value of K is different as

$$K = 1 - \exp\left[-\gamma\left(y_{t-d} - c\right)^2\right]. \tag{5}$$

The Lagrange multiplier (LM) test cannot be used directly to test whether a series has LSTAR or ESTAR behavior as the values of parameters in these models are unidentified under the null hypothesis which is that the model is in linear form. As a result, Terasvirta [36] creates a background for determining whether a series is best modelled as an LSTAR or ESTAR process. Terasvirta [36] proposed an auxiliary regression test to detect the presence of LSTAR behavior:

$$e_t = \partial_0 + \partial_1 y_{t-1} + \cdots + \partial_p y_{t-p} + \partial_{11} y_{t-1} y_{t-d} + \cdots + \partial_{1p} y_{t-p} y_{t-d} + \partial_{21} y_{t-1} y_{t-d}^2 + \cdots + \partial_{2p} y_{t-1} y_{t-d}^2 \tag{6}$$
$$+ \partial_{31} y_{t-1} y_{t-d}^3 + \cdots + \partial_{3p} y_{t-1} y_{t-d}^3 + \varepsilon_t.$$

For more detail, see [36].

2.5. Threshold Cointegration and Asymmetric Adjustment.

The Engel–Granger's [8] methodology for PPP begins by proposing a long-run equilibrium relationship of the form:

$$E_t = \beta_0 + \beta_1 p_t - \beta_2 p_t^* + \mu_t, \tag{7}$$

where E_t is the log of the nominal exchange rate; p_t defines the log of the domestic price level; p_t^* represents the foreign price levels; and μ_t is the disturbance term.

After estimating the (7) by OLS methodology, the next step in the Engel–Granger procedure is to test the stationarity of the linear combination of the nominal exchange rate and prices. Residuals of (7) are commonly used to measure the linear combination. Therefore, the stationarity of the residuals series conforms the existence of PPP theory in long run. Stationarity of residual series is tested by applying ADF auxiliary regression, i.e., provided under

$$\Delta\mu_t = \rho\mu_{t-1} + \epsilon_t. \tag{8}$$

If the hypothesis $\rho = 0$ is rejected, then the sequence of μ_t is stationary. In such a case, the PPP holds the long-run exchange rate.

The standard cointegration test implicitly assumes a symmetric adjustment process. If the exchange rate adjustment is asymmetric, then (8) is unspecified. The errors from (7) are estimated in the form of a threshold autoregressive (TAR) defined by Enders and Siklos [28]. The mathematical form of the TAR model is given by

$$\Delta\mu_t = I_t \rho_1 \mu_{t-1} + (1 - I_t)\rho_2 \mu_{t-1} + \epsilon_t, \tag{9}$$

where I_t is the indicator function such that

$$I_t = \begin{cases} 0, & \text{for } \mu_{t-1} < \tau, \\ 1, & \text{for } \mu_{t-1} > \tau, \end{cases} \tag{10}$$

and τ is the value of the threshold.

Asymmetric adjustment is held if $\rho_1 \ne \rho_2$. When the term μ_{t-1} is positive, the value of adjustment is $\rho_1\mu_{t-1}$ which is positive, and when the term μ_{t-1} is negative, the value of adjustment $\rho_2\mu_{t-1}$ is negative. A sufficient but not necessity condition for stationarity is μ_t which is $-2 < (\rho_1, \rho_2) < 0$. If the variance of μ_t is sufficiently large, it is also possible for one value of ρ_j to be between minus two and zero and for the other value equal to zero [14]. For a nonstandard distribution, the value of F-statistic with the null hypothesis will be $\rho_1 = \rho_2 = 0$; for this purpose, Enders and Siklos [28] used the expression Φ_μ. If the null hypothesis of $\rho_1 = \rho_2 = 0$ is not accepted, it is quite possible to test for symmetric adjustment, i.e., $\rho_1 = \rho_2$ using a standard procedure of F-test. If $\rho_1 = \rho_2$, the adjustment is symmetric; a special case of the equation is the Engel–Granger test for cointegration (10).

2.6. Empirical Study.

For empirical study, the dataset is taken from the international monitoring fund (IMA) with

TABLE 2: Unit root test at first difference.

Variable	Model	$\pi_1 = 0$	$\pi_2 = 0$	$\pi_3 = \pi_4 = 0$	$\pi_5 = \pi_6 = 0$	$\pi_7 = \pi_8 = 0$	$\pi_9 = \pi_{10} = 0$	$\pi_{11} = \pi_{12} = 0$	Order of integer
EX_{Pak_USA}	C^*, t^{**}, d^{**}	-4.65^* (-2.82)	-5.32^* (-1.89)	23.20^* (3.05)	23.121^* (3.08)	23.39^* (3.14)	25.17^* (3.05)	20.23^* (3.07)	I(1)
EX_{Pak_China}	C^*, t^{**}, d^{**}	-4.79^* (-2.78)	-4.56^* (-1.89)	31.09^* (3.05)	25.901^* (3.08)	25.05^* (3.11)	21.46^* (3.05)	22.80^* (3.07)	I(1)
$EX_{Pak\_Germany}$	C^*, t^{**}, d^{**}	-5.85^* (-2.78)	-5.99^* (-1.89)	27.80^* (3.05)	29.12^* (3.08)	31.12^* (3.14)	26.44^* (3.05)	31.82^* (3.07)	I(1)
$CPI_{Germany}$	C^{**}, t^{**}, d^{**}	-5.80^* (-1.87)	-5.33^* (-1.89)	29.29^* (3.05)	32.32^* (3.11)	31.12^* (3.16)	26.00^* (3.07)	26.54^* (3.08)	I(1)
CPI_{Pak}	C^*, t^{**}, d^*	-3.49^* (-2.81)	-5.96^* (-2.97)	34.77^* (6.35)	27.31^* (6.37)	43.99^* (6.29)	20.90^* (6.36)	24.36^* (6.31)	I(1)
CPI_{USA}	C^*, t^*, d^*	-4.49^* (-2.76)	-6.50^* (-2.79)	45.49^* (6.35)	45.05^* (6.37)	29.85^* (6.29)	48.35^* (6.36)	16.16^* (6.31)	I(1)
CPI_{China}	C^*, t^{**}, d^{**}	-2.10^* (-1.87)	-3.82^* (-1.89)	12.17^* (3.05)	17.71^* (3.11)	11.98^* (3.16)	20.40^* (3.07)	18.85^* (3.08)	I(1)

the URL given at the end of the paper. The foremost and necessary step in the implementation of the cointegration test is to examine the stationarity of individual variables by using the Beaulieu and Miron test. Table 2 reflects the result of the unit root test. The results describe those variables are nonstationary at the level and stationary at first difference.

All variables are in the log form and integrated of order one I (1). The * indicates significant results at the 5% level of significance while ** shows insignificant results. The second column shows the existence of constant (c), trend (t), and dummies (d) in the model. The critical values enclosed in parenthesis are taken from Beaulieu and Miron [21] and recently cited by Philip Hans Franses and Bart Hobijn [37].

The next step is to check whether the behavior of the Pak rupee exchange rate against the US dollar, China yuan, and German euro series is linear or nonlinear; we have considered the STAR model. Furthermore, whether the series behave like ESTAR or LSTAR model, for this purpose, STAR model is used. The identification of linearity vs nonlinearity is carried out through the Akaike information criteria (AIC). Table 3 defines the result.

On the basis of the above analysis (Table 3), it is determined that the exchange rate series shows asymmetric behavior; in this case, linear cointegration procedure is misleading. A proper way to introduce by Enders and Siklos [28] is that the residuals from a linear combination of the exchange rate and prices are estimated in the form of threshold autoregressive (TAR) model. As the threshold, level of $\widehat{\mu}_t$ series is unknown. Therefore, we had followed Chan's [38] methodology for identifying the unknown threshold. Chan [38] illustrated that the consistent estimate of the threshold will be the smallest residual sum of square (RSS). The results of threshold cointegration for the exchange rate of Pakistan against the US dollar, Chinese yuan, and German euro are presented in Table 4.

The critical values with alpha = 10% are taken from Enders and Siklos [28].

Table 4 explicates the estimated values of the speed of adjustment under each regime. The result shows that the

TABLE 3: Linear vs nonlinear behavior of exchange rates.

Variable name	Lag	Linear vs nonlinear	ESTAR vs LSTAR
EX_{Pak_USA}	2	Nonlinear	ESTAR
EX_{Pak_China}	2	Nonlinear	LSTAR
$EX_{Pak\_Germany}$	1	Nonlinear	LSTAR

TABLE 4: Estimated equations using threshold cointegration test and consistent estimates (based country: Pakistan).

Country	ρ_1	ρ_2	$\rho_1 = \rho_2$	ϕ_μ	Lag	Flag	Threshold value
USA	−0031	−0.032	2.432	6.67	2	TAR	0.5991
China	−0.002	−0.081	7.010	6.67	12	TAR	0.00041
Germany	−0.0151	−0.026	1.895	6.67	0	TAR	−0.03257

exchange rates in the case of the USA and Germany do not converge from lower regime to upper regime and indicates the existence of linear cointegration. While in the case of China, it shows convergence from one regime to another and nonlinear cointegration. It is concluded that the linear form of PPP holds in the case of the exchange rate of Pakistan against the USA dollar and German euro while the nonlinear form of PPP holds in the case of China. The consistent estimated values of the threshold are obtained.

To study a long-run equilibrium relationship of variables, one can use an asymmetric error-correction model. In the case of Pakistan and China, the estimated long-run relationship is

$$EX_{Pak\_China} = -2.0919 + 0.702706p_t - 0.277305p_t^*,$$

$$\text{St. error: } (0.078) \ (0.014) \ (0.028).$$

$$(11)$$

Using the long-run relationship of the nominal exchange rate and prices in a case of China, the estimated error-correction equation with the consistent estimates is given by

$$
\begin{aligned}
\Delta EX_{Pak-China} = &-0.036716ec_{t-1}^+ - 0.065466ec_{t-1}^- + 0.042\Delta P_{t-1} + 0.130\Delta P_{t-2} + 0.238\Delta P_{t-3} - 0.214\Delta P_{t-4} \\
&- 0.052\Delta P_{t-1}^* - 0.134\Delta P_{t-2}^* - 0.029\Delta P_{t-3}^* - 0.014\Delta P_{t-4}^* + 0.093\Delta EX_{t-1} + 0.044\Delta EX_{t-2} \\
&+ 0.024\Delta EX_{t-3} + 0.039\Delta EX_{t-4}, \\
&\text{Adj} - R^2 = 0.07101, \\
&\text{Auto}_{\chi^2_{(12)}} = 0.2143, \\
&\text{Hetro}_{\chi^2_{(12)}} = 0.99,
\end{aligned}
$$

$$(12)$$

where

$$ec_{t-1}^+ = (1 - I_t)\left(e_t + 2.0919 - 0.702706p_t + 0.277305p_t^*\right),$$

$$(13)$$

and I_t is the indicator function and is defined as

$$I_t = \begin{cases} 1, & \text{if } \Delta\widehat{\mu}_{t-1} \geq 0.00041, \\ 0, & \text{if } \Delta\widehat{\mu}_{t-1} < 0.00041. \end{cases}$$

$$(14)$$

The coefficients of ec_{t-1}^+ define the speed adjustment of positive deviations from PPP while ec_{t-1}^- determines the speed adjustment of negative deviations from PPP. The t-

statistics 2.52545 and −3.30908 conclude that both coefficients are significant at a 5% level. The point estimates imply that the exchange rate adjusts by 3.67% of a positive gap from long-run PPP and by 6.54% of a negative gap. The point is that positive deviations from PPP are eliminated faster than negative deviations.

3. Conclusion and Policy Implication

The goal of the paper is to estimate the long-run relationship in PPP by using threshold cointegration along with an asymmetric adjustment in three major trading partners of Pakistan; these are the United States, China, and Germany. The study used time series monthly data over the period 1982 to 2013. To check the order of integration of variables, we considered a seasonal unit root test. Accordingly, all variables, i.e., nominal exchange rate, domestic price level, and foreign price level are stationary at the first difference for all countries. In order to check whether the behavior of the exchange rate series is linear or nonlinear, we use STAR model and found nonlinear behavior in the case of all exchange rate series. Further, we specify whether the nonlinear behavior of the exchange rate series is according to ESTAR or LSTAR and selected ESTAR behavior in favor of LSTAR for Pak-USA and LSTAR for Pak-China and Pak-Germany exchange rates. The asymmetric behavior of the exchange rates series allowed executing the threshold cointegration suggested by Enders and Siklos [28].

The findings of PPP suggest that it holds between Pakistan and China, while in the case of Pakistan vs USA, it does not hold. Thus, Pakistan needs to increase its exports to China; trade and investment with China will bring advantage to Pakistan. The trade performance of Pakistan can be improved if the policy makers monitor the exchange rate fluctuation in major trading partners. Trading will be more beneficial if the exchange rate varied with respect to major trade partners rather than only with the US dollar.

The research study may be conducted to cover the data during COVID-19 and record the fluctuations in the exchange rate of the top three trade partners of Pakistan. A future research study may be conducted with the nonlinear cointegration technique with the ARDL approach. Similarly, the same study may be extended to add some other trade partners.

Data Availability

The data used to support this study are available at https://data.imf.org/?sk=4c514d48-b6ba-49ed-8ab9-52b0c1a0179b&sId=14091512409.

Disclosure

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Conflicts of Interest

The authors declare that they have no conflicts of interest.

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