Research Article

Chaotic Dynamics Analysis Based on Financial Time Series

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It is a common phenomenon in the field of financial research to study the dynamic of financial market and explore the complexity of financial system by using various complex scientific methods. In this paper, the chaotic dynamic properties of financial time series are analyzed. Firstly, the nonlinear characteristics of the data are discussed through the empirical analysis of agriculture index data; the daily agriculture index returns can be decomposed into the different scales based on wavelet analysis. Secondly, the dynamic system of some nonlinear characteristic data is established according to the Taylor series expansion form, and the corresponding dynamic characteristics are analyzed. Finally, the bifurcation diagram of the system shows complicated bifurcation phenomena, which provides a perspective for the analysis of chaotic phenomena of economic data.

1. Introduction

With the development and progress of the society, the financial system we are facing is becoming more and more complex, and the fundamental reason for this complexity is the nonlinearity of the financial system. For financial data, it is usually difficult to establish mathematical models. It is a common phenomenon in the field of financial research to study the dynamics of financial market and explore the complexity of financial system with various complex scientific methods [1–4]. In the past, there have been some different research directions through different methods to study financial problems, such as economic physics, chaos economics, and dynamic economics [5–9]. For many years, researchers have devoted themselves to the study of the relationship between dynamics and time data [10–12]. In 1996, Chen et al. used the Hankel matrix method to discuss dynamic system identification of time data [13, 14]. In 2003, Lu et al. used the least square method to reconstruct the dynamic system, based on noise observation data [15]. In 2007, Yu et al. discussed dynamic system synchroniztion and parameter identification of time data [16]. In 2013, Liu et al. reconstructed time series data through recursive graph of power system [17]. In [18], the authors discussed the evolution process of the dynamic system through feedback control method.

Most of the existing research studies about the identification methods of nonlinear dynamic characteristics of data mainly analyze some properties of data from the macro level. The hidden economic relations (regression, cointegration, causality, etc.) in financial data are a dynamic evolution process. Once the dynamic mechanism of financial data evolution is established, it is of great significance to understand and predict the evolution of financial data.

As far as we know, there are few studies on the internal structure of financial data from the micro scale of complex dynamic system, and financial data in the index market have also not been tested by chaotic dynamics at different time scales. In this paper, we firstly analyze the nonlinear characteristics of the data; then the dynamic system is established according to the Taylor series expansion form, and the dynamic properties of the data are also analyzed.

The remainder of the paper is organized as follows: Section 2 describes the data and empirical analysis. In Section 3, the nonlinear dynamic characteristics of the data are verified. Finally, the conclusion is given in Section 4.
2. Data and Empirical Analysis

The Shanghai Stock Exchange and Shenzhen Stock Exchange are two stock exchanges in China. Agriculture listed companies is the representative of the advanced agriculture productivity. The agricultural products index is issued by Shenzhen Stock Exchange, which is composed of 26 index components. It is a very liquid component of China’s agricultural stock market. The index reflects the average value of the stock price of the listed agricultural companies in the Shenzhen stock market.

The data, daily observation on returns to the agriculture index from February 2, 2009 to December 24, 2015 (a total of 1680 observations) were collected from the Wind database in our empirical study.

Let \( p_t \) be the closing price of index on day \( t \). As known to all, historical prices are nonstationary, which can be solved by calculating the returns. The daily price return \( r_t \) can be calculated as its logarithmic difference; that is,

\[
r_t = \log \left( \frac{p_{t+1}}{p_t} \right).
\]

The agriculture index is displayed in Figure 1. The graphical representation of returns is illustrated as Figure 2. To decompose a given time series on a scale-by-scale basis, the wavelet analysis is introduced. The early research on the application of wavelet method in economics and finance was carried out by Ramsey and Usikov [19] and Ramsey and Zhang [20]. More recent contributions, among others, are Lee [21, 22], Lin and Stevenson [23], Gencay and Selcuk [24–27], Hong and Kao [28], and In and Kim [29]. In recent years, wavelet has been applied more and more in the field of economy and finance [30–32].

Wavelet analysis is an improvement of Fourier analysis. It provides a powerful method to analyze the time series of signals, images, and other types of data, which contain different powers at different frequencies. This method is called the wavelet multiscale method, which means to decompose the given time series scale-by-scale. The main advantage of wavelet analysis is that it can decompose data into multiple time scales, and it can process nonstationary data, locate in time, and distinguish signals based on the time scale of analysis.

Krishnan et al. proposed the use of the continuous Wavelet transform with a Daubechies mother wavelet of order 4 (DB4) as the mother wavelet and showed its efficacy using simulated data from the ASCE benchmark structure [33]. They also noted that the DB4 wavelet was a good choice as it has a Discrete Wavelet Transform (DWT) counterpart, which would be more suitable for sensor level processing. It is implemented using the DB4 mother wavelet in this study, and the DWT appears to be better suited to smart sensing applications.

Considering the sample size and the length of the wavelet filter, we settle on the MODWT based on the Daubechies external phase wavelet filter of length 4 (Db(4)). As we take advantage of the daily return on data, the daily price returns can be divided into 1–6 scales.

\[
r = A_6 + D_6 + D_5 + D_4 + D_3 + D_2 + D_1.
\]

Wavelet scale 1 means 2–4 day dynamics, scale 2 means 4 to 8 days of dynamic, scale 3 means 8 to 16 days of dynamics, and scale 4 means 16 to 32 days of dynamic. Also scale 5 is associated with 32–64 day dynamics, scale 6 is associated with 64–128 day dynamics, and scale 7 corresponds with 128–256 day dynamics, that is, approximately 1 year. The recomposed crystals \( D_1 \) and \( D_6 \) mean the return on the market portfolio at scales 1 and 6, respectively. As we see, \( D_1 \) depicts the high frequency fluctuations of the market portfolio, whereas \( D_6 \) depicts its long-term behavior (see Figure 3).

Descriptive statistics of the returns in different scales are presented in Table 1.

![Image](image1.png)

Chaotic dynamics are surveyed in different scales by the Lyapunov exponent \( \lambda \).

Shintani and Linton obtained the asymptotic distribution of \( \lambda \) based on a central limit theorem from a functional Markov process [34]. Ahmed [35] approximated the variance of the largest Lyapunov exponent as

\[
\Sigma = \frac{1}{M} \sum_{j=M+1}^{j=M-1} \left[ \xi \left( \frac{j}{M} \right) \right] \sum_{t=1}^{M} \tilde{\eta}_{t-j}.\]

The test statistic is asymptotically normal:

\[
\bar{W} = \sqrt{M} \cdot \hat{\lambda}_M \longrightarrow \text{Asymptotically } N(0, \Sigma).
\]

Let null hypothesis be \( H_0: \lambda \geq 0 \), and its rejection \( H_1: \lambda < 0 \). The rejection of the null hypothesis provides a strong evidence of no chaotic dynamics. The details are omitted here.

The results are summarized in Table 2. In fact, we may not accept chaos at a certain level of significance (for example, \( p \) is equal to 0.05) for negative \( \lambda \). As to the raw signal for the Chinese agriculture index, we can see that lambda is -0.4649 and the \( p \) value is almost equal to zero. The test has rejected the null hypothesis of chaotic dynamics for the Chinese agriculture index. Result has shown that returns are stochastic and not chaotic.

When we decompose a given agriculture index time series on a scale-by-scale basis, different chaos properties can be found. Lambda is negative and equal to -0.1264, -0.0326, and -0.0046 in \( D_3, D_3, \) and \( D_6, \) respectively. Associated to the \( p \) value, the test has rejected the null hypothesis of chaotic dynamics at the shortest scale (2–4 day dynamics). The test cannot reject the null hypothesis in the case of \( D_7 \). Lambda is positive and equal to 0.0213, 0.0025, 0.0199, and 0.4725 in \( D_2, D_3, D_6, \) and \( A_6, \) respectively. So at the intermediate time scale and long time scale, the test cannot reject the null hypothesis.

3. Chaos Dynamic Characteristics of Agriculture Index Time Series

Generally, when Lyapunov exponent is positive, the system exhibits chaotic behavior. In this section, we use scale 2, that is, \( D_2, \) data as an example to further verify the nonlinear
**Figure 1**: The Chinese agriculture index from February 2, 2009 to December 24, 2015.

**Figure 2**: The Chinese daily agriculture index returns from February 2, 2009 to December 24, 2015.

**Figure 3**: Six-level decomposition of the agriculture index return.
dynamic characteristics of the data. Of course, a similar analysis can be made for that the other Lyapunov exponents is positive.

In the following, we discuss the growth and change of agricultural index, and set its simplest Taylor series expansion form as

\[
\frac{dx}{dt} = a_1 x + a_2 x^2 .
\]  

(5)

When \( x = 0 \), \((dx/dt) = 0\), so, equation (5) can be rewritten as

\[
\frac{dx}{dt} = a_1 x + a_2 x^2 = a_1 x \left( 1 - \frac{a_2}{a_1} x \right).
\]  

(6)

Equation (6) is a typical Logistic equation, which is a nonlinear feedback system, and it can be expressed by the following difference equation:

\[
x_i = a_2 x_{i-1} \left( 1 - a_2 x_{i-1} \right).
\]  

(7)

In the paper, the evolution process of agricultural index market is analyzed based on Logistic dynamic model. Also, the time series data scale 2, that is, \( D_2 \), is regarded as a series of discrete values of the logistic dynamic model, and the differential equation model is obtained:

\[
x_i = 0.6584 x_{i-1} - 0.1765 x_{i-1}^2.
\]  

(8)

If equation (8) has only one parameter to change,

\[
x_i = a x_{i-1} - 0.1765 x_{i-1}^2.
\]  

(9)

Equation (9) will show the complicated nonlinear phenomena.

According to the stability criterion of the fixed point of the difference equation, if the critical state of equation (9) exists, that is, \(|\lambda| = |f'(x)| = 1\), there are two states happening. On one hand, if \(|\lambda| = |f'(x)| < 1\), the trend will be stable. On the other hand, if \(|\lambda| = |f'(x)| > 1\), the trend will be unstable.

For equation (9), when \( x = ax - 0.1765x^2 \), there are two fixed points, \( x_1 = 0 \), \( x_2 = (a - 1)/0.1765 \).

Obviously, \( x_1 = 0 \) is a vanishing fixed point, and it does not make any sense.

From \( \lambda = f'(x) = 2 - a \), if \(|2 - a| = 1\), that is, \( a = 1 \) or \( a = 3 \), the system is in the critical state. If \(|2 - a| > 1\), \( a < 1 \) or \( a > 3 \), the system is unstable and monotonous divergent.

In equation (8), \( a = 0.6584 < 1 \), so system (8) is unstable. Instability is a prerequisite for the generation of chaos in the system, which is exactly consistent with \( D_2 \) data’s Lyapunov positive exponent.

The bifurcation diagram would be far better to summarize all of the possible behaviors as the parameter varies on one diagram. For \( a \in [-2, 4] \), the bifurcation diagram of system (9) shows the complicated bifurcation phenomena (see Figure 4).

From Figure 4, when the parameter \( a \) lies in \([-1.57, -1] \cup [3, 5.7] \), the trajectory of ordinate fluctuates greatly with the increase of parameters, and the system undergoes period doubling evolution. If the parameter \( a \) lies in the range \([-2, -1.57) \cup (3.57, 4] \), the system shows chaotic properties.
4. Conclusion

In recent decades, chaos theory has attracted the attention of many financial analysts and economists. Multiscale wavelet decomposition can analyze time and frequency at the same time. It is a valuable method to study and forecast the complex dynamics of economic time series. In this paper, daily price returns have been decomposed into different scales based on the wavelet method. Chaos test has shown that the rate of return is random and not chaotic in the shortest scale, but in the medium- and long-term scales, the test cannot reject the null hypothesis, and the financial data have shown some chaotic characteristics. We have taken scale 2 with 4–8 day dynamics as an example to further verify the nonlinear dynamic characteristics of the data. Based on the logistic model to describe the evolution of the index, the bifurcation diagram of the system has been done and shown the complicated bifurcation phenomena. As the bifurcation behavior in the economic system will increase the instability of the system. Therefore, the control of the bifurcation of the economic system will help to reduce the risk of the system, which will be our next research topic.

Data Availability

The data used to support the results of this study can be obtained from the first author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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