

Research Article

Chimera State in the Network of Fractional-Order FitzHugh–Nagumo Neurons

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The fractional calculus in the neuronal models provides the memory properties. In the fractional-order neuronal model, the dynamics of the neuron depends on the derivative order, which can produce various types of memory-dependent dynamics. In this paper, the behaviors of the coupled fractional-order FitzHugh–Nagumo neurons are investigated. The effects of the coupling strength and the derivative order are under consideration. It is revealed that the level of the synchronization is decreased by decreasing the derivative order, and the chimera state emerges for stronger couplings. Furthermore, the patterns of the formed chimeras rely on the order of the derivatives.

1. Introduction

Fractional-order models have attracted much attention from scientists in different fields such as physics and electronics [1–5]. Considering the fractional derivative, a memory feature is added to the systems. Therefore, the fractional-order model can provide a more precise description of the real phenomena than the integer-order [6]. Furthermore, the fractional calculus has found wide applications in controlling the integer-order systems [7]. The fractional derivative also plays important role in demonstrating different firing patterns of the neurons [8]. Consequently, several fractional neuron models have been presented [9–11].

The complicated interactions among the neurons cause the neural system to act as a complex network [12]. The emergence of collective behaviors is an important

characteristic of complex networks [13]. Some examples of collective behaviors are synchronization [14], chimera state [15], and solitary state [16]. Synchronization is an important phenomenon in many applications [17]. Many studies have focused on the synchronization of chaotic systems [18-23]. Furthermore, synchronization manages many neural functions and participates in many brain disorders [24]. In special cases, synchrony and asynchrony are observed simultaneously in a specific region of the brain. For example, the unihemispheric sleep, the neural bump state, and the epileptic seizure disease can be mentioned [25]. This particular condition is called the chimera state [26]. After the foundation of chimera state in 2002 [27], it became the focus of many researchers in a variety of dynamical systems such as the mechanical [28], optical [29], and chemical [30] oscillators and neuronal models [25, 31, 32]. Furthermore, these studies have represented the chimeras with different

spatiotemporal patterns and properties, including the amplitude chimera [33] and traveling chimera [34].

In neuronal studies, the chimeras were under consideration from different perspectives such as the neurons' dynamics, network topology, and coupling scheme. Santos et al. [35] investigated the chimera in 2D networks with regular and fractal topologies and found the spiral chimeras with multiple asynchronous cores. Wang et al. [36] reported the existence of chimera state in the hyperchaotic neurons with hyperchaotic dynamics. Blondeau Soh et al. [37] represented that shifting the neighbors in the coupling leads the network towards the chimera state. Provata and Venetis [38] studied a neuronal network with power-law coupling and showed that the chimera exists in the weak couplings with large exponents. Li et al. [39] considered two unidirectionally coupled layers of neurons and showed different collective behaviors in the master layer and the induced imperfect chimera state in the slave layer.

Among the chimera studies in neuronal networks, a few have considered the fractional models. Vázquez-Guerrero et al. [40] showed that the network of fractional Hindmarsh–Rose neurons is capable of representing chimera state. They also presented a fractional adaptive controller to obtain the synchronization. In another study, they designed an observer to synchronize the chimera state in coupled fractional neurons [41]. He [42] investigated the magnetic Hindmarsh–Rose model with fractional derivative and showed that analyzing the complexity of the network can help in recognition of its dynamical behavior. In this paper, we study the dynamical behaviors of a network of fractionalorder FitzHugh–Nagumo systems. The effects of the order of derivatives and the coupling strength on the chimera state are under consideration.

2. The Model

Recently, the scientists have focused on proposing new models for describing the neural behaviors with considering different aspects of neurons [43, 44]. Here, we use the FitzHugh–Nagumo (FHN) model with considering the fractional derivative as follows [45]:

$$\frac{d^{q}u}{dt^{q}} = u - \frac{u^{3}}{3} - v + I,$$
(1)
$$\frac{d^{q}v}{dt^{q}} = 0.08 (u + 0.7 - 0.8v),$$

where *u* and *v* are the membrane voltage and the recovery variable and *I* is the external excitation current fixed at 0.5. The fractional derivative order is denoted by *q*, and d^q/dt^q is the Caputo–Fabrizio (CF) fractional operator defined by

$$\frac{d^{q}}{dt^{q}}u(t) = \frac{1}{\Gamma(1-q)} \int_{t_{0}}^{t} \frac{\dot{x}(\tau)}{(t-\tau)^{q}} d\tau, \quad 0 < q < 1,$$
(2)

where Γ is the Gamma function. The dynamics of the model relies on the values of the derivative order. Figure 1(a) represents the bifurcation of the model according to *q*. To consider the spiking firing for the model, the range of

Complexity

0.7 < q < 1 is selected in all simulations. The time series and phase spaces of the model for q = 1, 0.9, 0.8, and 0.7 are shown in Figures 1(b)–1(e). It is observed that by decreasing q, the amplitude of the oscillations decreases and the period increases.

We consider the network of fractional FHN neurons with the following equations:

$$\frac{d^{q}u_{i}}{dt^{q}} = u_{i} - \frac{u_{i}^{3}}{3} - v_{i} + I + d\sum_{j=1}^{N} G_{ij} [b_{uu}(u_{j} - u_{i}) + b_{uv}(v_{j} - v_{i})],$$

$$\frac{d^{q}v_{i}}{dt^{q}} = 0.08 (u_{i} + 0.7 - 0.8v_{i}) + d\sum_{j=1}^{N} G_{ij} [b_{vu}(u_{j} - u_{i}) + b_{vv}(v_{j} - v_{i})],$$
(3)

where *d* is the coupling strength and *G* is the Laplacian matrix of connections. The network has a ring structure with nonlocal coupling as shown in Figure 2 (each neuron is connected to its 40 nearest neighbors, and N = 100). The coupling between variables is through a rotational matrix as follows [46]:

$$B = \begin{pmatrix} b_{uu} & b_{uv} \\ b_{vu} & b_{vv} \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix},$$
(4)

with $\phi = (\pi/2) - 0.1$ being the coupling phase.

3. Results

The network is solved numerically by using the Adam–Bashforth method based on the algorithm proposed in [47] with random initial conditions. To identify different behaviors, the strength of incoherence (SI) is used [48]. To find this measure, at first, the variables are transformed into new ones as $x_i = u_i - u_{i+1}$, i = 1, ..., N. Then, the network is divided into M = N/n groups of n neurons, and the local standard deviation is computed as follows:

$$\sigma(m) = \left\langle \sqrt{\frac{1}{n} \sum_{j=n(m-1)+1}^{mn} \left[x_i - \langle x \rangle \right]^2} \right\rangle_t, \quad m = 1, \dots, M,$$
(5)

where $\langle x \rangle = 1/N \sum_{i=1}^{N} x_i(t)$. Finally, the SI is computed by $s_m = \theta(\delta - \sigma(m)),$

$$SI = 1 - \frac{\sum_{m=1}^{M} s_m}{M},$$
(6)

where θ is the Heaviside function and δ is a threshold set at 0.23, here, and n = 4. The value of SI determines the behavior of the network by SI = 0, 0 < SI < 1, and SI = 1 for synchronization, chimera, and asynchronization, respectively.

The network of equation (3) with integer-order (q = 1) represents different dynamical behaviors with varying the coupling strength (*d*). However, the dynamical changes occur in very small coupling strengths. The patterns of the neurons by varying *d* values are demonstrated in Figure 3. The left panel represents the space-time plots, and the right



FIGURE 1: (a) The bifurcation diagram of the model; inter spike intervals (ISI) versus the derivative order (*q*). (b–e) The time series and phase spaces of the model for different *q* values: (b) q = 1; (c) q = 0.9; (d) q = 0.8; (e) q = 0.7.



FIGURE 2: The ring structure of the N = 100 neurons with nonlocal coupling. Each neuron is connected to its 40 nearest neighbors. The connections for i = 1 are illustrated.

panel represents the time snapshots of the neurons. By increasing the coupling strength, the initial asynchronous pattern of the neurons changes to the chimera state. For $d = 10^{-7}$, a chimera state is created (shown in Figure 3(a)), and it can be seen that there are synchronous and asynchronous neurons in the network. With an increment in the

coupling strength, more neurons locate in the synchronous group. The behavior of the network for $d = 1.7 \times 10^{-5}$ is illustrated in Figure 3(b). In this case, most neurons are synchronous, while a few oscillate differently. This behavior is called the solitary state [49, 50]. As the coupling becomes stronger, different neurons are attracted to the synchronous group and a complete synchronization is observed (Figure 3(c)).

The behavior of the neurons is considerably influenced when the derivative of the network's equations changes to the fractional. To investigate this, the coupling strength is considered to be fixed at $d = 2 \times 10^{-5}$, where the integerorder network shows the solitary state (similar to Figure 3(b)) and the fractional order is changed. With decreasing the fractional order (q), firstly, the network synchronization is enhanced. This is shown in Figure 4(a) for q = 0.9. However, more decrement of q disturbs the synchronous behavior of the network. Figure 4(b) represents the chimera state for q = 0.8 with a similar coupling strength value. When q decreases to q = 0.7, the network becomes completely asynchronous (Figure 4(c)). The waveforms of the neurons in each case are depicted in the right panel of Figure 4.

For different fractional orders, the range of the coupling strength for the appearance of different dynamical behaviors is different. As the fractional order decreases, the chimera



FIGURE 3: The patterns of the network (left panel) and time snapshots (right panel) with integer-order derivative: (a) chimera state for $d = 10^{-7}$ (SI = 0.74); (b) solitary state for $d = 1.7 \times 10^{-5}$ (SI = 0.39); (c) synchronization for $d = 5 \times 10^{-5}$ (SI = 0).

state is formed in higher coupling strengths. Figure 5(a) shows the chimera state in q = 0.9 for $d = 4.6 \times 10^{-6}$. In this case, there are several groups containing a few synchronous neurons. For q = 0.8, the chimera is observed for $d = 3 \times 10^{-5}$ and more neurons are involved in the synchronous cluster (Figure 5(b)). When q decreases to q = 0.7, in some time intervals, some neurons become synchronous. Therefore, the chimera for this derivative is nonstationary. The chimera for q = 0.7 is shown in Figure 5(c). To illustrate the coherent and incoherent clusters better, the local order parameter is computed and shown in the right column of Figure 5. This parameter can be obtained as $L_k = |1/2p\sum_{|k-l| \le p} \exp(j\phi_l)|$, $k = 1, \ldots, N$, where ϕ_l is the geometric phase of *l*th oscillator calculated by $\phi_l = a \tan(y_l/x_l)$. The size of the spatial window is denoted by *p*. When $L_k = 1$, the *k*th oscillator belongs to a coherent group.

Figure 6 represents the strength of incoherence of the network for different fractional orders. For q = 1, the network is in chimera state until $d = 5 \times 10^{-5}$ and becomes synchronous for larger coupling strengths (Figure 6(a)). A similar pattern is observed for q = 0.95 (Figure 6(b)). When q decreases to q = 0.9, the synchronization occurs for very smaller coupling strengths ($d = 2.2 \times 10^{-5}$). For q = 0.85, the network's dynamical behavior returns to the q = 1 manner. With more decrement of q, a stronger coupling is needed for the synchronization. Figure 6(e) shows that q = 0.8 has the larger chimera region. For q = 0.7 and q = 0.75, a large asynchronization region is



FIGURE 4: Upper panel: the patterns of the network for $d = 2 \times 10^{-5}$ and different derivative orders. The integer-order network exhibits a solitary state at this coupling. (a) Synchronization for q = 0.9 (SI = 0). (b) Chimera for q = 0.8 (SI = 0.8). (c) Asynchronization for q = 0.7 (SI = 0).

observed. The variation of SI according to q is illustrated in Figure 7. It is observed that for low coupling strength, the integer network is in a chimera state. The chimera state is preserved until q = 0.835, and then all neurons become asynchronous. For strong couplings, the integer-order

network is synchronous. With decreasing q, the synchronization remains until q = 0.84. For q < 0.84, the chimera state is formed. However, in the range 0.8 < q < 0.823, the synchronization may appear in the network determined by the initial conditions.



FIGURE 5: The chimera patterns in the fractional-order network. The left and right columns show the spatiotemporal patterns and the local order parameters (p = 2), respectively: (a) q = 0.9 and $d = 4.6 \times 10^{-6}$ (SI =); (b) q = 0.8 and $d = 3 \times 10^{-5}$ (SI = 0.48); (c) q = 0.7 and $d = 10^{-4}$ (SI = 0.42). By varying the derivative order, the chimera is formed with different patterns.





FIGURE 6: The strength of incoherence according to coupling strength for various values of derivative order: (a) q = 1; (b) q = 0.95; (c) q = 0.95; (d) q = 0.85; (e) q = 0.85; (f) q = 0.75; (g) q = 0.75.



FIGURE 7: The strength of incoherence according to derivative order (q) for weak and strong coupling strengths: (a) $d = 10^{-5}$; (b) $d = 10^{-4}$.

4. Conclusion

In this paper, a network of coupled fractional-order Fitz-Hugh–Nagumo neurons was studied. The dynamical behavior of the network was investigated under the variation of the coupling strength and the derivative order. The bifurcation diagram of the fractional system with respect to the derivative order revealed that the dynamics of the model is dependent on the fractional order. Consequently, by changing the value of the derivative order, various collective behaviors of the neurons can be found. The integer-order neurons experience asynchronization, chimera, and synchronization with increasing the coupling strength, respectively. In the fractional-order network, decreasing the derivative order for the constant coupling strength, resulted in the lower synchrony level in the network. Therefore, the fractional-order network has the same state transition; however, it occurs in higher coupling strengths. Thus, the chimera or synchronous states appear for stronger couplings. Furthermore, the pattern of the chimera was changed with varying the derivative order. For q = 0.9, some small synchronous clusters were formed, while in q = 0.8, a large

cluster of synchronous neurons was observed. For lower q values (q = 0.7), the position of the synchronous cluster was time-dependent and the nonstationary chimera state appeared.

Data Availability

The data used to support the findings of the study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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