Hindawi
Complexity
Volume 2021, Article ID 2505043, 11 pages
https://doi.org/10.1155/2021/2505043

Research Article

Robust Observer Design for Discrete Descriptor Systems with Packet Losses

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Received 15 May 2020; Accepted 4 August 2020; Published 31 January 2021

Academic Editor: Carlos Aguilar-Ibanez

This paper considers observer design for discrete-time descriptor systems with packet losses. By taking packet loss into consideration, the error dynamic of the proposed observer becomes a stochastic switched system. Consequently, the proposed observer is synthesized in a stochastic switched system framework. Sufficient conditions for the stochastic stability with a prescribed robust performance of the error dynamic system are derived and converted into linear matrix inequalities. Not only can the proposed observer deal with packet losses, but it also attenuates the effect of process disturbance and measurement noise. A numerical simulation of a truck-trailer is given to demonstrate the effectiveness of the proposed method.

1. Introduction

Descriptor systems, which are also referred to as singular systems, differential-algebraic equation systems, or generalized state-space systems, appear in many practical systems such as electrical circuits [1, 2], power systems [3], mechanical systems [4], and robots [5, 6]. Moreover, the descriptor system approach has recently been used for synchronization [7] and fault diagnosis [8, 9]. Therefore, in the past decades, the studies on descriptor systems have attracted considerable attention. Many results on descriptor systems have been reported in the literature, such as the analysis and design [10, 11], robust estimation and control [12–14], and fault diagnosis and fault-tolerant control [15–17].

It is known that state feedback is very important in control design [18–20]. However, it may be too expensive or even impossible to directly measure all of the states in many applications. In these situations, state estimation from output measurements is necessary to implement the control algorithm. Moreover, the estimated output provided by observers can be used to generate residuals that contain information about fault. Therefore, observers have also been widely used in fault detection and diagnosis [21, 22]. Therefore, the observer design problem is of important practical significance and has been extensively investigated in the literature; see [23–26], just to name a few. In the past decades, many significant results have been reported on observer design for descriptor systems. Observer design for discrete linear descriptor systems was studied in [27]. For the continuous-time linear descriptor systems, full-order and reduced-order observer design methods were proposed in [28]. By using the linear matrix inequality (LMI) technique, Lu and Ho in [29] proposed full-order and reduced-order observer design methods for continuous Lipschitz descriptor systems. Observer design for descriptor systems with unknown disturbance has also been considered in the literature. The unknown input observer design method for continuous linear descriptor systems was considered in [30, 31]. For continuous-time Lipschitz nonlinear descriptor systems with unknown input, Koenig in [32] proposed an unknown input observer design method via convex optimization. In [33], Darouach et al. presented an H_∞ observer design method for a class of Lipschitz nonlinear descriptor systems. It is noted that most of the existing works focus on continuous-time descriptor systems while the results on observer design for discrete descriptor systems are limited. In [34], Wang et al. proposed an LMI-based approach to design observers for discrete-time linear and Lipschitz nonlinear descriptor systems and the methodology in [34] has been used to design fault diagnosis observers for linear time-invariant descriptor systems [35] and linear parameter-varying descriptor systems [36].

It should be noted that all the aforementioned methods assume that the communication link between the measurements and observer is perfect. However, it is not the case in practice, especially in networked control systems that are often subject to packet dropouts and other nonideal phenomena. For example, the GPS signal may be intermittently available due to system malfunctions or obstacles between the receiver and satellite signal [37]. In the past few years, networked control systems have attracted much attention, but to the best of the authors' knowledge, no work has been done on observer design for descriptor systems with packet losses. The main contribution of this paper lies in the following aspects. First, a novel observer is proposed to deal with packet losses in the descriptor systems. Second, the convergence of the proposed observer is guaranteed by using a stochastic switched system framework. Moreover, the H_∞ technique is used in the design of the proposed observer such that the effect of process disturbance and measurement noise is attenuated.

The notation used throughout the paper is standard. The superscript T stands for matrix transposition. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ denote an n-dimensional and $m \times n$-dimensional Euclidean space, respectively. $\mathbb{C}$ represents the one-dimensional complex space. I_n denotes the $n \times n$ identity matrix; 0 represents the zero matrix with appropriate dimensions. For a square matrix M, $\lambda_{\min}(M)$ denotes the minimum eigenvalue of matrix M. For a real symmetric matrix P, $P > 0$ and $P < 0$ mean that P is positive definite and negative definite, respectively. $\mathscr{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$, and $\|x\|_2 = \sqrt{\sum_{k=0}^{\infty} x^T(k)x(k)}$ stands for the $\mathscr{L}_2$ norm. In symmetric block matrices, we use an asterisk (*) to represent a term that is induced by symmetry. In addition, $\mathscr{P}(A)$ denotes the probability if the event A occurs, and $\mathscr{E}\{x\}$ and $\mathscr{E}\{x|y\}$ represent expectation of x and expectation of x conditional on y, respectively.

2. Problem Formulation

Consider the following discrete descriptor system:

$$\begin{cases} Ex(k+1) = Ax(k) + Bu(k) + Dd(k), \\ y(k) = \sigma(k)Cx(k) + v(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^p$ is the input vector, $d(k) \in \mathbb{R}^l$ and $v(k) \in \mathbb{R}^m$, respectively, represent the process disturbance and measurement noise, and $y(k) \in \mathbb{R}^m$ is the measured output vector. Without loss of generality, it is assumed that $d(k)$ and $v(k)$ are square summable sequences; i.e., $d(k), v(k) \in \mathscr{L}_2[0,\infty)$. The matrix $E \in \mathbb{R}^{n\times n}$ may be singular; i.e., $\operatorname{rank}(E) = r \le n$. $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$, $C \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{n\times l}$ are known constant matrices. In this paper, it is assumed that system (1) is observable, i.e.,

$$\operatorname{rank}\begin{bmatrix} E \\ C \end{bmatrix} = n, \tag{2}$$

$$\operatorname{rank}\begin{bmatrix} zE - A \\ C \end{bmatrix} = n, \quad \text{for any } z \in \mathbb{C}, |z| \ge 1. \tag{3}$$

In this paper, the considered descriptor system is in a network environment where the measured output is affected by packet losses, which is described by $\sigma(k) \in \{0,1\}$. It is assumed that $\sigma(k)$ is a stochastic variable with the probability distribution as follows:

$$\begin{cases} \mathscr{P}\{\sigma(k) = 1\} = \overline{\sigma}, \\ \mathscr{P}\{\sigma(k) = 0\} = 1 - \overline{\sigma}, \end{cases} \tag{4}$$

where $\overline{\sigma} \in (0,1)$ is a priori probability. There is no packet loss at the instant k if $\sigma(k) = 1$. Otherwise, the measurement is missed. Without loss of generality, this paper assumes the data packet has a timestamp so that $\sigma(k)$ is available in real time.

The aim of this paper is to design an observer to estimate the state $x(k)$ even in the presence of packet losses. Moreover, the state estimate should be robust against the disturbance and measurement noise. The proposed observer takes the form

$$\begin{cases} E\hat{x}(k+1) = A\hat{x}(k) + Bu(k) + L(\hat{y}(k) - C\hat{x}(k)), \\ \hat{y}(k) = \sigma(k)y(k) + (1-\sigma(k))C\hat{x}(k), \end{cases} \tag{5}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the state estimate vector, $\hat{y}(k) \in \mathbb{R}^m$ is introduced to deal with the packet losses, and $L \in \mathbb{R}^{n\times m}$ is the gain matrix to be synthesized.

Remark 1. The observer in (5) is a generalization of the Luenberger observer to the descriptor system with packet losses.

If we define the estimation error as

$$e(k) = x(k) - \hat{x}(k), \tag{6}$$

and subtract 5 from 1, the error dynamic can be obtained as follows:

$$\begin{aligned} Ee(k+1) = {} & (A - \sigma(k)LC)e(k) + Dd(k) - \sigma(k)Lv(k) \\ & \sigma(k)(A - LC)e(k) + (1-\sigma(k))Ae(k) \\ & + Dd(k) - \sigma(k)Lv(k). \end{aligned} \tag{7}$$

Note that the introduction of the stochastic variable $\sigma(k)$ makes the error dynamic system (7) stochastic instead of deterministic. Therefore, before proceeding further, it is necessary to introduce the notion of stochastic stability. Similar to stochastic stability in [38], we propose the following definition of stochastic stability for discrete-time stochastic descriptor systems.

Definition 1. A discrete-time stochastic descriptor system $Ex(k+1) = g(x(k))$ is said to be stochastically stable if there exists a finite $Q \geq 0$ independent of $x(0)$ such that the following holds:

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} x^T(k)x(k)|x(0)\right\} \leq x^T(0)Qx(0), \tag{8}$$

for any initial condition $x(0)$.

Based on the definition of stochastic stability, the robust observer design problem in this paper is stated as follows.

Consider the descriptor system in (1). Design an observer in the form of (5) such that

(1) The error dynamic system in (7) is stochastically stable.

(2) The state estimation error $e(k)$ is stochastically robust to the process disturbance and measurement noise, i.e.,

$$\mathcal{E}\left\{\|e\|_2^2\right\} \leq \gamma_d^2\mathcal{E}\left\{\|d\|_2^2\right\} + \gamma_v^2\mathcal{E}\left\{\|v\|_2^2\right\} + V(0), \tag{9}$$

where $\gamma_d > 0$ and $\gamma_v > 0$ are given scalars and $V(0)$ is a quadratic function of $e(0)$.

3. Main Results

In this section, an observer design method for descriptor systems (1) is formulated as an LMI feasibility problem. First, the following theorem is proposed to guarantee the stochastic stability and stochastic robustness of the error dynamic system in (7).

Theorem 1. *Consider the descriptor system (1) and observer (5). The error dynamic system in (7) is stochastically stable and satisfies the robust performance condition (9) if there exist matrices $P \in \mathbb{R}^{n\times n}$ and $L \in \mathbb{R}^{n\times m}$ satisfying*

$$E^T PE \geq 0, \tag{10}$$

$$\begin{bmatrix} \Phi + I_n & * & * \\ \overline{\sigma}D^T P(A - LC) + (1-\overline{\sigma})D^T PA & D^T PD - \gamma_d^2 I_l & * \\ -\overline{\sigma}L^T P(A - LC) & -\overline{\sigma}L^T PD & \overline{\sigma}L^T PL - \gamma_v^2 I_m \end{bmatrix} < 0, \tag{11}$$

where

$$\Phi = \overline{\sigma}(A - LC)^T P(A - LC) + (1-\overline{\sigma})A^T PA - E^T PE. \tag{12}$$

Proof. We first prove the stochastic stability of the error dynamic system in (7). Consider the following Lyapunov candidate:

$$V(k) = e^T(k)E^T PEe(k), \tag{13}$$

with P satisfying (10), and define

$$\mathcal{E}\{\Delta V\} = \mathcal{E}\left\{e^T(k+1)E^T PEe(k+1)|et(k)\right\} - e^T(k)E^T PEe(k). \tag{14}$$

If we consider the nominal part of the error dynamic in (7) (i.e., $d(k) = 0$ and $v(k) = 0$), we obtain

$$\begin{aligned} \mathcal{E}\{\Delta V\} &= \mathcal{E}\left\{e^T(k)[\sigma(k)(A - LC) + (1-\sigma(k))A]^T P[\sigma(k)(A - LC) + (1-\sigma(k))A]e(k)|e(k)\right\} \\ &\quad - e^T(k)E^T PEe(k) \\ &= \overline{\sigma}e^T(k)(A - LC)^T P(A - LC)e(k) + (1-\overline{\sigma})e^T(k)A^T PAe(k) - e^T(k)E^T PEe(k) \\ &= e^T(k)\Phi e(k). \end{aligned} \tag{15}$$

If $\Phi < 0$, we obtain

$$\mathcal{E}\{\Delta V\} \leq -\lambda_{\min}(-\Phi)e^T(k)e(k). \tag{16}$$

It follows that

$$\begin{aligned} &\mathcal{E}\left\{e^T(k+1)E^T PEe(k+1)\right\} - \mathcal{E}\left\{e^T(k)E^T PEe(k)\right\} \\ &\quad \leq -\lambda_{\min}(-\Phi)\mathcal{E}\left\{e^T(k)e(k)\right\}. \end{aligned} \tag{17}$$

Summing up both sides of (17) from $k = 0, 2, \ldots, \infty$ leads to

$$\begin{aligned} &\mathcal{E}\left\{e^T(\infty)E^T PEe(\infty)\right\} - \mathcal{E}\left\{e^T(0)E^T PEe(0)\right\} \\ &\quad \leq -\lambda_{\min}(-\Phi)\mathcal{E}\left\{\sum_{k=0}^{\infty} e^T(k)e(k)\right\}, \end{aligned} \tag{18}$$

which implies

$$\mathcal{E}\left\{\sum_{k=0}^{\infty} e^T(k)e(k)\right\} \le \frac{1}{\lambda_{\min}(-\Phi)}\Big(\mathcal{E}\big\{e^T(0)E^T PEe(0)\big\} - \mathcal{E}\big\{e^T(\infty)E^T PEe(\infty)\big\}\Big) \le \frac{1}{\lambda_{\min}(-\Phi)}\mathcal{E}\big\{e^T(0)E^T PEe(0)\big\}. \quad (19)$$

Then, we obtain

$$\mathcal{E}\sum_{k=0}^{\infty} e^T(k)e(k)|ten(0)\} \le x^T(0)Qx(0), \quad (20)$$

where $Q = (1/\lambda_{\min}(-\Phi))E^T PE$. It is easy to see that the (1, 1) block of (11) gives

$$\Phi + I_n < 0, \quad (21)$$

which implies that $\Phi < 0$. Thus, according to Definition 1, it is known that the error dynamic system in (7) is stochastically stable in mean square.

Next, we prove that the performance defined in (9) is guaranteed. To this end, a criterion function is introduced as

$$J = \mathcal{E}\left\{\sum_{k=0}^{\infty}\left[\Delta V(k) + e^T(k)e(k) - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\right]\right\}. \quad (22)$$

By using the fact that $\Delta V(k) = V(k+1) - V(k)$, we obtain

$$J = \mathcal{E}\left\{V(\infty) - V(0) + \sum_{k=0}^{\infty}\left[e^T(k)e(k) - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\right]\right\}. \quad (23)$$

Note that $V(\infty) \ge 0$. Consequently, if $J < 0$ holds, we have

$$\mathcal{E}\left\{\|e\|_2^2 - \gamma_d^2\|d\|_2^2 - \gamma_v^2\|v\|_2^2\right\} - V(0) < 0. \quad (24)$$

That is, criterion (9) is satisfied. On the other hand, it is clear that $J < 0$ holds if

$$\mathcal{E}\left\{\Delta V(k) + e^T(k)e(k) - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\right\} < 0. \quad (25)$$

Therefore, criterion (9) is satisfied if (25) holds. Using (7), it gives

$$\begin{aligned}
&\mathcal{E}\left\{\Delta V(k) + e^T(k)e(k) - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\right\} \\
&= \mathcal{E}\big\{[\sigma(k)(A-LC)e(k) + (1-\sigma(k))Ae(k) + Dd(k) - \sigma(k)Lv(k)]^T P[\sigma(k)(A-LC)e(k) \\
&\quad + (1-\sigma(k))Ae(k) + Dd(k) - \sigma(k)Lv(k)] - e^T(k)E^T PEe(k) + e^T(k)e(k) \\
&\quad - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\big\} \\
&= \mathcal{E}\big\{\sigma^2(k)e^T(k)(A-LC)^T P(A-LC)e(k) + 2\sigma(k)(1-\sigma(k))(A-LC)^T PAe(k) \\
&\quad + 2\sigma(k)e^T(k)(A-LC)^T P(Dd(k) - \sigma(k)Lv(k)) + (1-\sigma(k))^2 e^T(k)A^T PAe(k) \\
&\quad + 2(1-\sigma(k))e^T(k)A^T P(Dd(k) - \sigma(k)Lv(k)) + d^T(k)D^T PDd(k) - 2\sigma(k)d^T(k)D^T PLv(k) \\
&\quad + \sigma^2(k)v^T(k)L^T PLv(k) - e^T(k)E^T PEe(k) + e^T(k)e(k) - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\big\}.
\end{aligned} \quad (26)$$

From the probability distribution of $\sigma(k)$ in (4), it is obtained that

$$\begin{aligned}\mathscr{E}\{\sigma(k)\sigma(k)\} &= \overline{\sigma},\\ \mathscr{E}\{\sigma(k)(1-\sigma(k))\} &= 0,\\ \mathscr{E}\left\{(1-\sigma(k))^2\right\} &= 1-\overline{\sigma}.\end{aligned} \tag{27}$$

Substituting (27) into (26) yields

$$\begin{aligned}&\mathscr{E}\left\{\Delta V(k) + e^T(k)e(k) - \gamma_d^2 d^T(k)d(k) - \gamma_v^2 v^T(k)v(k)\right\}\\ &\quad= \overline{\sigma}e^T(k)(A-LC)^T P(A-LC)e(k) + (1-\overline{\sigma})e^T(k)A^T PAe(k) - e^T(k)E^T PEe(k)\\ &\qquad + e^T(k)e(k) + 2\overline{\sigma}e^T(k)(A-LC)^T P(Dd(k) - Lv(k))\\ &\qquad + 2(1-\overline{\sigma})e^T(k)A^T PDd(k) + d^T(k)D^T PDd(k) - \gamma_d^2 d^T(k)d(k)\\ &\qquad - 2\overline{\sigma}d^T(k)D^T PLv(k) + \overline{\sigma}v^T(k)L^T PLv(k) - \gamma_v^2 v^T(k)v(k)\\ &\quad\triangleq \begin{bmatrix} e(k)\\ d(k)\\ v(k)\end{bmatrix}^T \begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13}\\ * & \Omega_{22} & \Omega_{23}\\ * & * & \Omega_{33}\end{bmatrix}\begin{bmatrix} e(k)\\ d(k)\\ v(k)\end{bmatrix},\end{aligned} \tag{28}$$

where

$$\begin{aligned}\Omega_{11} &= \overline{\sigma}(A-LC)^T P(A-LC) + (1-\overline{\sigma})A^T PA - E^T PE + I_{n+q},\\ \Omega_{12} &= \overline{\sigma}(A-LC)^T PD + (1-\overline{\sigma})A^T PD,\\ \Omega_{13} &= -\overline{\sigma}(A-LC)^T PL,\\ \Omega_{22} &= D^T PD - \gamma_d^2 I_l,\\ \Omega_{23} &= -\overline{\sigma}D^T PL,\\ \Omega_{33} &= \overline{\sigma}L^T PL - \gamma_v^2 I_m.\end{aligned} \tag{29}$$

Now, (25) is satisfied if (11) holds, and then criterion (9) is fulfilled. This completes the proof. □

Remark 2. Without loss of generality, it is assumed that (E, A) is regular, which guarantees the existence and the uniqueness of a solution to (1). According to Lemma 2.1 in [12], there exist two nonsingular matrices M_1 and N_1 such that $M_1EN_1 = \mathrm{diag}(I, J)$, where J is a nilpotent matrix. Consequently, we can find a matrix P satisfying (10). However, the conditions in Theorem 1 are not strict LMIs. In order to facilitate the design of the observer (5), further procedures are needed to transform the conditions in (10) and (11) into LMIs.

Before proposing the main result, we recall the following useful lemma.

Lemma 1 (see [39]). *For matrices X, Y, and $J > 0$ with appropriate dimensions, the following inequality holds:*

$$XY + Y^T X^T \le XJX^T + Y^T J^{-1} Y. \tag{30}$$

Based on Theorem 1 and Lemma 1, the main result is proposed in the following theorem.

Theorem 2. *The error dynamic system in (7) is stochastically stable with the robust performance (9) if there exist matrices $P \in \mathbb{R}^{n\times n}$, $G \in \mathbb{R}^{n\times n}$, $M \in \mathbb{R}^{(n+l+m)\times n}$, and $L \in \mathbb{R}^{n\times m}$ and a scalar $\epsilon > 0$ such that*

$$E_L^T PE_L > 0, \tag{31}$$

$$\begin{bmatrix}\Psi + M\overline{A} + \overline{A}^T M^T & * & * & *\\ -M^T + G\overline{A} - W\overline{C} & \overline{\sigma}P - G - G^T & * & *\\ M^T & 0 & \varepsilon\left(I_n - G - G^T\right) & *\\ -W\overline{C} & 0 & 0 & -\dfrac{1}{\varepsilon}I_m\end{bmatrix} < 0, \tag{32}$$

where

$$\Psi = \begin{bmatrix}(1-\overline{\sigma})A^T PA - E^T PE + I_n & * & *\\ (1-\overline{\sigma})D^T PA & (1-\overline{\sigma})D^T PD - \gamma_d^2 I_l & *\\ 0 & 0 & -\gamma_v^2 I_m\end{bmatrix}, \tag{33}$$

$$\overline{A} = [\,AD0\,], \tag{34}$$

$$\overline{C} = [\,C0I\,], \tag{35}$$

and $E_L \in \mathbb{R}^{n\times r}$ is a matrix satisfying

$$E_L E_R = E, \tag{36}$$

with $E_R \in \mathbb{R}^{r\times n}$and $rank(E_L) = rank(E_R) = r$. Moreover, if the LMIs in (31) and (32) are solvable, then the matrix L can be determined by $L = G^{-1}W$. Proof. First, (31) implies that

$$E_R^T E_L^T P E_L E_R \geq 0. \quad (37)$$

That is, the condition in (10) is satisfied.

By the definitions of Ψ, $\overline{A}$, *and* $\overline{C}$, *inequality (11) can be written as*

$$\Psi + \overline{\sigma}(\overline{A} - L\overline{C})^T P (\overline{A} - L\overline{C}) < 0. \quad (38)$$

Moreover, (38) holds if there exist a matrix M and a nonsingular matrix G such that

$$\begin{bmatrix} \Psi + M(\overline{A} - L\overline{C}) + (\overline{A} - L\overline{C})^T M^T & * \\ -M^T + G(\overline{A} - L\overline{C}) & \overline{\sigma}P - G - G^T \end{bmatrix} < 0. \quad (39)$$

This can be easily shown by pre- and postmultiplying 38 with $\begin{bmatrix} I & (\overline{A} - tLn\overline{C})^T \end{bmatrix}$ and its transpose, respectively.

Then, (39) is written as

$$\begin{bmatrix} \Psi + M\overline{A} + \overline{A}^T M^T & * \\ -M^T + G(\overline{A} & -tLn\overline{C}) \ \overline{\sigma}P - G - G^T \end{bmatrix} + \begin{bmatrix} MG^{-1} \\ 0 \end{bmatrix} \begin{bmatrix} -(GL\overline{C})^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} -(GL\overline{C})^T \\ 0 \end{bmatrix} \begin{bmatrix} MG^{-1} \\ 0 \end{bmatrix}^T < 0. \quad (40)$$

Using Lemma 1, we obtain

$$\begin{aligned} &\begin{bmatrix} \Psi + M\overline{A} + \overline{A}^T M^T & * \\ -M^T + G(\overline{A} - L\overline{C}) & \overline{\sigma}P - G - G^T \end{bmatrix} + \begin{bmatrix} MG^{-1} \\ 0 \end{bmatrix} \begin{bmatrix} -(GL\overline{C})^T \\ 0 \end{bmatrix}^T + \begin{bmatrix} -(GL\overline{C})^T \\ 0 \end{bmatrix} \begin{bmatrix} MG^{-1} \\ 0 \end{bmatrix}^T \\ &\leq \begin{bmatrix} \Psi + M\overline{A} + \overline{A}^T M^T & * \\ -M^T + G(\overline{A} - L\overline{C}) & \overline{\sigma}P - G - G^T \end{bmatrix} + \begin{bmatrix} MG^{-1} \\ 0 \end{bmatrix} \epsilon^{-1} \begin{bmatrix} MG^{-1} \\ 0 \end{bmatrix}^T \\ &\quad + \begin{bmatrix} -(GL\overline{C})^T \\ 0 \end{bmatrix} \epsilon \begin{bmatrix} -(GL\overline{C})^T \\ 0 \end{bmatrix}^T. \end{aligned} \quad (41)$$

Since G is a nonsingular matrix, by using (41) and Schur complement lemma, it is shown that (40) holds if

$$\begin{bmatrix} \Psi + M\overline{A} + \overline{A}^T M^T & * & * & * \\ -M^T + G(\overline{A} - L\overline{C}) & \overline{\sigma}P - G - G^T & * & * \\ M^T & 0 & -\varepsilon G^T G & * \\ -GL\overline{C} & 0 & 0 & -\frac{1}{\varepsilon} I_m \end{bmatrix} < 0. \quad (42)$$

Note that the inequality $(I_n - G)^T (I_n - G) \geq 0$ implies that

$$-G^T G \leq I_n - G - G^T. \quad (43)$$

Therefore, inequality (42) holds if

$$\begin{bmatrix} \Psi + M\overline{A} + \overline{A}^T M^T & * & * & * \\ -M^T + G(\overline{A} - L\overline{C}) & \overline{\sigma}P - G - G^T & * & * \\ M^T & 0 & \varepsilon\left(I_n - G - G^T\right) & * \\ -GL\overline{C} & 0 & 0 & -\frac{1}{\varepsilon} I_m \end{bmatrix} < 0. \quad (44)$$

Let $W = GL$, then (44) becomes (32). This completes the proof.

Remark 3. It is noted that the condition in (39) is a non-convex problem, which is inconvenient to solve. By using Lemma 1, this condition is converted into the matrix inequality in (32). By choosing a scalar ϵ in advance, (32) becomes a linear matrix inequality, which can be solved efficiently.

4. Simulation Results

In this section, a truck-trailer model from [40] is used to show the state estimation performance of the proposed method. The dynamic equation of the truck-trailer is given by

$$\begin{cases} \dot{x}_1(t) = \dfrac{v}{l}\tan\left(u_0(t)\right) - \dfrac{\theta}{L_0}\sin\left(x_1(t)\right), \\ \dot{x}_2(t) = \dfrac{v}{L_0}\sin\left(x_1(t)\right), \\ \dot{x}_3(t) = v\cos\left(x_1(t)\right)\left(\sin\dfrac{x_2(t) + x_2(t - \Delta t)}{2}\right), \end{cases} \quad (45)$$

where Δt is the sampling period. The discrete-time version of (45) is

$$\begin{cases} x_1(k+1) = x_1(k) + \dfrac{v\Delta t}{l}\tan(u_0(k)) - \dfrac{\theta(k)t}{L_0}\sin(x_1(k)), \\ x_2(k+1) = x_2(k) + \dfrac{v\Delta t}{L_0}\sin(x_1(k)), \\ x_3(k+1) = x_3(k) + v\Delta t\cos(x_1(k))\left(\sin\dfrac{x_2(k+1)+x_2(k)}{2}\right), \end{cases} \tag{46}$$

where $x_1(k)$ is the angle difference between truck and trailer, $x_2(k)$ is the angle of trailer, $x_3(k)$ is the vertical position of the rear end of the trailer, and $u_0(k)$ is the steering angle. $l = 2.8$ m is the length of truck, $L_0 = 5.5$ m is the length of trailer, $\Delta t = 0.2$ s is the sampling time, and $v = -1$ m/s represents the reversing speed.

Under the assumption that $x_1(k)$ is small, the truck-trailer model (46) can be simplified as

$$\begin{cases} x_1(k+1) = \left(1 - \dfrac{v\Delta t}{L_0}\right)x_1(k) + \dfrac{v\Delta t}{l}\tan(u_0(k)), \\ x_2(k+1) = x_2(k) + \dfrac{v\Delta t}{L_0}x_1(k), \\ x_3(k+1) = x_3(k) + v\Delta t\cdot\sin\left(\dfrac{x_2(k+1)+x_2(k)}{2}\right). \end{cases} \tag{47}$$

By introducing the variables

$$x_4(k) = \frac{v\Delta t}{2}x_2(k),\ u(k) = \tan(u_0(k)), \tag{48}$$

the simplified track-trailer model (47) becomes

$$Ex(k+1) = Ax(k) + Bu(k) + Dd(k), \tag{49}$$

where

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A = \begin{bmatrix} 1-\dfrac{v\Delta t}{L} & 0 & 0 & 0 \\ \dfrac{v\Delta t}{L} & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & -\dfrac{v\Delta t}{2} & 0 & 1 \end{bmatrix},$$

$$B = \begin{bmatrix} \dfrac{v\Delta t}{L} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 \\ 0 \\ v\Delta t \\ 0 \end{bmatrix},$$

$$d(k) = \sin\left(\frac{x_2(k+1)+x_2(k)}{2}\right) - \frac{x_2(k+1)+x_2(k)}{2}. \tag{50}$$

Remark 4. It can be seen that rank$(E) = 3$. This implies that the truck-trailer is a descriptor system, rather than a regular one. Therefore, the observer design method for a descriptor system is necessary in this situation. In the simulation, the measurement equation takes the following form:

$$y(k) = \sigma(k)Cx(k) + v(k), \tag{51}$$

with

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{cases} \mathscr{P}\{\sigma(k) = 1\} = 0.4, \\ \mathscr{P}\{\sigma(k) = 0\} = 0.6. \end{cases} \tag{52}$$

It is assumed that the measurements are corrupted by zero-mean Gaussian noises, with the standard deviations of noises in the measurements of x_2 and x_3 being 0.0087 rad (0.5°) and 0.05 m, respectively.

By choosing $\epsilon = 0.1$, $\gamma_d = 3$, and $\gamma_v = 5$ and solving the LMIs in (31) and (32), we obtain

$$L = \begin{bmatrix} -0.4281 & 0.1738 & 0.0153 \\ 0.2867 & -0.1343 & -0.0068 \\ -0.2284 & 0.6497 & -0.0533 \\ 0.0245 & 0.0008 & 0.0035 \end{bmatrix}. \tag{53}$$

Note that the state estimation $\hat{x}(k)$ should be used in the controller since the actual state $x(k)$ is not available. The control input $u(k)$ in the simulation is given by $u(k) = K\hat{x}(k)$, with $K = [\,3\ \ -6\ \ 0.7\,]$. In the simulation, the initial conditions are $x(0) = [-0.1745, 1.2217, 3, -0.1222]^T$ and $\hat{x}(0) = [0, 1.138, 3.5, -0.105]^T$. For the packet loss scenario shown in Figure 1, the state estimation result is depicted in Figure 2. It can be seen from Figure 2 that the designed observer is able to provide accurate state estimates despite the presence of packet losses. Although the initial

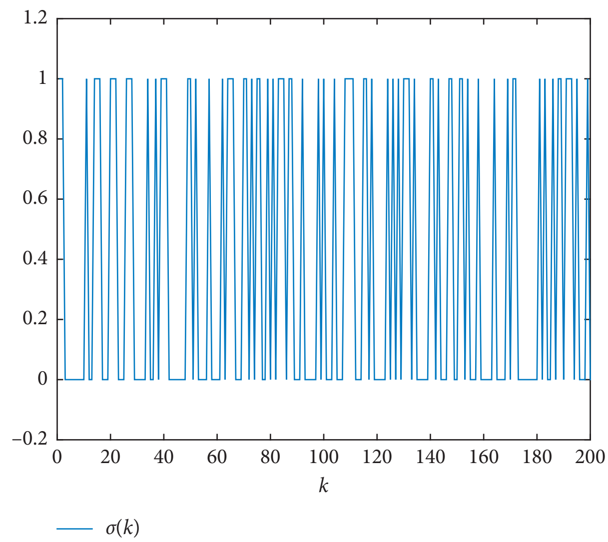

Figure 1: The sequence of $\sigma(k)$.

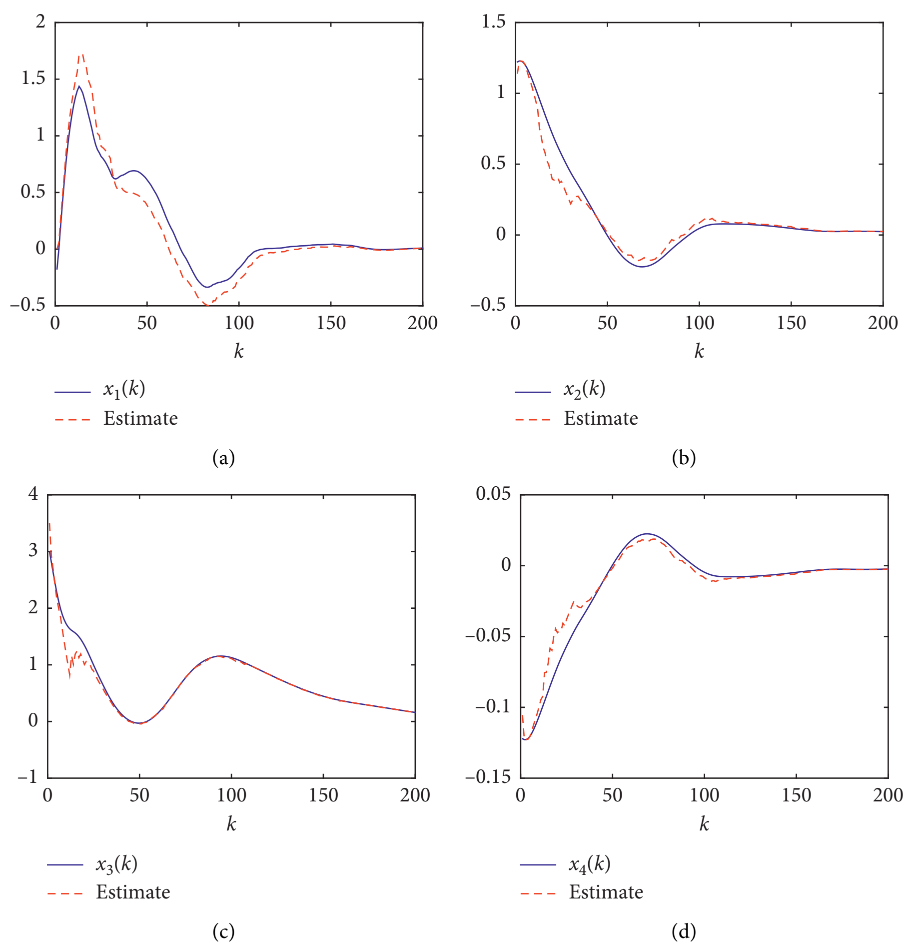

Figure 2: The state estimation responses by the proposed method.

estimation error and long-time packet losses at the beginning of the simulation render some estimation error, the state estimates quickly converge to the actual states.

The observer in [34] can also be used to estimate the states of system (1) if it is slightly modified as follows:

$$\begin{cases} z(k+1) = TA\hat{x}(k) + TBu(k) + L(\hat{y}(k) - C\hat{x}(k)), \\ \hat{x}(k) = z(k) + N\hat{y}(k), \\ \hat{y}(k) = \sigma(k)y(k) + (1-\sigma(k))C\hat{x}(k), \end{cases} \tag{54}$$

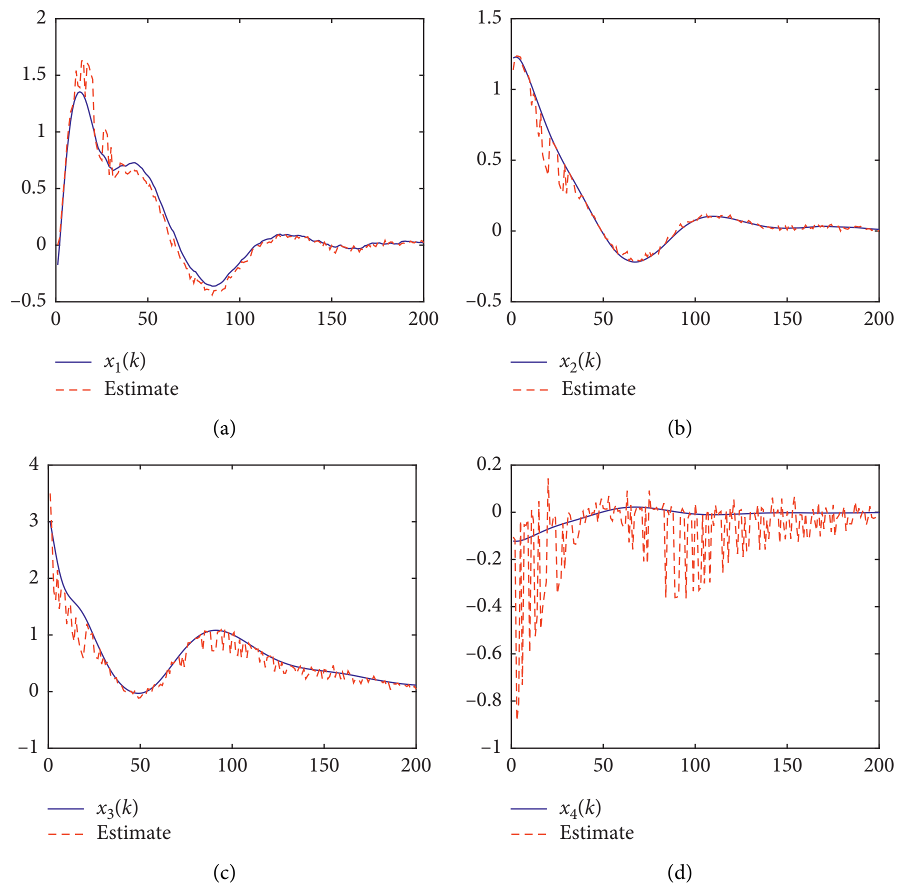

Figure 3: The state estimation responses by the method in [34].

where T and N should satisfy $TE + NC = I$. Using the design method presented in [34], we can obtain the following parameters:

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.6667 & 0 \\ 0 & 0 & -0.3333 & 1 \end{bmatrix},$$
$$N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.3333 & 0.6667 \\ 0 & 0.3333 & 0.6667 \end{bmatrix}, \qquad (55)$$
$$L = \begin{bmatrix} -1.4658 & 0 & 0 \\ 1.05 & 0 & 0 \\ 0 & 0.6667 & 0.6667 \\ 0.1 & -0.3333 & 0.6667 \end{bmatrix}.$$

The state estimation results are shown in Figure 3. From Figures 2 and 3, it can be seen that the method in [34] can obtain a little better estimation results of $x_1(k)$ and $x_2(k)$ than the proposed method, but the estimation results of $x_3(k)$ and $x_4(k)$ by the proposed method are much better than that provided by the method in [34]. We can conclude that the proposed method can obtain more accurate estimation results than the method in [34].

5. Conclusion

This paper proposed an observer design method for descriptor systems with packet losses. To deal with packet losses, the observer changes its structure depending on whether the measurements are successfully received. As a result, the error dynamic becomes a stochastic switched system. With the definition of stochastic stability, sufficient conditions to ensure the stochastic stability and stochastic robust performance of the error dynamic were derived and transformed into linear matrix inequalities. The simulation results of a truck-trailer system are given to demonstrate that the proposed method has sufficient performance. Future work will focus on extending the proposed method to nonlinear systems, such as Lipschitz nonlinear systems and Takagi–Sugeno systems.

Data Availability

Only simulation data are included in this research.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Yan Liu and Jiazhong Xu suggested the approach. Weifeng Zhong and Bo You implemented it and analyzed the results. Qiang Fan completed simulations for comparison in the revised manuscript. Yan Liu was a major contributor in writing the manuscript. All authors read and approved the final manuscript.

Acknowledgments

This work was partially supported by the National Natural Science Foundation of China (Grant nos. 61801476 and 61971413), Nature Science Foundation of Jiangsu Province (Grant nos. BK20170387 and BK20180221), Suzhou Industry Technological Innovation Livelihood (Grant no. SS201866), and Foundation of Key Laboratory in Science and Technology Development Project of Suzhou (Grant no. SZS201818).

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