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Research Article

Design of Multiple Dependent State Sampling Plan Application for COVID-19 Data Using Exponentiated Weibull Distribution

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The proposed sampling plan in this article is referred to as multiple dependent state (MDS) sampling plans, for rejecting a lot based on properties of the current and preceding lot sampled. The median life of the product for the proposed sampling plan is assured based on a time-truncated life test, when a lifetime of the product follows exponentiated Weibull distribution (EWD). For the proposed plan, optimal parameters such as the number of preceding lots required for deciding whether to accept or reject the current lot, sample size, and rejection and acceptance numbers are obtained by the approach of two points on the operating characteristic curve (OC curve). Tables are constructed for various combinations of consumer and producer's risks for various shape parameters. The proposed MDS sampling plan for EWD is demonstrated using the coronavirus (COVID-19) outbreak in China. The performance of the proposed sampling plan is compared with the existing single-sampling plan (SSP) when the quality of the product follows EWD.

1. Introduction

The demand for high-quality products by consumers has always put pressure on the manufacturing industry, which has consequently resulted in time to time stretching its efficiency of production processes, to cater for consumer demand. The quality of the product management is based on the continuous manufacturing processes, and the final product could be tested for quality characteristics. For the continuous production processes, management is always done by using quality control charts, while sampling inspection is used for final product inspection by both consumers and producers, which, in contemporary literature, is referred to as acceptance sampling plan (ASP).

The theme of sampling plans (SPs) is to decide to reject or accept a lot at the minimum inspection costs, while satisfying both consumers and producers' risks simultaneously. To arrive at this decision, a random sample was taken from a randomly selected lot, from produced lots by the industry. The two risks are considered simultaneously to have a more reliable decision on the disposition of the lot coming for inspection, at the minimum cost in terms of finances and time consumed to inspect all products. This reduces the possibility of the cost incurred of acceptance of bad lots by a consumer (consumer's risk β) and rejection of a good lot by the producers (producers' risk α).

The levels of quality connected to consumer and producer's risks have been referred to by quality control expertise as limiting quality level (LQL) and acceptable quality level (AQL), respectively. The field of ASP has experienced several improvements so far depending on the nature of the interest of the author and researcher. This paper intends to concentrate on time-truncated life that has proven to be more effective and gained popularity among acceptance sampling techniques. Normally, life test is used in the inspection of a lifetime of a particular product and sets the limit of time, which ultimately aids in the provision of life assurance of the product to both consumer and producer of the product [1].

The ASPs using time-truncated life tests have been used to study manufacturing industries product's reliability. Several SPs have been proposed in the literature using life tests for various distributions (for quality characteristics) under different situations. Aslam et al. [2] discussed extensively a tightened-normal-tightened group acceptance sampling plan with the assumption that the percentile life of the product is taken as product's quality. Aslam et al. [3] established a group sampling plan centred on truncated life tests considering an inverse Gaussian distribution. In the literature, the impact of misspecification of the model parameters for group sampling plan has also been explored. In addition, Gui [4] worked on ASP considering truncated life tests for half exponential power life distribution. AL-Omari [5] worked on an ASP for truncated life tests based on three parameter-kappa distributions. A single- and double-sampling plan for variable sampling inspection was proposed [6] under the Weibull distribution based on sudden death testing. Balamurali et al. [7] dealt with Weibull distributed lifetime assuring mean life for optimal design of multiple deferred state sampling plans. In addition, Balamurali et al. [7] took more effort to design multiple deferred state sampling plans for the generalized inverted exponential distribution. Based on the literature that we have gone through, there is no work that has been done for multiple deferred state sampling plans based on exponentiated Weibull distribution (EWD), hence the interest of this article.

2. Exponentiated Weibull Distribution (EWD)

The EWD originated from Weibull distribution, and the said distribution fits lifetime data, with exception of data with empirical hazard rates with non-monotone shapes, which are commonly encountered in survival analysis, and this makes the Weibull model useless in analysing data [8]. Generalization and extension of Weibull distribution was the result of such limitations, which therefore gave flexible alternatives in terms of modelling. For more details, please refer, Marshall and Olkin [9]; the extended Weibull distribution, the new extended Weibull distribution by Peng and Yan [10]; Lee et al. [11] the beta Weibull distribution, the modified Weibull distribution by Sarhan and Zaindin [12]. Others are the (P-A-L) extended Weibull distribution by Al-Zahrani et al. [13]; the additive Weibull distribution by [14], the generalized Weibull distribution by [15, 16] proposed EWD, the Kumaraswamy Weibull distribution as proposed by Cordeiro, et al. [17] and the generalized modified Weibull distribution established by Carrasco et al. [18].

EWD was introduced by Gupta and Kapoor [19]; the distribution is characterized by shape and scale parameters which make it look like the gamma or Weibull family. The two-parameter gamma and Weibull distributions are the popular distributions used for analysing lifetime data. The gamma distribution is widely applied apart from survival analysis. Its survival function cannot be obtained in a closed form unless the shape parameter is an integer. On its part, the Weibull distribution's survival function and failure rate have simple forms. This characteristic has made the Weibull distribution much useful in analysing lifetime data. In this paper, we consider the exponentiated Weibull family that was introduced by Mudholkar and Srivastava [16], which has one scale parameter and two shape parameters. Let us assume that lifetime of quality characteristic follows EWD with two shape parameters. Then, EWD has the following cumulative distribution function (CDF):

$$F(t;\delta,\gamma,\lambda)=\left(1-\exp\left(-\left(\frac{t}{\lambda}\right)^{\gamma}\right)\right)^{\delta}, \tag{1}$$

where $t>0, \delta>0, \gamma>0, \lambda>0$.

The probability density function for such a variable is given as follows:

$$f(t;\delta,\gamma,\lambda)=\left(\frac{\delta\gamma}{\lambda}\right)\left(\frac{t}{\lambda}\right)^{\gamma-1}\exp\left(-\left(\frac{t}{\lambda}\right)^{\gamma}\right)\left(1-\exp\left(-\left(\frac{t}{\lambda}\right)^{\gamma}\right)\right)^{\delta-1}, \tag{2}$$

where $t>0, \delta>0, \gamma>0, \lambda>0$.

For known shape parameters (δ,γ), the CDF depends only on λt and the q^{th} quantile (t_q) of the product; when its products' lifetime follows the EWD, it is given as

$$t_q=\lambda\left[-\ln\left(1-q^{1/\delta}\right)\right]^{(1/\gamma)}. \tag{3}$$

But for median, quantile $(q)=0.5$.

$$t_{0.5}=\lambda\left[-\ln\left(1-0.5^{1/\delta}\right)\right]^{(1/\gamma)}. \tag{4}$$

The failure probability of products before the experiment time t_0 under the EWD is given as follows:

$$p=\left(1-\exp\left(-\left(\frac{t_0}{\lambda}\right)^{\gamma}\right)\right)^{\delta}. \tag{5}$$

The termination time (t_0) can be expressed as t_q^0), $t_0=at_q^0$, where t_q^0 is time truncation; also, the scale parameter can be written as

$$\lambda_0=\frac{t_q^0}{\left[-\ln\left(1-(1/2)^{1/\delta}\right)\right]^{(1/\gamma)}}. \tag{6}$$

For simplicity, $\lambda_0=(t_q^0/\eta)$.

Hence, the failure probability of the product given in equation (3) can be expressed by substituting values for λ and t_0 as follows:

$$p=\left(1-\exp\left(-\left(\frac{t_0}{t_q^0}\right)^{\gamma}(a\eta)^{\gamma}\right)\right)^{\delta}. \tag{7}$$

Then, in expanded form, it becomes

$$p=\left[1-\exp\left(\left(\frac{t_q}{t_q^0}\right)^{\gamma}\left(a\ \ln\left(1-q^{1/\delta}\right)\right)^{\gamma}\right)\right]^{\delta}. \tag{8}$$

When the quantile ratio (t_q/t_q^0) is greater than one, the above failure probability can be considered as AQL (p_1), and when it is equal to one, this probability is called the LQL (p_2). In this study, the assumption is that the shape parameters (δ,γ) of the EWD are known. The shape parameters can be estimated from a previous production process, and generally, the manufacturer maintains the history of the product.

3. Multiple Dependent State Sampling Plans (MDSSPs)

The MDSSPs are regarded as one of the conditional SPs under special-purpose SP. Baker and Wortham introduced the MDS sampling concept. These are the modifications of the chain-sampling plan proposed by Dodge as MDS-1 (c_1, c_2) [20, 21]. This can be applied for continuous production, and lots are submitted for serial inspection in the production order. The sample size can be reduced by implementing the MDSSP since the decision regarding the disposition of the current lot is made using the results of samples drawn from both current and successive lots. Several scholars have investigated MDSSP in various situations. Few to mention are Govindaraju and Subramani who discussed the selection of MDSSP for given AQL and LQL [22]. Balamurali and Jun [23] studied the MDSSP based on measurement data. Balamurali et al. [24] investigated the MDSSP using Bayesian methodology. For more details on MDSSP, one may refer to Vaerst [25]; Soundararajan and Vijayaraghavan [26]; Subramani and Haridoss [27]; and Aslam et al. [28]. Recently, the concept of MDS sampling has been used in control chart design. Aslam et al. [29, 30] studied an attribute control chart for monitoring the manufacturing process based on an MDS sampling approach. In this work, we followed the methods of Rao et al. [31]. MDSSP is proposed to utilize the median life of the product based on a time-truncated life test, when the lifetime of the product (quality characteristics of the product) follows a EWD. The operating procedure of the MDSSP for EWD is given in the next section. The proposed plan and existing plans' performances are compared to reveal the best among them.

4. Designing MDSSPs for EWD

In this section, the operating procedure and designing methodology of the MDSSP are discussed for EWD.

4.1. Operating Procedure. In this section, the operating procedure and designing methodology of the MDSSP are discussed for EWD. The following are the various steps in the operating procedure to obtain the parameters.

(1) From a current lot, draw a random sample of n units. Fix time t_0 and put all units simultaneously through a life test.
(2) Record the number of failed units before t_0 (fixed time) and call it d.
(3) The decision to reject or accept the lot is as follows: if $d > c_2$, reject the current lot, if $d \leq c_1$, accept the lot, and end the test. If $c_1 < d \leq c_2$, accept the current lot provided that successive m (previous m) lots will be accepted with the condition $d \leq c_1$.

The proposed MDSSP is characterized by four parameters, namely, c_1, c_2, n, and m, where n is the sample size, m is the number of previous lots needed to make a decision, c_2 is the maximum number of failed items for conditional acceptance, and c_1 is the maximum number of failed items for unconditional acceptance.

Note that the attribute MDSSP is the generalization to a SSP, and it also reduces to SSP when either $m \longrightarrow \infty$ $c_1 = c_2 = c$ or. The operating characteristic (OC) function of the MDSSP for EWD for time-truncated life test is given by

$$P_a(p) = p(d \leq c_1) + p(c_1 < d \leq c_2)(p(d \leq c_1))^m. \tag{9}$$

For the lot acceptance, probability at failure probability p considering a binomial distribution is obtained using the following equation:

$$P_a(p) = \sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p^d (1-p)^{n-d} \left(\sum_{d=0}^{c_1} \binom{n}{d} p^d (1-p)^{n-d} \right)^m. \tag{10}$$

4.2. Designing Methodology. SSPs are designed to focus on minimizing the average sample number (ASN). A preferable SSP is the one with a minimum ASN. This is because for the minimum ASN, the corresponding inspection time and inspection cost will be reduced. In this article, we attempt to minimize the ASN of the proposed MDSSP for EWD under truncated life tests. This is achieved through optimization problem that minimizes the ASN to obtain optimal parameters for proposed design as follows:

$$\begin{aligned} &\text{minimize ASN}(p) = n \\ &\qquad P_a(p_1) \geq 1 - \alpha, \\ &\text{subjected to } P_a(p_2) \leq \beta, \\ &\qquad n > 1, m > 1, c_2 > c_1 \geq 0, \end{aligned} \tag{11}$$

where failure probability is p_1 at the producer's risk and the failure probability at the consumer's risk is p_2. The quality level is expressed as the ratio of its true lifetime quantile ratio given as t_0/t_q^0. The lifetime quantile ratio concept helps the producer to enhance product quality. The probabilities of acceptance of the lot at AQL and LQL under an MDS sampling plan are, respectively, obtained using the following equations:

$$P_a(p_1) = \sum_{d=0}^{c_1} \binom{n}{d} p_1^d (1-p_1)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p_1^d (1-p_1)^{n-d} \left(\sum_{d=0}^{c_1} \binom{n}{d} p_1^d (1-p_1)^{n-d} \right)^m,$$
$$P_a(p_2) = \sum_{d=0}^{c_1} \binom{n}{d} p_2^d (1-p_2)^{n-d} + \sum_{d=c_1+1}^{c_2} \binom{n}{d} p_2^d (1-p_2)^{n-d} \left(\sum_{d=0}^{c_1} \binom{n}{d} p_2^d (1-p_2)^{n-d} \right)^m. \quad (12)$$

The producer wishes that his product should be accepted when it is of good quality. Therefore, we consider the quantile ratio $t_0/t_q^0 = 2, 4, 6, 8, 10$ at the producer's risk. The consumer wants to reject the product if it is at a poor quality level. Therefore, we consider the mean ratio $t_0/t_q^0 = 1$ at the consumer's risk.

The optimal parameters of the proposed MDS sampling plan for EWD with known shape parameters $(\delta, \gamma) = (2, 2), (2, 1.5), (1.5, 1.5)$, and $(1.5, 2)$ under truncated life tests are given in Tables 1–5 by assuming that the consumer's risk $\beta = 0.25, 0.10, 0.05$, and 0.01 and producer's risk $\alpha = 0.05$ at 50$^{\text{th}}$ percentile and 25$^{\text{th}}$ percentile. Also, the optimal parameters of the proposed MDS sampling plan for EWD with estimated shape parameters $\gamma = 0.9525$ and $\delta = 4.4859$ for the real-time data related to the coronavirus outbreak in China are given in Table 6. The termination ratio is considered as $a = 0.5$, $a = 0.7$, and $a = 1.0$. From the results in Tables 1–6, we observed the following:

(1) When other parametric combinations are fixed, we noticed that as the termination ratio increases from 0.5 to 1.0, the required sample size n decreases.

(2) It is interesting to note that as consumer's risk decreases, the sample sizes increase when other parametric combinations are fixed.

(3) Furthermore, it is observed that shape parameters also influence sample size in the proposed design.

(4) In addition, we have noticed that sample size decreases when quantile value increases (i.e., from 25$^{\text{th}}$ percentile to 50$^{\text{th}}$ percentile) assuming that all other parametric combinations are fixed.

(5) Results revealed that the increase in quantile ratio also increases the producer's probability of `lot acceptance when other parametric combinations are fixed.

5. Application of Proposed Sampling Plan for COVID-19 Data

In this section, real data of COVID-19 mortality rates from Mexico (see https://covid19.who.int/) are given to test the EWD's goodness of fit. The data represent 108-day COVID-19 mortality rate data belonging to Mexico which were recorded from 4 March to 20 July 2020. These data were used by Almongy et al. [32] for the application of a new extended Rayleigh distribution. These data formed rough mortality rate. The data are as follows: 1.041, 1.205, 1.402, 1.800, 1.815, 1.867, 1.923, 2.058, 2.065, 2.070, 2.077, 2.326, 2.352, 2.438, 2.500, 2.506, 2.601, 2.838, 2.926, 2.988, 3.027, 3.029, 3.215, 3.218, 3.219, 3.228, 3.233, 3.257, 3.286, 3.298, 3.327, 3.336, 3.359, 3.395, 3.440, 3.499, 3.537, 3.632, 3.751, 3.778, 3.922, 4.089, 4.120, 4.292, 4.344, 4.424, 4.557, 4.648, 4.661, 4.697, 4.730, 4.909, 4.949, 5.143, 5.242, 5.317, 5.392, 5.406, 5.442, 5.459, 5.854, 5.985, 6.015, 6.105, 6.122, 6.140, 6.182, 6.327, 6.370, 6.412, 6.535, 6.560, 6.625, 6.656, 6.697, 6.814, 6.968, 7.151, 7.260, 7.267, 7.486, 7.630, 7.840, 7.854, 7.903, 8.108, 8.325, 8.551, 8.696, 8.813, 8.826, 9.284, 9.391, 9.550, 9.935, 10.035, 10.043, 10.158, 10.383, 10.685, 10.855, 11.665, 12.042, 12.878, 13.220, 14.604, 14.962, 16.498.

To fit the EWD, the shape parameters are estimated using the maximum likelihood approach, and they are $\gamma = 0.9525$ and $\delta = 4.4859$. The Kolmogorov–Smirnov test statistic value is 0.0684, and p value is 0.6925. Figure 1 shows the empirical and theoretical PDFs, empirical and theoretical CDFs, Q-Q plots, and P-P plots to highlight the goodness of fit of EWD. Hence, EWD yields a good fit for COVID-19 mortality rate data.

Suppose that the experimenter would like to use the proposed multiple dependent state sampling plans to implement the median life percentile of the product where the product lifetime follows an EWD with the shape parameters $\gamma = 0.9525$ and $\delta = 4.4859$. If the medical practitioner assume that the median mortality rate of the person suffering from COVID-19 is 2.0, the medical practitioner expected that the median mortality rate would be 4.0. The consumer's risk is 0.05 if the actual median mortality rate is 2.0 and the producer's risk is 0.10 if the actual median mortality rate is 4.0. With these constraints, the optimal parameters selected from Table 6 are $n = 29$, $c_1 = 1$, $c_2 = 3$, and $m = 2$ with values of $\gamma = 0.9525$ and $\delta = 4.4859$, $t_q^0 = 1.5$, $\alpha = 0.05$, $\beta = 0.10$, and $t_q/t_q^0 = 2$ at $a = 0.5$. The multiple dependent state sampling plans are established as follows.

A sample of 29 mortality rates due to COVID-19 will be selected at random for the group of people, and their mortality rate is 2.0. If the mortality rate before 2.0 is less than or equal to 1 case, then the group of people will be accepted and the group of people will be rejected if it is greater than 3 cases. There will be a disposition of the present group of people deferred until the 2 preceding groups of people will be tested in case of the number of cases between 1 and 3. In this real example, there are seven cases before the mortality rate of 2.0 due to COVID-19 in Mexico. Hence, we reject the present group of people. Thus, medical

Table 1: Optimal parameters of the proposed MDSSP for EWD with $\delta = 2, \gamma = 2$ at 50^{th} percentile.

β	t_q/t_q^0	$a = 0.5$					$a = 0.7$					$a = 1.0$				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	20	0	10	3	0.9710	7	0	6	2	0.9690	3	0	2	1	0.9616
	4	20	0	10	3	0.9998	7	0	6	2	0.9998	3	0	2	1	0.9997
	6	20	0	10	3	1.0000	7	0	6	2	1.0000	3	0	2	1	1.0000
	8	20	0	10	3	1.0000	7	0	6	2	1.0000	3	0	2	1	1.0000
	10	20	0	10	3	1.0000	7	0	6	2	1.0000	3	0	2	1	1.0000
0.10	2	35	0	1	1	0.9566	12	0	2	1	0.9546	7	1	3	1	0.9928
	4	32	0	10	3	0.9996	11	0	1	1	0.9997	4	0	1	1	0.9994
	6	32	0	10	3	1.0000	11	0	1	1	1.0000	4	0	1	1	1.0000
	8	32	0	10	3	1.0000	11	0	1	1	1.0000	4	0	1	1	1.0000
	10	32	0	10	3	1.0000	11	0	1	1	1.0000	4	0	1	1	1.0000
0.05	2	66	1	2	3	0.9877	22	1	2	1	0.9873	8	1	3	1	0.9882
	4	42	0	10	2	0.9996	14	0	1	1	0.9995	5	0	2	1	0.9993
	6	42	0	10	2	1.0000	14	0	1	1	1.0000	5	0	2	1	1.0000
	8	42	0	10	2	1.0000	14	0	1	1	1.0000	5	0	2	1	1.0000
	10	42	0	10	2	1.0000	14	0	1	1	1.0000	5	0	2	1	1.0000
0.01	2	93	1	2	1	0.9782	30	1	2	1	0.9686	11	1	5	1	0.9687
	4	64	0	10	2	0.9990	21	0	2	1	0.9992	7	0	2	1	0.9986
	6	64	0	10	2	1.0000	21	0	2	1	1.0000	7	0	2	1	0.9999
	8	64	0	10	2	1.0000	21	0	2	1	1.0000	7	0	2	1	1.0000
	10	64	0	10	2	1.0000	21	0	2	1	1.0000	7	0	2	1	1.0000

Table 2: Optimal parameters of the proposed MDSSP for EWD with $\delta = 2, \gamma = 1.5$ at 50^{th} percentile.

β	t_q/t_q^0	$a = 0.5$					$a = 0.7$					$a = 1.0$				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	21	1	11	4	0.9839	10	1	4	2	0.9854	5	1	2	1	0.9720
	4	11	0	10	3	0.9973	5	0	2	2	0.9974	3	0	2	1	0.9964
	6	11	0	10	3	0.9997	5	0	2	2	0.9997	3	0	2	1	0.9996
	8	11	0	10	3	1.0000	5	0	2	2	1.0000	3	0	2	1	0.9999
	10	11	0	10	3	1.0000	5	0	2	2	1.0000	3	0	2	1	1.0000
0.10	2	30	1	11	3	0.9600	14	1	2	1	0.9501	7	1	3	1	0.9503
	4	18	0	10	2	0.9953	8	0	7	2	0.9936	4	0	1	1	0.9916
	6	18	0	10	2	0.9995	8	0	7	2	0.9994	4	0	1	1	0.9991
	8	18	0	10	2	0.9999	8	0	7	2	0.9999	4	0	1	1	0.9998
	10	18	0	10	2	1.0000	8	0	7	2	1.0000	4	0	1	1	1.0000
0.05	2	39	1	3	1	0.9588	22	2	12	2	0.9822	11	2	5	1	0.9775
	4	23	0	10	2	0.9925	10	0	9	2	0.9901	5	0	2	1	0.9905
	6	23	0	10	2	0.9993	10	0	9	2	0.9990	5	0	2	1	0.9990
	8	23	0	10	2	0.9999	10	0	9	2	0.9998	5	0	2	1	0.9998
	10	23	0	10	2	1.0000	10	0	9	2	0.9999	5	0	2	1	0.9999
0.01	2	65	2	12	2	0.9610	29	2	4	1	0.9568	17	3	8	1	0.9771
	4	35	0	10	2	0.9835	16	0	2	1	0.9873	7	0	2	1	0.9820
	6	35	0	10	2	0.9983	16	0	2	1	0.9987	7	0	2	1	0.9980
	8	35	0	10	2	0.9997	16	0	2	1	0.9998	7	0	2	1	0.9996
	10	35	0	10	2	0.9999	16	0	2	1	0.9999	7	0	2	1	0.9999

Table 3: Optimal parameters of the proposed MDSSP for EWD with $\delta = 1.5, \gamma = 1.5$ at 50th percentile.

β	t_q/t_q^0	$a = 0.5$					$a = 0.7$					$a = 1.0$				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	17	1	5	2	0.9603	9	1	8	2	0.9534	7	2	3	2	0.9720
	4	8	0	7	4	0.9828	5	0	1	1	0.9889	3	0	2	1	0.9867
	6	8	0	7	4	0.9968	5	0	1	1	0.9980	3	0	2	1	0.9976
	8	8	0	7	4	0.9991	5	0	1	1	0.9994	3	0	2	1	0.9990
	10	8	0	7	4	0.9997	5	0	1	1	0.9998	3	0	2	1	0.9997
0.10	2	32	2	12	2	0.9663	17	2	12	2	0.9577	10	2	6	1	0.9557
	4	14	0	10	2	0.9739	7	0	6	2	0.9715	4	0	1	1	0.9697
	6	14	0	10	2	0.9952	7	0	6	2	0.9947	4	0	1	1	0.9942
	8	14	0	10	2	0.9986	7	0	6	2	0.9985	4	0	1	1	0.9983
	10	14	0	10	2	0.9995	7	0	6	2	0.9994	4	0	1	1	0.9994
0.05	2	40	2	6	1	0.9535	24	3	6	2	0.9705	13	3	12	2	0.9612
	4	18	0	1	1	0.9684	9	0	8	2	0.9554	5	0	2	1	0.9655
	6	18	0	1	1	0.9942	9	0	8	2	0.9914	5	0	2	1	0.9934
	8	18	0	1	1	0.9883	9	0	8	2	0.9975	5	0	2	1	0.9981
	10	18	0	1	1	0.9994	9	0	8	2	0.9990	5	0	2	1	0.9993
0.01	2	69	4	7	1	0.9746	36	4	6	1	0.9558	20	4	10	1	0.9555
	4	28	0	3	1	0.9508	20	1	3	1	0.9968	11	1	5	1	0.9952
	6	27	0	1	1	0.9873	14	0	2	1	0.9895	7	0	2	1	0.9875
	8	27	0	1	1	0.9963	14	0	2	1	0.9969	7	0	2	1	0.9963
	10	27	0	1	1	0.9986	14	0	2	1	0.9988	7	0	2	1	0.9986

Table 4: Optimal parameters of the proposed MDSSP for EWD with $\delta = 1.5, \gamma = 2$ at 50th percentile.

β	t_q/t_q^0	$a = 0.5$					$a = 0.7$					$a = 1.0$				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
0.25	2	26	1	2	2	0.9881	11	1	3	2	0.9911	5	1	2	1	0.9840
	4	13	0	1	3	0.9980	6	0	5	2	0.9981	3	0	2	1	0.9981
	6	13	0	1	3	0.9998	6	0	5	2	0.9998	3	0	2	1	0.9998
	8	13	0	1	3	1.0000	6	0	5	2	1.0000	3	0	2	1	1.0000
	10	13	0	1	3	1.0000	6	0	5	2	1.0000	3	0	2	1	1.0000
0.10	2	37	1	5	2	0.9796	15	1	4	2	0.9756	7	1	3	1	0.9725
	4	22	0	10	2	0.9967	9	0	8	2	0.9959	4	0	1	1	0.9954
	6	22	0	10	2	0.9997	9	0	8	2	0.9996	4	0	1	1	0.9996
	8	22	0	10	2	0.9999	9	0	8	2	0.9999	4	0	1	1	0.9999
	10	22	0	10	2	1.0000	9	0	8	2	1.0000	4	0	1	1	1.0000
0.05	2	45	1	11	2	0.9620	18	1	5	2	0.9566	8	1	3	1	0.9568
	4	28	0	10	2	0.9947	11	0	1	2	0.9927	5	0	2	1	0.9948
	6	28	0	10	2	0.9995	11	0	1	2	0.9990	5	0	2	1	0.9995
	8	28	0	10	2	0.9999	11	0	1	2	1.0000	5	0	2	1	0.9999
	10	28	0	10	2	1.0000	11	0	1	2	0.9999	5	0	2	1	1.0000
0.01	2	79	2	4	1	0.9817	32	2	3	1	0.9540	14	2	6	1	0.9711
	4	43	0	1	1	0.9909	17	0	10	2	0.9863	7	0	2	1	0.9901
	6	43	0	1	1	0.9991	17	0	10	2	0.9986	7	0	2	1	0.9990
	8	43	0	1	1	0.9998	17	0	10	2	0.9998	7	0	2	1	0.9998
	10	43	0	1	1	1.0000	17	0	10	2	0.9999	7	0	2	1	1.0000

Table 5: Optimal parameters of the proposed MDSSP for EWD with $\delta = 1.5, \gamma = 2$ at 25th percentile.

β	t_q/t_q^0	$a = 0.5$					$a = 0.7$					$a = 1.0$				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
	2	45	0	2	1	0.9500	26	1	11	3	0.9912	10	1	2	3	0.9842
	4	34	0	1	3	0.9981	13	0	1	3	0.9980	5	0	4	3	0.9979
0.25	6	34	0	1	3	0.9998	13	0	1	3	0.9998	5	0	4	3	0.9998
	8	34	0	1	3	1.0000	13	0	1	3	1.0000	5	0	4	3	1.0000
	10	34	0	1	3	1.0000	13	0	1	3	1.0000	5	0	4	3	1.0000
	2	94	1	11	3	0.9756	37	1	4	2	0.9797	15	1	2	1	0.9658
	4	56	0	1	2	0.9964	22	0	10	2	0.9997	9	0	1	1	0.9967
0.10	6	56	0	1	2	0.9997	22	0	10	2	0.9967	9	0	1	1	0.9997
	8	56	0	1	2	0.9999	22	0	10	2	0.9999	9	0	1	1	0.9999
	10	56	0	1	2	1.0000	22	0	10	2	1.0000	9	0	1	1	1.0000
	2	115	1	5	2	0.9675	45	1	11	2	0.9626	18	1	3	1	0.9678
	4	72	0	1	2	0.9942	28	0	4	2	0.9947	11	0	1	1	0.9950
0.05	6	72	0	1	2	0.9995	28	0	4	2	0.9995	11	0	1	1	0.9995
	8	72	0	1	2	0.9999	28	0	4	2	0.9999	11	0	1	1	0.9999
	10	72	0	1	2	1.0000	28	0	4	2	1.0000	11	0	1	1	1.0000
	2	203	2	3	1	0.9659	79	2	3	1	0.9614	31	2	5	1	0.9813
	4	111	0	10	2	0.9893	43	0	1	1	0.9909	17	0	2	1	0.9919
0.01	6	111	0	10	2	0.9990	43	0	1	1	0.9991	17	0	2	1	0.9992
	8	111	0	10	2	0.9998	43	0	1	1	0.9998	17	0	2	1	0.9999
	10	111	0	10	2	1.0000	43	0	1	1	1.0000	17	0	2	1	1.0000

Table 6: Optimal parameters of the proposed MDSSP for EWD with $\delta = 4.4859, \gamma = 0.9525$ at 50th percentile.

β	t_q/t_q^0	$a = 0.5$					$a = 0.7$					$a = 1.0$				
		n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$
	2	14	0	2	1	0.9516	9	1	8	4	0.9828	5	1	2	1	0.9683
	4	11	0	2	2	0.9995	5	0	4	2	0.9987	3	0	2	1	0.9974
0.25	6	11	0	2	2	1.0000	5	0	4	2	0.9999	3	0	2	1	0.9998
	8	11	0	2	2	1.0000	5	0	4	2	1.0000	3	0	2	1	1.0000
	10	11	0	2	2	1.0000	5	0	4	2	1.0000	3	0	2	2	1.0000
	2	29	1	3	2	0.9834	13	1	11	2	0.9676	9	2	8	2	0.9812
	4	17	0	2	2	0.9987	8	0	1	1	0.9977	4	0	1	1	0.9938
0.10	6	17	0	2	2	0.9999	8	0	1	1	0.9999	4	0	1	1	0.9996
	8	17	0	2	2	1.0000	8	0	1	1	1.0000	4	0	1	1	1.0000
	10	17	0	2	2	1.0000	8	0	1	1	1.0000	4	0	1	1	1.0000
	2	35	1	4	2	0.9715	17	1	4	1	0.9582	11	2	5	1	0.9730
	4	22	0	10	2	0.9979	10	0	1	1	0.9964	5	0	2	1	0.9930
0.05	6	22	0	10	2	0.9999	10	0	1	1	0.9998	5	0	2	1	0.9995
	8	22	0	10	2	1.0000	10	0	1	1	1.0000	5	0	2	1	0.9999
	10	22	0	10	2	1.0000	10	0	1	1	1.0000	5	0	4	2	1.0000
	2	50	1	4	1	0.9526	28	2	5	1	0.9737	17	3	8	1	0.9716
	4	34	0	1	1	0.9963	15	0	2	1	0.9944	7	0	2	1	0.9866
0.01	6	34	0	1	1	0.9998	15	0	2	1	0.9997	7	0	2	1	0.9991
	8	34	0	1	1	1.0000	15	0	2	1	1.0000	7	0	2	1	0.9999
	10	34	0	1	1	1.0000	15	0	2	1	1.0000	7	0	2	1	1.0000

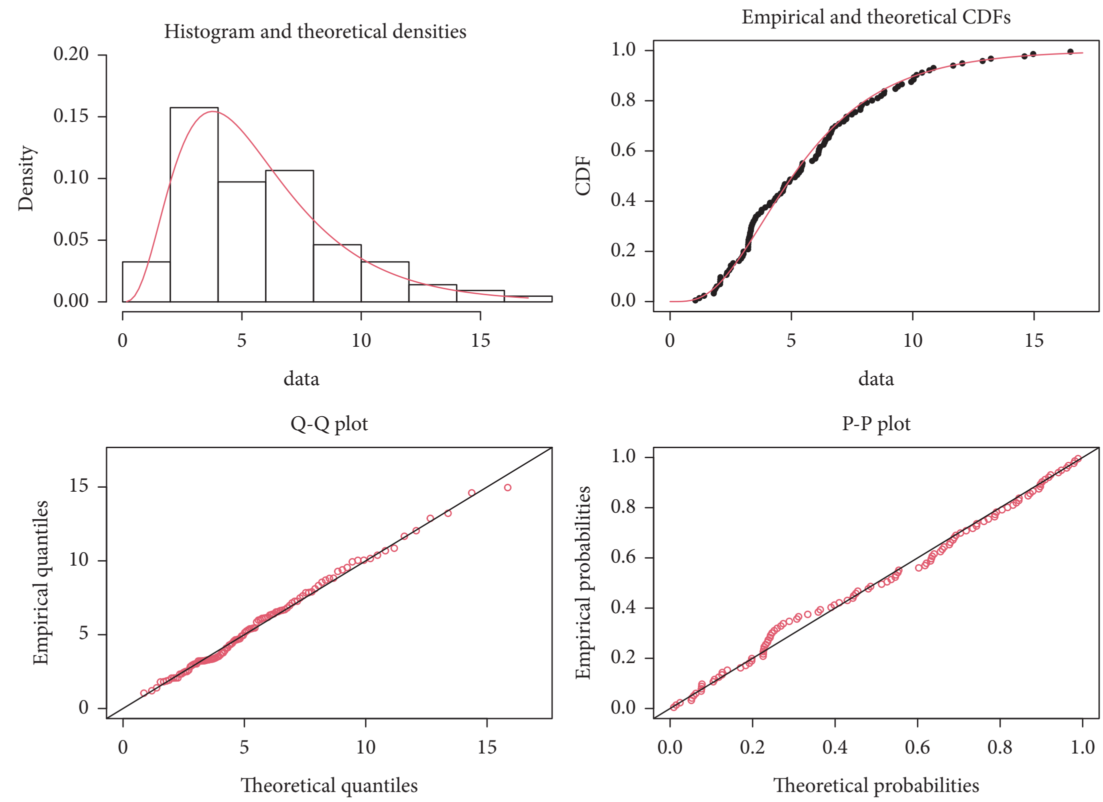

FIGURE 1: The empirical and theoretical PDFs, empirical and theoretical CDFs, Q-Q plots, and P-P plots for the EWD for COVID-19 mortality rate data.

TABLE 7: Comparison of optimal parameters of the proposed MDSSP and SSP for EWD with $\delta = 1.5, \gamma = 1.5$ at 50$^{\text{th}}$ percentile.

		$a = 0.5$								$a = 1.0$							
β	t_q/t_q^0	MDSSP					SSP			MDSSP					SSP		
		n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$	n	c_1	c_2	m	$P_a(p_1)$	n	c	$P_a(p_1)$
	2	17	1	5	2	0.9603	31	3	0.9661	7	2	3	2	0.9720	12	4	0.9680
	4	8	0	7	4	0.9828	16	1	0.9912	3	0	2	1	0.9867	5	1	0.9853
0.25	6	8	0	7	4	0.9968	8	0	0.9713	3	0	2	1	0.9976	2	0	0.9668
	8	8	0	7	4	0.9991	8	0	0.9818	3	0	2	1	0.999	2	0	0.9823
	10	8	0	7	4	0.9997	8	0	0.9907	3	0	2	1	0.9997	2	0	0.9891
	2	32	2	12	2	0.9663	48	4	0.9580	10	2	6	1	0.9557	17	5	0.9562
	4	14	0	10	2	0.9739	23	1	0.9823	4	0	1	1	0.9697	7	1	0.9707
0.1	6	14	0	10	2	0.9952	14	0	0.9503	4	0	1	1	0.9942	7	1	0.9944
	8	14	0	10	2	0.9986	14	0	0.9735	4	0	1	1	0.9983	4	0	0.9648
	10	14	0	10	2	0.9995	14	0	0.9839	4	0	1	1	0.9994	4	0	0.9784
	2	40	2	6	1	0.9535	63	5	0.9603	13	3	12	2	0.9612	21	6	0.9590
	4	18	0	1	1	0.9684	28	1	0.9743	5	0	2	1	0.9655	8	1	0.9619
0.05	6	18	0	1	1	0.9942	28	1	0.9953	5	0	2	1	0.9934	8	1	0.9927
	8	18	0	1	1	0.9883	18	0	0.9661	5	0	2	1	0.9981	5	0	0.9562
	10	18	0	1	1	0.9994	18	0	0.9793	5	0	2	1	0.9993	5	0	0.9731
	2	69	4	7	1	0.9746	95	7	0.9634	20	4	10	1	0.9555	30	8	0.9578
	4	28	0	3	1	0.9508	39	1	0.9527	11	1	5	1	0.9952	14	2	0.9833
0.01	6	27	0	1	1	0.9873	39	1	0.9911	7	0	2	1	0.9875	11	1	0.9861
	8	27	0	1	1	0.9963	39	1	0.9974	7	0	2	1	0.9963	11	1	0.9959
	10	27	0	1	1	0.9986	27	0	0.9691	7	0	2	1	0.9986	7	0	0.9625

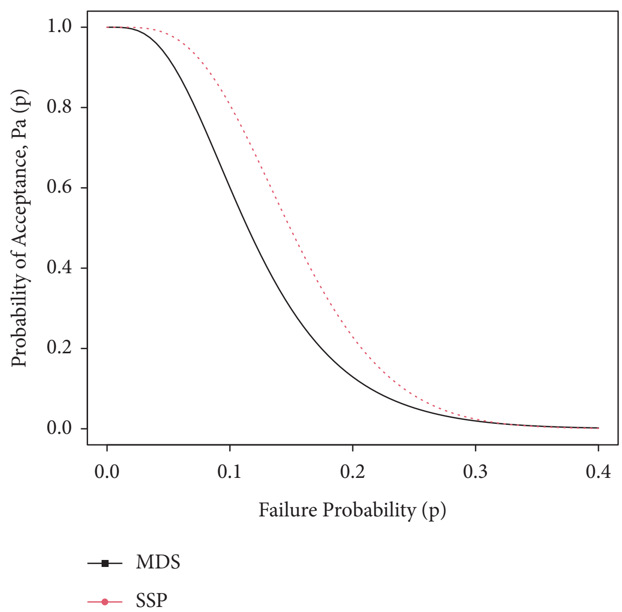

FIGURE 2: OC curves of MDSP and SSP designs.

practitioners could suggest to the government or medical institutions that the median mortality rate due to COVID-19 is at an unacceptable level.

6. Comparative Study

A comparitive study is made between single and MDSSP when quality control follows EWD. The OC curve is used to show the efficiency of the plan. The curve has displayed the difference in probabilities of accepting a good lot as well as rejecting the bad lot. Table 7 reveals the efficiency of the proposed MDSSP over the SSP while assuming the underlying distribution of data to follow exponentiated Weibull distribution considering quantile ratio $t_0/t_q^0 = 2, 4, 6, 8, 10$ for each consumer's risk $\beta = 0.25, 0.10, 0.05, 0.01$ while keeping producer's risk at $\alpha = 0.05$. The comparison is basically on the sample size n and probability of acceptance $P_a(p_1)$. The acceptance sample size for the proposed MDSSP is smaller than the existing single-sampling plan for several set parameters (see Table 7). For quantile ratio 2, the plan parameters for MDSP are $n = 17$, $c_1 = 1$, $c_2 = 5$, and $m = 2$, whereas for SSP, the plan parameters are $n = 31$ and $c_1 = 3$ with the corresponding probability of acceptance of 0.972 and 0.968. The acceptance sample size is smaller for MDSP and relatively larger for SSP, while the acceptance probability is larger for MDSP and relatively larger than that of SSP. As the quantile ratio increased, the acceptance sample size decreased for both sampling plans. Figure 2 depicts the operating characteristic (OC) curve for comparison of MDSP with plan parameters $n = 17$, $c_1 = 1$, $c_2 = 5$, and $m = 2$ and SSP with $n = 31$ and $c_1 = 3$. It is noticed that the MDS plan is reasonably and greatly efficient than SSP in terms of sample size.

7. Conclusions

In this article, multiple deferred state sampling plans were developed under the assumption that the lifetime of the product follows an exponentiated Weibull distribution when lifetime tests are truncated. The optimal parameters of the proposed sampling plan are obtained by satisfying the respective consumer and producer's risks simultaneously. A comparative study of the proposed MDSSP has been performed using OC curves along with a single-sampling plan. We conclude that the proposed MDS sampling plan is more effective than the existing single-sampling plans to secure the consumer and producer with less inspection.

Data Availability

The datasets used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

GSR was responsible for methodology and computations. AKF and JKP wrote the paper and collected the data. All authors read and approved the final manuscript.

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