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Research Article

Finite-Time Admissibility and Controller Design for T-S Fuzzy Stochastic Singular Systems with Distinct Differential Term Matrices

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The finite-time admissibility analysis and controller design issues for extended T-S fuzzy stochastic singular systems (FSSSs) with distinct differential term matrices and Brownian parameter perturbations are discussed. When differential term matrices are allowed to be distinct in fuzzy rules, such fuzzy models can describe a wide class of nonlinear stochastic systems. Using fuzzy Lyapunov function (FLF), a new and relaxed sufficient condition is proposed via strict linear matrix inequalities (LMIs). Different from the existing stability conditions by FLF, the derivative bounds of fuzzy membership functions are not required in this condition. Based on admissibility analysis results, a design method for parallel distribution compensation (PDC) controller of FSSSs is given to guarantee the finite-time admissibility of the closed-loop system. Finally, the feasibility and effectiveness of the proposed methods in this article are illustrated with three examples.

1. Introduction

T-S fuzzy systems have gained a lot of attentions in the recent years mostly due to fuzzy systems being the omnipotent simulator of nonlinear systems [1]. Meanwhile, the related domestic and foreign research achievements of the T-S fuzzy system are abundant and creative [2–4]. Stability is the fundamental characteristic of the system, which is the precondition to make sure the control system can run normally. Based on the common Lyapunov function (CLF), the stability conditions of such systems are proposed via LMIs in [5]. Then, considering the conservatism of the CLF in the stability analysis, a fuzzy Lyapunov function is given in [6] to study this kind of systems. Singular systems are a class of more general dynamic systems with extensive application background and hence have attracted much attention [7, 8]. So, T-S fuzzy singular systems with distinct derivative term matrices [9] are generalized from normal systems, which can describe the actual system accurately. Afterwards, considering the system with time delay, the PDC controller design method of these systems is given without all the subsystems which are regular and impulse-free in [10]. In [11], PDC and proportional-derivative state feedback controller for fuzzy singular systems with different derivative matrices are designed. In [12], the dissipativity analysis and controller design for a class of fuzzy descriptor systems with different derivative matrices are studied. In [13], under the presence of uncertainties, the novel fuzzy sliding-mode controller is given to maintain the system states onto the predefined fuzzy manifold.

In practical systems, randomness and nonlinearity are inevitable [14–16]. In this situation, it is necessary to study the stability of nonlinear stochastic singular systems. Considering the complexity of nonlinear stochastic singular systems, an effective method to use the fuzzy model as the approximators of the original system has achieved the analysis and control of such systems. The issues of robust stabilization for fuzzy stochastic singular systems are studied by using integral sliding-mode controller [17]. In [18], the sliding-mode controller design method of fuzzy stochastic singular systems with different local input matrices is given and it still has strong anti-interference. In [19], based on augmented singular sliding-mode observer, fault tolerant control for T-S fuzzy stochastic singular systems is discussed. In [20], using the suitable singular stochastic Lyapunov function, the sufficient conditions are firstly given to ensure the stochastically admissibility and dissipativity of the considered system. In [21], the integral sliding-mode controller is given for fuzzy singularly perturbed descriptor systems under nonlinear perturbation.

Different from Lyapunov asymptotic stability concept, finite-time stability issues focus on the behavior of the systems in a finite-time horizon. During the actual system operation, when the system is asymptotically stable, it may be useless because of the bad system instantaneous performance. So, related research studies on finite-time stability have received a lot of attention. In [22], the finite-time H_∞ controller design method is given for the nonlinear delay system presented by a fuzzy model. The reliable finite-time H_∞ control issues of singular nonlinear delay markov systems with bounded transition probabilities are considered in [23]. For the case of time-varying and unknown transition probabilities, finite-time H_∞ filtering T-S fuzzy nonhomogeneous Markov systems are addressed in [24]. Sufficient conditions are given to ensure that the closed-loop system is finite-time bounded with the H_∞ performance in [25]. In [26], considering unknown nonlinearities and random interference terms, a novel adaptive control method for the finite-time stability of nonlinear stochastic systems is proposed. In [27], the sufficient conditions of finite-time stability are given for the stochastic singular biological economic systems based on CLF. At present, the research on finite-time stability of fuzzy singular stochastic systems with Brownian motion is not sufficient, especially distinct differential term matrices in fuzzy rules.

Based on the above analysis, in this paper, the finite-time admissibility issues of T-S fuzzy singular stochastic systems with distinct differential term matrices and Brownian parameter perturbations are discussed via FLF. Firstly, the innovative sufficient condition is proposed to ensure that these systems are finite-time admissible via strict LMIs. Then, a novel design approach of fuzzy PDC controller is given, which avoids solving bilinear matrix inequalities. Finally, the effectiveness and feasibility of the improved method are verified by three examples. The contributions of this article are summarized as follows:

(i) The finite-time admissibility analysis and controller design for T-S fuzzy singular stochastic systems with distinct differential term matrices and Brownian parameter perturbations are studied. And this kind of systems can describe a wide range of nonlinear singular stochastic systems, and many actual systems are described more simply and naturally by such systems. Unlike existing methods, a relaxed finite-time admissibility condition is given based on FLF. Meanwhile, considering it is hard to obtain the bounds of derivatives for fuzzy membership functions, fuzzy Lyapunov matrices are redesigned to eliminate the derivatives of membership functions appearing in the admissibility condition.

(ii) Fuzzy PDC controller is further investigated to guarantee closed-loop finite-time admissibility. Using FLF to analyze stability, it is unavoidable to solve the bilinear matrix problem in the design process of PDC controller. Under the proposed approach, the controller design becomes simple and effective in form of strict LMIs.

2. Notations

$\mathrm{Det}(\mathscr{A})$: determinant of the matrix $\mathscr{A}$

$\mathfrak{R}^n$: n-dimensional real Euclidean space

$\mathfrak{R}^{m\times n}$: $m\times n$ matrices with real elements

Deg: degree of the polynomial

$\mathscr{A}^+$: Moore–Penrose pseudo inverse of $\mathscr{A}$

$\mathscr{Q}\succcurlyeq 0\,(\mathscr{Q}\prec 0)$: positive (negative) semidefinite matrix

$\mathscr{Q}\preccurlyeq 0\,(\mathscr{Q}<0)$: positive (negative) matrix

$\mathrm{Rank}(\mathscr{A})$: rank of the matrix $\mathscr{A}$

The off-diagonal blocks of the symmetric matrix can be abbreviated as $*$, i.e.,

$$\begin{bmatrix} \mathscr{W}_1 & \mathscr{W}_2 \\ \mathscr{W}_2^{\mathrm{T}} & \mathscr{W}_3 \end{bmatrix} = \begin{bmatrix} \mathscr{W}_1 & \mathscr{W}_2 \\ * & \mathscr{W}_3 \end{bmatrix} = \begin{bmatrix} \mathscr{W}_1 & * \\ \mathscr{W}_2^{\mathrm{T}} & \mathscr{W}_3 \end{bmatrix}. \tag{1}$$

3. Preliminaries

Consider the following fuzzy singular models with distinct derivative matrices and Brownian parameter perturbations:

R_ι: **If** $\varsigma_1(t)$ is $\mathbb{M}_{1\iota}$, $\varsigma_2(t)$ is $\mathbb{M}_{2\iota}, \ldots,$ **AN D** $\varsigma_\kappa(t)$ is $\mathbb{M}_{\kappa\iota}$, **THEN**

$$\begin{aligned} \widehat{\mathscr{E}}_\iota \mathrm{d}x(t) &= \left(\widehat{\mathscr{A}}_\iota \widehat{x}(t) + \widehat{\mathscr{B}}_\iota u(t)\right)\mathrm{d}t + \widehat{\mathscr{J}}_\iota x(t)\mathrm{d}\varpi(t), \\ z(t) &= \widehat{\mathscr{C}}_\iota \widehat{x}(t) + \widehat{\mathfrak{D}}_\iota u(t), \quad \iota = 1,2,\ldots,N_r, \end{aligned} \tag{2}$$

where $\widehat{x}(t)\in\mathfrak{R}^n$ denotes state vectors, $u(t)\in\mathfrak{R}^m$ and $z(t)\in R^l$ denote input/output vectors, and $\widehat{\mathscr{E}}_\iota$, $\widehat{\mathscr{A}}_\iota$, $\widehat{\mathscr{B}}_\iota$, $\widehat{\mathscr{C}}_\iota$, and $\widehat{\mathscr{D}}_\iota$ are known matrices, and

$$\mathrm{rank}(\mathscr{E}_\iota) = n_q \le n, \tag{3}$$

$\mathbb{M}_{\kappa\iota}$, $\kappa = 1,2,\ldots,p$, are fuzzy sets, and $\varsigma_1(t), \varsigma_2(t), \ldots, \varsigma_p(t)$ denote premise variables which may be the functions of system states.

Consider the derivative matrices which met the basic assumptions.

Assumption 1 (see [10, 20]).

$$\widehat{\mathscr{E}}_\iota \mathbb{V} = [E_{1\iota}, 0],$$
$$\operatorname{Rank}\left(\widehat{\mathscr{E}}_\iota, \widehat{\mathscr{J}}_\iota\right) = \operatorname{Rank}\left(\widehat{\mathscr{E}}_\iota\right), \quad \iota = 1, 2, \ldots, n_r, \tag{4}$$

where $\mathscr{E}_{1\iota} \in \mathfrak{R}^{n\times n_q}$ are full column rank matrices and $\mathbb{V}$ is an invertible matrix.

Using Assumption 1, we can find two invertible matrices $\mathbb{U}_\iota, \iota = 1, 2, \ldots, n_r$ and $\mathbb{V}$, such that

$$\mathbb{U}_\iota \widehat{\mathscr{E}}_\iota \mathbb{V} = \begin{bmatrix} \mathscr{I}_{n_q} & 0 \\ 0 & 0 \end{bmatrix} \doteq \mathscr{E}. \tag{5}$$

Then, considering $x(t) = \mathbb{V}^{-1}\widehat{x}(t)$, the fuzzy rules are transformed as R_ι: **IF** $\varsigma_1(t)$ is $\mathbb{M}_{1\iota}$, $\varsigma_2(t)$ is $\mathbb{M}_{2\iota}$, . . .**AND** $\varsigma_\kappa(t)$ is $\mathbb{M}_{\kappa\iota}$, **THEN**

$$\mathscr{E}\mathrm{d}x(t) = (\mathscr{A}_\iota x(t) + \mathscr{B}_\iota u(t))\mathrm{d}t + \mathscr{J}_\iota x(t)\mathrm{d}\varpi(t),$$
$$z(t) = \mathscr{C}_\iota x(t) + \mathscr{D}_\iota u(t), \tag{6}$$

where $\mathscr{E}$ is the same as (5) and

$$\mathscr{A}_\iota = \mathbb{U}_\iota \widehat{\mathscr{A}}_\iota \mathbb{V} \doteq \begin{bmatrix} \mathscr{A}_{\iota 11} & \mathscr{A}_{\iota 12} \\ \mathscr{A}_{\iota 21} & \mathscr{A}_{\iota 22} \end{bmatrix},$$
$$\mathscr{B}_\iota = \mathbb{U}_\iota \widehat{\mathscr{B}}_\iota,$$
$$\mathscr{J}_\iota = \mathbb{U}_\iota \widehat{\mathscr{J}}_\iota \mathbb{V} \doteq \begin{bmatrix} \mathscr{J}_{\iota 1} & \mathscr{J}_{\iota 2} \\ 0 & 0 \end{bmatrix}, \tag{7}$$
$$\mathscr{C}_\iota = \widehat{\mathscr{C}}_\iota \mathbb{V},$$
$$\mathscr{D}_\iota = \widehat{\mathscr{D}}_\iota.$$

Then, the overall fuzzy model can be given as follows:

$$\mathscr{E}\mathrm{d}x(t) = \sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\left[(\mathscr{A}_\iota x(t) + \mathscr{B}_\iota u(t))\mathrm{d}t + \mathscr{J}_\iota x(t)\mathrm{d}\varpi(t)\right],$$
$$z(t) = \sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\left[\mathscr{C}_\iota x(t) + \mathscr{D}_\iota u(t)\right], \tag{8}$$

where $h_\iota(\varsigma(t)) = \left(\left(\prod_{\kappa=1}^{p} \mathbb{M}_{\kappa\iota}(\varsigma_\kappa(t))\right)/\left(\sum_{\iota=1}^{n_r}\prod_{\kappa=1}^{p} \mathbb{M}_{\kappa\iota}(\varsigma_\kappa(t))\right)\right)$, and we can directly find

$$h_\iota(\varsigma(t)) \geq 0, \iota = 1, 2, \ldots, n_r, \quad \sum_{\iota=1}^{n_r} h_\iota(\varsigma(t)) = 1. \tag{9}$$

Assumption 2. $0 \leq h_\iota(\varsigma(t))h_\kappa(\varsigma(t)) \leq \alpha_{\iota\kappa}$, where $\alpha_{\iota\kappa}$ are the known constant.

Remark 1. By the shape of membership functions, the boundary $\alpha_{\iota\kappa}$ can be obtained accurately. In general case, $\alpha_{\iota\kappa} = \begin{cases} 1, & \iota = \kappa \\ 0.25, & \iota \neq \kappa \end{cases}$.

For simplicity, system (7) is rewritten by

$$\begin{cases} \mathscr{E}\mathrm{d}t = \mathscr{A}_\varsigma x(t)\mathrm{d}t + \mathscr{B}_\varsigma u(t)\mathrm{d}t + \mathscr{J}_\varsigma x(t)\mathrm{d}\varpi(t), \\ z(t) = \mathscr{C}_\varsigma x(t) + \mathscr{D}_\varsigma u(t). \end{cases} \tag{10}$$

The definition for unforced systems is given as follows:

$$\mathscr{E}\mathrm{d}t = \mathscr{A}_\varsigma x(t)\mathrm{d}t + \mathscr{J}_\varsigma x(t)\mathrm{d}\varpi(t). \tag{11}$$

Definition 1

(i) System (11) is regular at $[\,0\ \ T\,]$ if $\exists s_0 \in \mathbb{C}$ satisfies

$$\operatorname{Det}\left(s_0\mathscr{E} - \mathscr{A}_\varsigma\right) \neq 0, \quad \forall t \in [\,0\ \ T\,]. \tag{12}$$

(ii) System (11) is impulse-free at $[\,0\ \ T\,]$ when

$$\operatorname{Deg}\left[\operatorname{Det}\left(s\mathscr{E} - \mathscr{A}_\varsigma\right)\right] = \operatorname{rank}(\mathscr{E}), \quad \forall t \in [\,0\ \ T\,]. \tag{13}$$

(iii) System (11) is stochastic finite-time stable (FTS) with respect to $(\mathfrak{c}_1, \mathfrak{c}_2, T, \mathscr{R})$, where $\mathscr{R} > 0$, $0 < \mathfrak{c}_1 < \mathfrak{c}_2$, if

$$\mathbf{E}\left\{x^{\mathrm{T}}(0)\mathscr{E}^{\mathrm{T}}R\mathscr{E}x(0)\right\} \leq \mathfrak{c}_1 \Longrightarrow \mathbf{E}\left\{x^{\mathrm{T}}(t)\mathscr{E}^{\mathrm{T}}R\mathscr{E}x(t)\right\} \leq \mathfrak{c}_2, \quad \forall \in [0, T]. \tag{14}$$

(iv) System (11) is stochastic finite-time admissible if the system is regular, impulse-free, and stochastic finite-time stable.

Lemma 1 (see [28]). *Given the piecewise continuous matrix $\mathscr{A}(t) \in \mathfrak{R}^{n\times n}$; if there exist the norm-bounded time-varying matrix $\mathscr{P}(t) \in \mathfrak{R}^{n\times n}$ and a scalar $\alpha > 0$ such that*

$$\mathscr{A}(t)^T\mathscr{P}(t) + \mathscr{P}(t)^T\mathscr{A}(t) \leq -\alpha I, \quad \forall t \in [0, \infty), \tag{15}$$

then the following holds:

(i) $\mathscr{A}(t)$ is invertible

(ii) $\mathscr{A}^{-1}(t)$ is bounded

Lemma 2 (see [29]). *Given a matrix $\Phi_{ij} \in \mathfrak{R}^{n\times n}$, then*

$$\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_\iota(\varsigma)h_\kappa(\varsigma)\varsigma_{\iota\kappa} < 0, \tag{16}$$

is fulfilled if the following conditions are hold:

$$\Phi_{\iota\iota} < 0, \quad \iota = 1, 2, \ldots, n_r,$$
$$\frac{2}{n_r - 1}\Phi_{\iota\iota} + \Phi_{\iota\kappa} + \Phi_{\kappa\iota} < 0, \quad 1 \leq \iota \neq \kappa \leq n_r. \tag{17}$$

Lemma 3. *When $\mathscr{E} = \begin{bmatrix} \mathscr{I}_{n_r} & 0 \\ 0 & 0 \end{bmatrix}$, the following statements are true:*

(i) Matrix $\mathscr{X}$ satisfies

$$\mathscr{E}\mathscr{X} = \mathscr{X}^{\mathrm{T}}\mathscr{E}^{\mathrm{T}} \geq 0, \tag{18}$$

if and only if

$$\mathscr{X} = \begin{bmatrix} \mathscr{X}_1 & 0 \\ \mathscr{X}_2 & \mathscr{X}_3 \end{bmatrix}, \tag{19}$$

where $\mathcal{X}_1 \geq 0 \in \Re^{n_r \times n_r}$. *Meanwhile, if* $\mathcal{X}$ *is the invertible matrix, we can obtain that* $\mathcal{X}_1 > 0$ *and* $Det(\mathcal{X}_3) \neq 0$.

(ii) *Now,* $\mathcal{X}$ *satisfying (18) is given as*

$$\mathcal{X} = \mathcal{P}\mathcal{E}^T + S\mathcal{Y}, \quad (20)$$

where $\mathcal{P} = diag\{\mathcal{X}_1, \Theta\}$, $\mathcal{Y} = [\mathcal{X}_2 \ \ \mathcal{X}_3]$, $S = \begin{bmatrix} 0 \\ I \end{bmatrix}$, *and* Θ *is an arbitrary matrix.*

(iii) *If* $\mathcal{X}$ *is invertible, matrices Q and R are two positive definite matrices,* $\mathcal{X}$ *and* $\mathcal{E}$ *satisfy (18), and* $\mathcal{P}$ *is a diagonal matrix, and the following equality holds:*

$$\mathcal{E}^T\mathcal{X}^{-1} = \mathcal{E}^T R^{1/2} Q R^{1/2}\mathcal{E} = \mathcal{E}^T\mathcal{P}^{-1}\mathcal{E}. \quad (21)$$

Then, the positive definite matrix $Q = R^{-1/2}\mathcal{P}^{-1}R^{-1/2}$ is a solution of (21).

Proof. The proof of this lemma is similar to the proof process in literature [30], which is omitted here. □

Lemma 4 (Gronwall inequality (see [31])). *If nonnegative function* $v(t)$ *satisfies*

$$v(t) \leq \alpha + \beta\int_0^t v(s)\mathrm{d}s, \quad 0 \leq t \leq T, \quad (22)$$

where constants $\alpha, \beta \geq 0$, *we have*

$$v(t) \leq \alpha e^{bt}, \quad 0 \leq t \leq T. \quad (23)$$

4. Main Result

4.1. Admissibility Analysis. The admissibility theorem for systems (11) is given as follows.

Theorem 1. *System (11) is stochastic finite-time admissibility if there exist symmetric matrices* $0 < \mathcal{P} \in \Re^{n_q \times n_q}$, $\Theta \in \Re^{(n-n_q)\times(n-n_q)}$, $0 < Q_{\iota\kappa} \in \Re^{2n\times 2n}$, *and* $\mathcal{Y}_\iota \in \Re^{(n-n_1)\times n}$, *such that*

$$\Theta_{\iota\iota} < 0, \quad \iota = 1, 2, \ldots, n_r, \quad (24)$$

$$\frac{2}{n_r - 1}\Theta_{\iota\iota} + \Theta_{\iota\kappa} + \Theta_{\kappa\iota} < 0, \quad 1 \leq \iota \neq \kappa \leq n_r, \quad (25)$$

$$\lambda_{\min}(Q)I \leq Q \leq \lambda_{\max}(Q)I, \quad (26)$$

$$\lambda_{\max}(Q)\mathfrak{c}_1^2 e^{aT} - \mathfrak{c}_2^2\lambda_{\min}Q < 0, \quad (27)$$

where

$$\begin{aligned}
\Theta_{\iota\kappa} &= \Delta_{\iota\kappa} + \Psi - Q_{\iota\kappa},\\
\Delta_{\iota\kappa} &= \begin{bmatrix} \mathcal{X}_\kappa^{\mathrm{T}}\mathcal{A}_\iota^{\mathrm{T}} + \mathcal{A}_\iota\mathcal{X}_\kappa - \alpha\mathcal{E}\mathcal{X}_\kappa & * \\ \mathcal{E}\mathcal{E}^+ J_\iota\mathcal{X}_\kappa & -\mathcal{X} \end{bmatrix},\\
\mathcal{X}_\iota &= \mathcal{X}\mathcal{E}^{\mathrm{T}} + \mathcal{S}Y_\iota,\\
\Psi &= \sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r}\alpha_{\iota\kappa}Q_{\iota\kappa},\\
\mathcal{X} &= \mathrm{diag}\{\mathcal{P}, \Theta\},\\
Q &= R^{-1/2}\mathcal{X}^{-1}R^{-1/2},\\
S &= \begin{bmatrix} 0 \\ I \end{bmatrix}.
\end{aligned} \quad (28)$$

Proof. Using Lemma 2 and (24) and (25), we have

$$\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_\iota(\varsigma(t))h_\kappa(\varsigma(t))\Theta_{\iota\kappa} < 0. \quad (29)$$

Then,

$$\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_\iota(\varsigma(t))h_\kappa(\varsigma(t))\Theta_{\iota\kappa} = \sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_\iota(\varsigma(t))h_\kappa(\varsigma(t))\Delta_{\iota\kappa} + \Psi + \sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_\iota(\varsigma(t))h_\kappa(\varsigma(t))Q_{\iota\kappa} < 0. \quad (30)$$

Further, by $h_\iota(\varsigma(t))h_\kappa(\varsigma(t)) \leq \alpha_{\iota\kappa}$ and $Q_{\iota\kappa} > 0$, we get

$$\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_\iota(\varsigma(t))h_\kappa(\varsigma(t))\Delta_{\iota\kappa} = \begin{bmatrix} \mathcal{X}_\varsigma^{\mathrm{T}}\mathcal{A}_\varsigma^{\mathrm{T}} + \mathcal{A}_\varsigma\mathcal{X}_\varsigma - \alpha\mathcal{E}\mathcal{X}_\varsigma & * \\ \mathcal{E}\mathcal{E}^+\mathcal{J}_\varsigma\mathcal{X}_\varsigma & -\mathcal{X} \end{bmatrix} < 0, \quad (31)$$

where $\mathcal{X}_\varsigma = \sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathcal{X}_\iota$.

Then, we have

$$\mathcal{A}_\varsigma^{\mathrm{T}}\mathcal{X}_\varsigma + \mathcal{X}_\varsigma^{\mathrm{T}}\mathcal{A}_\varsigma - \alpha\mathcal{E}\mathcal{X}_\varsigma < 0. \quad (32)$$

Next, based on Lemma 3, $\mathcal{X}_\iota$ can be constructed as

$$\mathcal{X}_\iota = \begin{bmatrix} \mathcal{P} & 0 \\ \mathcal{X}_{2\iota} & \mathcal{X}_{2\iota} \end{bmatrix}. \quad (33)$$

So, we get

$$\begin{aligned}
\mathcal{X}_\varsigma &= \begin{bmatrix} \mathcal{P} & 0 \\ \sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))X_{2\iota} & \sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))X_{3\iota} \end{bmatrix}\\
&= \begin{bmatrix} P & 0 \\ \mathcal{X}_{2\varsigma} & \mathcal{X}_{3\varsigma} \end{bmatrix}.
\end{aligned} \quad (34)$$

Then, according to (32) and (34), we have

$$\begin{bmatrix} \star & \star \\ \star & \mathcal{X}_{3\varsigma}^{\mathrm{T}}(t)\mathcal{A}_{22\varsigma} + \mathcal{A}_{22\varsigma}^{\mathrm{T}}\mathcal{X}_{3\varsigma} \end{bmatrix} < 0, \quad (35)$$

where $\star$ means that it will not affect the proof.

Considering

$$\|\mathscr{X}_{3\varsigma}\| \leq \sum_{\iota=1}^{n_r} \|\mathscr{X}_{3\iota}\|, \tag{36}$$

it can be deduced that $\mathscr{A}_{22\varsigma}$ and $\mathscr{X}_{3\varsigma}$ are nonsingular based on Lemma 1. So, system (11) is regular and impulse-free.

Then, let us choose the candidate Lyapunov function as

$$\mathscr{V}(x(t)) = x^T(t)\mathscr{E}^T\left(\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{X}_\iota\right)^{-1} x(t) = x^T(t)\mathscr{E}^T\mathscr{X}_\varsigma^{-1}x(t). \tag{37}$$

Since $\mathscr{P} > 0$ and $\mathscr{X}_{3\varsigma}$ are nonsingular, we can get $\mathscr{X}_\varsigma = \begin{bmatrix} P & 0 \\ \mathscr{X}_{2\varsigma} & \mathscr{X}_{3\varsigma} \end{bmatrix}$ is nonsingular.

Then,

$$\mathscr{E}^T\mathscr{X}_\varsigma^{-1} = \mathscr{X}_\varsigma^{-1}\mathscr{E} \geq 0. \tag{38}$$

By the Itô formula, the stochastic derivative of along the trajectory of system (11) can be obtained as

$$d\mathscr{V}(x(t)) = \mathscr{L}\mathscr{V}(x(t))dt + 2x^T(t)\mathscr{X}_\varsigma^{-T}\mathscr{J}_\varsigma x(t)d\varpi(t), \tag{39}$$

where

$$\mathscr{L}\mathscr{V}(x(t)) = x^T(t)\mathscr{E}^T\frac{d\mathscr{X}_\varsigma^{-1}}{dt}x(t) + \frac{dx^T(t)}{dt}\mathscr{E}^T\mathscr{X}_\varsigma^{-1}x(t) + x^T(t)\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\frac{dx(t)}{dt}. \tag{40}$$

By $\mathscr{X}_\varsigma\mathscr{X}_\varsigma^{-1} = I$, we get $(d/dt)[\mathscr{X}_\varsigma\mathscr{X}_\varsigma^{-1}] = 0$.

Next,

$$\frac{d\mathscr{X}_\varsigma^{-1}}{dt} = -\mathscr{X}_\varsigma^{-1}\frac{d}{dt}\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{X}_\iota\mathscr{X}_\varsigma^{-1} = -\mathscr{X}_\varsigma^{-1}\sum_{\iota=1}^{n_r}\dot{h}_\iota(\varsigma(t))\mathscr{X}_\iota\mathscr{X}_\varsigma^{-1}. \tag{41}$$

So,

$$\begin{aligned}
\mathscr{L}V(x(t)) &= -x^T(t)\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{X}_\iota\mathscr{X}_\varsigma^{-1}x(t) + \frac{dx^T(t)}{dt}\mathscr{E}^T\mathscr{X}_\varsigma^{-1}x(t) + x^T(t)\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\frac{dx(t)}{dt} \\
&= -x^T(t)\mathscr{X}_\varsigma^{-T}\mathscr{E}\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{X}_\iota\mathscr{X}_\varsigma^{-1}x(t) + \frac{dx^T(t)}{dt}\mathscr{E}^T\mathscr{X}_\varsigma^{-1}x(t) + x^T(t)\mathscr{X}_\varsigma^{-T}\mathscr{E}\frac{dx(t)}{dt} \\
&= -x^T(t)\mathscr{X}_\varsigma^{-T}\mathscr{E}\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{X}_\iota\mathscr{X}_\varsigma^{-1}x(t) + x^T(t)\left\{\mathscr{A}_\varsigma^T\mathscr{X}_\varsigma^{-1} + \mathscr{X}_\varsigma^{-T}\mathscr{A}_\varsigma + \mathscr{J}_\varsigma^T(\mathscr{E}^+)^T\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\mathscr{E}^+\mathscr{J}_\varsigma\right\}x(t).
\end{aligned} \tag{42}$$

Further, we can obtain

$$\mathscr{L}\mathscr{V}(x(t)) - \alpha\mathscr{V}(x(t)) = -\mathscr{X}_\varsigma^{-T}\mathscr{E}\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{X}_\iota\mathscr{X}_\varsigma^{-1} + \mathscr{A}_\varsigma^T\mathscr{X}_\varsigma^{-1} + \mathscr{X}_\varsigma^{-T}\mathscr{A}_\varsigma + \mathscr{J}_\varsigma^T(\mathscr{E}^+)^T\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\mathscr{E}^+\mathscr{J}_\varsigma - \alpha\mathscr{E}^T\mathscr{X}_\varsigma^{-1}. \tag{43}$$

When $\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t)) = 1$, we have

$$\sum_{\iota=1}^{n_r} \dot{h}_\iota(\varsigma(t)) = 0. \tag{44}$$

Then, one has

$$\sum_{\iota=1}^{n_r} \dot{h}_\iota(\varsigma(t))\mathscr{E}\mathscr{X}_\iota = \sum_{\iota=1}^{n_r} \dot{h}_\iota(\varsigma(t))\mathscr{E}\mathscr{P}\mathscr{E}^T = 0. \tag{45}$$

So, $\mathscr{L}\mathscr{V}(x(t)) - \alpha\mathscr{V}(x(t)) < 0$ holds if and only if the following inequality holds:

$$\mathscr{A}_\varsigma^T\mathscr{X}_\varsigma^{-1} + \mathscr{X}_\varsigma^{-T}\mathscr{A}_\varsigma + \mathscr{J}_\varsigma^T\left(\mathscr{E}^+\right)^T\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\mathscr{E}^+\mathscr{J}_\varsigma - \alpha\mathscr{E}^T\mathscr{X}_\varsigma^{-1} < 0. \tag{46}$$

Multiplying inequality (46) on the left by $\mathscr{X}_\varsigma^{\mathrm{T}}$ and right $\mathscr{X}_\varsigma$, respectively, one has

$$\mathscr{X}_\varsigma^T\mathscr{A}_\varsigma^T + \mathscr{A}_\varsigma\mathscr{X}_\varsigma + \mathscr{X}_\varsigma^T\mathscr{J}_\varsigma^T\left(\mathscr{E}^+\right)^T\mathscr{E}^T\mathscr{X}_\varsigma^{-1}\mathscr{E}^+\mathscr{J}_\varsigma\mathscr{X}_\varsigma - \alpha\mathscr{E}\mathscr{X}_\varsigma < 0. \tag{47}$$

According to Lemma 3, we obtain $\mathscr{E}^T\mathscr{X}_\varsigma^{-1} = \mathscr{E}^T\mathscr{X}^{-1}\mathscr{E}$. Then, inequality (47) is rewritten as

$$\mathscr{X}_\varsigma^T\mathscr{A}_\varsigma^T + \mathscr{A}_\varsigma\mathscr{X}_\varsigma + \mathscr{X}_\varsigma^T\mathscr{J}_\varsigma^T\left(\mathscr{E}^+\right)^T\mathscr{E}^T\mathscr{X}^{-1}\mathscr{E}\mathscr{E}^+\mathscr{J}_\varsigma\mathscr{X}_\varsigma - \alpha\mathscr{E}\mathscr{X}_\varsigma < 0. \tag{48}$$

Using Schur complement Lemma with (31), we have

$$\mathscr{L}\mathscr{V}(x(t)) - \alpha\mathscr{V}(x(t)) < 0. \tag{49}$$

Integrating (49) and then taking the mathematical expectation, we get

$$\mathbf{E}(\mathscr{V}(x(t))) < \mathscr{V}(x(0)) + \alpha\int_0^t \mathbf{E}(\mathscr{V}(x(s)))\mathrm{d}s. \tag{50}$$

By Lemma 4, we have

$$\mathbf{E}(\mathscr{V}(x(t))) < \mathscr{V}(x(0))e^{\alpha t}. \tag{51}$$

Because

$$\begin{aligned}
\mathbf{E}(\mathscr{V}(x(t))) &= \mathbf{E}\left(x^T(t)\mathscr{E}^T\mathscr{X}_\varsigma^{-1}x(t)\right),\\
&= \mathbf{E}\left(x^T(t)\mathscr{E}^TR^{1/2}Q^{-1}R^{1/2}\mathscr{E}x(t)\right)\\
&\geq \lambda_{\min}(Q)\mathbf{E}\left(x^T(t)\mathscr{E}^TR\mathscr{E}x(t)\right).\\
\mathbf{E}(\mathscr{V}(x(0)))e^{\alpha t} &= \mathbf{E}\left(x^T(0)\mathscr{E}^TR^{1/2}Q^{-1}R^{1/2}\mathscr{E}x(0)\right)e^{\alpha t},\\
&\leq \lambda_{\max}(Q)\mathbf{E}\left(x^T(t)\mathscr{E}^TR\mathscr{E}x(t)\right)e^{\alpha t}\\
&\leq \lambda_{\max}(Q)\mathfrak{c}_1^2e^{\alpha T},
\end{aligned} \tag{52}$$

the following inequality can be obtained:

$$\mathbf{E}\left(x^T(t)\mathscr{E}^TR\mathscr{E}x(t)\right) \leq \frac{\lambda_{\max}(Q)}{\lambda_{\min}(Q)}\mathfrak{c}_1^2e^{\alpha T}. \tag{53}$$

Then, by (27), we have

$$\mathbf{E}\left(x^T(t)\mathscr{E}^TR\mathscr{E}x(t)\right) \leq \mathfrak{c}_2^2. \tag{54}$$

So, this system is stochastic finite-time stable. The proof is completed. □

Remark 3. Based on FLF and Itô stochastic system theory, the complete proof of the finite-time admissibility for fuzzy stochastic singular systems is firstly discussed, which provides a good base for the analysis and control of this kind of systems. Since the membership function is also related to the state of the system, it is difficult to obtain the boundary information of the membership function. In order to overcome this difficulty, in Theorem 1, through redesigning $\mathscr{X}_\iota = \mathscr{X}\mathscr{E}^T + \mathscr{S}Y_\iota$, we get

$$\sum_{\iota=1}^{n_r} \dot{h}_\iota(\varsigma(t))\mathscr{E}\mathscr{X}_\iota = \sum_{\iota=1}^{n_r} \dot{h}_\iota(\varsigma(t))\mathscr{E}\mathscr{P}\mathscr{E}^T = 0. \tag{55}$$

4.2. Controller Design. Fuzzy PDC controller for the T-S FSSs (11) is designed as follows. R_ι: **IF** $\varsigma_1(t)$ is $\mathbb{M}_{1\iota}$, $\varsigma_2(t)$ is $\mathbb{M}_{2\iota}$, ...**AND** $\varsigma_\kappa(t)$ is $\mathbb{M}_{\kappa\iota}$, **THEN**

$$u(t) = -\mathscr{F}_\iota x(t), \quad \iota = 1, 2, \ldots, n_r, \tag{56}$$

where $\mathscr{F}_\iota$ is the feedback gain matrix. Then, the over fuzzy PDC controller is given as

$$u(t) = -\sum_{\iota=1}^{n_r} h_\iota(\varsigma(t))\mathscr{F}_\iota x(t) = -\mathscr{F}_\varsigma x(t). \tag{57}$$

The closed-loop system is

$$\mathscr{E}\mathrm{d}(t) = \left[\mathscr{A}_\varsigma(t) - \mathscr{B}_\varsigma\mathscr{F}_\varsigma\right]x(t)\mathrm{d}t + \mathscr{J}_\varsigma x(t)\mathrm{d}\varpi(t). \tag{58}$$

Theorem 2. *System (11) is stochastic finite-time admissibile if there exist symmetric matrices* $0 < \mathscr{P} \in \mathfrak{R}^{n_q\times n_q}$, $\Theta \in \mathfrak{R}^{(n-n_q)\times(n-n_q)}$, *and* $0 < Q_{\iota\kappa} \in \mathfrak{R}^{3n\times 3n}$ *and matrices* $\mathscr{Y}_\iota \in \mathfrak{R}^{(n-n_1)\times n}$, $M \in \mathfrak{R}^{n\times n}$, *and* $S_\iota \in \mathfrak{R}^{m\times n}$, *such that*

$$\Theta_{\iota\iota} < 0, \quad \iota = 1, 2, \ldots, n_r, \tag{59}$$

$$\frac{2}{n_r - 1}\Theta_{\iota\iota} + \Theta_{\iota\kappa} + \Theta_{\kappa\iota} < 0, \quad 1 \leq \iota \neq \kappa \leq n_r, \tag{60}$$

$$\lambda_{\min}(Q)I \leq Q \leq \lambda_{\max}(Q)I, \tag{61}$$

$$\lambda_{\max}(Q)\mathfrak{c}_1^2e^{\alpha T} - \mathfrak{c}_2^2\lambda_{\min}Q < 0, \tag{62}$$

where

$$
\begin{aligned}
&\Theta_{\iota\kappa} = \overline{\Delta}_{\iota\kappa} + \Psi - Q_{\iota\kappa},\\
&\overline{\Delta}_{\iota\kappa} = \begin{bmatrix} -\Upsilon - \Upsilon^{T} - \alpha\mathscr{E}\mathscr{X}_{\iota} & * & * \\ \mathscr{X}_{\iota} - \mu\Upsilon + M & \mu\left(M^{\mathrm{T}} + M\right) & * \\ \mathscr{E}\mathscr{E}^{+}J_{\iota}\mathscr{X}_{\kappa} & 0 & -\mathscr{X} \end{bmatrix},\\
&\Upsilon = M\mathscr{A}_{\iota}^{\mathrm{T}} + S_{\kappa}^{\mathrm{T}}\mathscr{B}_{\iota}^{\mathrm{T}},\\
&\mathscr{X}_{\iota} = \mathscr{X}\mathscr{E}^{\mathrm{T}} + \mathscr{S}Y_{\iota},\\
&\Psi = \sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r}\alpha_{\iota\kappa}Q_{\iota\kappa},\\
&\mathscr{X} = \mathrm{diag}\{\mathscr{P}, \Theta\},\\
&Q = R^{-1/2}\mathscr{X}^{-1}R^{-1/2},\\
&S = \begin{bmatrix} 0 \\ I \end{bmatrix}.
\end{aligned}
\tag{63}
$$

Furthermore, the feedback gains are given by $\mathscr{F}_{\iota} = S_{\iota}M^{-T}, \kappa = 1, 2, \ldots, r$.

Proof. Similar as the proof method of Theorem 1, we can obtain

$$
\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_{\iota}(\varsigma)h_{\kappa}(\varsigma)\overline{\Delta}_{\iota\kappa} = \begin{bmatrix} -\Sigma - \Sigma^{\mathbb{T}} - \alpha\mathscr{E}\mathscr{X}_{\varsigma} & * & * \\ \varsigma_{\varsigma} - \mu\Sigma + M^{\mathrm{T}} & \mu\left(M^{\mathrm{T}} + M\right) & * \\ \mathscr{E}\mathscr{E}^{+}J_{\varsigma}\mathscr{X}_{\varsigma} & 0 & -\mathscr{X} \end{bmatrix} < 0,
\tag{64}
$$

where $\Sigma = M(\mathscr{A}_{\varsigma} - B_{\varsigma}F_{\varsigma})^{T}$.

Then, pre- and postmultiplying (62) by $\begin{bmatrix} I & \mathscr{A}_{\varsigma} - F_{\varsigma}\mathscr{B}_{\varsigma} & 0 \\ 0 & 0 & I \end{bmatrix}$ and its transpose matrix, we have

$$
\sum_{\iota=1}^{n_r}\sum_{\kappa=1}^{n_r} h_{\iota}(\varsigma)h_{\kappa}(\varsigma)\overline{\Delta}_{\iota\kappa} = \begin{bmatrix} -\Xi - \Xi^{\mathbb{T}} - \alpha\mathscr{E}\mathscr{X}_{\varsigma} & * \\ \mathscr{E}\mathscr{E}^{+}J_{\varsigma}\mathscr{X}_{\varsigma} & -\mathscr{X} \end{bmatrix} < 0, \tag{65}
$$

where $\Sigma = (\mathscr{A}_{\varsigma} - \mathscr{B}_{\varsigma}F_{\varsigma})\mathscr{X}_{\varsigma}$. Based on Theorem 1, system (58) is admissible. This completes the proof. □

5. Illustrative Examples

Example 1. Consider the following fuzzy stochastic singular model:

$$
\mathscr{E}\mathrm{d}x(t) = \sum_{\iota=1}^{2} h_{\iota}(\varsigma(t))\left(\mathscr{A}_{\iota}x(t)\mathrm{d}t + \mathscr{J}_{\iota}x(t)\mathrm{d}\varpi(t)\right), \tag{66}
$$

where

$$
\begin{aligned}
&\mathscr{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
&\mathscr{A}_1 = \begin{bmatrix} -10 & 2 & 1 \\ 2 & -5 & 1-b \\ 1 & -a & -4 \end{bmatrix},\\
&\mathscr{A}_2 = \begin{bmatrix} -1 & 1 & 0 \\ 4 & -3 & 2 \\ 1-0.5b & -1 & -a \end{bmatrix},\\
&\mathscr{J}_1 = \begin{bmatrix} 0.1 & 0.2 & -0.1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
&\mathscr{J}_2 = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\\
&h_1(\varsigma(t)) = 1 - \sin^2(x_1),\\
&h_2(\varsigma(t)) = 1 + \sin^2(x_1).
\end{aligned}
\tag{67}
$$

Choosing the parameter $\alpha = 0.5, T = 5, c_1 = 1$, and $c_2 = 100$, the finite-time admissibility of this model is checked by comparing Theorem 1 and Theorem 10 in [27], for the pairs (a, b), where $a \in [-5, 4]$ and $b \in [1, 10]$. The feasible region is depicted in Figure 1. From Figure 1, it can be directly seen that Theorem 1 provides the larger feasible region.

Example 2. The 2-rule FSSSs with distinct differential term matrices $\mathscr{E}_{\iota}$ are given as

$$
\sum_{\iota=1}^{2} h_{\iota}(\varsigma(t))\mathscr{E}_{\iota}\mathrm{d}x(t) = \sum_{\iota=1}^{2} h_{\iota}(\varsigma(t))\left[\left(\mathscr{A}_{\iota}x(t) + \mathscr{B}_{\iota}u(t)\right)\mathrm{d}t + \mathscr{J}_{\iota}x(t)\mathrm{d}\varpi(t)\right], \tag{68}
$$

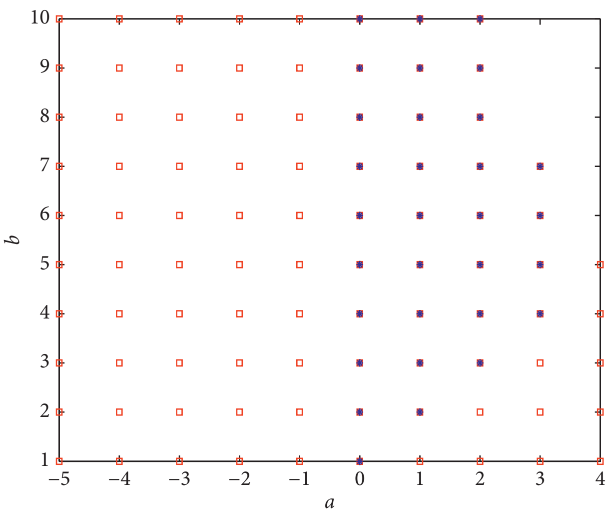

Figure 1: Feasible region of Theorem 1 (□) and [27] (∗).

where

$$
\begin{aligned}
\mathscr{E}_1 &= \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{E}_2 &= \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{A}_1 &= \begin{bmatrix} 1.2 & 1.6 & 1.5 \\ 0.2 & 1 & 1 \\ 2 & 0.5 & 0 \end{bmatrix}, \\
\mathscr{A}_2 &= \begin{bmatrix} -0.5 & 0.2 & 0.6 \\ -0.1 & 0.7 & 1.1 \\ 0.2 & 0.5 & 0.1 \end{bmatrix}, \\
\mathscr{J}_1 &= \begin{bmatrix} 0.9 & 0 & 0.1 \\ 0.1 & -0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{J}_2 &= \begin{bmatrix} 0.5 & 0.2 & 0.4 \\ 0.7 & 0.7 & 0.4 \\ 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{B}_1 &= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \\
\mathscr{B}_2 &= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \\
h_1(\varsigma(t)) &= 1 - \cos^2(x_1), \\
h_2(\varsigma(t)) &= 1 + \cos^2(x_1).
\end{aligned}
\tag{69}
$$

Choosing the parameter $\alpha = 0.1, T = 5, c_1 = 1, c_2 = 8$, and $\mu = 1$, the PDC controller is solved by Theorem 2, and the obtained controller gains are

$$
\begin{aligned}
F_1 &= [\,1.9158 \;\; 1.8431 \;\; 0.7171\,], \\
F_2 &= [\,0.3489 \;\; 1.1485 \;\; 0.6251\,].
\end{aligned}
\tag{70}
$$

Selecting the initial state $x_1(0) = 0.9$ and $x_2(0) = -0.4$ to verify the effectiveness of the proposed controller, the dynamic corresponding diagram of the open-loop system and closed-loop system is shown in Figures 2 and 3, respectively. From Figure 3, we can find that the system under the proposed controller is finite-time stable.

Example 3. Consider the DC motor actuated pendulum system in [17].

$$
\begin{aligned}
\left(J + ml^2\right)\ddot{d}(t) &= k_m i(t) + mgl \sin(\mathrm{d}(t)) - k_c \dot{\mathrm{d}}(t), \\
Li(t) &= u_c(t), \\
u(t) &= Ri(t) + k_f \dot{\mathrm{d}}(t) + u_c(t),
\end{aligned}
\tag{71}
$$

where $\mathrm{d}(t)$ is the angular displacement of the pendulum; B $\dot{\mathrm{d}}(t)$ is the angular velocity of the pendulum; $u_c(t)$, $i(t)$, and $u(t)$ are the inductance voltage, electric current, and armature circuit applied voltage, respectively; parameter $J = 0.126 \times 10^{-3}\,\mathrm{kg \cdot m^2}$ is the rotor mechanical inertia; $l = 0.2\,\mathrm{m}$ and $m = 0.1\,\mathrm{kg}$ are the pendulum length and mass, respectively; $g = 9.8\,\mathrm{m/s^2}$ is the gravity constant; $k_m = 0.0446\,\mathrm{NM/A}$ is the motor torque constant; $L = 92\,\mathrm{mH}$ is the armature circuit inductance; $R = 0.1\,\Omega$ is the armature circuit resistance; $k_f = 0.076\,\mathrm{V \cdot s/rad}$ is the back electromotive force constant; and $k_c = 1 \times 10^{-3}\,\mathrm{N \cdot m \cdot s/rad}$ is the viscous friction coefficient.

Considering the influence of random factors on the system, we assume that the viscous friction coefficient is affected by the white noise. Let

$$
x(t) = \left[\, \mathrm{d}(t) \;\; \dot{\mathrm{d}}(t) \;\; i(t) \;\; u_c(t) \,\right].
\tag{72}
$$

This system is rewritten as

$$
\begin{aligned}
\mathrm{d}x_1(t) &= x_2(t)\mathrm{d}t, \\
dx_2(t) &= \frac{k_m x_3(t) + mgl \sin x_1(t) - k_c x_2(t)}{J + ml^2} \\
&\quad - \frac{\overline{k}_c x_2(t)}{J + ml^2}\,\mathrm{d}\varpi(t), \\
\mathrm{d}x_3(t) &= \frac{x_4(t)}{L\mathrm{d}t}, \\
0 &= Rx_3(t) + k_f x_2(t + x_4(t) - u(t)).
\end{aligned}
\tag{73}
$$

Then, based on sector nonlinearity approach, the parameters of the fuzzy stochastic singular system can be obtained as

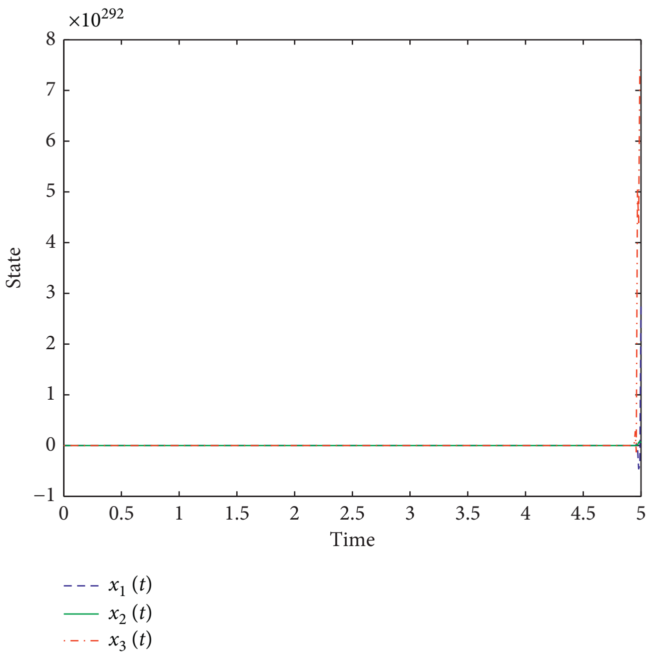

FIGURE 2: State response curve (open-loop system).

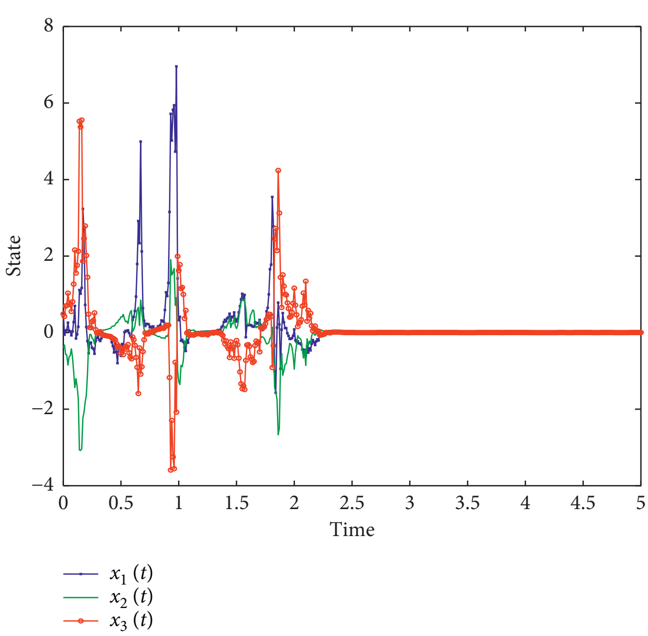

FIGURE 3: State response curve (closed-loop system).

$$\mathscr{E}dx(t) = \sum_{\iota=1}^{2} h_\iota(\varsigma(t))[(\mathscr{A}_\iota x(t) + \mathscr{B}_\iota u(t))dt + \mathscr{J}_\iota x(t)d\varpi(t)], \tag{74}$$

where

$$\begin{gathered}
\mathscr{E} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{A}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -47.5521 & -0.2424 & 10.8095 & 0 \\ 0 & 0 & 0 & 0.0109 \\ 0 & 0.0760 & 0.10007 & 1 \end{bmatrix}, \\
\mathscr{A}_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -33.5393 & -0.2424 & 10.8095 & 0 \\ 0 & 0 & 0 & 0.0109 \\ 0 & 0.0760 & 0.1000 & 1 \end{bmatrix}, \\
\mathscr{J}_1 = \mathscr{J}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\
\mathscr{B}_1 = \mathscr{B}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \\
\mathscr{B}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}. \\
h_1 = \frac{\sin(x_1(t)) - 0.7053x_1(t)}{0.2947x_1(t)}, \\
h_2 = 1 - h_1.
\end{gathered} \tag{75}$$

According to Theorem 2, choosing $\alpha = 0.1, T = 10, c_1 = 1, c_2 = 10$, and $\mu = 1$, the controller gains F_1 and F_2 are given by

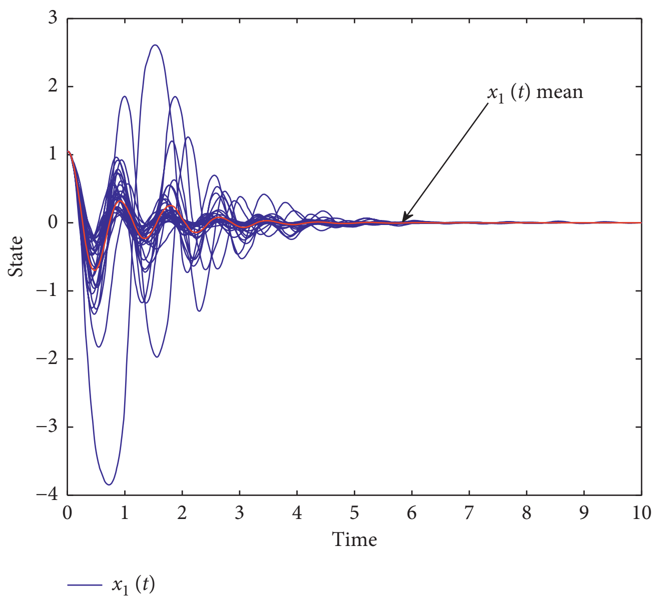

Figure 4: State x_1 (closed-loop system).

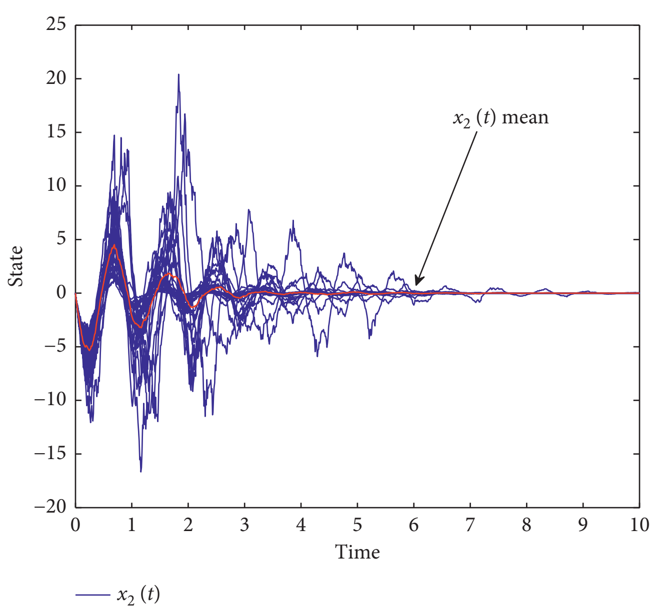

Figure 5: State x_2 (closed-loop system).

$$
\begin{aligned}
F_1 &= \begin{bmatrix} 954.8922 & -86.4795 & -375.2858 & -2.2567 \end{bmatrix},\\
F_2 &= \begin{bmatrix} 560.3136 & -67.8014 & -343.4330 & -2.2135 \end{bmatrix}.
\end{aligned} \tag{76}
$$

Choosing the initial states $x_0 = [\pi/3 \;\; 0 \;\; 0 \;\; -0.79]$, the resultant stochastic state responses (30 tests) and mean of the closed-loop system under PDC controller are shown in Figures 4–7 . So, this system under PDC controller is finite-time stable.

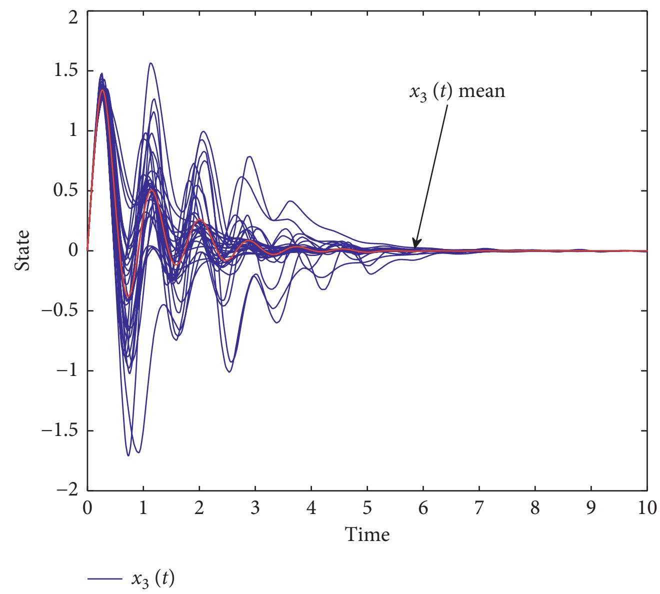

Figure 6: State x_3 (closed-loop system).

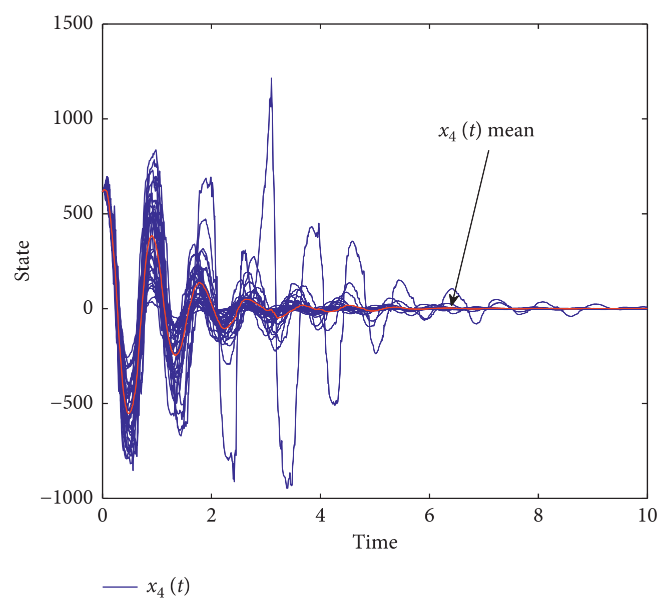

Figure 7: State x_4 (closed-loop system).

6. Conclusions

This paper studies finite-time admissibility analysis and controller design issues for FSSSs with distinct differential term matrices and Brownian parameter perturbations. Based on FLF approach and Itô-type stochastic system theory, the finite-time admissibility criterion of T-S fuzzy stochastic singular systems is proposed. The boundary information of derivative of fuzzy membership function is not needed in this admissibility condition. Furthermore, the PDC controller design method is given to ensure finite-time admissibility of the closed-loop system. Finally, simulation examples are used to test the feasibility and practicability of the proposed results.

It is important to point out that the other control problems of T-S FSSSs with distinct differential term matrices and Brownian parameter perturbations can also follow the proposed method in this paper, e.g., observer design, output feedback controller design, robust controller design, and so on. Meanwhile, if the differential term matrices do not satisfy the assumption condition, the corresponding problems will become very difficult and full of challenging. These topics are for future study.

Data Availability

The data used to support the findings of this study are included in this paper.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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References

[1] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its applications to modeling and control," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. SMC-15, no. 1, pp. 116–132, 1985.

[2] W. Zhou, Y. Wang, C. K. Ahn, J. Cheng, and C. Chen, "Adaptive fuzzy backstepping-based formation control of unmanned surface vehicles with unknown model nonlinearity and actuator saturation," *IEEE Transactions on Vehicular Technology*, vol. 69, no. 12, pp. 14749–14764, 2020.

[3] Y. Wei, Y. Wang, C. K. Ahn, and D. Duan, "IBLF-based finite-time adaptive fuzzy output-feedback control for uncertain MIMO nonlinear state-constrained systems," *IEEE Transactions on Fuzzy Systems*, p. 1, 2020.

[4] Y. Wei, P. Zhou, Y. Liang, Y. Wang, and D. Duan, "Adaptive finite-time neural backstepping control for multi-input and multi-output state-constrained nonlinear systems using tangent-type nonlinear mapping," *International Journal of Robust and Nonlinear Control*, vol. 30, no. 14, pp. 5559–5578, 2020.

[5] H. O. Wang, K. Tanaka, and M. F. Griffin, "An approach to fuzzy control of nonlinear systems: stability and design issues," *IEEE Transactions on Fuzzy Systems*, vol. 4, no. 1, pp. 4–23, 1996.

[6] K. Tanaka, T. Hori, and H. O. Wang, "A multiple Lyapunov function approach to stabilization of fuzzy control systems," *IEEE Transactions on Fuzzy Systems*, vol. 11, no. 4, pp. 582–589, 2003.

[7] S. Xu and J. Lam, *Robust Control and Filtering of Singular Systems*, Springer, Berlin, Germany, 2006, https://link.springer.com/book/10.1007/11375753.

[8] G. Duan, *Analysis and Design of Descriptor Linear Systems*, Springer, Berlin, Germany, 2010, https://link.springer.com/book/10.1007/978-1-4419-6397-0.

[9] T. Taniguchi, K. Tanaka, and H. O. Wang, "Fuzzy descriptor systems and nonlinear model following control," *IEEE Transactions on Fuzzy Systems*, vol. 8, no. 4, pp. 442–452, 2000.

[10] C. Chong Lin, Q. G. Qing-Guo Wang, and T. H. Tong Heng Lee, "Stability and stabilization of a class of fuzzy time-delay descriptor systems," *IEEE Transactions on Fuzzy Systems*, vol. 14, no. 4, pp. 542–551, 2006.

[11] C.-P. Huang, "Stability analysis and controller synthesis for fuzzy descriptor systems," *International Journal of Systems Science*, vol. 44, no. 1, pp. 23–33, 2013.

[12] Q. Zhang, L. Qiao, B. Zhu, and H. Zhang, "Dissipativity analysis and synthesis for a class of T-S fuzzy descriptor systems," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 47, no. 8, pp. 1774–1784, 2017.

[13] Y. Wang, Y. Xia, H. Li, and P. Zhou, "A new integral sliding mode design method for nonlinear stochastic systems," *Automatica*, vol. 90, pp. 304–309, 2018.

[14] V. Anishchenko, V. Astakhov, and A. Neiman, *Nonlinear Dynamics of Chaotic and Stochastic Systems: Tutorial and Modern Developments*, Springer-Verlag, Berlin, Germany, 2007, https://www.springer.com/gp/book/9783540381648.

[15] H. R. Karimi, M. Zapateiro, and N. Luo, "A linear matrix inequality approach to robust fault detection filter design of linear systems with mixed time-varying delays and nonlinear perturbations," *Journal of the Franklin Institute*, vol. 347, no. 6, pp. 957–973, 2010.

[16] J. Huang, X. Yang, and K. Peng, "Double layer distributed monitoring based on sequential correlation information for large-scale industrial processes in dynamic and static states," *IEEE Transactions on Industrial Informatics*, vol. 17, no. 9, pp. 6419–6428, 2021.

[17] J. Li, Q. Zhang, X.-G. Yan, and S. K. Spurgeon, "Robust stabilization of T-S fuzzy stochastic descriptor systems via integral sliding modes," *IEEE Transactions on Cybernetics*, vol. 48, no. 9, pp. 2736–2749, 2018.

[18] Q. Zhang, J. Zhang, and Y. Wang, "Robust sliding-mode control for fuzzy stochastic singular systems with different local input matrices," *IEEE Access*, vol. 6, pp. 29391–29406, 2018.

[19] R. Li and Y. Yang, "Fault tolerant control for T-S fuzzy stochastic singular systems," *IEEE Transactions on Fuzzy Systems*, p. 1, 2020.

[20] S. Xing, F. Deng, and L. Qiao, "Dissipative analysis and control for nonlinear stochastic singular systems," *IEEE Access*, vol. 6, pp. 43070–43078, 2018.

[21] Y. Wang, X. Xie, M. Chadli, S. Xie, and Y. Peng, "Sliding mode control of fuzzy singularly perturbed descriptor systems," *IEEE Transactions on Fuzzy Systems*, p. 1, 2020.

[22] H. Liu and X. Lin, "Finite-time H_∞control for a class of nonlinear system with time-varying delay," *Neurocomputing*, vol. 149, Article ID 1481C1489, 2015.

[23] Y.-C. Ma, Y.-F. Liu, and H. Chen, "Reliable finite-time $H\infty$ control of uncertain singular nonlinear Markovian jump

systems with bounded transition probabilities and time-varying delay H_∞ control of uncertain singular nonlinear Markovian jump systems with bounded transition probabilities and time-varying delay," *International Journal of Systems Science*, vol. 48, no. 11, pp. 2249–2261, 2017.

[24] J. Wang, S. Ma, C. Zhang, and M. Fu, "Finite-time H_infinity filtering for nonlinear singular systems with nonhomogeneous markov jumps H_∞ filtering for nonlinear singular systems with nonhomogeneous markov jumps," *IEEE Transactions on Cybernetics*, vol. 49, no. 6, pp. 2133–2143, 2019.

[25] F. Wang, B. Chen, Y. Sun, Y. Gao, and C. Lin, "Finite-time fuzzy control of stochastic nonlinear systems," *IEEE Transactions on Cybernetics*, vol. 50, no. 6, pp. 2617–2626, 2020.

[26] W. Guan and F. Liu, "Finite-time H_∞ memory state feedback control for uncertain singular TS fuzzy time-delay system under actuator saturation," *Advances in Difference Equations*, vol. 52, pp. 1–19, 2016.

[27] S. Y. Xing, Q. L. Zhang, and Y. Zhang, "Finite-time stability analysis and control for a class of stochastic singular biological economic systems based on T-S fuzzy model," *Abstract and Applied Analysis*, vol. 2013, Article ID 946491, 10 pages, 2013.

[28] Y. Yuan, Q. Zhang, and D. Zhang, "Admissible conditions of fuzzy descriptor systems based on fuzzy Lyapunov function approach," *International Journal of Information and Systems Sciences*, vol. 4, no. 2, pp. 219–232, 2009.

[29] H. D. Tuan, P. Apkarian, T. Narikiyo, and Y. Yamamoto, "Parameterized linear matrix inequality techniques in fuzzy control system design," *IEEE Transactions on Fuzzy Systems*, vol. 9, no. 3, pp. 324–332, 2001.

[30] Y. Zhang, C. Liu, and X. Mu, "Robust finite-time $H\infty$ control of singular stochastic systems via static output feedback H_∞ control of singular stochastic systems via static output feedback," *Applied Mathematics and Computation*, vol. 218, no. 9, pp. 5629–5640, 2012.

[31] B. Oksendal, *Stochastic Differential Equations: An Introduction with Applications*, Springer-Verlag, New York, NY, USA, 2000.