Research Article

Iterative Learning Tracking Control of Nonlinear Multiagent Systems with Input Saturation

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A tracking control algorithm of nonlinear multiple agents with undirected communication is studied for each multiagent system affected by external interference and input saturation. A control design scheme combining iterative learning and adaptive control is proposed to perform parameter adaptive time-varying adjustment and prove the effectiveness of the control protocol by designing Lyapunov functions. Simulation results show that the high-precision tracking control problem of the nonlinear multiagent system based on adaptive iterative learning control can be well realized even when the input is saturated. Finally, the validity of the proposed algorithm is verified by numerical analysis.

1. Introduction

A multiagent control system is formed by multiple agents with independent action capabilities to complete a complex task through local cooperation and interaction [1–4]. In recent years, multiagent control has achieved great breakthroughs due to the rapid development of coordinated control of dynamic systems. Multiagent systems have a wide range of applications in many engineering fields, such as formation control of various robots, positioning of distributed sensor networks, and multisatellite collaborative control [5–10].

The problem of consistency tracking is the basis of research on coordinated control, and it is the most important and basic problem of multiagent control. This problem has very important engineering value and theoretical significance. A number of scholars have carried out in-depth research on consistency tracking and achieved great results in consistency tracking [11, 12], distributed tracking positioning [13], swarm [14], and other group behavior control problems.

According to the consistency problem, multiagent control system can be divided into leader and nonleader control systems. However, in practical engineering applications, most of the systems are nonlinear regardless of with or without leader control system. Therefore, research on the consistency of nonlinear multiagent control system has more important practical significance. In [15], a fuzzy logic system is used as a feedforward compensation device to study the consistency problem of multiple agents with unknown mixed nonlinearity regardless of direction. In [16], a distributed group tracking algorithm for a class of nonlinear multiple agents is proposed to solve the problem of distributed group tracking. On the basis of [16], [17] investigated the problem of distributed robust asymptotic finite-time tracking control. This protocol does not need the navigator to input accurate boundary and does not need to know the exact boundary of the dynamic following agent in contrast to other control protocols. In [18, 19], an adaptive distributed control strategy is proposed to deal with the influence of uncertainty and nonlinear dynamics in a nonlinear multi-intelligent system. Another work [20] evaluated the consistency problem of the Lupin clustering of nonlinear agents through adaptive sliding mode variable control based on [21–23] which considered the control problem of switched nonlinear multiagent systems and...
achieved the consensus tracking under the presented control strategies. However, our results are still different compared with the above two articles. Compared with the event-triggered control problem of multiagent systems and the finite-time control problem of multiagent systems, our results are only studied from the consensus control problem. The main reason is that the consensus control problem is the basic problem of the multiagent systems. Besides, we introduce the iterative learning control method to solve the consensus problem, which can achieve control effects from two perspectives: time domain and iteration domain.

In an actual control system, the controller controls the object through the actuator. However, the system itself determines that the output of the actuator and its rate of change cannot be arbitrarily changed, so the saturation of the input amplitude is inevitable [24–26]. The performance of multiagent control may be degraded, and it may even become unstable due to strict constraints on speed, overload, and so forth. Many scholars have endeavored to solve the input saturation problem. In [27], a new PI protocol is designed, in which a more rigorous adaptive coupling saturation function is constructed to deal with the multiagent system with relative state saturation constraints. The study in [28] developed a distributed low-gain feedback control, which solves the problem of semiglobal and global multiagent control of input saturation. The study in [29] established a sliding mode based on literature [28] to distribute controller for handling input-saturated second-order multiagent distributed robust global control problems. The study in [30] continued to improve the semiglobal multiagent control of input saturation by using low-gain and inverse saturation methods based on literature [28]. The study in [31] used Lyapunov theory to design an adaptive iterative learning algorithm with time-varying coupled-gain full-saturation parameter update to solve the problem of multiagent coordinated learning control with input saturation. However, most scholars have rarely investigated the relationship between input saturation and nonlinearity.

Iterative learning is similar to the process and characteristics of human learning. Iterative learning constantly corrects the current signal by the deviation between the output signal and the given signal of the controlled system, thereby improving the tracking performance of the controlled system; as such, it is widely used in uncertain, nonlinear, and complex intelligent systems. In [32–34], distributed iterative learning for nonlinear follower is studied. In [35, 36], an iterative control protocol is designed for each follower for the unknown input gain function of the system, so each follower can follow the leader with high accuracy. The authors of [37–40] investigated the first-order, second-order, and even higher-order models of multiple agents based on iterative learning. Based on previous research, we use adaptive iterative control to solve the high-precision tracking problem of nonlinear multiple agents in undirected communication.

Inspired by the above analysis, we discuss the iterative learning control approach for tracking control of leader-following nonlinear multiagent systems with input saturation. Each following agent is modeled as first-order nonlinear dynamics, and external disturbance is considered. The main contributions of this paper are summarized as follows:

(i) A class of multiagent systems with nonlinear dynamics and external disturbance is addressed in this work, which are more general than those in previous studies [17–21]. The design of control protocol will be further complicated due to the introduction of nonlinear dynamics.

(ii) In comparison with [31, 32], the present work discusses an adaptive control protocol with iterative learning control for nonlinear multiagent systems. Moreover, the adaptive updating laws for time-varying parameters are designed.

(iii) A new Lyapunov function is structured to check the validity of the proposed control protocol. The tracking problem of leader-following nonlinear multiagent systems with input saturation can be comprehensively solved using the presented iterative learning control protocol.

The rest of the paper is designed as follows: Section 2 introduces basic knowledge and related theorems about graph theory. Section 3 reports the consensus of the proposed methods and analysis-related issues. Section 4 presents a nonlinear multiagent adaptive iterative control protocol with input saturation and analyzes system convergence. Section 5 discusses the simulation results. Section 6 provides the conclusion and summarizes the full text.

2. Preliminaries

2.1. Graph Theory. On the basis of graph theory, information topology among agents can be described as an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, with the set of nodes $\mathcal{V} = \{v_1, \ldots, v_n\}$ and the set of edges $\mathcal{E} = \{(i, j), i, j \in \mathcal{V}, \text{ and } i \neq j\}$. The weighted adjacency matrix is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$, in which $a_{ij} = a_{ji} > 0$ if $(i, j) \in \mathcal{E}$; otherwise, $a_{ij} = a_{ji} = 0$. $a_{ii}$ is assumed. The set of neighbors of node $i$ is defined as $\mathcal{N}_i = \{v_j: (v_j, v_i) \in \mathcal{E}\}$. The Laplacian matrix of $\mathcal{G}$ is denoted by $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}(d_1, \ldots, d_n)$ with $d_i = \sum_{j=1}^{n} a_{ij}$. Graph $\mathcal{G}$ is connected if a path exists between any two nodes.

Augmented graph $\mathcal{G}$ consists of $n$ following agents whose information topology graph is $\mathcal{G}$ and one leader agent is concerned. Let $b_i$ denote the connection matrix between agent $i$ and the leader. If agent $i$ obtains the information of the leader, then $b_i > 0$; otherwise, $b_i = 0$. Hence, the connection matrix between the leader and following agents can be defined as $\mathcal{B} = \text{diag}(b_1, \ldots, b_n)$, $\mathcal{H} = \mathcal{L} + \mathcal{B}$ is a matrix associated with $\mathcal{G}$.

Lemma 1 (see [31]). If graph $\mathcal{G}$ is connected, then the symmetric matrix $\mathcal{H}$ associated with $\mathcal{G}$ is positive definite.

2.2. Useful Definitions and Lemmas. Some useful definitions and lemmas are provided as follows to facilitate subsequent analysis.
Definition 1 (see [32]). Convergent series sequence \( \{\Delta_k\} \) is denoted by \( \Delta_k = (c/k^{m}) \), where \( k \in Z^{+}, \ c > 0 \), and \( m (\in Z^{+}) \geq 2 \) are the parameters to be designed.

**Lemma 2** (see [32]). For a given sequence \( \{c/k^{m}\} \), where \( k \in Z^{+}, \ c > 0 \), and \( m (\in Z^{+}) \geq 2 \), \( \lim_{k \to \infty} \sum_{j=1}^{k} (c/j^{m}) \leq 2c \).

**Lemma 3** (see [33]). For any \( b (\in R) > 0 \) and \( \zeta > 0 \), the hyperbolic tangent function satisfies \( 0 \leq |b| - b \tanh (b/\zeta) \leq q \zeta \), where \( q = 0.2785 \).

### 3. Problem Formulation

Considering a class of leader-following first-order nonlinear multiagent systems with input saturation, the dynamics of the \( i \)th agent as the \( k \)th iteration are described as

\[
\dot{x}_{k}^{i}(t) = f(x_{k}^{i}(t), t) + \text{sat}(u_{k}^{i}(t)) + d_{k}^{i}(t),
\]

where \( x_{k}^{i}(t) \in R \) and \( u_{k}^{i}(t) \in R \) are the position and control input of the \( i \)th agent, respectively; \( f(x_{k}^{i}(t), t) \) represents the continuous nonlinear function; \( d_{k}^{i}(t) \) is the unknown but bounded external disturbance; that is, \( \|d_{k}^{i}(t)\| \leq \|d_{k}^{n}\| \) with \( d_{k}^{n} \) as an unknown positive constant; \( k \) denotes the iteration number; and \( t \in [0, T] \). \text{sat}(u_{k}^{i}(t)) \) is the saturation function defined as

\[
\text{sat}(u_{k}^{i}(t)) = \begin{cases} 
\text{sign}(u_{k}^{i}(t))\pi, & |u_{k}^{i}(t)| > \pi, \\
\dot{x}_{k}^{i}(t), & |u_{k}^{i}(t)| \leq \pi,
\end{cases}
\]

where \( \pi > 0 \) is the upper bound of the saturation function and it is prespecified. The dynamics of the leader are given as

\[
\dot{x}_{0}(t) = f(x_{0}(t), t) + u_{0}(t),
\]

where \( x_{0}(t) \in R \) and \( u_{0}(t) \in R \) are the state and control input of the leader agent, respectively. In addition, we assume that the information of the leader is only accepted by a portion of following agents.

On the basis of multiagent systems (1) and (3), the tracking error of the \( i \)th following agent is defined as

\[
e_{k}^{i}(t) = x_{0}(t) - x_{k}^{i}(t).
\]

**Definition 2.** The consensus tracking problem of leader-following nonlinear multiagent systems with input saturation (1) and (3) is achieved if \( \lim_{k \to \infty} x_{k}^{i}(t) = x_{0}(t) \) for \( i = 1, \ldots, n \) over the interval \([0, T]\).

**Assumption 1.** The alignment initial condition, that is, \( x_{k}^{i}(0) = x_{k}^{i-1}(T) \) for \( i = 1, \ldots, n \), is considered, and the trajectory of the leader is spatially closed; namely, \( x_{0}(0) = x_{0}(T) \). \( e_{k}^{i}(0) = e_{k}^{i-1}(T) \) for \( i = 1, \ldots, n \) can be easily obtained.

**Assumption 2.** Function \( f(x(t), t) \) is uniform global Lipschitz in \( x \) over the interval \([0, T]\); that is, constant \( \kappa_{f} \) exists, such that

\[
\|f(x_{1}(t), t) - f(x_{2}(t), t)\| \leq \kappa_{f}\|x_{1}(t) - x_{2}(t)\|.
\]

**Remark 1.** For the initial condition setting, three cases are mostly applied in existing literature. The first one is the alignment initial condition, similar to Assumption 1 in this study. The second one is the resetting initial condition, that is, \( x_{k}^{i}(0) = x_{0}^{i} \) for each iteration, where \( x_{0}^{i} \) is a given constant. The last one is that the initial values are different for each iteration, that is, \( x_{k}^{i}(0) = x_{k}^{0} \), where \( x_{k}^{0} \) is a constant of change. The alignment initial condition is considered for the convenience of analysis. For the two other cases, the analysis approach is similar to that for the alignment initial condition.

The control objective of this work is to design an appropriate control scheme \( u_{k}^{i}(t) \) for \( i = 1, \ldots, n \) and adaptive updating laws such that all following agents can track the trajectory of the leader over the interval \([0, T]\) as the iteration number \( k \) tends to infinity.

### 4. Control Protocol Design and Convergence Analysis

In this section, the tracking problem of leader-following nonlinear multiagent systems with input saturation is introduced. The adaptive control protocol is designed on the basis of the iterative learning control approach. The adaptive updating laws are also presented. A new Lyapunov function is developed to analyze the convergence of the proposed control protocol.

#### 4.1. Distributed Iterative Learning Control Protocol Design

In view of multiagent systems (1) and (3), the consensus tracking error is described as

\[
e_{k}^{i}(t) = \sum_{j \in S_{i}} a_{ij}(x_{k}^{j}(t) - x_{k}^{i}(t)) + b_{i}(x_{0}(t) - x_{k}^{i}(t)).
\]

Considering the definition of \( e_{k}^{i}(t) \), the vector form of Equation (6) can be expressed as

\[
\dot{e}_{k}^{i}(t) = \mathcal{H}(1_{n}x_{0}(t) - x_{k}^{i}(t)) = \mathcal{H}e_{k}^{i}(t),
\]

where \( e_{k}^{i}(t) = [e_{k}^{1}(t), \ldots, e_{k}^{n}(t)]^{T} \), \( e_{k}^{i}(t) = 1_{n}x_{0}(t) - x_{k}^{i}(t) \), \( x_{k}^{i}(t) = [x_{k}^{1}(t), \ldots, x_{k}^{n}(t)]^{T} \), \( \mathcal{H} = \mathcal{L} + \mathcal{B} \), and \( 1_{n} = [1, \ldots, 1]^{T} \).

**Remark 2.** We only investigate the states of each agent as \( x_{k}^{i}(t) \in R \) and \( x_{0}(t) \in R \). For the case of \( x_{k}^{i}(t) \in R^{p} \) and \( x_{0}(t) \in R^{p} \), we can easily obtain the results by utilizing the Kronecker product. Thus, Equation (7) can be written as

\[
\dot{e}_{k}^{i}(t) = (\mathcal{H} \otimes I_{p})e_{k}^{i}(t),
\]

where \( \otimes \) represents the Kronecker product. All related results can be changed by applying the Kronecker product operation.

On the basis of Equation (7), the derivative of \( \dot{e}_{k}^{i}(t) \) is as follows:
\[ \hat{\xi}^k(t) = \mathcal{H} \hat{\xi}^k(t) = \mathcal{H} \left( I_n \hat{x}_0(t) - \hat{\xi}^k(t) \right). \]  \tag{9} 

Substituting \( \hat{x}_0(t) \) and \( \hat{\xi}^k(t) \) into Equation (9) yields
\[ \dot{\xi}^k(t) = \mathcal{H} \left( I_n f(x_0(t), t) - f(\hat{\xi}^k(t), t) + I_n u_0(t) \right) - \text{sat}(u^k(t) - \hat{d}^k(t)), \]  \tag{10} 

where \( f(\hat{\xi}^k(t), t) = \left[ f(x_1^k(t), t), \ldots, f(x_n^k(t), t) \right]^T \), \( \text{sat}(u^k(t)) = [\text{sat}(u_1^k(t)), \ldots, \text{sat}(u_n^k(t))]^T \), and \( d^k(t) = [d_1^k(t), \ldots, d_n^k(t)]^T \).

Hence, the adaptive iterative learning control protocol for each following agent is designed as
\[ u^k_i(t) = u_0(t) + a \left( \dot{\xi}^k_i(t) + v^k_i(t) \right) + \rho^k_i(t) \text{tanh} \left( \frac{\rho^k_i(t) \hat{x}^k_i(t)}{\Delta_k} \right), \]  \tag{11} 

and the adaptive updating laws for \( v^k_i(t) \) and \( \rho^k_i(t) \) are designed as
\[
\begin{cases} 
\dot{v}^k_i(t) = a \alpha^k_i(t) - \left( \frac{\partial u^k_i(t)^T \delta u^k_i(t)}{2 \|v^k_i(t)\|^2} \right), \\
v^k_i(0) = v^k_{i-1}(T), \\
\dot{\rho}^k_i(t) = \lambda_1 \rho^k_{i-1}(t), \\
\rho^k_i(0) = \rho^k_{i-1}(T) > 0, \quad \rho^k_i(t) > 0, \\
\end{cases} \tag{13} 
\]

where \( a > 0 \) and \( \lambda_1 > 0 \) are the constants to be designed, \( v^k_i(t) \) and \( \rho^k_i(t) \) are the time-varying parameters, and \( \delta u^k_i(t) = u^k_i(t) - \text{sat}(u^k_i(t)) \) with \( u^k_i(t) = [u_1^k(t), \ldots, u_n^k(t)]^T \), \( \text{sat}(u^k_i(t)) = [\text{sat}(u_1^k(t)), \ldots, \text{sat}(u_n^k(t))]^T \), and \( v^k_i(t) = [v_1^k(t), \ldots, v_n^k(t)]^T \).

The vector form of the control protocol (11) is written as
\[ u^k(t) = I_n u_0(t) + a \left( \dot{x}^k(t) + v^k(t) \right) + \sigma^k(t), \]  \tag{14} 

where \( \sigma^k(t) = [\sigma_1^k(t), \ldots, \sigma_n^k(t)]^T \) with \( \sigma_i^k(t) = \rho_i^k(t) \text{tanh} \left( (\rho_i^k(t) \hat{x}_i^k(t)) / \Delta_k \right) \).

Remark 3. In the control protocol (11), time-varying parameters \( v^k_i(t) \) and \( \rho^k_i(t) \) are considered. The purpose of designing \( v^k_i(t) \) is to compensate saturation error \( \delta u^k_i(t) \), and the purpose of designing \( \rho^k_i(t) \) is to eliminate unknown disturbance \( \hat{d}^k_i(t) \). By introducing \( v^k_i(t) \) and \( \rho^k_i(t) \), the desired tracking control problem of leader-following nonlinear multiagent systems with input saturation (11) and (3) can be comprehensively solved over the interval [0, T].

4.2. Convergence Analysis. Theorem 1 presents the main result of this study.

Theorem 1. Leader-following first-order nonlinear multiagent systems with input saturation (1) and (3) under Assumptions 1 and 2 are considered. Let graph \( \mathcal{T} \) be connected and the adaptive iterative learning control protocol (11) and adaptive updating laws (12) and (13) be adopted; then, all following agents can track the trajectory of the leader agent over the interval [0, T] as the iteration number \( k \) tends to infinity; namely, \( \lim_{k \to \infty} \hat{x}_i^k(t) = x_0(t) \) for \( i = 1, \ldots, n \).

Proof. A Lyapunov function candidate is defined as
\[ V^k(t) = \frac{1}{2} \left( \dot{x}^k(t) \right)^T \mathcal{H}^{-1} \dot{x}^k(t) + \frac{1}{2} \left( v^k(t) \right)^T v^k(t) \]
$$+ \sum_{i=1}^{n} \frac{1}{2 \lambda_i} (\rho_i^k(t) - \rho_0)^2, \]  \tag{15} 

where \( \rho_0 > 0 \) is a constant to be confirmed later.

Hence, the difference between \( V^k(t) \) and \( V^{k-1}(t) \) is given as
\[ \Delta V^k(t) = V^k(t) - V^{k-1}(t), \]
\[ = \frac{1}{2} \left( \dot{x}^k(t) \right)^T \mathcal{H}^{-1} \dot{x}^k(t) - \frac{1}{2} \left( \dot{x}^{k-1}(t) \right)^T \mathcal{H}^{-1} \dot{x}^{k-1}(t) \]
\[ + \frac{1}{2} \left( v^k(t) \right)^T v^k(t) - \frac{1}{2} \left( v^{k-1}(t) \right)^T v^{k-1}(t) \]
\[ + \sum_{i=1}^{n} \frac{1}{2 \lambda_i} (\rho_i^k(t) - \rho_0)^2 - \sum_{i=1}^{n} \frac{1}{2 \lambda_i} (\rho_i^{k-1}(t) - \rho_0)^2. \]  \tag{16} 

Notably,
\[ \frac{1}{2} \left( \dot{x}^k(t) \right)^T \mathcal{H}^{-1} \dot{x}^k(t) = \frac{1}{2} \left( \dot{x}^k(0) \right)^T \mathcal{H}^{-1} \dot{x}^k(0) \]
\[ + \int_{0}^{t} \left( \dot{x}^k(t) \right)^T \mathcal{T} \mathcal{H}^{-1} \dot{x}^k(t) \, dt. \]  \tag{17} 

Substituting Equation (10) into Equation (17) yields
\[ \frac{1}{2} \left( \dot{x}^k(t) \right)^T \mathcal{H}^{-1} \dot{x}^k(t) = \frac{1}{2} \left( \dot{x}^k(0) \right)^T \mathcal{H}^{-1} \dot{x}^k(0) \]
\[ + \int_{0}^{t} \left( \dot{x}^k(t) \right)^T \left( I_n f(x_0(t), \tau, \tau) - f \left( x^k(t), \tau \right) \right) \, d\tau \]
\[ + \int_{0}^{t} \left( \dot{x}^k(t) \right)^T \left( I_n u_0(t) - \text{sat}(u_k(t)) \right) \, d\tau \]
\[ - \int_{0}^{t} \left( \dot{x}^k(t) \right)^T da^k(t) \, d\tau. \]  \tag{18} 

Considering Assumption 2, we have
\[ \int_{0}^{t} \left( \dot{x}^k(t) \right)^T \left( I_n f(x_0(t), \tau, \tau) - f \left( x^k(t), \tau \right) \right) \, d\tau \leq \kappa \| \mathcal{T} \| \sum_{i=1}^{n} \left\| c^0_i(t) \| \right\| c^0_i(t) \| \, d\tau, \]  \tag{19} 

and
\[ = \kappa \int_{0}^{t} \left( \dot{x}^k(t) \right)^T \left( \mathcal{H} \mathcal{T}^{-1} \dot{x}^k(t) \right) \, d\tau, \]
where $\rho = \kappa \|H\|$.

Given that $sat(u^k(t)) = u^k(t) - \delta u^k(t)$ and considering Equation (14), we have

$$
\int_0^t (e^k(\tau)\nabla (1,u_0(\tau) -sat(u^k(\tau)))) \, d\tau \\
= \int_0^t (e^k(\tau)\nabla (1,u_0(\tau) - u^k(\tau) + \delta u^k(\tau))) \, d\tau \\
= -\alpha \int_0^t (e^k(\tau)\nabla e^k(\tau)) \, d\tau - \alpha \int_0^t (e^k(\tau)\nabla \delta u^k(\tau)) \, d\tau \\
- \int_0^t (\delta u^k(\tau)) \, d\tau \\
+ \int_0^t (\delta u^k(\tau)) \, d\tau \\
\leq -\left(\alpha - \frac{1}{2}\right) \int_0^t \int_0^t -\sum_{i=1}^n \int_0^t \rho^k_i(\tau)e_i^k(\tau) \tanh \left(\frac{\rho^k_i(\tau)e_i^k(\tau)}{\Delta \kappa}\right) \, d\tau,
$$

where $\rho^k_i(\tau) \leq \rho^k_i(\tau) - \rho^k_i(0)$ is applied.

Meanwhile,

$$
- \int_0^t (e^k(\tau)\nabla e^k(\tau)) \, d\tau = -\sum_{i=1}^n \int_0^t e_i^k(\tau) \nabla e_i^k(\tau) \, d\tau \\
\leq \sum_{i=1}^n \int_0^t \nabla e_i^k(\tau) \, d\tau.
$$

Substituting Equations (19)–(21) into Equation (18) yields

$$
\frac{1}{2} (e^k(t)\nabla H - e^k(t)) \\
\leq \frac{1}{2} (e^k(0)\nabla H - e^k(0)) \\
- \int_0^t (e^k(\tau)\nabla e^k(\tau)) \, d\tau - \alpha \int_0^t (e^k(\tau)\nabla \delta u^k(\tau)) \, d\tau \\
+ \frac{1}{2} \int_0^t (\delta u^k(\tau)) \, d\tau \\
- \sum_{i=1}^n \int_0^t \rho^k_i(\tau)e_i^k(\tau) \tanh \left(\frac{\rho^k_i(\tau)e_i^k(\tau)}{\Delta \kappa}\right) \, d\tau + \sum_{i=1}^n \int_0^t \nabla e_i^k(\tau) \, d\tau,
$$

where $B = (\alpha - (1/2)I_n - \kappa (H\nabla H))^{-1}$.

Analogically, we obtain

$$
\frac{1}{2} (u^k(t)\nabla H - u^k(t)) = \frac{1}{2} (u^k(0)\nabla H - u^k(0)) \\
+ \int_0^t (u^k(\tau)\nabla \delta u^k(\tau)) \, d\tau \\
- \frac{1}{2} \int_0^t (\delta u^k(\tau)) \, d\tau \\
- \sum_{i=1}^n \int_0^t \rho^k_i(\tau)e_i^k(\tau) \tanh \left(\frac{\rho^k_i(\tau)e_i^k(\tau)}{\Delta \kappa}\right) \, d\tau + \sum_{i=1}^n \int_0^t \nabla e_i^k(\tau) \, d\tau,
$$

where adaptive updating laws $\dot{u}_i^k(t)$ and $\dot{\rho}_i^k(t)$ are adopted. Therefore, Equations (22)–(24) yield the following:
\[ \Delta V^k(t) \leq \frac{1}{2} \left( \dot{e}^k(0) \right)^T \mathcal{H}^{-1} \dot{e}^k(0) - \frac{1}{2} \left( \dot{e}^{k-1}(t) \right)^T \mathcal{H}^{-1} \dot{e}^{k-1}(t) + \frac{1}{2} \left( \nu^k(0) \right)^T \nu^k(0) - \frac{1}{2} \left( \nu^{k-1}(t) \right)^T \nu^{k-1}(t) \\
+ \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^k_i(0) - \rho_0)^2 - \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^k_i(t) - \rho_0)^2 - \int_0^t (\dot{e}^k(r))^T B e^k(r) dr + \sum_{i=1}^{n} \int_0^t \rho^k_i(r) |\dot{e}^k_i(r)| dr \\
- \sum_{i=1}^{n} \int_0^t \rho^k_i(r) \tanh \left( \frac{\rho^k_i(r) |\dot{e}^k_i(r)|}{\Delta_k} \right) dr - \sum_{i=1}^{n} \int_0^t (\rho_0 - d_i^k) |\dot{e}^k_i(r)| dr, \] 

(25)

and \( \rho^k_i(t) \geq 0 \) from the adaptive updating law (13); then, Lemma 3 shows that

\[ \sum_{i=1}^{n} \int_0^t \rho^k_i(r) |\dot{e}^k_i(r)| dr - \sum_{i=1}^{n} \int_0^t \rho^k_i(r) \tanh \left( \frac{\rho^k_i(r) |\dot{e}^k_i(r)|}{\Delta_k} \right) dr \leq nTq_{\Delta_k}. \] 

(26)

Furthermore, \( \rho_0 \) is sufficiently large such that \( \rho_0 > \max \{d_i^k\} \) for \( i = 1, \ldots, n \). Let \( \theta_{\min}(B) \) represent the minimum singular value of \( B \). Based on (25) and (26), we obtain

\[ \Delta V^k(t) \leq \frac{1}{2} \left( \dot{e}^k(0) \right)^T \mathcal{H}^{-1} \dot{e}^k(0) - \frac{1}{2} \left( \dot{e}^{k-1}(t) \right)^T \mathcal{H}^{-1} \dot{e}^{k-1}(t) + \frac{1}{2} \left( \nu^k(0) \right)^T \nu^k(0) - \frac{1}{2} \left( \nu^{k-1}(t) \right)^T \nu^{k-1}(t), \]

\[ + \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^k_i(0) - \rho_0)^2 - \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^k_i(t) - \rho_0)^2 - \theta_{\min}(B) \int_0^t (\dot{e}^k(r))^T \dot{e}^k(r) dr + nTq_{\Delta_k}. \] 

(27)

On the basis of Equation (27),

\[ V^k(t) = V^{k-1}(t) + \Delta V^k(t), \]

\[ \leq \frac{1}{2} \left( \dot{e}^k(0) \right)^T \mathcal{H}^{-1} \dot{e}^k(0) + \frac{1}{2} \left( \dot{e}^{k-1}(T) \right)^T \mathcal{H}^{-1} \dot{e}^{k-1}(T) + \frac{1}{2} \left( \nu^k(0) \right)^T \nu^k(0) + \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^k_i(0) - \rho_0)^2 - \theta_{\min}(B) \int_0^t (\dot{e}^k(r))^T \dot{e}^k(r) dr + nTq_{\Delta_k}. \] 

(28)

Considering Assumption 1, \( \dot{e}^k(0) = \dot{e}^{k-1}(T) \) can be easily obtained. We also achieve \( \nu^k_i(0) = \nu^{k-1}_i(T) \) and \( \rho^k_i(0) = \rho^{k-1}_i(T) \) from Equations (12) and (13), respectively. We obtain

\[ V^k(t) \leq \frac{1}{2} \left( \dot{e}^{k-1}(T) \right)^T \mathcal{H}^{-1} \dot{e}^{k-1}(T) + \frac{1}{2} \left( \nu^{k-1}(T) \right)^T \nu^{k-1}(T) + \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^{k-1}_i(T) - \rho_0)^2 - \theta_{\min}(B) \int_0^t (\dot{e}^k(r))^T \dot{e}^k(r) dr + nTq_{\Delta_k}, \]

\[ = V^{k-1}(T) - \theta_{\min}(B) \int_0^T (\dot{e}^k(r))^T \dot{e}^k(r) dr + nTq_{\Delta_k}. \] 

(29)

Let \( t = T \). On the basis of Equation (29), we acquire

\[ V^k(T) \leq V^{k-1}(T) - \theta_{\min}(B) \int_0^T (\dot{e}^k(r))^T \dot{e}^k(r) dr + nTq_{\Delta_k}. \] 

(30)
The dynamics of the five following agents are described as

\[ \dot{x}_i^k(t) = x_i^k(t) \sin \left( x_i^k(t) \right) + \text{sat}(u_i^k(t)) + d_i^k(t), \]

where the disturbance of the ith following agent is \( d_i^k(t) = a_i \sin(\alpha_i i) \) for \( i = 1, 2, 3, 4, 5 \), where \( a_i \) and \( \alpha_i \) are arbitrary positive real numbers and \( a_i, \alpha_i \in [0, 2] \).

The dynamics of the leader agent are given as

\[ \dot{x}_0(t) = x_0(t) \sin(x_0(t)) - 2 \cos(2\pi t). \]

Let the initial states of the leader agent and five following agents be \( x_0(0) = 0.1, x_1(0) = -0.5, x_2(0) = 1.0, x_3(0) = -0.7, x_4(0) = 0.6, \) and \( x_5(0) = 1.2 \), respectively. The simulation time \( t \in [0, 2] \) and iteration number \( k_{\text{max}} = 50 \). Other parameters are selected as follows: \( c = 1, m = 2, \varpi = 5, \alpha = 1.5, \) and \( \beta_0 = 3, \beta_1 = 0.5, \beta_2 = 1.5, \beta_3 = 1.0, \beta_4 = 2.5, \) and \( \nu(0) = [0, 0.1, 0.1, 0.1, 0.1, 0.1]^T \) and \( \rho(0) = [0.2, 0.2, 0.2, 0.2, 0.2]^T \).

Figures 2–4 display the simulation results for 50 iterations considering the adaptive iterative learning control protocol (11) and adaptive updating laws (12) and (13).

From Figure 3, the control inputs are constrained, but the tracking problem is solved. From another perspective, the designed control protocol is implied to be effective. Figure 4 depicts the response of saturation inputs at the 50th iteration.

A comparative simulation result is presented to further clarify the effectiveness of the design method. Considering the designed control protocol of this paper (Scheme 1), the control law of [7] (Scheme 2) and the control law without input saturation (Scheme 3) are presented. The simulation results are shown in Figure 5.

Figure 5 indicates that the tracking control problem can be solved using Schemes 2 and 3. However, compared with the control protocol of this paper (Scheme 1), the control protocol designed in this article can obtain better control effects. Despite the fact that the control input of the multiagent system is saturated, better control effects can be achieved by applying the proposed control protocol.
Figure 1: Communication topology.

Figure 2: Tracking trajectories of five agents.

Figure 3: Maximum absolute errors of five agents.
Saturation control inputs of the last iteration

![Graph showing saturation control inputs over time](image)

**Figure 4:** Saturation control inputs of the last iteration.

![Graph showing x0-x1 and x0-x2 over time](image)

(a) (b)

**Figure 5:** Continued.
Remark 4. In the process of simulation analysis, the undirected graph is considered in our paper. In fact, similar results can also be obtained applying the undirected graph; only the analysis process is different. Through the simulation analysis, it is not difficult to find that the iterative learning control method has better advantages in solving complex nonlinear problems. Therefore, some complex control issues with iterative learning control method will be considered in our follow-up work.

Remark 5. In the simulation analysis part, the alignment initial condition, that is, $x_0^{(i)}(0) = x_0(0), i = 1, 2, \ldots, n$ for $x_0^{(i)}(0) = x_0(0), i = 1, 2, \ldots, n$, is considered. In fact, the same initial state and different initial states can also be considered, that is, $x_0^{(i)}(0) = x_0(0), i = 1, 2, \ldots, n$ or $x_0^{(i)}(0) \neq x_0(0), i = 1, 2, \ldots, n$, which are also the direction and measure for improvement. Although the paper did not analyze the other two situations, similar results can be obtained by applying the redesigned control strategy.

6. Conclusion

In this work, an adaptive iterative learning control is designed for high-accuracy tracking of input saturated nonlinear multi-intelligence systems. A new Lyapunov function is constructed and simulated, which strictly proves that the tracker can accurately monitor the leader’s trajectory in time and iteration domains. Even under the conditions of input saturation and system nonlinearity, the proposed control protocol can ensure that each follower
Complexity

Appendix

Proof of the boundedness of $V^1(t)$

On the basis of the definition of $V^k(t)$, we obtain

$$V^1(t) = \frac{1}{2}(\dot{e}^1(t))^T \mathcal{H}^{-1} \dot{e}^1(t) + \frac{1}{2}(\dot{v}^1(t))^T \dot{v}^1(t) + \sum_{i=1}^{n} \frac{1}{2\lambda_i} (\rho^i_1(t) - \rho_0)^2. \quad (A.1)$$

Hence, the derivative of $V^1(t)$ is as follows:

$$\dot{V}^1(t) = (\dot{e}^1(t))^T \mathcal{H}^{-1} \dot{e}^1(t) + (\dot{v}^1(t))^T \dot{v}^1(t) + \sum_{i=1}^{n} \frac{1}{\lambda_i} (\rho^i_1(t) - \rho_0) \dot{\rho}^i_1(t). \quad (A.2)$$

By substituting $\dot{e}^1(t)$, $\dot{v}^1(t)$, and $\dot{\rho}^i_1(t)$ into $\dot{V}^1(t)$, we have

$$\dot{V}^1(t) = (\dot{e}^1(t))^T (1_n f(x_0(t),t) - f(x^1(t),t)) - (\dot{e}^1(t))^T d^1(t) + (\dot{e}^1(t))^T (1_n u_0(t) - \text{sat}(u^1(t))) + \alpha (\dot{e}^1(t))^T v^1(t) - \frac{1}{2} (\delta u^1(t))^T \delta u^1(t) + \sum_{i=1}^{n} \rho^i_1(t) |e^i_1(t)| - \sum_{i=1}^{n} \rho^i_0 |e^i_1(t)|. \quad (A.3)$$

Notably,

$$\langle e^1(t) \rangle^T (1_n f(x_0(t),t) - f(x^1(t),t)) \leq \kappa_f \| \mathcal{H} \| \sum_{i=1}^{n} \| e^i_1(t) \| \| e^i_1(t) \|, \quad (A.4)$$

$$\leq \kappa (e^1(t))^T (\mathcal{H} \mathcal{H}^T)^{-1} e^1(t),$$

$$- (e^1(t))^T d^1(t) \leq \sum_{i=1}^{n} d^i_1 |e^i_1(t)|,$$

$$\langle e^1(t) \rangle^T (1_n u_0(t) - \text{sat}(u^1(t))) = -\alpha (e^1(t))^T e^1(t) - \alpha (e^1(t))^T v^1(t) - (e^1(t))^T \sigma^1(t) + (e^1(t))^T \delta u^1(t) \leq -\left(\alpha - \frac{1}{2}\right) (e^1(t))^T e^1(t) - \alpha (e^1(t))^T v^1(t) + \frac{1}{2} (\delta u^1(t))^T \delta u^1(t) - \sum_{i=1}^{n} \rho^i_1(t) e^i_1(t) \tanh \left(\frac{\rho^i_1(t) e^i_1(t)}{\Delta_1}\right),$$

where $\kappa = \kappa_f \| \mathcal{H} \|$. Therefore, we have

$$\dot{V}^1(t) \leq - (e^1(t))^T B e^1(t) - \sum_{i=1}^{n} (\rho_0 - d^i_1) |e^i_1(t)| + \sum_{i=1}^{n} \rho^i_1(t) |e^i_1(t)| - \sum_{i=1}^{n} \rho^i_0(t) e^i_1(t) \tanh \left(\frac{\rho^i_1(t) e^i_1(t)}{\Delta_1}\right). \quad (A.5)$$
where $B = (a - (1/2))I_n - \kappa (HH^T)^{-1}$. $\rho_0$ is sufficiently large such that $\rho_0 > \max_{i \in S} d_i^2_0$. Considering the fact that

$$\sum_{i=1}^{n} \rho_i^1 (t) e_i^1 (t) + \sum_{i=1}^{n} \rho_i^2 (t) e_i^2 (t) \tanh \left( \frac{\rho_i^1 (t) e_i^1 (t)}{\Delta_1} \right) \leq \rho_0 \Delta_1,$$

(A.6)

$V^1 (t)$ becomes

$$V^1 (t) \leq - \theta_{\min} (B) (e^1 (t))^T e^1 (t) + \rho_0 \Delta_1 \leq \rho_0 \Delta_1,$$

(A.7)

where $\theta_{\min} (B)$ is the minimum singular value of $B$. The following result can be derived:

$$V^1 (t) = V^1 (0) + \int_0^t V^1 (\tau) d\tau \leq V^1 (0) + \int_0^t \rho_0 \Delta_1 d\tau,$$

$$= \frac{1}{2} (e^1 (0))^T H^{-1} e^1 (0) + \frac{1}{2} (v^1 (0))^T v^1 (0) + \sum_{i=1}^{n} \frac{1}{2k_i} (\rho_i^1 (0) - \rho_0)^2 + \int_0^t \rho_0 \Delta_1 d\tau,$$

$$\leq V^0 (T) + nT q \Delta_1$$

(A.8)

Thus, the boundedness of $V^1 (t)$ is obtained.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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